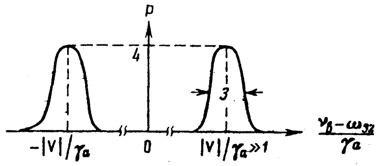

FIG. 2. Spectral distributions of the probability of the emission of light per unit time as a result of the 3→2 transition in the presence of a very strong field, calculated on the assumption that $\gamma_a = \gamma_b$. The ordinate gives $P = 9\,dw_{32}(\nu_b)(d\nu_b/2\pi)^{-1}$.

difference $\delta E_{32} = \delta E_3 - \delta E_2$, which is of the same order of magnitude as the separation between the resonances.

We shall conclude by pointing out the numerical criteria of strong fields. In the one-photon resonance case at optical frequencies a field can be regarded as strong if $\mathcal{E} > 10^2$ V/cm[4] and it is easy to estimate that in the two-photon resonance case considered here the critical field is considerably higher: $\mathcal{E} > 10^6$ V/cm.

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Transition radiation and transition scattering produced in a vacuum in the presence of a strong electromagnetic field

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In the presence of a strong electromagnetic field (in particular, a constant magnetic field), the vacuum behaves, as is well known, like a medium with permittivity and permeability that depend on the strong-field intensities. Transition radiation and transition scattering can therefore take place in vacuum. The article considers the transition radiation produced when a charge crosses the boundary between a strong magnetic field and a field-free region. The problems solved are those of transition scattering of sufficiently long strong electromagnetic waves by an immobile charge with frequency doubling, and of scattering without a change of frequency in the presence of a strong magnetic field. The same problems are considered also for a moving charge (in all considered cases the scattering takes place also for a charge with mass $M \to \infty$).

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Transition radiation is a rather common phenomenon which occurs when a charge or some other source (having no natural frequency) moves with constant velocity in or near an inhomogeneous medium. If the properties of the medium (the refractive index etc.) vary periodically in space and (or) in time, then the transition radiation acquires distinct features and can be called resonant transition radiation or transition scattering. The use of the last term is quite natural when one deals with a charge that is immobile relative to the medium and scatters a permittivity wave.[1] An effect analogous to transition scattering takes place in vacuum when a gravitational wave is incident on an immobile electric charge or dipole (electric or magnetic).[2]

We consider in this article transition radiation and transition scattering produced likewise in vacuum, but in the presence of a strong electromagnetic field. The gist of the matter that in a strong field electrodynamics becomes, as is well known, nonlinear even in vacuum, since the field gives rise to a vacuum polarization that is analogous to some extent to polarization of a medium. Transition radiation should therefore take place in an inhomogeneous strong field, and when a sufficiently strong wave is incident on a charge, transition scattering should take place. Of course, in a consistent quantum-electrodynamic calculation the transition effects are taken into account in the corresponding problems, but this calls for cumbersome computations. An exam-

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ple is the transition radiation that should appear when a charged particle crosses the boundary between a region with a strong magnetic field and a region without a field. Yet the use of classical theory of transition radiation makes it possible to solve this problem without difficulty. Thus, the use of transition-radiation and transition-scattering theory with allowance for the quantum-electrodynamic expressions for the vacuum polarization is an adequate for the calculation of the corresponding cross sections or energies of the transition radiation and scattering. This is precisely the approach used in the present article.

1. Let a strong magnetic field B_0 directed along the z axis be constant in time and homogeneous in the half-space $y<0$; in the region $y>0$ the field is equal to zero. If course under anywhere near real conditions the function $B_0(y)$ changes not jumpwise at $y=0$, but over a certain interval Δy. We shall assume below, however, the transition to be abrupt ($\Delta y \to 0$). The condition when this assumption is followed will be indicated. A particle with charge q and mass M moves along the y axis (for the sake of argument, in the positive direction). The particle velocity $\mathbf{v}$ is assumed constant. This is possible only under the assumption that the constancy of the particle velocity is maintained by some external source, or else by assuming that $M\to\infty$. The transition radiation of interest to us, which occurs when the particle crosses from the region $y<0$ into the region $y>0$, exists in "pure form" precisely at constant $\mathbf{v}$, and accordingly the effect does not vanish as $M\to\infty$ (the same pertains to Cerenkov radiation, but for simplicity we disregard the interference between the transition and Cerenkov radiation). Thus, the foregoing assumptions correspond to the essence of the problem. The solution of this problem breaks up into two parts. It is first necessary to find the permittivity and permeability of the vacuum in the presence of the field B_0. Second, it is necessary to know the solution of the problem of the transition radiation for a ferrodielectric with corresponding permittivity and permeability, and, of course, with symmetry axes chosen to fit the formulated problem.

The magnetic field B_0 will be assumed to be relatively weak in the sense of satisfaction of the following inequality ($-e$ and m are the charge and mass of the electron)

$$B_0 \ll B_c = \frac{m^2c^3}{e\hbar} \approx 4.4\cdot 10^{13}\text{ G}. \tag{1}$$

Then, and at sufficiently low frequencies ω of the propagating waves, the solution of the electrodynamic problems can be based on the Lagrangian (see, e.g.,[3] Sec. 126, where the induction B is designated H):

$$L=L_0+L',$$
$$L_0=(\mathbf{E}^2-\mathbf{B}^2)/8\pi,\quad L'={}^1/_2\varkappa\{(\mathbf{E}^2-\mathbf{B}^2)^2+7(\mathbf{EB})^2\},$$
$$\varkappa=\frac{1}{4\pi}\frac{\alpha}{45\pi B_c^2},\quad \alpha=\frac{e^2}{\hbar c}\approx\frac{1}{137}. \tag{2}$$

The polarization and the magnetization **P** and **M** of the vacuum are given by

$$\mathbf{P}=\frac{\partial L'}{\partial \mathbf{E}}=\varkappa\{2(\mathbf{E}^2-\mathbf{B}^2)\mathbf{E}+7(\mathbf{EB})\mathbf{B}\},$$
$$\mathbf{M}=\frac{\partial L'}{\partial \mathbf{B}}=\varkappa\{-2(\mathbf{E}^2-\mathbf{B}^2)\mathbf{B}+7(\mathbf{EB})\mathbf{E}\}. \tag{3}$$

Since P and M depend on E and B in nonlinear fashion, we can introduce different material tensors. We confine ourselves to the particular case when

$$\mathbf{B}=\mathbf{B}_0+\mathbf{B}_1,\quad \mathbf{E}=\mathbf{E}_1,\quad B_1\ll B_0,\quad E_1\ll B_0, \tag{4}$$

i.e., we have a strong magnetic field B_0 and a weak field, say a wave field (E_1, B_1). Then

$$\mathbf{P}=\mathbf{P}_1=\varkappa\{-2B_0^2\mathbf{E}_1+7(\mathbf{E}_1\mathbf{B}_0)\mathbf{B}_0\},\quad \mathbf{M}=\mathbf{M}_0+\mathbf{M}_1,$$
$$\mathbf{M}_0=2\varkappa B_0^2\mathbf{B}_0,\quad \mathbf{M}_1=2\varkappa\{B_0^2\mathbf{B}_1+2(\mathbf{B}_0\mathbf{B}_1)\mathbf{B}_0\}, \tag{5}$$

and it is convenient to introduce the material tensors for the weak field

$$\varepsilon_{ij}=\delta_{ij}+\delta\varepsilon_{ij},\quad \mu_{ij}=\delta_{ij}+\delta\mu_{ij},$$
$$P_{1,i}=\frac{\delta\varepsilon_{ij}}{4\pi}E_{1,j},\quad M_{1,i}=\frac{\delta\mu_{ij}}{4\pi}B_{1,j}. \tag{6}$$

In connection with the last expression it should be noted that, by definition, $M_{1,i}=\delta\mu_{ij}H_j/4\pi$, $\mathbf{H}_1=\mathbf{B}_1-4\pi\mathbf{M}_1$. We, however, are interested only in the case when

$$|\delta\varepsilon_{ij}|\ll 1,\quad |\delta\mu_{ij}|\ll 1. \tag{7}$$

Under these conditions we can replace H_1 in (6) by B_1. Recognizing that the field B_0 is assumed directed along the z axis, we have

$$\delta\varepsilon_{zz}\equiv\delta\bar\varepsilon=20\pi\varkappa B_0^2=\frac{5\alpha}{45\pi}\frac{B_0^2}{B_c^2},$$
$$\delta\varepsilon_{xx}=\delta\varepsilon_{yy}\equiv\delta\varepsilon=-8\pi\varkappa B_0^2=-\frac{2\alpha}{45\pi}\frac{B_0^2}{B_c^2},$$
$$\delta\mu_{xx}=\delta\mu_{yy}\equiv\delta\mu=8\pi\varkappa B_0^2=\frac{2\alpha}{45\pi}\frac{B_0^2}{B_c^2},$$
$$\delta\mu_{zz}\equiv\delta\bar\mu=24\pi\varkappa B_0^2=\frac{6\alpha}{45\pi}\frac{B_0^2}{B_c^2},$$
$$\varepsilon_{zz}\equiv\bar\varepsilon=1+\delta\bar\varepsilon,\quad \varepsilon_{xx}=\varepsilon_{yy}\equiv\varepsilon=1+\delta\varepsilon,$$
$$\mu_{zz}\equiv\bar\mu=1+\delta\bar\mu,\quad \mu_{xx}=\mu_{yy}\equiv\mu=1+\delta\mu \tag{8}$$

(All the components $\delta\epsilon_{ij}$ and $\delta\mu_{ij}$, except those written out, are equal to zero.) The weak field of the wave propagating in the "medium" (vacuum) with material constants (8) is written in the form

$$\mathbf{E}_1=\mathbf{E}_{1,k}e^{i(\mathbf{kr}-\omega t)},\quad \mathbf{B}_1=\mathbf{B}_{1,k}e^{i(\mathbf{kr}-\omega t)},\quad k=\{\mathbf{k},\omega\}. \tag{9}$$

Obviously, this field satisfies the ordinary Maxwell's equations, by virtue of which

$$B_{1,i}=c\omega^{-1}[\mathbf{k}\times\mathbf{E}_1]_i,\quad \varepsilon_{ij}E_{1,j}=-c\omega^{-1}[\mathbf{k}\times\mathbf{H}_1]_i,\quad B_{1,i}=\mu_{ij}H_{1,j} \tag{10}$$

(where i and j are the vector indices; of course, Eqs. (10) are equally valid for the fields E_1 and the amplitudes $E_{1,k}$). Substituting in (10) the tensors (8), we obtain the dispersion equation (the dependences of ϵ_{ij} and μ_{ij} on the coordinates are disregarded here—the field will be "joined together" on the boundary $y=0$). From the dispersion equation we obtain the refractive index n, defined by the relation $|\mathbf{k}|=\omega n/c$.

In the case (8) the indices n depend only on the angle

θ between B_0 and k. The normal waves are polarized (we are referring to the directions of the vectors $E_{1,k}$) perpendicular to B_0 and in the (k, B_0) plane. The direction of B_0 (the z axis) is the optical axis. For this direction, the refractive index is

$$n(\theta=0)=(\varepsilon\mu)^{1/2}\approx 1+{}^1/_2(\delta\varepsilon+\delta\mu)=1.$$

For waves propagating across the field B_0 (angle $\theta = \pi/2$),

$$\begin{aligned} n_{\parallel}&=(\tilde{\varepsilon}\mu)^{1/2}\approx 1+\frac{\delta\tilde{\varepsilon}+\delta\mu}{2}=1+\frac{7\alpha}{90\pi}\frac{B_0^2}{B_c^2},\\ n_{\perp}&=(\varepsilon\tilde{\mu})^{1/2}\approx 1+\frac{\delta\varepsilon+\delta\tilde{\mu}}{2}=1+\frac{2\alpha}{45\pi}\frac{B_0^2}{B_c^2} \end{aligned} \tag{11}$$

where the symbol $\parallel$ pertains to a wave polarized along B_0, and $\perp$ to a wave polarized in the perpendicular direction[1]); expressions (11) agree (to the degree of accuracy with which they were derived) with those given in [4,5].

If the particle (charge) moves along B_0, then in the ultrarelativistic limit it radiates in practice by virtue of some acceleration (which we assume to take place) only in the same direction. This should pertain also to Cerenkov radiation, because of exceedingly small deviation of the refractive indices $n(\theta)$ from unity. In this approximation, however (and in fact in a much more general approximation),[5] we have $n(0) = 1$. Therefore when the source moves along the field B_0 the polarization of the vacuum exerts no influence on its radiation (this pertains, more accurately, to the changes due to the influence of the refractive index n, which are the only significant ones in the relativistic case at $|n - 1| \ll 1$). Here, however, the field B_0 is assumed to be homogeneous, corresponding to a homogeneous medium. As to the transition radiation produced when a boundary between two media is crossed, its intensity depends not only on n but also on ϵ_{ij} and μ_{ij}. However, both the transition radiation and other radiative effects are much stronger under the considered conditions at $\theta \neq 0$ than as $\theta \to 0$.

Both in the calculation of Cerenkov radiation in a strong field,[7] and in the case of transition radiation, it is necessary to integrate over the frequencies. It is therefore necessary to know the frequency region in which expressions of the type (11) are applicable, and how these expressions change at higher frequencies. The initial Lagrangian (2) is suitable only at sufficiently low frequencies and, specifically, formulas (1) and the more general ones at $\theta \neq \pi/2$ are valid under the condition[5]

$$\lambda=\frac{3\hbar\omega}{2mc^2}\frac{B_0}{B_c}\sin\theta\ll 1, \tag{12}$$

where in the case of (11) we must put $\sin\theta = 1$.

At $\lambda \ll 1$, and in practice also at $\lambda < 1$, the radiation absorption due to production of the electron-positron pairs is negligibly small. To the contrary, at $\lambda \gtrsim 1$ the absorption must be taken into account, and accordingly the constants ϵ and μ contain imaginary parts. To be sure, the product $\epsilon\mu = 1$ and at the angle $\theta = 0$ we have $n(0) = (\mathrm{Re}\,\epsilon\mu)^{1/2} = 1$. Using [4,5], we have at $\lambda \gg 1$

$$\begin{aligned} \tilde{\varepsilon}\mu&=1+\frac{2}{3}(\varepsilon\tilde{\mu}-1)=1-\frac{2\alpha}{7\pi^{1/2}}\frac{6^{2/3}(\Gamma(2/3))^2}{\Gamma(1/6)}\left(\frac{mc^2}{\hbar\omega}\right)^{4/3}\left(\frac{B_0}{B_c}\right)^{2/3}\\ &\times(1-i3^{1/2})=1-0.0013\left(\frac{mc^2}{\hbar\omega}\right)^{4/3}\left(\frac{B_0}{B_c}\right)^{2/3}(1-i3^{1/2}),\quad \lambda\gg 1. \end{aligned} \tag{13}$$

At $\theta = \pi/2$, the refractive indices are approximately equal, $n_\perp^2 = \mathrm{Re}\,\tilde{\epsilon}\mu$ and $n_\parallel^2 = \mathrm{Re}\,\epsilon\tilde{\mu}$, where $\tilde{\epsilon}\mu$ and $\epsilon\tilde{\mu}$ are given by (13). This conclusion agrees with[5].

Under cosmic conditions, a certain plasma is always present, for which (we confine ourselves to the nonrelativistic case; the contribution of a relativistic plasma with the same concentration N is less significant)

$$\varepsilon=1-\frac{\omega_p^2}{\omega^2},\qquad \omega_p^2=\frac{4\pi e^2 N}{m}$$

According to (13), in the case of vacuum the decrease of ϵ with increasing frequency is slower than for a plasma. The refractive index of vacuum begins to change significantly, roughly speaking, at a frequency ω_c determined from the condition $\lambda(\theta) = 1$, i.e., at the frequency (at $\theta = \pi/2$)

$$\omega_c=\frac{2mc^2}{3\hbar}\frac{B}{B_0}\sim 10^{21}\frac{B}{B_0}. \tag{14}$$

At this frequency, the role of the plasma (its influence on the refractive index) is small if

$$\omega_p\ll 10^{-2}\frac{mc^2}{\hbar}\sim 10^{19}\,\mathrm{sec}^{-1},\qquad N=\frac{m\omega_p^2}{4\pi e^2}\ll 3\cdot 10^{28}\,\mathrm{cm}^{-3}. \tag{15}$$

Near pulsars this condition is always satisfied. At lower frequencies the role of the plasma, of course, increases and can be easily estimated. The influence of the magnetic field B_0 on the plasma permeability is determined by the frequency $\omega_B = eB_0/mc = 1.76 \cdot 10^7 B_0$. Even at $B_0 \sim 10^{12}$G the frequency $\omega_B \sim 10^{19}\,\mathrm{sec}^{-1}$ is small in comparison with the frequency ω_c, so that the plasma can usually be regarded as isotropic.

2. Electrodynamic problems involving Cerenkov and transition radiation in a medium, particularly in an anisotropic medium, with the magnetic permeability taken into account, were solved in a large number of papers (see, e.g.,[6]). The calculations and their results are given in a form that is convenient and general enough for us in[8]; here we need only the final formulas for ultrarelativistic particles ($v \to c$, i.e., $\mathscr{E}/Mc^2 \gg 1$, where $\mathscr{E}$ is the particle energy). For the case in question, when the charge leaves the region with the field and goes to the region without a field, only the forward transition radiation (relative to the particle velocity v) is significant in the ultrarelativistic approximation. The energy radiated into a wave of type $\parallel$ (the vector E directed along the z axis, i.e., along the field B_0) is here

$$\begin{aligned} W_\parallel&=\int_0^\infty W_\parallel(\omega)\,d\omega=\frac{q^2}{2\pi^2 c}\int_0^\infty d\omega\int_0^{2\pi}\sin^2\varphi\,d\varphi\int_0^\infty \psi^2\,d\psi^2\\ &\times\left|\frac{1}{(Mc^2/\mathscr{E})^2+\psi^2+(1-\varepsilon\tilde{\mu})}-\frac{1}{(Mc^2/\mathscr{E})^2+\psi^2}\right|^2\\ &=\frac{q^2}{2\pi c}\int_0^\infty d\omega\int_0\frac{\psi^2\,d\psi^2\,|\varepsilon\tilde{\mu}-1|^2}{|(Mc^2/\mathscr{E})^2+\psi^2+(1-\varepsilon\tilde{\mu})|^2[(Mc^2/\mathscr{E})^2+\psi^2]^2}, \end{aligned} \tag{16}$$

where ψ is the angle between k and the particle velocity

v (i.e., the y axis; φ is the azimuthal angle that determines the projections on the axes x and z near **k**); in the integration with respect to ψ only small angles are significant, so that the upper limit of the integration is set only arbitrarily equal to ∞.

For waves of type $\perp$ we have

$$W_\perp=\int_0^\infty W_\perp(\omega)d\omega$$
$$=\frac{q^2}{2\pi c}\int_0^\infty d\omega\int_0^\infty\frac{\psi^2d\psi^2|\bar{\varepsilon}\mu-1|^2}{|(Mc^2/\mathscr{E})^2+\psi^2+(1-\bar{\varepsilon}\mu)|^2[(Mc^2/\mathscr{E})^2+\psi^2]^2}. \quad (17)$$

We use for $\bar{\epsilon}\mu$ and $\epsilon\bar{\mu}$ first the expressions (11). Then in the particle-energy region

$$\mathscr{E}\ll\mathscr{E}_c=Mc^2(45\pi/2\alpha)^{1/2}(B_c/B_0)$$
$$Mc^2/\mathscr{E}_c\sim|1-\varepsilon\bar{\mu}|^{1/2}\sim|1-\bar{\varepsilon}\mu|^{1/2}, \quad (18)$$

we can assume in (16) and (17) that $(Mc^2/\mathscr{E})^2$ is much larger than $|1-\bar{\epsilon}\mu|$ or $|1-\epsilon\bar{\mu}|$. In this case

$$W(\omega)=W_\parallel(\omega)+W_\perp(\omega)=\frac{q^2}{2\pi c}[(\bar{\varepsilon}\mu-1)^2+(\varepsilon\bar{\mu}-1)^2]\int_0^\infty\frac{\psi^2d\psi^2}{[\psi^2+(Mc^2/\mathscr{E})^2]^4}$$
$$=\frac{q^2}{12\pi c}[(\bar{\varepsilon}\mu-1)^2+(\varepsilon\bar{\mu}-1)^2]\left(\frac{\mathscr{E}}{Mc^2}\right)^4=\frac{13\alpha^2q^2}{4860\pi^2c}\left(\frac{B_0}{B_c}\right)^2\left(\frac{\mathscr{E}}{Mc^2}\right)^4. \quad (19)$$

where, of course, account is taken of condition (1); the pole that can appear in principle in the integrand of (16) and (17) corresponds to the Cerenkov condition (see below). The flat frequency spectrum (19) extends all the way to frequencies $\omega\sim\omega_c$ [see (14)], after which $W(\omega)$ begins to decrease with frequency. This yields an estimate of the total radiative energy

$$W=\int_0^\infty W(\omega)d\omega=s\frac{13}{4860\pi^3}\frac{\alpha^2q^2}{\hbar c}mc^2\left(\frac{B_0}{B_c}\right)^3\left(\frac{\mathscr{E}}{Mc^2}\right)^4$$
$$=0.43s\frac{q^2}{\hbar c}mc^2\frac{B_c}{B_0}\left(\frac{\mathscr{E}}{\mathscr{E}_c}\right)^4,$$
$$\frac{\mathscr{E}}{Mc^2}\ll\frac{\mathscr{E}_c}{Mc^2}=\left(\frac{45\pi}{2\alpha}\right)^{1/2}\frac{B_c}{B_0}, \quad (20)$$

where the numerical factor s, generally speaking, is of the order of unity (its exact value can be obtained with the aid of very cumbersome formulas that are given in[5]; if expression (20) is "joined" with the value (22) pertaining to the case $\mathscr{E}\gg\mathscr{E}_c$, then $s\approx7.4$.

At $\mathscr{E}\gg\mathscr{E}_c$ the term $\epsilon\bar{\mu}-1$ or $\bar{\epsilon}\mu-1$ predominates in the denominators of (16) and (17), until these terms drop as a result of the dispersion that is taken into account by formulas (13). Assuming $|\bar{\epsilon}\mu-1|^{1/2}\sim Mc^2/\mathscr{E}$ we obtain from (7) an estimate for the highest radiated frequencies:

$$\omega_{max}\sim\frac{mc^2}{\hbar}\frac{B_c}{B_0}\left(\frac{\mathscr{E}}{\mathscr{E}_c}\right)^{3/2}\sim\omega_c\left(\frac{\mathscr{E}}{\mathscr{E}_c}\right)^{3/2},\quad\mathscr{E}\gg\mathscr{E}_c. \quad (21)$$

As is clear from (16) and (17), at frequencies $\omega\ll\omega_{max}$ (and in practice also at $\omega\lesssim\omega_{max}$) the spectrum $W(\omega)$ is independent of frequency, and then (at $\omega\gg\omega_{max}$) it begins to decrease like $\omega^{-8/3}$. The contribution to the integrated radial energy from the frequency region $\omega\lesssim\omega_c\sim mc^2B_c/\hbar B_0$ turns out to be negligible. Consequently, we can use expressions (13) when integrating by parts in (16) and (17). As a result of integration over the angles and frequencies, we obtain

$$W=g\frac{q^2\alpha^{1/3}mc^2}{\pi\hbar c}\left(\frac{B_0}{B_c}\right)^{1/3}\left(\frac{\mathscr{E}}{Mc^2}\right)^{2/3}\approx3.2\frac{q^2mc^2}{\hbar c}\frac{B_c}{B_0}\left(\frac{\mathscr{E}}{\mathscr{E}_c}\right)^{2/3},$$
$$g=\frac{2^{2/3}\pi^{1/3}(\Gamma(2/3))^{4/3}}{7^{1/3}(\Gamma(1/3))^{2/3}}(2^{2/3}+3^{2/3})(2-3^{1/3})^{1/2}\approx0.41, \quad (22)$$
$$\mathscr{E}\gg\mathscr{E}_c=Mc^2\left(\frac{45\pi}{2\alpha}\right)^{1/2}\frac{B_c}{B_0}.$$

Of course, at $\mathscr{E}\sim\mathscr{E}_c$ formulas (20) in (22) give approximately the same results.

$$W\left(\mathscr{E}\sim\mathscr{E}_c=Mc^2\left(\frac{45\pi}{2\alpha}\right)^{1/2}\right)\sim\frac{q^2mc^2}{\hbar c}\frac{B_c}{B_0}. \quad (23)$$

The foregoing results are limited in the sense that we took into account only vacuum polarization of electron-positron types. Inasmuch as for muons the field $B_{c,\mu}$ is $(m_\mu/m)^2\sim4000$ times stronger than $B_c\equiv B_{c,m}$ (see (1); m_μ is the muon mass), this limitation plays no role from the practical point of view. As is clear from (15), at the frequencies $\omega\gtrsim\omega_c$, meaning all the more in the frequency region (21), the role of the plasma is also insignificant.

The boundary of the magnetic field was assumed above to be abrupt, whereas physically this condition cannot be satisfied and the formulas used for ϵ_{ij} and μ_{ij} are valid, in any case, if the thickness of the "boundary" is

$$\Delta y\gg\hbar/mc. \quad (24)$$

It is known from the theory of transition radiation[8] that its intensity is determined by the dimension L_f of the zone in which the radiation is formed near the boundary, with

$$L_f\sim\frac{4\pi c}{\omega}\left(\frac{\mathscr{E}}{Mc^2}\right)^2, \quad (25)$$

where no account is taken of the influence of the plasma, and the angle ψ is set equal to zero; more accurately, it is assumed that

$$\left(\frac{Mc^2}{\mathscr{E}}\right)^2\gg\psi^2+\frac{\omega_p^2}{\omega^2}.$$

It is obvious [see (14) and (21)]

$$L_f(\omega_c)\sim\frac{4\pi\hbar}{mc}\frac{B_0}{B_c}\left(\frac{\mathscr{E}}{Mc^2}\right)^2,$$
$$L_f(\omega_{max})\sim\frac{4\pi\hbar}{mc}\frac{B_0}{B_c}\left(\frac{\mathscr{E}}{Mc^2}\right)^2\left(\frac{\mathscr{E}_c}{\mathscr{E}}\right)^{3/2}, \quad (26)$$
$$L_f(\mathscr{E}=\mathscr{E}_c)\sim\frac{4\pi\hbar}{mc}\frac{45\pi}{2\alpha}\frac{B_c}{B_0}\sim10^5\frac{\hbar}{mc}\frac{B_c}{B_0}.$$

Thus, at least at $\mathscr{E}\gtrsim\mathscr{E}_c$, the length $L_f\gg\hbar/mc$ and the conditions $L_f\gg\Delta y\gg\hbar/mc$ are compatible.

We compare now transition radiation with synchrotron radiation and Cerenkov radiation. The synchrotron radiation power is (the angle between **v** and $\mathbf{B}_0$ is $\pi/2$, and $\mathscr{E}/Mc^2\gg1$)

$$Q_s=\frac{2q^4B_0^2}{3M^2c^3}\left(\frac{\mathscr{E}}{Mc^2}\right)^2=\frac{2q^4m^4c^2}{3M^2\hbar^2\alpha}\left(\frac{B_0}{B_c}\right)^2\left(\frac{\mathscr{E}}{Mc^2}\right)^2. \quad (27)$$

It is reasonable to compare the transition-radiation energy with the energy radiated via the synchrotron mechanism over the length of the formation zone, i.e., within the time L_f/c. This energy is

$$W_s \sim Q_s \frac{L_f}{c} \sim \frac{4\pi q^4 m^3}{M^2 \hbar^2 \alpha}\left(\frac{B_0}{B_c}\right)^3\left(\frac{\mathcal{E}}{Mc^2}\right)^4, \quad \mathcal{E} \ll \mathcal{E}_c = \left(\frac{45\pi}{2\alpha}\right)^{1/2}\frac{B_c}{B_0}Mc^2 \tag{28}$$

and

$$W_s \sim \frac{4\pi q^4 m^3}{M^2 \hbar^2 \alpha}\left(\frac{B_0}{B_c}\right)^3\left(\frac{\mathcal{E}}{Mc^2}\right)^4\left(\frac{\mathcal{E}_c}{\mathcal{E}}\right)^{4/3} \quad \mathcal{E} \gg \mathcal{E}_c. \tag{29}$$

Comparing (20) and (28), we see that the transition radiation can exceed the synchrotron radiation only for particles with very large mass $M \gg m(45\pi Z/2\alpha)$ (here $q = |e|Z$).[2)] If furthermore we compare (29) with (22), then we arrive at an analogous conclusion. In fact the situation is more complicated, since it is necessary to distinguish between the formation zones in both media (in this case—in the field and in the region outside the field). Therefore the ratio of the synchrotron radiation to the transition radiation changes under certain conditions in favor of the latter. On the whole, there is no doubt that in the considered example (motion of a charge across a magnetic field) the synchrotron radiation is generally predominant.

The Cerenkov-radiation power under the same conditions (for more details see[8]) is

$$Q_{\text{Cer}} = \frac{q^2}{2c}\int_0^\infty \omega\, d\omega\left\{\delta\mu+\delta\bar{\mu}+\delta\varepsilon+\delta\bar{\varepsilon}-2\left(\frac{Mc^2}{\mathcal{E}}\right)^2\right\}. \tag{30}$$

Substituting (8) we obtain (see also[7])

$$Q_{\text{Cer}} = \frac{q^2}{c}\int_c^\infty \omega\, d\omega\left\{\frac{11\alpha}{90\pi}\left(\frac{B_0}{B_c}\right)^2-\left(\frac{Mc^2}{\mathcal{E}}\right)^2\right\} \tag{31}$$

Obviously, radiation is possible only under the condition

$$\frac{\mathcal{E}}{Mc^2} > \left(\frac{90\pi}{11\alpha}\right)^{1/2}\frac{B_c}{B_0} = \left(\frac{4}{11}\right)^{1/2}\frac{\mathcal{E}_c}{Mc^2}. \tag{32}$$

When this condition is satisfied, we can roughly estimate the power by integrating in (31) up to the frequency $\omega_c \sim (mc^2/\hbar)(B_c/B_0)$ above which the refractive index of the vacuum begins to decrease with frequency. Such an estimate yields

$$Q_{\text{Cer}} \sim \frac{11\alpha q^2}{180\pi c}\left(\frac{B_0}{B_c}\right)^2\omega_c^2 \sim 10^{-4}\frac{q^2}{c}\left(\frac{mc^2}{\hbar}\right)^2. \tag{33}$$

The energy radiated over the formation zone is [see (25) and (26)]

$$W_{\text{Cer}} = Q_{\text{Cer}}\frac{L_f}{c} \sim 10^{-3}\frac{q^2}{\hbar c}mc^2\frac{B_0}{B_c}\left(\frac{\mathcal{E}}{Mc^2}\right)^2\left(\frac{\mathcal{E}_c}{\mathcal{E}}\right)^{4/3}. \tag{34}$$

Comparing this expression with (22), we see that the transition radiation exceeds the Cerenkov radiation if

$$\frac{\mathcal{E}}{Mc^2} \gg 3\cdot 10^{-6}\frac{\mathcal{E}_c}{Mc^2}. \tag{35}$$

By virtue of (32) the condition (35) is always satisfied.

4. If an ultrarelativistic particle moves along the field B_0 then, as already noted, the refractive index is $n(0) = 1$ and there is neither synchrotron nor Cerenkov radiation. On the other hand, transition radiation is produced if, of course, there is a boundary between the region with the field and without the field. By virtue of the equation div B = 0 it is not easy to produce a boundary along the field, although within certain limits this is possible if external currents are available. It must furthermore be recognized that the boundary need not necessarily be perpendicular to the particle velocity, i.e., in this case it need not be located in the plane (x, y). On the other hand, for a boundary perpendicular to the field, the transition radiation (in the case of motion along the field) is much weaker than in the case considered above. The point is that in an approximation such as (16) and (17) the radiated energy depends only on n (according to (11), Eqs. (16) and (17) contain only the quantities $n_\parallel^2$ and $n_\perp^2$). We therefore need to use for the calculations more exact formulas, which are given in [8]. We confine ourselves here to the result for the case $\ln(\mathcal{E}/Mc^2) \gg 1$:

$$W(\omega) = \frac{2q^2}{\pi c}(\varepsilon-1)^2 \ln\frac{\mathcal{E}}{Mc^2} = \frac{8q^2\alpha^2}{2025\pi^3 c}\left(\frac{B_0}{B_c}\right)^4 \ln\frac{\mathcal{E}}{Mc^2}. \tag{36}$$

The total radiated energy can be estimated by integrating up to the frequency ω_c [see (14)]:

$$W \sim \int_0^{\omega_c} W(\omega)\, d\omega \sim \frac{8\alpha^2}{2025\pi^3}\frac{q^2}{\hbar c}mc^2\left(\frac{B_0}{B_c}\right)^3 \ln\frac{\mathcal{E}}{Mc^2}. \tag{37}$$

At $q = e$ we obtain

$$W \sim 3\cdot 10^{-17}\left(\frac{B_0}{B_c}\right)^3 \ln\frac{\mathcal{E}}{Mc^2}\ [\text{erg}]. \tag{38}$$

At pulsar surfaces the field B_0 is usually estimated at 10^{12} G, but it is possible that in some cases it is stronger by one order of magnitude. Thus, the parameter B_0/B_c for pulsars can quite readily reach a value 0.1. As to the particle concentration near pulsars, the estimates here are less reliable. We confine ourselves to the remark that the condition $B_0^2/8\pi \gg N\mathcal{E}$ (N is the concentration of particles with energy $\mathcal{E}$), at which the pressure of the field predominates, it is satisfied up to concentrations $N \sim 10^{26}$ cm^{-3} at $B_0 \sim 5\times 10^{12}$ G and $\mathcal{E} \sim 10^9$ eV $\sim 10^{-3}$ erg. Even at $N \sim 10^{20}$ the total pulsar radiation power (38) for a particle flux Nc through an area $S \sim 10^{12}$ cm^2 amounts to (at $B_0/B_c \sim 0.1$ and $\ln(\mathcal{E}/Mc^2) \sim 1$)

$$Q \sim NcSW \sim 3\cdot 10^{22}\ \text{erg/sec}. \tag{39}$$

At the same parameters we have for motion across the field in accordance with (23) with $q = e$

$$Q \sim 10^{-10}NcS \sim 3\cdot 10^{32}\ \text{erg/sec}. \tag{40}$$

We do not regard these estimates at all as realistic (as applied to pulsars).[3)] They indicate, however, that transition radiation can play a substantial role in pulsar physics. There are also some prospects of producing in the laboratory, conditions in which the parameters B_0/B_c and $\mathcal{E}/Mc^2$ are large enough for nonlinear phenomena to manifest themselves in vacuum (the same holds for certain atomic nuclei). It must be remembered here that the transition radiation, taken in its broader meaning, takes place whenever a medium (or the vacuum in a strong field) is inhomogeneous in space and (or) in time. It is advantageous, as already noted in the introduction, to make partial use of classical theory of transition radiation.

We note in conclusion that in semiconductors it is possible to simulate within certain limits the nonlinear phenomena that occur in a vacuum, by using much weaker fields ($B_0 \sim 10^5 - 10^6$ G). Roughly speaking, what is done here is replacing the "gap" $2mc^2 \approx 10^6$ eV in vacuum by the width of the forbidden band in the superconductors $\mathcal{E}_g \sim 1$ eV. In particular, in a strong constant magnetic field a semiconductor has rather unique electrodynamic properties (see, e.g., [11]). It is possible that interest attaches in this connection to an analysis of certain features of the transition radiation and scattering in semiconductors situated in strong fields. Incidentally, the foregoing pertains not so much specifically to semiconductors as in general to nonlinear media (for more details see [8]).

5. We dwell now on transition radiation, which is the simplest mechanism of conversion of waves of one type into waves of another type by a charge (or by some other source of polarization of the medium), without requiring (sometimes an important factor) a change in the motion of the charge itself.

We assume below for simplicity that the charge q is immobile (pinned). Its electric field, without allowance for the nonlinearity of the vacuum can be written in the form

$$\mathbf{E}^q = q\mathbf{r}/r^3,$$
$$\mathbf{E}_k^q = \frac{1}{(2\pi)^4}\int \mathbf{E}^q(\mathbf{r},t)\,e^{-i(\omega t-\mathbf{kr})}\,d\mathbf{r}\,dt = \frac{4\pi q}{i(2\pi)^3}\frac{\mathbf{k}}{k^2}\delta(\omega),\quad k=\{\mathbf{k},\omega\}. \tag{41}$$

Let there be incident on this charge an electromagnetic wave whose field is equal to

$$\mathbf{E}^w = E_0\mathbf{e}\cos(\omega_0 t - \mathbf{k}_0\mathbf{r}+\varphi_0),\quad \omega_0 = |\mathbf{k}_0|c,\quad \mathbf{e}^2=1,\quad k=\{\mathbf{k},\omega\},$$
$$\mathbf{E}_k^w = {}^1/_2 E_0\mathbf{e}[e^{-i\varphi_0}\delta(\omega-\omega_0)\delta(\mathbf{k}-\mathbf{k}_0)+e^{i\varphi_0}\delta(\omega+\omega_0)\delta(\mathbf{k}+\mathbf{k}_0)], \tag{42}$$

if the wave is coherent and monochromatic. If the incident waves have a broad spectrum, are not correlated in phase, and are not polarized, then

$$\langle E^{w}_{i,k}E^{w}_{j,k'}\rangle = {}^1/_2|E|_{\mathbf{k}}^2(\delta_{ij}-k_ik_j/k^2)\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'),$$
$$|E|_k^2 = {}^1/_4|E|_{\mathbf{k}}^2\delta(\omega-|\mathbf{k}|c)+{}^1/_4|E|_{-\mathbf{k}}^2\delta(\omega+|\mathbf{k}|c). \tag{43}$$

The magnetic field $\mathbf{B}^w$ and the time-average energy $\overline{W}$ of the waves are respectively, for the coherent case,

$$\mathbf{B}^w = E_0[\mathbf{n}_0\mathbf{e}]\cos(\omega_0 t-\mathbf{k}_0\mathbf{r}+\varphi_0),\quad \mathbf{n}_0=\mathbf{k}_0/|k_0|,$$
$$\overline{W} = \frac{1}{8\pi}\{(\mathbf{E}^w)^2+(\mathbf{B}^w)^2\} = \frac{E_0^2}{8\pi}. \tag{44}$$

For an incoherent field, the average energy is

$$\langle W\rangle = \int\frac{|E|_{\mathbf{k}}^2\,d\mathbf{k}}{8\pi}. \tag{45}$$

The values corresponding to a random quasimonochromatic field in (43) and (5) are

$$|E|_{\mathbf{k}}^2 = E_0^2\delta(\mathbf{k}-\mathbf{k}_0),\quad \langle W\rangle = E_0^2/8\pi. \tag{46}$$

Consider now a process in which the incident wave of frequency ω_0 is converted into a scattered wave with frequency $2\omega_0$. In quantum language this corresponds to absorption of two quanta $\hbar\omega_0$ with emission of one quantum of energy $2\hbar\omega_0$. In the case of the spectrum, we are dealing here with doubling of each of the frequencies $\omega = c|\mathbf{k}|$. Assuming that $(\mathbf{E}^w)^2 = (\mathbf{B}^w)^2$ and $(\mathbf{E}^w\cdot\mathbf{B}^w)=0$, it is necessary for the process in question to retain in formulas (3) for $\mathbf{P}$ and $\mathbf{M}$ the terms that are linear in the field $\mathbf{E}^q$ of the charge and quadratic in the field $\mathbf{E}^w$ of the wave

$$\mathbf{P} = \varkappa\{4(\mathbf{E}^q\mathbf{E}^w)\mathbf{E}^w+7(\mathbf{E}^q\mathbf{B}^w)\mathbf{B}^w\},$$
$$\mathbf{M} = \varkappa\{-4(\mathbf{E}^q\mathbf{E}^w)\mathbf{B}^w+7(\mathbf{E}^q\mathbf{B}^w)\mathbf{E}^w\}. \tag{47}$$

For the wave (42) we obtain hence

$$\mathbf{P} = {}^1/_2\varkappa\{4\mathbf{e}(\mathbf{e}\mathbf{E}^q)+7[\mathbf{n}_0\mathbf{e}]([\mathbf{n}_0\mathbf{e}]\mathbf{E}^q)\}(1+\cos(2\omega_0 t-2\mathbf{k}_0\mathbf{r}+2\varphi_0)),$$
$$\mathbf{M} = {}^1/_2\varkappa\{-4[\mathbf{n}_0\mathbf{e}](\mathbf{e}\mathbf{E}^q)+7\mathbf{e}([\mathbf{n}_0\mathbf{e}]\mathbf{E}^q)\}(1+\cos(2\omega_0 t-2\mathbf{k}_0\mathbf{r}+2\varphi_0)). \tag{48}$$

Thus, the polarization and the magnetization have oscillating terms with frequency $2\omega_0$ and wave vector $2\mathbf{k}_0$. These alternating polarization and magnetization produce transition scattering by the charge q. The intensity of the scattering can be easily obtained by the method described in [1], being equal to the power radiated by a current of density

$$\delta\mathbf{j} = c\,\mathrm{rot}\,\mathbf{M}+\frac{\partial\mathbf{P}}{\partial t}. \tag{49}$$

The radiation power for the scattered wave (set over both polarizations), is

$$Q = \int_0^\pi Q(\theta)2\pi\sin\theta\,d\theta,\quad Q(\theta) = \frac{139\pi q^2}{8}\varkappa^2 k_0^2 c\sin^2\theta,$$
$$Q = \frac{139}{48}\frac{q^2\omega_0^2}{c}\left(\frac{\alpha}{45\pi}\right)^2\left(\frac{E_0}{E_c}\right)^4 \tag{50}$$

where E_c is equal to B_c in the cgs esu, i.e.,

$$E_c = 4.4\cdot 10^{13}\text{ cgs esu} = 1.32\cdot 10^{16}\text{ V/cm}$$

The result (50) coincides in fact with that obtained in[12] on the basis of a cumbersome quantum electrodynamic calculation (the calculations in[12] were carried out also for the region of high frequencies ω).

For arbitrary nonmonochromatic waves, the Fourier components $\delta\mathbf{j}$ of the current density are given by

$$\delta j_{i,k} = \int\Sigma_{i,j,l,s}(k,k_1,k_2)E_{j,k_1}E_{l,k_2}E_{s,k-k_1-k_2}dk_1\,dk_2,$$
$$k=\{\mathbf{k},\omega\},\quad dk = d\mathbf{k}\,d\omega, \tag{51}$$

$$\Sigma_{i,j,l,s}(k,k_1,k_2) = {}^1/_2(\Sigma_{i,j,l,s}(k,k_1,k_2)+\Sigma_{i,l,j,s}(k,k_2,k_1)),$$
$$\Sigma_{i,j,l,s}(k,k_1,k_2) = -i\varkappa\left\{4\delta_{ls}\left(\delta_{ij}\omega+e_{iqr}k_q e_{rmj}\frac{k_{1,m}}{\omega_1}\right)+\right.$$
$$\left.+7e_{sql}\frac{k_{2,q}}{\omega_2}\left(e_{irj}\omega\frac{k_{1,r}}{\omega_1}-e_{irj}k_r\right)\right\}, \tag{52}$$

where e_{ils} is a unit antisymmetrical tensor of third rank. The radiation power of the current (51) at the frequency $\omega = c(|\mathbf{k}_1| + |\mathbf{k}_2|)$ for random waves, averaged over the phases with the aid of (43), is of the form

$$Q = \int Q_{\mathbf{k}_1,\mathbf{k}_2}(|\mathbf{k}_1|+|\mathbf{k}_2|)^2 c|E|_{\mathbf{k}_1}^2 |E|_{\mathbf{k}_2}^2 d\mathbf{k}_1 d\mathbf{k}_2, \quad (53)$$

$$Q_{\mathbf{k}_1,\mathbf{k}_2} = \int d\Omega \frac{\pi}{16} q^2 \varkappa^2 \frac{n_s' n_{s'}'}{|\mathbf{n}'|^4} (\delta_{ii'} - n_i n_{i'})(\delta_{jj'} - n_{0j} n_{0j'})(\delta_{ll'} - n_{0l} n_{0l'})$$
$$\times\{4\delta_{ls}(\delta_{ij} + e_{iqr} n_q e_{rmj} n_{1m}) + 4\delta_{js}(\delta_{il} + e_{iqr} n_q e_{rml} n_{2m}) + 7 e_{sql} n_{2q} e_{irj}(n_{1r} - n_r)$$
$$+ 7 e_{sqj} n_{1q} e_{irl}(n_{2r} - n_r)\}\{4\delta_{l's'}(\delta_{i'j'} + e_{i'q'r'} n_{q'} e_{r'm'j'} n_{1m'})$$
$$+ 4\delta_{j's'}(\delta_{i'l'} + e_{i'q'r'} n_{q'} e_{r'm'l'} n_{2m'}) + 7 e_{s'q'l'} n_{2q'} e_{i'r'j'}(n_{1r'} - n_{r'})$$
$$+ 7 e_{s'q'j'} n_{1q'} e_{i'r'l'}(n_{2r'} - n_{r'})\}, \quad (54)$$

where $\mathbf{n} = \mathbf{k}/k$, $\mathbf{n}_1 = \mathbf{k}_1/k_1$, $\mathbf{n}_2 = \mathbf{k}_2/k_2$, $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors of the scattered wave, $\mathbf{k}$ is the wave vector and $d\Omega$ is the solid angle of the scattered waves; finally

$$\mathbf{n}' = \mathbf{n} - \mathbf{n}_1 \frac{|\mathbf{k}_1|}{|\mathbf{k}_1|+|\mathbf{k}_2|} - \mathbf{n}_2 \frac{|\mathbf{k}_2|}{|\mathbf{k}_1|+|\mathbf{k}_2|}.$$

In the case of a quasimonochromatic random wave it is necessary to substitute (46) in (53) and (54). The result coincides with (50). This means that the result (50) remains in force for a scattered wave packet of sufficiently general form in the case when its spectral width is $\Delta\omega \ll \omega_0$ and the total energy is equal to $E_0^2/8\pi$. The latter is important for a possible experimental confirmation of the effect with the aid of, for example, scattering of intense laser radiation by massive ions.

In connection with the nonlinearities of the vacuum, the effect most frequently discussed was the so-called Delbrück scattering (see [3], Sec. 125, and [13]). This effect can be considered by using the nonlinear polarizations P and M, if two of the fields in the terms cubic in the field in (3) are taken to be the charge field $\mathbf{E}^q$, and one is taken to be the wave field $\mathbf{E}^w$, i.e., if it is assumed that $P, M \propto (E^q)^2 E^w$. Calculation of the intensity of the radiation of the scattered field with the aid of such P and M leads to integrals that diverge at large values of $\mathbf{k}_1$ (of the momenta $\hbar\mathbf{k}_1$) of the field of the scattering charge. This demonstrates that the Lagrangian L', which is suitable under the condition $k_1 \ll mc/\hbar$, cannot be used to describe the process. Terminating the integration at $k_{\max} \approx mc/\hbar$, we obtain the correct estimate of the power of the Delbrück scattering [[3], formula (125.1)]:

$$Q \approx \frac{\alpha^2 q^2 \omega_0^2}{c} \frac{q^2}{\hbar c} \alpha \left(\frac{\hbar\omega_0}{mc^2}\right)^2 \left(\frac{E_0}{E_c}\right)^2. \quad (55)$$

If the wave of the field $E^w \approx E_0$ is weak (in the sense that $E_0 \ll E_c$), but

$$\left(\frac{E_0}{E}\right)^2 \gg \alpha^2 Z^2 \left(\frac{\hbar\omega_0}{mc^2}\right)^2 \quad q^2 = Z^2 e^2, \quad (56)$$

then when the frequency is doubled the scattering power (50) exceeds power (55) of the Delbrück scattering which takes place without a change of frequency. At sufficiently low frequencies, the condition (56) is readily satisfied.[4] It is important here that the power (2) can be correctly calculated on the basis of the use of the Lagrangian L', since the lengths that play an important role are those of order $\lambda_0 = 2\pi c/\omega_0$.

Thus, Delbrück scattering, just as the scattering $\omega_0 \to 2\omega_0$, can be regarded as transition scattering due to the nonlinearity of vacuum.

We make two additional remarks. First, formulas (53) and (54) describe scattering in the case of arbitrary frequency and angular distribution of the scattered wave. Thus, for example, in the case of a standing wave

$$|E|_{\mathbf{k}}^2 = \frac{E_0^2}{2}\{\delta(\mathbf{k}-\mathbf{k}_0) + \delta(\mathbf{k}+\mathbf{k}_0)\}. \quad (57)$$

We obtain from (53) and (54)

$$Q = \int Q(\theta) 2\pi \sin\theta \, d\theta, \quad Q(\theta) = \frac{E_0^4}{16} q^2 \varkappa^2 k_0^2 c\pi \sin^2\theta (260 + 139\cos^2\theta),$$
$$Q = \frac{263}{80} \frac{q^2\omega_0^2}{c} \left(\frac{\alpha}{45\pi}\right)^2 \left(\frac{E_0}{E_c}\right)^4. \quad (58)$$

Second, the intensity of scattering by a moving, and in particular, relativistic charge is obtained by simple recalculation from (50) and (58). The integral intensity can be calculated by using the arguments presented, for example, in [14], Sec. 73. As to the spectral and angular distribution, it can be obtained by the method described above, using for $\mathbf{E}^q$ the field of the moving charge. The highest frequencies are obtained in the case when the waves move opposite to each other

$$\mathbf{v} = -v\mathbf{n}_0, \quad \omega = \frac{2\omega_0(1+v/c)}{1+(v/c)(\mathbf{n}\mathbf{n}_0)}. \quad (59)$$

The maximum frequency (59) is radiated in the direction of motion of the particle $\mathbf{n} = -\mathbf{n}_0$. As $v \to c$ we have

$$\omega_{\max} \approx 8\omega_0 (\mathscr{E}/Mc^2)^2. \quad (60)$$

The intensity of the scattering of a coherent or quasimonochromatic random wave is determined by the relation

$$Q(\theta) = \frac{q^2\omega_0^2}{c} \frac{139}{128\pi} \left(\frac{\alpha}{45\pi}\right)^2 \left(\frac{E_0}{E_c}\right)^4 \frac{\sin^2\theta (1+v/c)^4}{(1+(v/c)\cos\theta)^5}. \quad (61)$$

Here E_0 is the amplitude of the wave field in a reference frame in which the particle has a velocity $\mathbf{v}$, and θ is the angle between $\mathbf{k}$ and $\mathbf{k}_0$ or the angle between $\mathbf{k}$ and $-\mathbf{v}$. The maximum scattering intensity takes place in directions close to the particle velocity, $\theta \sim Mc^2/\mathscr{E} = (1 - v^2/c^2)^{1/2}$, and the integral scattering intensity is equal to (at $v \to c$)

$$Q = \int_0^\pi Q(\theta) 2\pi \sin\theta \, d\theta = \frac{q^2\omega_0^2}{c} \frac{139}{3} \left(\frac{\alpha}{45\pi}\right)^2 \left(\frac{E_0}{E_c}\right)^4 \left(\frac{\mathscr{E}}{Mc^2}\right)^6. \quad (62)$$

Attention is called to the strong dependence of the scattering intensity on the particle energy.

6. We dwell also on the nonlinear effect in vacuum-scattering of a wave by a charge q in the presence of a strong magnetic field $\mathbf{B}_0$. We assume first that the charge is immobile and consider the scattering process when the frequency of the scattered wave is equal to the frequency of the incident wave. In this case

$$\mathbf{E} = \mathbf{E}^w + \mathbf{E}^q, \quad \mathbf{B} = \mathbf{B}_0 + \mathbf{B}^w. \quad (63)$$

Assuming the field of the charge to be weak, we take into account the terms that are linear in E^q in the expressions (3) for the polarization P and the magnetization M. In the corresponding expressions that are quadratic in the field e^w and B_0, the terms of order B_0^2 and $(E^w)^2$ describe the two types of processes which have already been discussed above, and the process of interest to us here is described by terms of order B_0E^w and B_0B^w. These parts of the polarization and magnetization are of the form

$$\delta \mathbf{P} = \varkappa\{-4(\mathbf{B}_0\mathbf{B}^w)\mathbf{E}^q + 7(\mathbf{E}^q\mathbf{B}_0)\mathbf{B}^w + 7(\mathbf{E}^q\mathbf{B}^w)\mathbf{B}_0\},$$
$$\delta \mathbf{M} = \varkappa\{-4(\mathbf{E}^q\mathbf{E}^w)\mathbf{B}_0 + 7(\mathbf{E}^q\mathbf{B}_0)\mathbf{E}^w + 7(\mathbf{E}^w\mathbf{B}_0)\mathbf{E}^q\}. \quad (64)$$

We present here the final formula for the scattering intensity of a wave propagating in the direction of the magnetic field $\mathbf{B}_0$ ($\mathbf{k}_0 = k_0\mathbf{B}_0/B_0$) in the case of a charge at rest:

$$Q = \frac{256\pi^2}{3}\omega_0^2\varkappa^2B_0^2E_0^2q^2 = \frac{16}{3}\left(\frac{\alpha}{45\pi}\right)^2\frac{\omega_0^2q^2}{c}\left(\frac{E_0}{E_c}\right)^2\left(\frac{B_0}{B_c}\right)^2. \quad (65)$$

For ultrarelativistic particles moving opposite to the wave in the magnetic field, we obtain

$$Q = \frac{128}{3}\left(\frac{\alpha}{45\pi}\right)^2\frac{\omega_0^2q^2}{c}\left(\frac{E_0}{E_c}\right)^2\left(\frac{B_0}{B_c}\right)^2\left(\frac{\mathscr{E}}{Mc^2}\right)^4. \quad (66)$$

The maximum frequency is radiated in a direction close to the particle velocity, and is equal to

$$\omega_{max} \approx 4\omega_0\left(\frac{\mathscr{E}}{Mc^2}\right)^2. \quad (67)$$

We emphasize that formulas (65) and (66) do not contain integration with respect to large k and are therefore exact within the framework of the assumptions.

Pulsars possibly accelerate ions to $\mathscr{E}/Mc^2 \sim 10^6$. Under these conditions, the discussed scattering with power (66) can be of interest. However, as already indicated, no estimates were made in the present article for realistic pulsar models.

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[1]In a medium with double (electric and magnetic) anisotropy the refractive indices of both normal waves depend on the angle θ (see, e.g., [6]). Therefore the designations "ordinary" and "extraordinary," which are customary for uniaxial nonmagnetic crystal (i.e., at $\mu_{ij} = \delta_{ij}$), can be applied to these waves only in a most arbitrary sense.

[2]This comparison of the integral intensity of the radiation is not always representative of the real situation, since the spectral compositions of the transition and synchrotron radiations are different. For synchrotron radiation the maximum frequency

$$\omega_{max} \sim \frac{qB_0}{Mc}\left(\frac{\mathscr{E}}{Mc^2}\right)^2 \approx Z\frac{B_0}{B_c}\frac{m}{M}\frac{mc^2}{\hbar}\left(\frac{\mathscr{E}}{Mc^2}\right)^2$$

is substantially lower than $\omega_c \sim mc^2B_c/\hbar B_0$ [see (14)] at $\mathscr{E} \ll \mathscr{E}_c(2\alpha M/45\pi mZ)^{1/2}$. If the last inequality is satisfied, then in the interval $\omega_{max} \leq \omega \leq \omega_c$, we have exclusively transition radiation, even if $M \ll m(45\pi Z/2\alpha)$.

[3]The particle concentration in the magnetosphere of pulsars has not yet been established with any degree of reliability. Frequently, however, the estimate $N \sim \Omega B_0/4\pi ec \sim 10^{12}$–$10^{13}$ cm^{-3} is used (here Ω is the angular velocity of the rotation of the neutron star, which for the known pulsars does not exceed $\Omega \sim 200$ sec^{-1}). Birefringence influences particularly strongly the radiation polarization (this pertains in particular to x-rays from pulsars[9,10]). We note also that transition radiation is produced in vacuum also in the case of an abrupt change of the magnetic field in time, for example in a "starquake" of a magnetized neutron star.

[4]We note that the estimate (55) can be obtained from (50) by setting in one of the factors $(E_0/E_c)^2$ in (50) the field E_0 equal to the field of the charge q at the distance $r_c \sim \hbar/mc$ from its center (then $E_0^2/E_c^2 \sim \alpha q^2/\hbar c$), and by taking into account the fact that the dimension of the area responsible for the scattering is small by a factor $(r_c/\lambda_0)^2 \sim (\hbar\omega_0/mc^2)^2$. This yields qualitatively the two small factors in (55).

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