Cyclotron resonance on the skipping orbits in an electron liquid

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(Submitted 29 November 1977)
Zh. Eksp. Teor. Fiz. **74**, 2138–2153 (June 1978)

A theory of cyclotron resonance for transitions between surface-electron levels is constructed which takes the interelectron interaction into account, as a result of which it becomes possible to eliminate the existing discrepancy between experiment and the theory of a gas of noninteracting conduction electrons, a theory which prohibits the corresponding resonances in metals and semimetals with convex Fermi surfaces. A method is suggested for the determination of the parameters characterizing the interelectron interaction.

PACS numbers: 76.40.+b

1. INTRODUCTION

In an earlier paper[1] we established the possibility of the propagation of surface quantum spin waves in the electron liquid of metals. The eigenfrequencies of such spin waves are close to the observed frequencies of resonance absorption of electrons glancing along the surface of a metal in a magnetic field.[2,3] At the same time because of the comparative weakness of the electron paramagnetism, the impedance resonances observed in a weak magnetic field are connected not with spin, but with cyclotron, waves. The theory of surface cyclotron waves and resonances in an electron gas is developed in Refs. 4–6, in which it is shown, in particular, that for the existence of such waves near the transition frequencies of the glancing electrons to be possible it is necessary that sections of the Fermi surface be cylindrical (see also Ref. 7). The latter assertion is, as shown in the present paper, connected with the use of the noninteracting particle model for the conduction electrons. Notice also that, according to Ref. 1, surface quantum spin waves can exist in metals whose Fermi surface does not have cylindrical sections when allowance is made for the interelectron interaction.

In the present paper we develop a theory of cyclotron resonance on the glancing orbits of the surface electrons in the electron liquid of metals with arbitrary Fermi surfaces. Notice that in the long-wave limit the frequency of the surface waves in an electron gas that are studied in Refs. 4–6 turns out to be lower than the frequency of the transition between surface levels. In contrast, the cutoff frequency predicted by our theory of surface waves in metals with convex Fermi surfaces is higher than the surface-transition frequency. Such waves do not exist in the gas model. The new possibility of the propagation of surface waves near the maxima of the transition frequency is due to the interaction between the conduction electrons. A comparison of the formulas of the theory developed by us with the experimental data on the oscillations in the impedance of a metal will allow us to draw conclusions about the strength of the interelectron interaction.

2. DERIVATION OF AN INTEGRAL EQUATION FOR THE GLANCING-ELECTRON DISTRIBUTION FUNCTION

The effects discussed below are essentially quantum effects. However, aiming at the most intelligible exposition, we do not in the present paper construct a theory directly on the basis of the quantum equation for the density matrix (cf. Ref. 1), but follow the quasi-classical approach used in the theory of cyclotron resonance on glancing electrons.[4] In this case with the aid of the quasiclassical equation it is easy, firstly, to introduce the parameters characterizing the Fermi-liquid interaction of the conduction electrons and, secondly, to consistently take the boundary conditions for the electrons on the metal surface into account. The subsequent quantization of the classical solution (which can be carried out, following, to a certain extent, the procedure used by Kaner and Makarov[4]) allows us to follow directly the themes of the new effects that manifest themselves in cyclotron resonances on the glancing orbits as a result of the allowance for the interelectron interaction.

In the theory of the electron liquid[8] the weakly-excited states are, upon the neglect of the paramagnetic effects, characterized by a deviation, δf, of the distribution function from the equilibrium value. Assuming the time dependence to be $\sim e^{-i\omega t}$, directing the constant magnetic field along the z axis, and assuming the metal occupies the half-space $y>0$, we can write for the case of a surface wave propagating along the direction of the constant magnetic field and the following kinetic equation:

$$\left[i(\omega-k_z v_z)-v_y\frac{\partial}{\partial y}-\frac{1}{\tau}+\Omega(p_z)\frac{\partial}{\partial\varphi}\right]\left[\delta f(y,\varepsilon,p_z,\varphi)-\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}\delta\varepsilon(y,\varepsilon,p_z,\varphi)\right]=\frac{\partial f_0(\varepsilon)}{\partial\varepsilon}[e\mathbf{v}\mathbf{E}(y)-i\omega\delta\varepsilon(y,\varepsilon,p_z,\varphi)]. \quad (2.1)$$

Here e is the electron charge, $\Omega(p_z)$ is the cyclotron frequency of the gyroscopic rotation of the electron, τ is the momentum relaxation time, φ is the angular var-

 0038-5646/78/061113-09$02.40

iable characterizing the position of the electron in its orbit in momentum space, ϵ is the electron energy, p_z is the component of the electron momentum along the direction of the magnetic field, while

$$\delta\varepsilon(y,\varepsilon,p_z,\varphi)=\int_0^\infty d\varepsilon'\int dp_z'\int d\varphi'\Phi(\varepsilon,p_z,\varphi;\varepsilon',p_z',\varphi')\delta f(y,\varepsilon',p_z',\varphi'). \tag{2.2}$$

Here $\Phi(\epsilon, p_z, \varphi; \epsilon', p_z', \varphi')$ is a function characterizing the interaction of the electrons. Setting $\delta\bar{f}=\delta f-\delta\epsilon\partial f_0/\partial\epsilon$, we shall seek the solution to Eq. (2.1) in the form $\delta f=g_0\partial f_0/\partial\epsilon$, $\delta\bar{f}=g(y,p_z,\varphi)\partial f_0/\partial\epsilon$, where $\partial f_0/\partial\epsilon=-2(2\pi\hbar)^{-3}\delta(\epsilon-\epsilon_F)$, ϵ_F is the Fermi energy. The function Φ determines the resolvent operator, R, of the equation

$$g(y,p_z,\varphi)=g_0(y,p_z,\varphi)+\frac{2}{(2\pi\hbar)^3}\int dp_z'd\varphi'\Phi(\varepsilon_F,p_z,\varphi;\varepsilon_F,p_z',\varphi')g_0(y,p_z',\varphi'), \tag{2.3}$$

with the aid of which we have:

$$g_0(y,p_z,\varphi)=\int dp_z'\,d\varphi'R(p_z,\varphi;p_z',\varphi')g(y,p_z',\varphi'). \tag{2.4}$$

In accordance with the formula (2.4) we obtain from (2.2) the expression

$$\delta\varepsilon(y,\varepsilon_F,p_z,\varphi)=-\int dp_z'd\varphi'\alpha(p_z,\varphi;p_z',\varphi'),$$

where we have introduced the notation

$$\alpha(p_z,\varphi;p_z',\varphi')=\frac{2}{(2\pi\hbar)^3}\int dp_z''d\varphi''\Phi(\varepsilon_F,p_z,\varphi;\varepsilon_F,p_z'',\varphi'')\times R(p_z'',\varphi'';p_z',\varphi'). \tag{2.5}$$

Below we shall omit the argument $\epsilon=\epsilon_F$. The function α characterized the effects of the interelectron interaction.

Let us set as our problem the search for the solutions to Eq. (2.1) that describe the resonance electrons glancing along, and undergoing small-angle scattering by, the metal surface. This means that in the characteristic equation

$$\Omega(p_z)dy=-v_y(p_z,\varphi)d\varphi \tag{2.6}$$

v_y should be considered to be small. Then, reckoning the angle φ from the zero value, at which $v_y(p_z,0)=0$, we can write for the glancing electrons the relation $v_y(p_z,\varphi)=v_y'(p_z)\varphi$. Correspondingly, we obtain for the orbits of the glancing electrons in accordance with the characteristic equation (2.6) the relation $y=\frac{1}{2}R(p_z)[\varphi_0^2-\varphi^2]$, where φ_0 is the value of the angle at which the electron collides with the surface, while $R(p_z)=v_y'(p_z)/\Omega(p_z)$ is the radius of the electron orbit.

In seeking the solutions to Eq. (2.1) we shall use the condition for specular reflection[9] of the glancing electrons: $g(0,p_z,\varphi)=g(0,p_z,-\varphi)$. Then from Eq. (2.1) follows:

$$g(y,p_z,\varphi)=[2i\Omega(p_z)]^{-1}\Big\{\operatorname{ctg}[\beta(p_z)\varphi_0(|y|,p_z,\varphi)\times\int_{-\varphi_0(|y|,p_z,\varphi)}^{\varphi_0(|y|,p_z,\varphi)}d\varphi'-i\Big[\int_{\varphi}^{\varphi_0(|y|,p_z,\varphi)}d\varphi'-\int_{-\varphi_0(|y|,p_z,\varphi)}^{\varphi}d\varphi'\Big]\Big\}$$
$$\times\exp[-i\beta(p_z)(\varphi-\varphi')]\{ev(p_z,0)E(\tfrac{1}{2}R(p_z)[\varphi_0^2(|y|,p_z,\varphi)-\varphi'^2])-i\omega\delta\varepsilon(\tfrac{1}{2}R(p_z)[\varphi_0^2(|y|,p_z,\varphi)-\varphi'^2],p_z,0)\}. \tag{2.7}$$

Here we have taken into account the smallness of φ in comparison with unity and have used the notation

$$\varphi_0(y,p_z,\varphi)=[\varphi^2+2y/R(p_z)]^{1/2},\quad \beta(p_z)=[\omega-k_zv_z(p_z,0)+i/\tau]/\Omega(p_z).$$

The formula (2.7) has been written in a form that allows us to see the possibility of its even continuation into the region of negative y values.

In what follows we shall use the Fourier expansion:

$$g(y,p_z,\varphi)=\frac{1}{\pi}\int_0^\infty dkG(k,p_z,\varphi)\cos ky,\quad E(y)=\frac{1}{\pi}\int_0^\infty dkE(k)\cos ky,$$
$$\delta\varepsilon(y,p_z,0)=\frac{1}{\pi}\int_0^\infty dk\delta\varepsilon(k,p_z)\cos ky.$$

With the aid of Eq. (2.7) we can write the following expression for the Fourier transform:

$$G(k,p_z,\varphi)=[R(p_z)/i\pi\Omega(p_z)]\int_\varphi^{\varphi_0}d\varphi_0\varphi_0\cos[kR(p_z)(\varphi_0^2-\varphi^2)/2]$$
$$\times\Big\{\operatorname{ctg}[\beta(p_z)\varphi_0]\int_{-\varphi_0}^{\varphi_0}d\varphi'-i\Big[\int_\varphi^{\varphi_0}d\varphi'-\int_{-\varphi_0}^{\varphi}d\varphi'\Big]\Big\}\exp[i\beta(p_z)(\varphi'-\varphi)]$$
$$\times\int_0^\infty dk'\cos[k'R(p_z)(\varphi_0^2-\varphi'^2)/2]\{ev(p_z,0)E(k')-i\omega\delta\varepsilon(k',p_z)\}. \tag{2.8}$$

In deriving this equation we neglected the contribution of the large values of y, which cannot correspond to glancing electrons, and therefore cannot lead to resonances at the frequencies of the surface transitions. Furthermore, we have made the change of variables $y\to\varphi_0$. Now following, to a certain extent, the procedure used in Ref. 4, we go over from the classical expression (2.8) to the corresponding quantum-mechanical expression, which allows us to separate out the effects connected with the transitions between the quantum levels of the surface electrons. To begin with, let us take into account the fact that

$$\exp[i\beta(p_z)(\varphi'-\varphi)]=\sum_s\frac{(-1)^s\sin[\beta(p_z)\varphi_0]}{\beta(p_z)\varphi_0-\pi s}\exp\Big[\frac{i\pi s}{\varphi_0}(\varphi'-\varphi)\Big].$$

Further, in the quantum theory we should allow for the quantization of the angle at which the electron collides with the metal surface:

$$\varphi_n(p_z)=\{3\pi(n-\tfrac{1}{4})|\hbar\Omega(p_z)/p_y'(p_z,\varphi=0)v_y'(p_z,\varphi=0)|\}^{1/3}.$$

Replacing in (2.8) φ_0 by such an expression, going over from integration over φ_0 to summation, and taking into account the fact that $\Delta\varphi_0=\Delta\varphi_n=\varphi_n\Delta n/3(n-\frac{1}{4})$, we obtain

$$G(k, p_z, \varphi)=[\hbar/ip_y'(p_z, 0)]\sum_{n,s}\cos[kR(p_z)(\varphi_n^2(p_z)-\varphi^2)/2]$$

$$\cdot\exp[-i\pi s\varphi/\varphi_n(p_z)]$$

$$\times\{\omega-s\omega_n(p_z)-k_zv_z(p_z, 0)+i/\tau\}^{-1}(-1)^s\varphi_n^{-2}(p_z)$$

$$\times\left\{\cos\beta\varphi_n(p_z)\int_{-\varphi_n(p_z)}^{\varphi_n(p_z)}d\varphi'-i\sin\beta\varphi_n(p_z)\left[\int_{\varphi}^{\varphi_n(p_z)}-\int_{-\varphi_n(p_z)}^{\varphi}\right]d\varphi'\right\}$$

$$\times\exp[i\pi s\varphi'/\varphi_n(p_z)]\int_0^\infty dk'\cos[k'R(p_z)(\varphi_n^2(p_z)-\varphi'^2)]$$

$$\times\{ev(p_z, 0)\mathbf{E}(k')-i\omega\delta\varepsilon(k', p_z)\}, \qquad (2.9)$$

where the frequency of the transition between the quasiclassical levels is given by the formula (cf. Ref. 10)

$$[E(p_z, n+s)-E(p_z, n)]/\hbar=s\omega_n(p_z)=s|\pi^2\Omega^2(p_z)p_y'(p_z)v_y'(p_z)/3(n-1/4)\hbar|^{1/3}.$$

For a further transformation of Eq. (2.9) we should use the fact that the resonances at the surface transitions occur only when one of the denominators of the sum on the right-hand side turns out to be small, so that $|\omega-s\omega_n(p_z, 0)-k_zv_z(p_z)|\ll\omega$. In this case it is important that such a denominator does not vanish, for otherwise there arises strong collisionless damping leading to considerable broadening, which virtually liquidates the resonance. A resonance is possible only when the variable frequency ω is close to the extremal value of the transition frequency $s\omega_n$ for $k_z=0$. In other words, this is possible when $|p_z-p_0|\ll p_0$, where for $k_z\neq 0$ the "extremal" momentum $p_0(k_z)$ is determined by the equation $(d/dp_z)[s\omega_n(p_z)+k_zv_z(p_z, 0)]=0$. We shall call the numbers n and s corresponding to the denominator that turns out to be small when $p_z=p_0$ resonance numbers. If n and s are not very large compared to unity, then the resonance term turns out to be unique. Let us emphasize that the denominators of the other terms ($m\neq n$, $r\neq s$) may turn out to be small, but the singularities corresponding to them occur far from the extremal momentum, and therefore their contribution turns out to be relatively small, although it can lead to such effects as collisionless damping.

Having it in mind to take such an effect into account, we can set $\beta\varphi_n=\pi s$ in all the terms of the formula (2.9). Then we obtain

$$G(k, p_z, \varphi)=[\hbar/ip_y'(p_0)]\Big\{\Psi_{ns}^*(k, p_0, \varphi)[\omega+i/\tau-s\omega_n(p_0)-k_zv_z(p_0, 0)$$

$$-(p_z-p_0)^2(s\omega_n''+k_zv_z'')/2]^{-1}\int_0^\infty dk'[ev(p_0, 0)\mathbf{E}(k')-i\omega\delta\varepsilon(k', p_0)]\int_{-\varphi_n(p_0)}^{\varphi_n(p_0)}d\varphi'$$

$$\Psi_{ns}(k', p_0, \varphi')-i\pi\sum_{r\neq s, m\neq n}\delta(\omega-r\omega_m(p_z)-k_zv_z(p_z, 0))\Psi_{mr}^*(k, p_z, 0)$$

$$\times\int_0^\infty dk'[ev(p_z, 0)\mathbf{E}(k')-i\omega\delta\varepsilon(k', p_z)]\int_{-\varphi_n(p_z)}^{\varphi_n(p_z)}d\varphi'\Psi_{mr}(k', p_z, \varphi')\Big\}, \qquad (2.10)$$

where

$$(s\omega_n''+k_zv_z'')=\{(d^2/dp_z^2)[s\omega_n(p_z)+k_zv_z(p_z, 0)]\}_{p_z=p_0},$$

$$\Psi_{mr}(k, p_z, \varphi)=\varphi_m^{-1}(p_z)\exp[i\pi r\varphi/\varphi_m(p_z)]\cos[kR(p_z)(\varphi_m^2(p_z)-\varphi^2)/2].$$

A direct computation yields the following expression for the quasiclassical matrix element:

$$\langle n+r|\cos ky|n\rangle=\int_{-\varphi_n(p_z)}^{\varphi_n(p_z)}d\varphi\Psi_{nr}^q(k, p_z, \varphi),$$

where

$$\Psi_{nr}^q(k, p_z, \varphi)=\Psi_{nr}(k, p_z, \varphi)[1+2r\varphi_n^2(p_z)/3(n-1/4)\varphi^2]^{-1/4}.$$

Bearing in mind the substantial contribution of the small angles φ, below instead of Ψ_{nr} we shall use its quantum analog. Furthermore, let us note that $\Psi_{nr}^q=0$ when $|\varphi|>\varphi_n(p_z)$.

For what follows it will be sufficient to find the function

$$G(k, p_z)=\int_{-\pi}^{\pi}d\varphi G(k, p_z, \varphi),$$

for which, according to (2.10) and the properties of Ψ_{ns}^q, we can write the following integral equation:

$$G(k, p_z)=[\hbar/ip_y'(p_0)]\Big\{I_{ns}(k, p_z)\int_0^\infty dk'I_{ns}(k', p_0)[ev(p_0, 0)\mathbf{E}(k')$$

$$+i\omega\int dp_z'\alpha(p_0, p_z')G(k', p_z')][\omega+i/\tau-s\omega_n(p_0)-k_zv_z(p_0, 0)$$

$$-(p_z-p_0)^2(s\omega_n''+k_zv_z'')/2]^{-1}-i\pi\sum_{r\neq s, m\neq n}I_{mr}(k, p_z)\int_0^\infty dk'I_{mr}(k', p_z)$$

$$\times[ev(p_z, 0)\mathbf{E}(k')+i\omega\int dp_z'\alpha(p_z, p_z')G(k', p_z')]\delta(\omega-r\omega_m(p_z)-k_zv_z(p_z, 0))\Big\}. \qquad (2.11)$$

Here we have used the notation $\alpha(p_z, 0; p_z', 0)\equiv\alpha(p_z, p_z')$,

$$I_{ns}(k, p_z)=\int_{-1}^{1}dx[1+2s/3(n-1/4)x^2]^{-1/4}$$

$$\times\cos\pi sx\cos[kR(p_z)\varphi_n^2(p_z)(1-x^2)/2]. \qquad (2.12)$$

The resonance properties of the solution of the quasiclassical equation (2.11) allow, as we shall see below, us to understand the laws governing cyclotron resonance on glancing electrons.

3. SOLUTION OF THE INTEGRAL EQUATION (2.11) FOR THE DISTRIBUTION FUNCTION OF THE GLANCING ELECTRONS

In this section we obtain the solution to Eq. (2.11) for $k_z=0$. In the previous papers devoted to the study of the properties of glancing electrons[4-6] the authors considered the model in which the metal is assumed to possess a cylindrical Fermi surface, which is characterized by the fact that the transition frequency does not depend on p_z. This means that the p_z integration that arises in the solution of the integral equation amounts to multiplication by the dimension of the cylindrical Fermi surface in the p_z direction. The nondependence of the transition frequency on p_z led to the result that no other frequency $r\omega_m$ with $r\neq s$ and $m\neq n$ satisfied the resonance condition $|\omega-r\omega_m|\ll\omega$. Consequently, in the model of a metal with a cylindrical Fermi surface the sum of the nonresonance terms did not contain terms whose denominators satisfied the condition for resonance. In accordance with (2.11), the impossibility of satisfying the resonance condition $|\omega-r\omega_m|\ll\omega$

for $r \neq s$ and $m \neq n$ implies the absence of collisionless damping in the model of a metal with a cylindrical Fermi surface.

The situation is different in the case of a metal with a convex noncylindrical Fermi surface, for which it is typical for the transition frequency $r\omega_m(p_z)$ to depend on p_z. Such a dependence leads to the variation of the quantity $r\omega_m(p_z)$ from zero at the reference point to the maximum value, $r\omega_m$, at the point corresponding to the maximum of the cross section of the Fermi surface. In those terms of the nonresonance sum for which the condition $rm^{-1/3} > sn^{-1/3}$ is satisfied, the dependence of $r\omega_m(p_z)$ on p_z leads to the result that the resonance condition

$$s\omega_n(p_0) - r\omega_m(p_1) = 0, \tag{3.1}$$

which corresponds to the appearance of collisionless Landau damping, is fulfilled at the point p_1. The formula (3.1) determines the quantity $p_1 = p_1(\omega, m, r)$. The point $p_0 = p_0(0)$ corresponds to the extremum of the resonance frequency $s\omega_n(p_z)$. The first term on the right-hand side of Eq. (2.11) gives the most important contribution to the integral over p_z from the region in the vicinity of the extremum of the denominator, which allows us to seek the solution in the form

$$G(k, p_z) = g_{ns}(k)\delta(p_z - p_0) + \sum_{r,m} q_{mr}(k)\delta(p_z - p_1(m,r)).$$

Bearing in mind that the collisionless damping is a weak effect, we can write for the function q_{mr} the following approximate expression:

$$q_{mr}(\kappa) = -[\pi\hbar/p_y'(p_1(m,r))]I_{mr}(k, p_1(m,r)) \int_0^\infty dk' I_{mr}(k', p_1(m,r)) \times [ev(p_1(m,r), 0)E(k') + i\omega\alpha(p_1(m,r), p_0)g_{ns}(k')]|r\omega_m'(p_1(m,r))|^{-1}. \tag{3.2}$$

Then for the function $g_{ns}(k)$ we obtain the following integral equation:

$$g_{ns}(k) = A_{ns}I_{ns}(k, p_0)\left\{\int_0^\infty dk' I_{ns}(k', p_0)g_{ns}(k') + \sum_{r,m} B(n, s; m, r) \times \int_0^\infty dk' I_{mr}(k', p_1(m,r))g_{mr}(k')\right\} + W_{ns}(k). \tag{3.3}$$

Here we have introduced the notation

$$A_{ns} = -\frac{\pi \cdot 2^{1/3}\hbar\omega\alpha}{s\omega_n''(p_0)p_y'(p_0)}\left[\frac{s\omega_n''(p_0)}{s\omega_n(p_0) - \omega - i/\tau}\right]^{1/2},$$

$$B(n, s; m, r) = -i\pi\hbar\omega N_{ns}^{mr}\frac{\alpha(p_0, p_1(m,r))\alpha(p_1(m,r), p_0)}{\alpha p_y'(p_1(m,r))|r\omega_m'(p_1(m,r))|},$$

$$W_{ns}(k) = \frac{A_{ns}}{i\omega\alpha}I_{ns}(k, p_0)\int_0^\infty dk'\left\{I_{ns}(k', p_0)ev(p_0, 0) - i\pi\hbar\omega \times \sum_{r,m} N_{ns}^{mr}\frac{\alpha(p_0, p_1(m,r))ev(p_1(m,r), 0)}{p_y'(p_1(m,r))|r\omega_m'(p_1(m,r))|}I_{mr}(k', p_1(m,r))\right\}E(k'),$$

where

$$N_{ns} = \int_0^\infty dk I_{ns}^2(k, p_0); \quad N_{ns}^{mr} = \int_0^\infty dk I_{ns}(k, p_0)I_{mr}(k, p_1(m,r)),$$

and $\alpha = \alpha(p_0, p_0)$. In this case $p_0 \equiv p_0(k_z = 0)$.

Bearing in mind the smallness of the coefficients B, we can write the solution to Eq. (3.3) in the following form:

$$g_{ns}(k) = W_{ns}(k) + A_{ns}I_{ns}(k, p_0)\left\{1 + A_{ns}\sum_{m,r} B(n, s; m, r)N_{ns}^{mr}\right\} \times \left[1 - A_{ns}N_{ns} - A_{ns}\sum_{m',r'} B(n, s; m', r')N_{ns}^{m'r'}\right]^{-1}\int_0^\infty dk' W(k')\left\{I_{ns}(k', p_0) + A_{ns}N_{ns}\sum_{m'',r''} B(n, s; m'', r'')I_{m''r''}(k', p_1(m'', r''))\right\}. \tag{3.4}$$

Hence it follows that the electron distribution has a resonant dependence on the frequency, while the value of the frequency of such a resonance is itself determined by the equation

$$1 = A_{ns}\left\{N_{ns} + \sum_{r\neq s, m\neq n} B(n, s; m, r)N_{ns}^{mr}\right\}. \tag{3.5}$$

We then have

$$\omega = s\omega_n(p_0)\left\{1 - \frac{2\omega_n(p_0)}{\omega_n''(p_0)}\left[\frac{\pi\alpha\hbar N_{ns}}{p_y'(p_0)}\right]^2\right\} - i\left(\frac{1}{\tau} + \nu_L(n, s)\right). \tag{3.6}$$

Here the contribution of the collisionless damping is characterized by the quantity

$$\nu_L(n, s) = -\frac{4\pi^2\hbar^2 s^2\omega_n^3(p_0)\alpha N_{ns}}{\omega_n''(p_0)[p_y'(p_0)]^2}\sum_{\substack{r\neq s\\ m\neq n}}\frac{\alpha(p_0, p_1(m,r))\alpha(p_1(m,r), p_0)[N_{ns}^{mr}]^2}{p_y'(p_1(m,r))|r\omega_m'(p_1(m,r))|}. \tag{3.7}$$

If the resonance frequency is close to the maximum value of the transition frequency, when $\omega_n''(p_0) < 0$, then, according to the solution (3.6), $\omega > s\omega_n(p_0)$. If, on the other hand, the resonance occurs in the vicinity of the minimum of the transition frequency, then $\omega < s\omega_n(p_0)$. In both cases the necessary condition for the existence of the solution (3.6) is

$$\omega_n''(p_0)p_y'(p_0)\alpha < 0, \tag{3.8}$$

which determines that sign of the function, α, characterizing the interelectron interaction for which the resonance (3.6) turns out to be possible.

According to our estimates, in the experiments that have been performed on cyclotron resonance on glancing electrons $\nu_L\tau \ll 1$. Therefore, below we shall focus our attention on the consideration of effects in which the collisionless damping is neglected. To begin with, with the aid of (2.12) we write

$$N_{ns} = \int_0^\infty dk I_{ns}^2(k, p_0) = [2\pi/R(p_0)\varphi_n'^2(p_0)]\int_0^1 dx[x^2 + 2s/3(n - {}^1/_4)]^{-1/2}\cos^2\pi s x \approx [\pi/R(p_0)\varphi_n'^2(p_0)]L_{ns}, \tag{3.9}$$

where $L_{ns} = \ln[6(n - \frac{1}{4})/s]$, $n \gg s$.

According to this, and without allowance for the collisionless damping, the formula (3.6) assumes the following form:

$$\omega=s\omega_n(p_0)\left\{1-\frac{2\pi^{1/2}\omega_n(p_0)\alpha^2}{3^{1/2}\omega_n''(p_0)}\left[\frac{\hbar\Omega(p_0)}{p_y'(p_0)v_y'(p_0)}\right]^{1/3}\frac{L_{ns}^2}{(n-1/4)^{1/3}}\right\}-\frac{i}{\tau}. \tag{3.10}$$

This expression will be used below to analyze the experimental data.

Let us give here without derivation the solution to Eq. (2.15) for $k_z\neq 0$. Here we neglect the collisionless damping. As a result

$$G(k,p_z)=[\hbar/ip_y'(p_0)][\omega-s\omega_n(p_0)-k_zv_z(p_0,0)-(p_z-p_0)^2\times(s\omega_n''(p_0)+k_zv_z''(p_0,0))/2+i/\tau]^{-1}\times I_{ns}(k,p_0)\int_0^\infty dk' I_{ns}(k',p_0)ev(p_0,0)E(k')\Lambda^{-1}. \tag{3.11}$$

Here $p_0=p_0(k_z)$, while

$$\Lambda=1+\frac{2\pi\alpha\hbar[s\omega_n(p_0)+k_zv_z(p_0,0)]N_{ns}}{[s\omega_n''(p_0)+k_zv_z''(p_0,0)][\Delta(k_z)]^{1/2}p_y'(p_0)}, \tag{3.12}$$

where we have used the notation

$$\Delta(k_z)=2\frac{s\omega_n(p_0(k_z))+k_zv_z(p_0(k_z),0)-\omega-i/\tau}{s\omega_n''(p_0(k_z))+k_zv_z''(p_0(k_z),0)}. \tag{3.13}$$

According to (3.11) the resonance frequency is determined by the equation:

$$[\Delta(k_z)]^{1/2}=A, \tag{3.14}$$

where

$$A=-\frac{2\pi^{1/2}\alpha L_{ns}}{3^{1/2}(n-1/4)^{1/3}}\left[\frac{\hbar\Omega(p_0)}{p_y'(p_0)v_y'(p_0)}\right]^{1/3}\frac{s\omega_n(p_0)+k_zv_z(p_0,0)}{s\omega_n''(p_0)+k_zv_z''(p_0,0)}. \tag{3.15}$$

The solution to Eq. (3.14) corresponds to the resonance frequency, provided

$$\alpha[s\omega_n(p_0)+k_zv_z(p_0,0)]/p_y'(p_0)[s\omega_n''(p_0)+k_zv_z''(p_0,0)]<0.$$

For a finite value of k_z the resonance should be identified with waves of frequency

$$\omega=s\omega_n(p_0(k_z))+k_zv_z(p_0(k_z),0)-i/\tau-1/2A^2[s\omega_n''(p_0(k_z))+k_zv_z''(p_0(k_z),0)]. \tag{3.16}$$

propagating along the metal surface. The admissible value of k_z is determined by the condition for $[\Delta(k_z)]^{1/2}$ for $\tau\to\infty$ to be real. In particular, for $s\omega_n''+k_zv_z''<0$ this condition has the form

$$k_zv_z(p_0(k_z),0)<\omega-s\omega_n(p_0(k_z)), \tag{3.17}$$

while in the opposite case

$$-k_zv_z(p_0(k_z),0)<s\omega_n(p_0(k_z))-\omega. \tag{3.18}$$

At small values of k_z, when a power series expansion is possible, these inequalities get simplified. If the resonance is connected with electrons of a noncentral cross section of the Fermi surface, then $v_z(p_0(0))\neq 0$ and $p_0(0)$ is determined, as usual, by the extremum of the surface-transition frequency $\omega_n'(p_0(0))=0$. Then, according to (3.17) and (3.18), we have $|k_zv_z(p_0(0))|<|\omega-s\omega_n(p_0(0))|$. If, on the other hand, the resonance is due to electrons of the central cross section, then $v_z(p_0(0))=0$ and $p_0(0)=0$, $p_0(k_z)=-k_z[v_z'(0,0)/\omega_n''(0)]$. Hence for k_z we have

$$2|\omega_n''(0)[\omega-s\omega_n(0)]|>[v_z'(0,0)]^2k_z^2.$$

In conclusion of this section, let us point out that the formula (3.11) determines the resonance contribution to the current due to the glancing electrons. Assuming that $\mathbf{v}(p_0,0)$ has only an x component, we have:

$$j_x^{res}=-2e(2\pi\hbar)^{-3}\int dp_zG(k,p_z)(\partial^2\sigma/\partial p_z\partial\varphi)_{\varphi=0}[v_x(p_z,0)/v(p_z,0)],$$

where σ is an element of area of the Fermi surface. According to (3.11)

$$-4\pi i\omega c^{-2}j_x^{res}=D\{[\Delta(k_z)]^{1/2}-A\}^{-1}I_{ns}(k,p_0)\int_0^\infty dk' I_{ns}(k',p_0)E_x(k'), \tag{3.19}$$

where

$$D=-\frac{e^2\omega}{\pi\hbar^2c^2}\left(\frac{\partial^2\sigma}{\partial p_z\partial\varphi}\right)_{\varphi=0,p_z=p_0}\frac{2v_x^2(p_0,0)}{v(p_0,0)p_y'(p_0)[s\omega_n''(p_0)+k_zv_z''(p_0)]}. \tag{3.20}$$

4. THE RESONANCE FREQUENCY FOR A METAL WITH AN ELLIPSOIDAL FERMI SURFACE (BISMUTH)

In the experiments described in Refs. 2, 11, and 12, the cyclotron resonance on skipping electrons in bismuth, which has an ellipsoidal Fermi surface, was studied in detail. In this case the theory allows significant progress to be made in the particularization of the general formulas of the preceding section. The energy spectrum of the electrons of a metal with an ellipsoidal Fermi surface is described by the formula

$$\varepsilon=(2m_x)^{-1}p_x^2+(2m_y)^{-1}p_y^2+(2m_z)^{-1}p_z^2, \tag{4.1}$$

where m_x, m_y, and m_z are constants having the dimensions of mass, while the Fermi energy ε_F determines the components of the Fermi momentum and velocity:

$$p_i^F=(2m_i\varepsilon_F)^{1/2},\ v_i^F=(2\varepsilon_F/m_i)^{1/2};\ i=x,\ y,\ z.$$

In accordance with (4.1), for the case when the p_z axis of the ellipsoid is oriented along the direction of the constant magnetic field, and

$$\Omega=|e|B/c(m_xm_y)^{1/2},\ \text{a}\ p_y'(p_z,0)=[2m_y(\varepsilon_F-p_z^2/2m_z)]^{1/2},$$

the transition frequency,

$$s\omega_n(p_z)=s[2\pi^2e^2B^2/3\hbar(n-1/4)m_xm_yc^2]^{1/2}(\varepsilon_F-p_z^2/2m_z)^{1/3}, \tag{4.2}$$

entering into the formula (3.10) has an extremum at the central cross section $p_z=0$. This allows us to write for it and for its second derivative such expressions:

$$s\omega_n = s[2\pi^2\varepsilon_F e^2 B^2/3(n-1/4)\hbar m_x m_y c^2]^{1/2},\quad s\omega_n'' = -s\omega_n/3m_z\varepsilon_F. \tag{4.3}$$

As a result, according to the formula (3.10), we have

$$\omega = s\omega_n\left\{1+\frac{2^{1/3}\pi^{4/3}m_z\alpha^2 L_{ns}^2}{3^{1/3}(n-1/4)^{4/3}}\left[\frac{\varepsilon_F\hbar^2e^2B^2}{m_x m_y c^2}\right]^{1/3}\right\} - \frac{i}{\tau}. \tag{4.4}$$

For an ellipsoidal Fermi surface it is useful to represent the quantity α by means of the parametrization, introduced in Ref. 13, of the function $\Phi(\mathbf{p}, \mathbf{p}')$. In this case it is necessary to take into account the fact that in our case, when the p_z axis of the ellipsoid is oriented along the magnetic field, $\Phi(\epsilon, p_z, \varphi; \epsilon', p_z', \varphi') = (m_x m_y)^{1/2} \times \Phi(\mathbf{p}, \mathbf{p}')$. In accordance with Ref. 13, we have:

$$\Phi(\mathbf{p},\mathbf{p}') = \pi^2\hbar^3(2\varepsilon_F m_x m_y m_z)^{-1/2}\sum_{l=0}^{\infty}(2l+1)A_l P_l(\cos\Theta). \tag{4.5}$$

Here the A_l are the parametrization coefficients, P_l is a Legendre polynomial, Θ is the angle between the vectors $\mathbf{w}$ and $\mathbf{w}'$, which are connected with $\mathbf{p}$ and $\mathbf{p}'$ by the relations $\mathbf{w} = \hat{T}\mathbf{p}$ and $\mathbf{w}' = \hat{T}\mathbf{p}'$. The tensor $\hat{T}$ then transforms the space of the momenta $\mathbf{p}$ into the space $\mathbf{w}$, in which the constant-energy surfaces are spheres. Assuming, in accordance with (4.1), that the axes of the coordinate system are directed along the principal axes of the ellipsoid, for the matrix $\hat{T}$ we have:

$$T = (m_x^2 + m_y^2 + m_z^2)^{1/4}\begin{pmatrix} m_x^{-1/2} & 0 & 0 \\ 0 & m_y^{-1/2} & 0 \\ 0 & 0 & m_z^{-1/2}\end{pmatrix}.$$

Let us, in accord with the parametrization (4.5), write Eq. (2.3) in the following form:

$$G(\theta,\psi) = G_0(\theta,\psi) + (1/4\pi)\int d\sigma_{w'}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}A_l P_l^m(\theta)P_l^m(\theta')(2l+1)$$
$$\times\exp[im(\psi-\psi')]G_0(\theta',\psi')(l-m)!/(l+m)!,$$

where θ, ψ and θ', ψ' are the polar and azimuthal angles of the vectors $\mathbf{w}$ and $\mathbf{w}'$, while P_l^m is the associated Legendre polynomial. Bearing in mind that

$$G_0(\theta,\psi) = G(\theta,\psi) - (1/4\pi)\int d\sigma_{w'}\sum_{l=0}^{\infty}\sum_{m=-l}^{l}(2l+1)[A_l/(1+A_l)]\cdot$$
$$\cdot P_l^m(\theta)P_l^m(\theta')\exp[im(\psi-\psi')]G(\theta',\psi')(l-m)!/(l+m)!$$

and taking into account the fact that $dp_z d\varphi = (2m_z\varepsilon_F)^{1/2} \times d\sigma_w$, we obtain for the function (2.7) the following expression:

$$\alpha(p_z,\varphi;p_z',\varphi') = (2m_z\varepsilon_F)^{-1/2}\sum_{l=0}^{\infty}(2l+1)A_l P_l(\cos\Theta)/4\pi(1+A_l).$$

Hence we have a relation connecting α and A_l:

$$\alpha = \alpha(0.0;0.0) = (2m_z\varepsilon_F)^{-1/2}\sum_{l=0}^{\infty}(2l+1)A_l/4\pi(1+A_l). \tag{4.6}$$

The relation (4.6) allows us to represent the formula (3.10) with the use of the constants A_l in the form

$$\omega = s\omega_n[p_x^F(n,s)/p_x^F]^{1/3} - i/\tau, \tag{4.7}$$

where

$$\frac{p_x^F(n,s)}{p_x^F} = \left\{1+\frac{\pi^{1/3}}{3^{1/3}2^{4/3}}\left(\frac{\hbar^2e^2B^2}{m_x m_y c^2\varepsilon_F}\right)^{1/3}\frac{L_{ns}^2}{(n-1/4)^{4/3}}\left[\sum_{l=0}^{\infty}\frac{(2l+1)A_l}{1+A_l}\right]^2\right\}^{1/3}$$
$$\approx 1 + \frac{3\hbar\omega L_{ns}^2}{64\varepsilon_F s(n-1/4)}\left[\sum_{l=0}^{\infty}\frac{(2l+1)A_l}{1+A_l}\right]^2. \tag{4.8}$$

The formulas (4.7) and (4.8) will be used in Sec. 6 to compare the theoretical results with the results of the experimental investigations described in Refs. 2, 11, and 12 and to obtain information about the magnitude of the Fermi-liquid interaction constant α.

5. THE RESONANCE PROPERTIES, DUE TO THE GLANCING ELECTRONS, OF THE IMPEDANCE OF A METAL

In this section we shall demonstrate how allowance for the interaction of electrons leads to a qualitatively new possibility of the existence of impedance resonances and surface electromagnetic waves, as compared with the theory[3-6] that neglects the electron interaction.

In deriving the expression for the impedance we use the fact that the volume and surface electrons make independent contributions to the current density that enters into the Maxwell equation. The motion of the volume electrons is virtually not affected by a weak magnetic field. Therefore, for the determination of the contribution made by the volume electrons to the total current, we should use the well-known results (see, for example, Refs. 7 and 14) obtained for a metal with an anisotropic Fermi surface. If we orient the coordinate axes along the principal axes of the tensor

$$B_{\alpha\beta} = \int_0^{2\pi} d\phi\, n_\alpha n_\beta K^{-1}(\phi),$$

where $K(\phi)$ is the Gaussian curvature of the Fermi surface and $n_i = v_i/v$, then we can obtain from the Maxwell equations two independent equations for the E_x and E_z components of the electric field. Assuming the electric field of the surface H-wave is polarized along the x direction, we write for the Fourier transform of the electric field the equation:

$$2E'(0) + (k_z^2 + k^2 - i\delta^{-3}k^{-1})E(k) = 4\pi i\omega c^{-2}j^{res}(k), \tag{5.1}$$

where $\delta = (c^2\pi\hbar^3/e^2B_{xx}\omega)^{1/3}$ is the depth of penetration of the field into a metal with an anisotropic Fermi surface and $E'(0)$ is the value of the derivative of the electric field at the point $y = 0$.

Noting that, when the Fermi-liquid interaction is taken into account, the expression, (3.19), for the resonance current differs from the corresponding expression obtained in the theory of noninteracting particles[4] by the resonance factor, we can write the following expression for the surface impedance:

$$Z(k_z)=\rho\frac{16\pi\omega\delta}{3\cdot 3^{1/2}c^2}e^{-i\pi/3}+\frac{2\pi^2 i\omega D\alpha_{ns}^2(k_z)}{c^2}\left\{[\Delta(k_z)]^{1/3}-A+\frac{\pi}{2}\beta_{ns}(k_z)D\right\}^{-1} \tag{5.2}$$

where (cf. Ref. 4)

$$\alpha_{ns}(k_z)=\frac{2}{\pi}\int_0^\infty I_{ns}(k,p_0)\,[k_z^2+k^2-i\delta^{-3}k^{-1}]^{-1}\,dk,$$

$$\beta_{ns}(k_z)=\frac{2}{\pi}\int_0^\infty I_{ns}^2(k,p_0)\,[k_z^2+k^2-i\delta^{-3}k^{-1}]^{-1}dk. \tag{5.3}$$

In the formula (5.2) the first term describes the impedance of the metal in the absence of a magnetic field. In the case of specular reflection of the nonresonance electrons from the surface $\rho=1$; in the case of diffuse reflection $\rho=\frac{3}{4}$. The impedance peak, corresponding to the vanishing of the denominator of the second term in (5.2), corresponds to the possibility of the excitation of a surface electromagnetic wave. The dispersion equation of such a surface wave is obtained by substituting the solution to Eq. (5.1) into the right-hand side of the expression

$$|k_z|^{-1}=\frac{1}{\pi}\int_0^\infty dk\,\frac{E(k)}{E'(0)}=\frac{c^2Z(k_z)}{4\pi i\omega}. \tag{5.4}$$

When the inequality $k_x\delta\ll 1$ is fulfilled, we can neglect in Eq. (5.4) the contribution connected with the volume terms as compared to the contribution of the resonance term. Bearing in mind the k_x values satisfying in the conditions for the anomalous skin effect the stronger inequality $|k_x v_x(p_0(k_x))|<|\omega-s\omega_n(p_0(k_x))|$, let us write the dispersion equation for the surface oscillations in the form

$$[\Delta(k_z)]^{1/3}\equiv\{2[s\omega_n(p_0(k_z))+k_z v_z(p_0(k_z))-\omega-i/\tau]/$$
$$/[s\omega_n''(p_0(k_z))+k_z v_z''(p_0(k_z))]\}^{1/3}=A-\pi D\beta_{ns}(k_z)/2. \tag{5.5}$$

In the $k_x=0$ case, when the condition

$$A>\pi D\,\mathrm{Re}\,\beta_{ns}(0)/2 \tag{5.6}$$

is fulfilled, from Eq. (5.5) we obtain

$$\mathrm{Re}\,\omega=s\omega_n-{}^1/_2 s\omega_n''[A-\pi D\,\mathrm{Re}\,\beta_{ns}(0)/2]^2+{}^1/_2 s\omega_n''[\mathrm{Im}\,\beta_{ns}(0)]^2(\pi D/2)^2, \tag{5.7}$$

$$\mathrm{Im}\,\omega=-1/\tau-s\omega_n''[A-\pi D\,\mathrm{Re}\,\beta_{ns}(0)/2]\,\mathrm{Im}\,\beta_{ns}(0)\pi D/2. \tag{5.8}$$

It follows from (5.8) that two terms contribute to the imaginary part of ω: first, the term equal to τ^{-1} and, secondly, the term proportional to $\mathrm{Im}\beta_{ns}(0)$. Let us, using the results of Ref. 4, write down the explicit expressions for $\beta_{ns}(0)$ in two limiting cases. If the "fine" glancing-electron level lies wholly inside the skin layer, i.e., if

$$y_n=\frac{1}{2}R\varphi_n^2\ll\delta,\quad \beta_{ns}(0)=\frac{y_n}{2(\pi s)^2}+i\,\frac{8[1-6/(\pi s)^2]^2 y_n^4\ln(\delta/y_n)}{\pi^5 s^4\delta^3}.$$

If the level is located at a depth much greater than the depth of the skin layer, i.e., if $y_n\gg\delta$, then $\beta_{ns}(0)=\pi\delta^2 e^{i\pi/3}/6\sqrt{3}y_n$. It follows from the formulas given that when the level is located at a depth, y_n, that coincides in order of magnitude with the depth of the skin layer, then $\mathrm{Im}\beta_{ns}(0)\approx\delta$.

The magnitude of the imaginary part of the frequency (5.8) is, when the inequality

$$\frac{1}{s\omega_n\tau}\gg\left[A-\frac{\pi}{2}D\,\mathrm{Re}\,\beta_{ns}(0)\right]\mathrm{Im}\,\beta_{ns}(0)\pi D\frac{|\omega_n''|}{2\omega_n}$$
$$\approx\frac{2\alpha\pi^{4/3}e^2v_x(p_0,0)\delta[\hbar\Omega(p_0)]^{1/3}L_{ns}\omega_n(p_0)}{3^{2/3}\hbar^2c^2p_y'(p_0)[p_y'(p_0)v_y''(p_0)]^{1/3}(n-{}^1/_4)^{2/3}|\omega_n''(p_0)|}\left(\frac{\partial^2\sigma}{\partial p_z\partial\varphi}\right)_{\varphi=0} \tag{5.9}$$

is fulfilled, determined by the momentum relaxation time.

In the $k_x\neq 0$ case, from Eq. (5.5) we obtain the following frequency spectrum and damping constant for the surface cyclotron oscillations:

$$\omega=s\omega_n(p_0)\left\{1-\frac{s\omega_n''(p_0)+k_z v_z''(p_0)}{2s\omega_n(p_0)}\left[A-\frac{\pi}{2}D\,\mathrm{Re}\,\beta_{ns}(0)\right]^2+\frac{k_z v_z(p_0)}{s\omega_n(p_0)}\right\}-\frac{i}{\tau}. \tag{5.10}$$

In the $\omega_n''>0$ case, when the value of $D\,\mathrm{Re}\beta_{ns}(0)<0$, the solution of the dispersion equation (5.5) is possible upon the fulfillment of the condition

$${}^1/_2\pi|D\,\mathrm{Re}\,\beta_{ns}(0)|>-A. \tag{5.11}$$

Such a solution exists, in particular, in the gas model of a metal (for $\omega<s\omega_n$), when $A=0$ (cf. Ref. 4). If, on the other hand, the Fermi-liquid interaction constant is sufficiently large in absolute value and negative, so that the inequality (5.11) is not fulfilled, then the propagation of cyclotron waves in the vicinity of the minimum of the surface-transition frequency, including those studied in Ref. 4, is forbidden. In the opposite case, when $\omega_n''<0$ and, according to (3.18), $D\,\mathrm{Re}\beta_{ns}(0)<0$, the waves studied in Ref. 4 cannot exist. On the other hand, the solution (5.10) to the dispersion equation (5.5) exists in our theory when the condition (5.6) is fulfilled. The frequency spectrum (5.10) lies above the surface-transition frequency, which causes the suppression of the collisionless Landau damping.

6. THE POSSIBILITIES OF A COMPARISON WITH EXPERIMENT

In discussing the possibilities of comparing our theory with experiment, it is first of all necessary to emphasize that, according to the results obtained in Ref. 4, in which the electron interaction is neglected, resonances in the surface impedance near the maximum of the frequency of the transition between the glancing-electron levels are forbidden because of the collisionless Landau damping. As shown in Ref. 4, in the gas model resonances are admissible only in those metals for which the transition frequency as a function of the momentum component along the direction of the constant magnetic field has a minimum. Accordingly, resonances should not have been observed in bismuth on the orbits of the glancing electrons, for the frequency of the tran-

sition between the surface levels in bismuth does not possess minima; on the contrary it has a maximum near the central cross section of the ellipsoidal Fermi surface. Therefore, the fact that our theory allows the elimination of such a disagreement that has existed up to the present between theory and experiment evidently constitutes at present the main qualitative agreement of the theory with experiment. Thus, according to our theory, the results of the investigations, described in Refs. 2, 11, and 12, of the oscillations in the surface impedance of bismuth are connected with the interelectron interaction, which is the cause of the shift of the transition frequency from the region of collisionless Landau damping. Here we can assert that the quantity

$$\sum_{l=0}^{\infty}\frac{(2l+1)A_l}{1+A_l} \tag{6.1}$$

turns out to be positive in the case of bismuth.

The qualitative estimates that we can try to obtain from a comparison of the theory with experiment cannot be accurate, first because the experimental results are not of sufficiently high accuracy and, secondly, although this is at present of less importance, because of the fact that the most accurate measurements have been performed at low values of the numbers n. Below we shall discuss the available possibilities, using apparently the most detailed experimental data, given in Refs. 11 and 12, for bismuth.

First of all, let us note that the positiveness of the quantity (6.1) for bismuth is necessary, according to (3.14), for the existence of resonance frequencies in the case when electrodynamic effects are neglected. Allowance for the latter leads to the necessity for the satisfaction of the inequality (5.6), which allows us to find a lower bound for the magnitude, (6.1), of the interelectron interaction. Using, in accord with Ref. 15, the following parameter values for the electronic spectrum of bismuth: $\epsilon_F=2.86\times10^{-14}$ erg, $m_x=0.57\times10^{-2}m$, $m_y=1.15\times10^{-2}m$, and $m_z=1.27m$, where m is the free-electron mass, we can, in accordance with the formulas (4.9), (3.19), and (5.3), write down the following inequality, which follows from the condition (5.6):

$$\sum_{l=0}\frac{(2l+1)A_l}{1+A_l}\geqslant 7.5\cdot10^{-2}\frac{(n-1/4)^{4/3}}{s^2L_{ns}B^{2/3}}, \tag{6.2}$$

where B is the magnetic-field intensity in oersteds. In deriving this relation to fit the data presented in Refs. 11 and 12, we assumed that the p_z axis of the ellipsoidal Fermi surface of the electrons in bismuth is oriented along the magnetic field. Consequently, according to the results obtained in the fourth section of our paper, the following expressions

$$A=\frac{(3\pi)^{1/2}(m_z\varepsilon_F)^{1/2}L_{ns}}{2^{11/4}(n-1/4)^{2/3}}\left(\frac{\hbar eB}{(m_xm_y)^{1/2}c\varepsilon_F}\right)^{1/2}\sum_{l=0}^{\infty}\frac{(2l+1)A_l}{1+A_l},$$

$$D=\frac{6\cdot2^{1/2}e^2m_z\varepsilon_F^{3/2}}{\pi\hbar^2c^2m_x^{1/2}},$$

$$\beta_{ns}(0)=\frac{R\varphi_n^2}{4\pi^2s^2}=\frac{3^{3/2}(n-1/4)^{2/3}}{2^{17/4}\pi^{4/3}s^2}\left(\frac{\hbar}{\varepsilon_F}\right)^{2/3}\left(\frac{\varepsilon_F}{m_y}\right)^{1/2}\left(\frac{(m_xm_y)^{1/2}c}{|e|B}\right)^{1/2},$$

which lead to the inequality (6.2), are valid.

For the resonances observed in the work published in Ref. 11 in the surface impedance of bismuth on the glancing-electron orbits, the right-hand side of the formula (6.2) is maximal for the resonance corresponding to the transition with the quantum numbers $n=3$ and $s=2$ for $B=2.11$ Oe. The right-hand side then turns out to be ~0.02. If we address ourselves to the analysis of the data given in Ref. 12, then among the impedance resonances distinguishable in Fig. 19 of that paper, to the maximum of the right-hand side of the formula (6.2) corresponds the $n=4$, $s=2$ transition in a magnetic field of intensity 1.25 Oe. These data correspond to a slightly higher value for the lower bound of the quantity (6.1), since in this case for the right-hand side of the formula (6.2) we obtain the value 0.04.

It seems to us that the quantitative data given in Ref. 11 for the locations of the impedance resonances of bismuth cannot be easily used for a detailed comparison with our theory since the experimental error in the determination of the locations of the resonances constitutes 2%. On the other hand, if we suppose that, within the limits of such an error, there does not arise any discrepancy between these data and our theory, then we can, in accordance with the formula (4.8), assert that for bismuth the quantity (6.1) does not exceed ten. The cause of the difficulty encountered in the comparison of the theory with the experiment described in Ref. 11 should be seen in the relatively short mean free time of the electrons ($\omega\tau\sim8$), which gives rise to the considerable broadening of the resonances. Bearing in mind that Fig. 19 in Ref. 12 exhibits a number of cyclotron-resonance peaks that at least several times exceeds the number of such peaks observed in the investigation published in Ref. 11, we can infer that experimenters possess, or at any rate can obtain, data on the locations of the impedance resonances due to the glancing orbits with an accuracy higher than the accuracy achieved in the work described in Ref. 11. Therefore, it seems to us that the upper limit for the quantity (6.1) in bismuth can be determined more accurately than is indicated by the estimate obtained in our paper.

APPENDIX

With the aid of the results obtained for cyclotron waves we can also write down a dispersion equation for the surface spin oscillations whose frequency is close to the frequency of the transition between the glancing-electron levels. Such a dispersion equation, which generalizes the results obtained in Ref. 1 to the case of an anisotropic Fermi surface, has the form

$$[(\omega+i/\tau-s\omega_n(p_0))/(-s\omega_n''(p_0))]^{1/2}=-\beta\pi^{4/3}2^{1/2}3^{-2/3}\times[\hbar\Omega(p_0)/p_y'(p_0)v_y'(p_0)]^{1/2}L_{ns}\omega_n(p_0)(n-1/4)^{-2/3}[\omega_n''(p_0)]^{-1}.$$

In this formula we have used the same notation used in Eq. (3.12) and β is the analog of the function α, with the difference that it is obtained as a result of the convolution of the resolvent operator of Eq. (2.3) with the spin-dependent Landau function describing the interelectron interaction.

For the frequency of the spin excitations we can write

the following expression:

$$\omega=s\omega_n(p_0)-\frac{1}{2}s\omega_n''(p_0)\left[\frac{2\pi^{1/2}\beta\omega_n(p_0)[\hbar\Omega(p_0)]^{1/2}L_{n,s}}{3^{3/2}\omega_n'''(p_0)[p_y'(p_0)v_y'(p_0)]^{1/2}(n-1/4)^{3/2}}\right]^2\frac{i}{\tau}. \quad \text{(A.1)}$$

Bearing in mind the overestimation of the integral (2.19) in Ref. 1, let us also write the formula (A.1) for a spherical Fermi surface in the case when $p_s^F\beta=(B_0/1+B_0)/4\pi$:

$$\omega=s\omega_n(0)+\Omega|B_0L_{n,s}/(1+B_0)|^2(\hbar\Omega/\varepsilon_F)^{1/2}\times s(n-1/4)^{-5/2}\pi^{4/2}2^{-13/2}\cdot 3^{-3/2}-i/\tau,$$

where $\Omega=|e|B/mc$, m being the electron mass on the spherical Fermi surface.

In order for the frequency shift to exceed the frequency decrement, the second term on the right-hand side of (A.1) should be greater than the third term. This implies that the inequality

$$1/\tau<|s\omega_n''/2|[2\pi^{4/3}\beta 3^{-3/2}\omega_n(p_0)[\hbar\Omega(p_0)]^{1/2}L_{n,s}/\omega_n''(p_0)\times[p_y'(p_0)v_y'(p_0)]^{1/2}(n-1/4)^{3/2}]^2.$$

should be fulfilled. In the case of bismuth, when it is assumed that $p_s^F\beta=0.1$, the latter inequality implies that, in fields of intensity $B=10$ Oe, $1/\tau<10^9$ sec^{-1}.

[1]V. P. Silin and O. M. Tolkachev, Zh. Eksp. Teor. Fiz. 70, 639 (1976) [Sov. Phys. JETP 43, 331 (1976)].
[2]M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 39, 212 (1960); Usp. Fiz. Nauk 96, 409 (1968) [Sov. Phys. JETP 12, 152 (1961); Sov. Phys. Usp. 11, 785 (1969)].
[3]T. W. Nêe and R. E. Prange, Phys. Lett. 25A, 583 (1967).
[4]É. A. Kaner and N. M. Makarov, Zh. Eksp. Teor. Fiz. 58, 1972 (1970) [Sov. Phys. JETP 31, 1063 (1970)].
[5]T. W. Nêe, J. E. Koch, and R. E. Prange, Phys. Rev. 174, 758 (1968).
[6]H. J. Fishbeck and J. Mertsching, Phys. Status Solidi 27, 345 (1968).
[7]A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metallov (Introduction to the Theory of Normal Metals), Nauka, M., 1972 (Eng. Transl. in: Solid State Physics, Suppl. 12, Academic Press, New York, 1972).
[8]V. P. Silin, Fiz. Met. Metallov. 29, 681 (1970) [Phys. Met. Metallogr. 29, No. 4, 7 (1970)].
[9]K. Fuchs, Proc. Camb. Philos. Soc. 34, 100 (1938).
[10]É. A. Kaner, N. M. Makarov, and I. M. Fuks, Zh. Eksp. Teor. Fiz. 55, 931 (1968) [Sov. Phys. JETP 28, 483 (1969)].
[11]J. F. Koch and J. D. Jensen, Phys. Rev. 184, 643 (1969).
[12]V. S. Edel'man, Adv. Phys. 25, 555 (1970).
[13]G. P. Alodzhants, Zh. Eksp. Teor. Fiz. 59, 1429 (1970) [Sov. Phys. JETP 32, 780 (1971)].
[14]V. P. Silin and A. A. Rukhadze, Élektromagnitnye svoĭstva plazmy i plazmopodobnykh sred (Electromagnetic Properties of a Plasma and Plasma-like Media), Atomizdat, 1961.
[15]V. S. Édel'man, Usp. Fiz. Nauk 102, 55 (1970) [Sov. Phys. Usp. 13, 583 (1971)].

Translated by A. K. Agyei

Theory of nonradiative processes in the "non-Condon" approximation

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(Submitted 13 February 1978)
Zh. Eksp. Teor. Fiz. 74, 2154–2166 (June 1978)

The exact solution of a model with shifted and distorted terms is obtained by expressing the transition probability in terms of a correlation function and applying the functional differentiation method. The nondiagonal terms are allowed for within the perturbation theory framework. It is shown that the dependence of the matrix element of a transition on the nuclear coordinates gives rise to an additional factor in the correlation function. In some specific cases an analytic expression can be obtained for this factor by solving an appropriate differential equation. The expression for the transition rate constant reduces to quadrature in the case of an arbitrary dependence of the matrix element of the transition on the nuclear coordinates if this matrix element can be represented as a Fourier integral. The validity criterion of the Condon approximation is obtained. The results may be used to describe various multiquantum processes.

PACS numbers: 32.50.+d, 33.50.Hv

In studies of multiquantum processes—such as radiative and nonradiative transitions in impurity centers in crystals or transitions in polyatomic molecules, neutron scattering by lattice vibrations, vibrational relaxation of impurity molecules, and chemical reactions—it is necessary to isolate two subsystems between which energy is exchanged in the course of a quantum transition. In such processes the matrix element of the first subsystem generally depends on the coordinates of the second subsystem. This dependence may become unimportant for large separations between the terms and then the Condon approximation may be used to describe multiquantum processes. This case has been investigated quite thoroughly.[1-5] However, there are certain optical phenomena which cannot be explained employing the Condon approximation.[6,7] It is also known that, in contrast to optical transitions, the influence of the dependence of the matrix element on the nuclear coordinates is much more important in nonradiative transitions since such transitions occur in the case when the nuclear configuration is far from equilibrium and the dependence of the matrix element near the term quasicrossing point is of resonant nature. Moreover, in the case of a strong electron-vibrational