Limiting velocities and types of magnetic moment waves

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The Landau-Lifshitz equations for a medium with an easy-axis anisotropy are used in an investigation of various types of nonlinear waves. Exact asymptotic expressions are obtained for the distribution of the magnetic moment in the region where a homogeneous magnetization is established and allowance is made for internal magnetic fields. It is shown that in addition to the solutions describing the steady motion of Bloch and Néel domain boundaries, there can be also solitary waves of two types. For waves of the first type (magnetic solitons) the establishment of a homogeneous magnetization is accompanied by precession of the magnetic moment about the anisotropy axis, whereas in the case of waves of the second type the precession is accompanied also by nutational motion of the magnetic moment vector. Waves of the second type separate the solutions of the magnetic soliton type from nonlinear spin waves. Expressions are obtained for the limiting velocities of the propagation of the new types of wave. The respective solutions are found numerically.

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1. Akhiezer and Borovik[1,2] investigated nonlinear magnetic moment waves and found an explicit solution corresponding to a moving Bloch or Néel domain boundary, as well as solutions corresponding to a "fast" solitary magnetic moment wave. In the case of a moving domain boundary, the explicit solution, characterized by an orientation of plane of rotation of the magnetic moment constant over all space, it was shown that the velocity of such a domain boundary has the upper limit

$$U_-=2|\gamma|\frac{(AK)^{1/2}}{M_s}[(1+\varepsilon)^{1/2}-1]\equiv 2|\gamma|\frac{(AK)^{1/2}}{M_s}u_-. \qquad (1.1)$$

Here, A and K are the exchange and anisotropy energy constants; M_s is the saturation magnetization; γ is the gyromagnetic ratio; $\epsilon=2\pi M_s^2/K$. A similar result, found by Walker, is given in Dillon's review paper.[3] However, the conclusion of the existence of a limiting velocity of a moving domain wall is based on the explicit analytic form of a particular solution of the Landau–Lifshitz equations, which does not make it possible to separate the class of solutions satisfying the above velocity limitation condition.

In the case of fast solitary magnetic moment waves (magnetic solitons) the unresolved questions include not only the separation of the relevant class of solutions of the Landau–Lifshitz equations but also the possibility of retention of this type of solution (a solitary wave or a magnetic soliton) if allowance is made for internal magnetic fields and for the propagation of a wave at an angle to the anisotropy axis. In fact, a fast solitary magnetic moment wave found by Akhiezer and Borovik[2] is an exact solution of the Landau–Lifshitz equations only for the case of a wave traveling along the anisotropy axis of a uniaxial ferromagnet. Only in this specific case do the Landau–Lifshitz equations admit the existence of two exact first integrals of the solutions and make it possible to analyze the problem completely in the phase plane. In all other cases, including the most interesting case of the propagation of a wave orthogonal to the anisotropy axis there are internal magnetic fields which result in the elimination of one of the two exact first integrals.

The ways of overcoming the above difficulties were pointed out in an earlier paper by the present authors.[4] Namely, we showed that the appearance of a lower velocity limit (1.1), discovered by Akhiezer, Borovik, and Walker is associated with a specific type of asymptotic boundary conditions for the behavior of the magnetic moment vector in the region where a homogeneous magnetization is established and these conditions determine the corresponding class of solutions of the Landau–Lifshitz equations. Moreover, a qualitative analysis of the problem of magnetic moment waves in the phase space revealed the possibility of retention of solutions in the form of fast solitary waves (magnetic solitons) even if allowance is made for internal magnetic fields. It was also shown that the limiting Walker velocity can be determined without an explicit solution, investigating only the asymptotic behavior of the magnetic moment distribution. However, the absence of general analytic expressions for the asymptotic behavior of the solutions in the region where a homogeneous magnetization is established made it impossible to find the exact upper limit to the velocity of fast solitary waves in the presence of internal magnetic fields and to prove the existence of specific types of magnetic solitons.

The explicit expression for the asymptotes obtained in the present paper have made it possible to show that there are two ways of establishing homogeneous states in a ferromagnet with an easy-axis anisotropy. In the first case the nature of the asymptotic boundary conditions is governed by zeros of the dependence of the precession frequency on the azimuthal angle and is characterized by the establishment of a plane of rotation of the magnetic moment constant in space, whereas in the second case these solutions are characterized by a period-average value of the ratio of the logarithmic derivative of the polar angle to the precession frequency. A change in the type of asymptotic boundary solutions occurs when the wave velocity passes through the limiting value given by Eq. (1.1). We can thus see that the limiting Walker velocity is simply the velocity at which there is a change in the nature of the magnetization switching.

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We shall find the dependence of the upper limiting velocity of the propagation of fast solitary waves on the characteristic parameter of a magnetic medium ϵ allowing for internal magnetic fields:

$$U_+=2|\gamma|\frac{(AK)^{1/2}}{M_s}[(1+\varepsilon)^{1/2}+1]\equiv 2|\gamma|\frac{(AK)^{1/2}}{M_s}u_+. \quad (1.2)$$

Moreover, an investigation of the asymptotes reveals the existence of a new characteristic velocity of fast magnetic moment waves, which is (see Fig. 2 below)

$$U_0=2|\gamma|\frac{(AK)^{1/2}}{M_s}[u_+^2(\varepsilon)-u_-^2(\varepsilon)]^{1/2}=4|\gamma|\frac{(AK)^{1/2}}{M_s}(1+\varepsilon)^{1/4}. \quad (1.3)$$

In the velocity range $U_-<U<U_0$ the asymptotics of simple waves corresponds to the establishment of a homogeneous state in the presence of precession of the magnetic moment around the anisotropy axis and monotonic variation of the polar angle of the magnetic moment vector. In the velocity range $U_0<U<U_+$ the establishment of a homogeneous state occurs in the presence of precession of the magnetic moment and of decaying oscillations of the polar angle θ. This type of solution separates solutions representing the magnetic solitons and nonlinear spin waves. It should be noted that such a solution vanishes in the limint $\epsilon\to 0$. As the parameter ϵ increases, the range of existence of fast solitary magnetic moment waves, characterized by the absence of oscillations of the polar angle θ in the region of establishment of a homogeneous state, becomes narrower (Fig. 2) and for $\epsilon>\epsilon_c=(\sqrt{2}+1)^4-1$ this type of solitary wave vanishes completely. A numerical calculation confirms the conclusion of the existence of solutions separating magnetic solitons from nonlinear spin waves. It is shown that solutions in the form of fast solitary waves of the magnetic moment are retained even if allowance is made for internal magnetic fields and for the propagation of the wave at right-angles to the easy magnetization axis.

The three characteristic velocities and the characteristic value of the parameter ϵ_c obtained in the present paper allow us essentially to determine completely all possible types of magnetic moment wave in a medium with the easy-axis anisotropy.

2. In the case of solutions corresponding to steady-state magnetic moment waves and, in particular, those describing the steady-state motion of domain boundaries, the Landau–Lifshitz equations for a ferromagnet with the easy-axis anisotropy become

$$d^2\theta/d\xi^2-(1+\varepsilon\cos^2\varphi+\omega^2)\sin\theta\cos\theta=u\omega\sin\theta,\quad \omega=d\varphi/d\xi,$$
$$\frac{d}{d\xi}(\omega\sin^2\theta)+\varepsilon\sin^2\theta\cos\varphi\sin\varphi=-u\frac{d\theta}{d\xi}\sin\theta. \quad (2.1)$$

Here, θ and φ are the polar and azimuthal angles of the magnetic moment vector in a spherical coordinate system with its polar axis along the easy magnetization direction; the variable $\xi=x-ut$ is reduced to the characteristic size of a Bloch boundary $\delta_0=(A/K)^{1/2}$, whereas the velocity u is reduced to the characteristic velocity $2|\gamma|(AK)^{1/2}/M_s$.

The system (2.1) has the first integral

$$\mathscr{H}=(d\theta/d\xi)^2-(1+\varepsilon\cos^2\varphi-\omega^2)\sin^2\theta, \quad (2.2)$$

and the condition for attainment of homogeneous-state solutions

$$\theta=0,\quad \theta=\pi, \quad (2.3)$$

corresponding to the magnetization along the anisotropy axis, is governed by the constant of the first integral which vanishes ($\mathscr{H}=0$).

We shall now determine the general asymptotic behavior of the solutions in the limits $\theta\to 0$ and $\theta\to\pi$. Introducing the logarithmic derivative of the polar angle θ

$$\frac{d}{d\xi}\ln\theta=\Gamma, \quad (2.4)$$

we find that in the limit $\theta\to 0$ and for arbitrary values of φ and ω, the system (2.1) leads to the equations

$$\Gamma^2+\omega^2=1+\varepsilon\cos^2\varphi,\quad d\Gamma/d\varphi=2(\omega+u/2). \quad (2.5)$$

Here, the new independent variable is the azimuthal angle φ. Eliminating the precession frequency $\omega(\varphi)$, we obtain

$$\Gamma^2+{}^1/_4(d\Gamma/d\varphi-u)^2=1+\varepsilon\cos^2\varphi, \quad (2.6)$$

which governs the dependence of the logarithmic derivative θ on the angle φ. In particular, if $\epsilon=0$, we find that

$$\Gamma^2=1-{}^1/_4u^2>0 \quad (2.7)$$

and the velocity of a simple wave has the upper and lower limits

$$0<u^2<u_{max}^2=4. \quad (2.8)$$

The exact solution of Eq. (2.6) can also be found for any value of the parameter of the medium $\epsilon\neq 0$. In fact, we can readily show that the solution of Eq. (2.6) is

$$\Gamma(\varphi)=\Gamma_0+\frac{\varepsilon}{u^2+4\Gamma_0^2}\left(\Gamma_0\cos 2\varphi-\frac{1}{2}u\sin 2\varphi\right). \quad (2.9)$$

The period-average value of Γ_0 is given by

$$\frac{\varepsilon^2/4}{u^2+4\Gamma_0^2}=1+\frac{1}{2}\varepsilon-\frac{1}{4}(u^2+4\Gamma_0^2). \quad (2.10)$$

Thus, there are two branches of the solutions, for one of which we have

$$u^2+4\Gamma_0^2=[(1+\varepsilon)^{1/2}+1]^2\equiv u_+^2(\varepsilon), \quad (2.11)$$

and for the other

$$u^2+4\Gamma_0^2=[(1+\varepsilon)^{1/2}-1]^2\equiv u_-^2(\varepsilon). \quad (2.12)$$

Then, $u_-(\epsilon)$ is identical with the upper limiting velocity of moving domain boundaries characterized by an orientation of the plane of rotation of the magnetic moment vector which is constant in space ($\varphi=\text{const}$).

For $\epsilon=0$ the velocities $u_\pm(\epsilon)$ are identical with the limiting values of the velocities of a solitary magnetic moment wave [see Eq. (2.8)]. Using Eqs. (2.11) and (2.12), we obtain

$$\Gamma_\pm(\varphi)=\Gamma_0+{}^1/_2 u_\mp(\varepsilon)\cos 2(\varphi+\delta_\pm), \tag{2.13}$$

$$\omega_\pm(\varphi)=-{}^1/_2 u-{}^1/_2 u_\mp(\varepsilon)\sin 2(\varphi+\delta_\pm). \tag{2.14}$$

Here, $\tan 2\delta=u/2\Gamma_0$. The selection of the upper or lower sign is governed by the selection of the branches for Γ_0 in accordance with Eqs. (2.11) or (2.12). The presence, in the $u<u_-(\epsilon)$ case, of two branches is explained by the fact that in the phase space there are two singularities of the four-dimensional saddle type and two separatrix solutions, corresponding to moving Bloch and Néel boundaries. The above explicit analytic expression is in full agreement with the results of a qualitative investigation of the asymptotes given in our earlier paper.[4]

In the limit as $\theta\to 0$, we have

$$\theta(\varphi)=\mathrm{const}\cdot\exp\left\{\int^{\varphi} d\Phi\,\frac{\Gamma(\Phi)}{\omega(\Phi)}\right\} \tag{2.15}$$

and, consequently, we can have two different waves of establishing a homogeneous magnetization state ($\theta\to 0$ or π). In the former case the asymptotic behavior of the solutions is governed by zeros of the function $\omega(\varphi)$ and gives

$$\theta(\varphi)=\mathrm{const}(\varphi-\varphi_c)^\alpha,\quad \alpha=\Gamma(\varphi_c)\Big/\frac{d\omega}{d\varphi_c}. \tag{2.16}$$

Here, φ_c is the angle at which the precession frequency vanishes. In this case the attainment of a state with a homogeneous magnetization establishes an orientation of the plane of rotation of the magnetic moment vector which is constant in space. It should be noted that in the case of the branch given by Eq. (2.12) only this method of attainment of a homogeneous state is possible. In fact, for all values $u<u_-(\epsilon)$ there is, according to Eq. (2.14), such a value of the angle φ_c for which the precession frequency vanishes:

$$\sin 2(\varphi_c+\delta_-)=-u/u_+(\varepsilon). \tag{2.17}$$

In contrast, for the branch given by Eq. (2.11), we find that zeros of the precession frequency $\omega(\varphi)$ are given by the equation

$$\sin 2(\varphi_c+\delta_+)=-u/u_-(\varepsilon), \tag{2.18}$$

which have no solutions if

$$u_-(\varepsilon)<u<u_+(\varepsilon). \tag{2.19}$$

In the last case the nature of establishment of a homogeneous state is governed by the period-average value

$$\frac{1}{\pi}\int_0^\pi d\Phi\,\frac{\Gamma(\Phi)}{\omega(\Phi)}\equiv\left\langle\frac{\Gamma}{\omega}\right\rangle_{av}. \tag{2.20}$$

Solitary waves disappear in the limit $\langle\Gamma/\omega\rangle_{av}\to 0$. The above explicit expressions for the asymptotic behavior of the solutions can be generalized in the case when a static magnetic field parallel to the anisotropy axis is present in the system.

In the plane $(2\Gamma_0, u)$ the two branches of the solutions (2.11) and (2.12) correspond to the two circles shown in Fig. 1. The asymptotic behavior of the solutions of the Landau–Lifshitz equations is characterized by the fact that for all the permissible values of Γ_0 and u there are such values of the azimuthal angle φ for which either the precession frequency vanishes or the logarithmic derivative of the polar angle θ becomes zero. For the solutions corresponding to the outer circle the values of Γ_0 and u correspond to the arcs AB and $A'B'$ on which the logarithmic derivative of the polar angle θ is constant in sign and the precession frequency vanishes for certain values of the azimuthal angle φ. For the values of Γ_0 and u corresponding to the arcs CD and $C'D'$ the precession frequency does not vanish but the logarithmic derivative of has zeros. In this case the asymptotic behavior of the solutions for the angle θ exhibits oscillations whose amplitude tends to zero in the limit as $\theta\to 0$ (or $\theta\to\pi$).

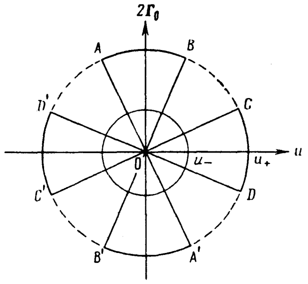

FIG. 1.

Consequently, an analysis of the asymptotic behavior demonstrates the existence of three characteristic velocities of magnetic moment waves whose dependences on the magnetic parameter ϵ is shown in Fig. 2. The transition from region I to region II results in the excitation of the precession of the magnetic moment in a wave and instead of solutions corresponding to the moving domain boundaries, we have solutions of the solitary wave type. Transition from region II to region III results not only in precession but also in nutational motion of the magnetic moment in the region where a homogeneous magnetization is established. Finally, transition from region III to region IV results in complete disappearance of the solutions of the solitary wave type and in the excitation of nonlinear spin waves. It should be noted that the existence of an intermediate region III is essentially associated with allowance for internal magnetic fields ($\epsilon\neq 0$).

3. In a numerical analysis of the problem of fast solitary waves the attention has been concentrated on the result needed in proofs. The system of equations (2.1) makes it possible to separate the class of solutions which are symmetric in respect of the angle $\theta(\xi)$ and

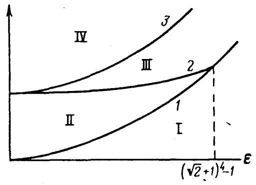

FIG. 2. Limiting velocities of magnetic moment waves of different types: 1) $u=u_-(\epsilon)$; 2) $u=[u_+^2(\epsilon)-u_-^2(\epsilon)]^{1/2}$; 3) $u=u_+(\epsilon)$.

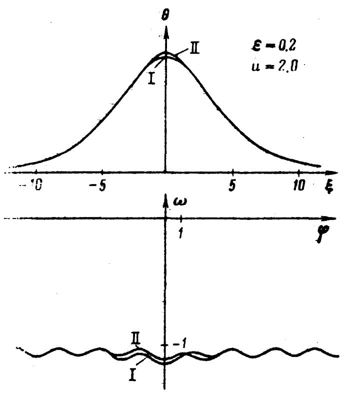

FIG. 3.

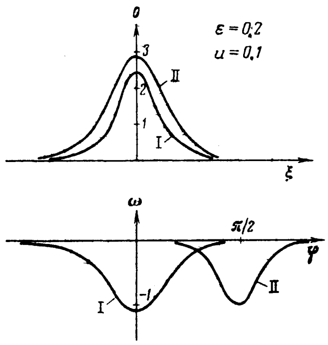

FIG. 5.

antisymmetric in respect of the angle $\varphi(\xi)$:

$$\theta(-\xi)=\theta(\xi), \quad \varphi(-\xi)=-\varphi(\xi)+2\varphi(0). \tag{3.1}$$

For this class of solutions the following conditions should be satisfied in the symmetry plane ($\xi=0$):

$$\begin{aligned} &\text{I. } \varphi(0)=0, \quad \omega^2(0)=1+\varepsilon, \\ &\text{II. } \varphi(0)=\pi/2, \quad \omega^2(0)=1; \end{aligned} \tag{3.2}$$

in this case we have $d\theta/d\xi\,|_{\xi=0}=0$.

According to the above asymptotic behavior of the solutions in the limit $\theta\to 0$ and, consequently, in the limit $|\xi|\to\infty$ the asymptotic boundary conditions for $\theta\ll\pi$ are determined completely by specifying the value $\varphi_\infty\in[0,\pi]$. For $u>u_-(\epsilon)$, the specification of φ_∞ determines, in accordance with Eqs. (2.9) and (2.12), the asymptotic values of the logarithmic derivative of $\Gamma(\varphi_\infty)$ and the precession frequencies $\omega(\varphi_\infty)$. Numerical calculation for different values of the velocities in the range

$$u_-(\varepsilon)<u<u_+(\varepsilon)$$

gave the values of $\varphi_\infty\in[0,\pi]$ which satisfied conditions I or II in Eq. (3.2). When the angle φ is varied, the resultant solutions first (for $\theta\ll\pi$) follow the asymptotic solution and then move away from it. For all the investigated values of the velocities there are always two values of φ_∞, which satisfy conditions I or II in Eq. (3.2). When the velocity is increased, max $\theta=\theta(0)$ decreases and the region of localization of a solitary wave becomes greater. Since an increase in the velocity is accompanied by a reduction in the value of $\Gamma(\varphi)/\omega(\varphi)$, the exact solution over an increasing range of the angle φ remains close to the asymptotic solutions found above. In the limit $u\to u_+(\epsilon)$ the asymptotes (2.9), (2.12), and (2.14) found here allow us to obtain essentially complete solution of the problem of a small-amplitude fast wave.

Figures 3, 4, and 5 give the dependences $\theta(\xi)$ and $\omega=\omega(\varphi)$ for the case $\epsilon=0.2$ and the wave velocities $u=2$, 1, and 0.1 [I is the solution corresponding to condition I of Eq. (3.2), II is the solution satisfying condition II of the same Eq. (3.2)]. The first of the velocities is close to the upper limit of the velocities of $u_+(\epsilon)$ waves and corresponds essentially to the case of a small-amplitude solitary wave and the second value is close to the lower limit $u_-(\epsilon)$. Figure 6 shows the dependences $\theta(\xi)$ and $\omega(\varphi)$ for $\epsilon=0.9$ and $u=2.35$. The range of existence of solitary waves of the first type is limited in respect of ϵ to the values in the range $\epsilon<\epsilon_c=(\sqrt{2}+1)^4-1$. Thus, the results of our numerical calculations demonstrate convincingly the existence, for $\epsilon\neq 0$, of two types of fast solitary magnetic moment waves in the velocity range $u_-(\epsilon)<u<u_+(\epsilon)$.

We shall conclude by pointing out that the method of finding asymptotic solutions proposed in the present paper can also be used in studies of nonlinear waves in ferromagnets with a more complex anisotropy, and also in the presence of a magnetic field perpendicular to the easy-magnetization axis.[5]

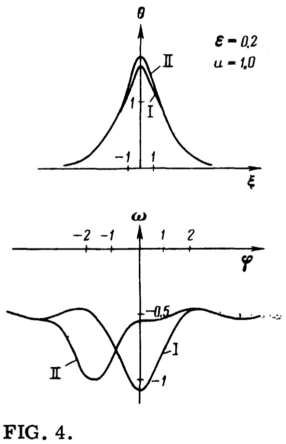

FIG. 4.

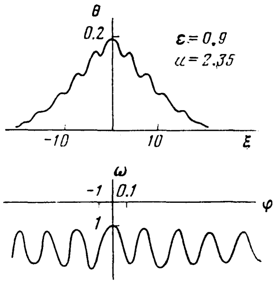

FIG. 6.

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Electrohydrodynamic instability and anisotropy of the electrical conductivity in the smectic *A* phase of a liquid crystal

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The phenomenon of electrohydrodynamic instability has been detected in the smectic *A* phase of a liquid crystal with positive dielectric anisotropy. The threshold, frequency, and contrast characteristics have been investigated both for this instability and for the electrically induced confocal-homotropic transition. The variation of the parameters of EHD instability and of the confocal and homotropic textures has been compared with the rules of variation of the electrical conductivity of the corresponding structures. A possible mechanism for the onset of instability is in many respects analogous to the Carr-Helfrich mechanism for nematic liquid crystals.

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The possibility of an onset of field-induced instability in the smectic *A* phase of liquid crystals (LC) is predicted by theory both in weak[1] and in sufficiently strong[2] electromagnetic fields. In the first case, because of the small value of the deformation, the effect has not yet been confirmed experimentally, whereas in fields of large intensity an electrically induced transition has been observed in a number of smectic liquid crystals (SLC).[3] The occurrence of electrohydrodynamic (EHD) instability in SLC is also predicted theoretically,[4] but experiments confirming this phenomenon are not, to our knowledge, reflected in the literature.

In the present paper, we report results of an investigation showing the presence of EHD instability in the smectic *A* phase, and we compare the behavior and peculiarities of the observed instability with the rules of variation of the electrical conductivity in SLC.

Chosen as object of investigation was 4-nitrophenyl-4-octyl oxybenzoate, which possesses both smetic and nematic mesophases and passes from the solid crystal following sequence of temperatures: SC 49 °C SLC-*A* (SC) state to the isotropic liquid (IL) state in the 61 °C NLC 68 °C IL. The smectic phase formed on cooling extended to ~33 °C. On slow transition from the isotropic phase to the nematic liquid crystal (NLC) phase, the LC molecules aligned themselves predominantly perpendicular to the surfaces of the electrodes, producing a homotropic orientation, which persisted also in the smectic state.

Application to the sample of a low-frequency (~20 Hz) voltage above a certain threshold value (U_{th}) causes the appearance in individual parts of the cell of nuclei of turbulent motion, which, after spreading, fill the whole field of view. The rate of spreading of the turbulence depends on the temperature of the sample and on the value of the applied voltage. Within the interval 35 °C < t < 55 °C, $0 < U - U_{th} < 15$ V the numerical values of these quantities satisfy the empirical relation

$$V[\mu\text{m/sec}] = (1{,}8 - t_{tr}/t)(U - U_{th}),$$

where t_{tr} is the temperature of the transition SLC-*A* → NLC.

The value of U_{th} increases with lowering of temperature and with increase of the frequency of the applied field (Fig. 1). Each temperature has its own "critical" frequency (f_{cr}), above which EHD instability does not occur. With increase of temperature, f_{cr} shifts toward high frequencies. It should be noted that the smectic phase formed on heating a solid crystal has a predominantly planar orientation of the molecules and that the U_{th} of the corresponding texture is somewhat below the threshold for the homotropic texture.

The optically transparent homotropic structure is destroyed in an electric field, and in the process, turbulence severely scatters the transmitted light. After the applied voltage is turned off, a stable confocal texture is formed, and a scattering condition persists. The relaxation time of such a state is very

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