Spontaneous coherent responses of many-level systems undergoing a phase transition of the super-radiative type

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It is shown that if a super-radiative or a super-nonradiative phase transition occurs in a three-level particle system it is possible to excite in it with the aid of external coherent sources essentially new types of spontaneous responses that exist only at temperatures below the critical temperature. The intensity of the generated responses (induction and echo signals) is proportional to $\epsilon^2E^2y_0/|^2$ (ϵE is the energy gap between the lower levels, $G_{k\sim 0}$ is the coupling constant, and y_0 is the order parameter). The frequency, wave vector, and nature of these signals may differ from those of the exciting fields, and this significantly simplifies the useful-signal extraction and also makes the observation of phase-stimulated analogs of forced inductions possible.

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The extensive development of work on the theory of coherent (collective) states of quantum systems that has followed the discovery of light and sound echoes[1,2] and optical and acoustic self-induced transparencies[3-5] has revealed a profound analogy between such seemingly different phenomena as superradiance and superconductivity, the Josephson effect, etc. And, finally, not quite long ago, in 1973, an analogy was established between the transition of a substance into an equilibrium collective (superradiant) state and second-order phase transitions. The possibility of such a phenomenon for systems of two-level particles interacting via a general photon field is validated in Ref. 6, where the now generally used term "superradiant" phase transition (SRPT) is introduced. The SRPT phenomenon consists in the appearance in a system of N centers at temperatures below the critical temperature, T_{cr}, of phased precessions of the transverse (in the representation in which the unperturbed Hamiltonian is diagonal) components of the dipole moments of these centers and of the macroscopic population of the resonance mode with the wave vector $\mathbf{k}_0$. The latter implies that, for $T<T_{cr}$

$$y_0=\lim_{N\to\infty}\langle a_{\mathbf{k}_0}{}^+a_{\mathbf{k}_0}\rangle/N\neq 0, \tag{1}$$

where $a^+_{\mathbf{k}_0}$ ($a_{\mathbf{k}_0}$) are the operators of creation (annihilation) of the quanta of the resonance mode.

The SRPT is a result of the interaction of the particles via their common electromagnetic field. Since impurity particles in solids interact not only via the photon field, but also via the fields of the elementary excitations of the medium itself (phonons, excitons, plasmons, etc.), there naturally arose the question whether a similar phase transition is possible in an equilibrium system of impurity centers and elementary-excitation fields that, to the centers, are resonant fields. As is shown in Refs. 7–9, the interaction of paramagnetic and paraelectric centers with the phonon field leads at temperatures $T<T_{cr}$ to a transition of these centers into a collective (coherent) state. As for the SRPT, there occurs a corresponding macroscopic population of the resonance phonon mode. The phenomenon was given the name of "super-nonradiative" phase transition (SNPT). It can be shown that such a capacity of the given quantum system (γ) for undergoing phase transitions into a spontaneous coherent state (SCS) is typical with respect to any boson field (δ), by means of which it is possible to excite the system into a state of superradiance. (Below we shall, for brevity, call such phase transitions SCS transitions.) Nevertheless, as is shown below, many-level quantum systems allow the observation of essentially new spontaneous coherent responses, which we have called phase-stimulated bosonic superradiance (PSBSR). The physical essence of the PSBSR phenomenon consists in the following.

Let there be stimulated in a system with a discrete spectrum a SCS transition with respect to a field that is resonant for the transition between the states $|\alpha\rangle$ and and $|\beta\rangle$ of the system. This can be secured by, for example, placing the system in an electromagnetic resonator of frequency $\omega_{\beta\alpha}=(E_\beta-E_\alpha)\hbar^{-1}$ for SRPT stimulation, or preparing the sample under an acoustic resonator of the same frequency for SNPT stimulation ($E_\beta>E_\alpha$; E_β and E_α are the energies of the β-th and α-th levels).

A SCS transition is accompanied by the appearance of a coherent superposition of the states $|\alpha\rangle\leftrightarrow|\beta\rangle$, i.e., its realization is analogous to the action on the system of some fictitious coherent stationary field of frequency $\omega_{\beta\alpha}$. But in this case, as shown in Ref. 10, an additional impulsive or steady coherent excitation by generators whose frequency is resonant for the transition $|\alpha\rangle\leftrightarrow|\sigma\rangle$ or $|\beta\rangle\leftrightarrow|\sigma\rangle$ ($E_\sigma>E_\beta$) leads to the generation of induction- or echo-type signals with frequencies different from $\omega_{\beta\alpha}$ and $\omega_{\sigma\alpha}$ or, correspondingly, from $\omega_{\beta\alpha}$ and $\omega_{\sigma\beta}$. This phase-stimulated bosonic radiation disappears at $T>T_{cr}$. The nature and intensity of the then generated boson field depends on the type of SCS transitions and the type of exciting field, as well as on the character of the relaxation processes in the system. Therefore, PSBSR is not only the most direct and convenient method for observing SRPT and SNPT, but is, in fact, an essentially new direction in the spectroscopy

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of interference states.

The theory of PSBSR can be constructed on the basis of the formalism developed in Refs. 9 and 10. Let us assume that there is no direct interaction between the particles. Then the Hamiltonian of the system can be represented in the form

$$H=H_b^{(0)}+H_n^{(0)}+H_1, \quad H_b^{(0)}=\sum_{j=1}^{N} H_0^j, \quad H_n^{0}=\sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}} \left(a_{\mathbf{k}}^+ a_{\mathbf{k}}+\frac{1}{2}\right),$$
$$H_1=\sum_{j=1}^{N} H_1^j, \quad H_1^j=\sum_{\mathbf{k},\alpha\beta} \left(\frac{A_{\mathbf{k},\alpha\beta}}{\sqrt{N}} a_{\mathbf{k}} P_{\alpha\beta} e^{i\mathbf{k}\mathbf{r}_j}+\frac{\dot{A}_{\mathbf{k},\beta\alpha}}{\sqrt{N}} a_{\mathbf{k}}^+ P_{\beta\alpha} e^{-i\mathbf{k}\mathbf{r}_j}\right), \tag{2}$$

where H_0^j is the unperturbed Hamiltonian of the j-th center; k is the wave vector of the boson field; α and β are the indices of the eigenstates of H_0^j, which, to the boson field k, are resonance states; $A_{\mathbf{k},\alpha\beta}$ is the constant of the interaction with this type of field; the $a_{\mathbf{k}}^+(a_{\mathbf{k}})$ are the creation (annihilation) operators for the quanta of the field; $P_{\alpha\beta}$ is the projection matrix, the $\alpha\beta$-th element of which is equal to unity, while the rest are each equal to zero; $\mathbf{r}_j$ is the radius vector of the location of the j-th particle.

The density matrix, $\rho(t)$, in the rotating-field approximation has the form

$$\rho^j(t)=\exp\{-i\hbar^{-1}H_1^j t\}\rho^j(0)\exp\{i\hbar^{-1}H_1^j t\}, \tag{3}$$

where $\rho^j(0)$ is the density matrix at the initial moment of time. (Below, to simplify the notation, we shall drop the index j.)

Since the $P_{\alpha\beta}$ are linearly independent matrices, any operator, Q, can be expanded in terms of them:

$$Q=\sum_{\alpha\beta} q_{\alpha\beta} P_{\alpha\beta}. \tag{4}$$

A similar expansion can be carried out for the density matrix

$$\rho(0)=\sum_{\alpha\beta} \rho_{\alpha\beta}(0) P_{\alpha\beta}, \tag{5}$$

where $q_{\alpha\beta}$ and $\rho_{\alpha\beta}$ are c numbers.

Substituting (5) into (3), we find

$$\rho(t)=\sum_{\alpha\beta} \rho_{\alpha\beta}(0)\exp\{-i\hbar^{-1}H_1 t\}P_{\alpha\beta}\exp\{i\hbar^{-1}H_1 t\}=\sum_{\alpha\beta} \rho_{\alpha\beta}(0)P_{\alpha\beta}(H_1,t). \tag{6}$$

Carrying out an expansion for $P_{\alpha\beta}(H_1,t)$ similar to (4) and (5), and substituting the obtained result into (6), we find

$$\rho(t)=\sum_{\alpha\beta,\gamma\delta} \rho_{\gamma\delta}(0)\, b_{\alpha\beta,\gamma\delta}(H_1,t)P_{\gamma\delta}. \tag{7}$$

Then

$$\langle Q(t)\rangle=\mathrm{Sp}\{\rho(t)Q(t)\}=\sum_{\alpha\beta,\gamma\delta} \rho_{\alpha\beta}(0)\, b_{\gamma\delta,\alpha\beta}(H_1,t)\, q_{\alpha\beta}. \tag{8}$$

The quantities $b_{\gamma\delta,\alpha\beta}(H_1,t)$ for a number of cases have been computed and tabulated by Solovarov.[11]

As is well known, for noninteracting centers, the intensity of the spontaneous radiation emitted per unit solid angle in the direction of k in the $|\alpha\rangle \to |\beta\rangle$ transition is equal to

$$I_{\alpha\beta}(\mathbf{k})=I_{\alpha\beta}^0(\mathbf{k})\left\{N\langle P_{\alpha\beta}\, P_{\beta\alpha}\rangle+\sum_{j\neq l}^{N}\langle P_{\alpha\beta}\rangle\langle P_{\beta\alpha}\rangle\exp[i\mathbf{k}(\mathbf{r}_j-\mathbf{r}_l)]\right\}, \tag{9}$$

where $I_{\alpha\beta}^0(\mathbf{k})$ is the intensity of the radiation emitted by an isolated center.

Let us, for example, consider the case of a system of three-level particles with state energies E, O, and ϵE (ϵ is the nonequidistance parameter). Let the SCS transition be stimulated with respect to $|2\rangle \to |3\rangle$. Then in (2) $\alpha=2, \beta=3$, and $A_{\mathbf{k},\alpha\beta}=G_{\mathbf{k}_0}$. In order to compute the PSBSR, it is necessary to compute the density matrix under SCS-transition conditions ($\rho(0)$). We can show, in much the same way as is done in Ref. 12, that for the model Hamiltonian (2) $\rho(0)$ can be represented in the form of a direct product of the density matrix for the field ($\rho_f(0)$) and the density matrix for the material ($\rho_0(0)$). Further, using the results of Ref. 12, we find that H_1 in (8) has the form

$$H_1=E(P_{11}-\varepsilon P_{33})+G_{\mathbf{k}_0}N^{-1/2}\langle a_{\mathbf{k}_0}\rangle P_{23}+G_{\mathbf{k}_0}^{*}N^{-1/2}\langle a_{\mathbf{k}_0}^+\rangle P_{32}. \tag{10}$$

The thermodynamic averages of the operators $a_{\mathbf{k}_0}^+$ and $a_{\mathbf{k}_0}$ can be expressed in the following manner in terms of the density of the boson resonance mode $\mathbf{k}_0$:

$$\langle a_{\mathbf{k}_0}\rangle=(Ny_0)^{1/2}e^{i\theta}, \quad \langle a_{\mathbf{k}_0}^+\rangle=(Ny_0)^{1/2}e^{-i\theta}, \tag{11}$$

where θ is an undetermined phase factor, while y_0 is the order parameter of the SCS transition, determinable from the equation

$$C_{\mathbf{k}_0}^{-2}\eta_0=2\,\mathrm{sh}\,(1/2\beta\varepsilon E\eta_0)\{\exp(-1/2\beta(\varepsilon+2)E)+2\,\mathrm{ch}\,(1/2\beta\varepsilon E\eta_0)\}^{-1},$$
$$C_{\mathbf{k}_0}=|G_{\mathbf{k}_0}|\varepsilon^{-1}E^{-1}, \quad \eta_0=(1+4C_{\mathbf{k}_0}^2 y_0)^{1/2}, \quad \beta=1/k_B T; \tag{12}$$

k_B is the Boltzmann constant and T_{cr} is determined from (12) with $\eta_0=1$.

Using the apparatus of matrix functions,[13] let us represent $\exp\{-\beta H_1\}$ in the form

$$\exp\{-\beta H_1\}=z_1\exp\{-E\beta\}+z_2\exp\{1/2\beta\varepsilon E(1+\eta_0)\}+z_3\exp\{1/2\beta\varepsilon E(1-\eta_0)\},$$
$$(z_1)_{11}=P_{11}, \quad (z_2)_{22}=a_{(-)}, \quad (z_2)_{33}=a_{(+)}, \quad (z_2)_{23}=(z_2)_{32}^{*}=b_{(-)},$$
$$(z_3)_{22}=a_{(+)}, \quad (z_3)_{33}=a_{(-)}, \quad (z_3)_{23}=(z_3)_{32}^{*}=b_{(+)},$$
$$a_{(\pm)}=(\eta_0\pm 1)/2\eta_0, \quad b_{(\pm)}=\pm A\varepsilon E/\eta_0, \tag{13}$$

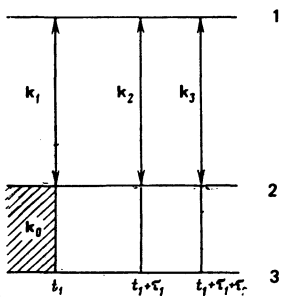

FIG. 1. Exciting-pulse train corresponding to the $|1\rangle \longleftrightarrow |2\rangle$ transition: $\mathbf{k}_0$ is the wave vector of the resonant mode exciting the SCS transition; $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the wave vectors of the exciting pulses.

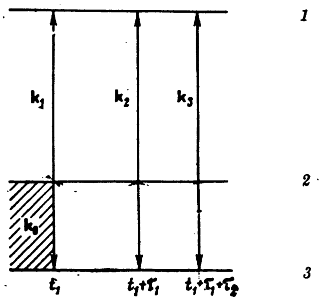

FIG. 2. Exciting-pulse train corresponding to the $|1\rangle \longleftrightarrow |3\rangle$ transition: $\mathbf{k}_0$ is the wave vector of the resonant mode that excites the SCS transition; $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ are the wave vectors of the exciting pulses.

where $A = |G_{\mathbf{k}_0}| y_0^{1/2} \exp\{i(\mathbf{k}_0 \cdot \mathbf{r}_j + \theta)\}$ is the same for all j, with the exception of the factor $\exp\{i\mathbf{k}_0 \cdot \mathbf{r}_j\}$.

With allowance for (12) and (13) we find the nonzero matrix elements of $\rho(0)$:

$$\rho_{11}=Z^{-1}e^{-\beta E},\quad \rho_{22}={}^1/_2(\sigma-C_{\mathbf{k}_0}^{-2}),\quad \rho_{33}={}^1/_2(\sigma+C_{\mathbf{k}_0}^{-2}),$$
$$\rho_{23}=\rho_{32}^*=-C_{\mathbf{k}_0}^{-1}y_0^{1/2}\exp\{i(\mathbf{k}_0\mathbf{r}_j+\theta)\},\tag{14}$$
$$Z=e^{-\beta E}+2\exp({}^1/_2\varepsilon E\beta)\,\mathrm{ch}\,({}^1/_2\beta\varepsilon E\eta_0),$$
$$\sigma=2\,\mathrm{ch}\,({}^1/_2\beta\varepsilon E\eta_0)\,[\exp(-{}^1/_2(\varepsilon+2)E\beta)+2\,\mathrm{ch}\,({}^1/_2\beta\varepsilon E\eta_0)]^{-1}.$$

In the absence of a SCS transition ($T > T_{cr}$) the nonzero elements of the density matrix are the following:

$$\rho_{11}(0)=Z_0^{-1}e^{-\beta E},\quad \rho_{22}(0)=Z_0^{-1},\quad \rho_{33}(0)=Z_0^{-1}e^{\beta\varepsilon E},$$
$$Z_0=e^{-\beta E}+1+e^{\beta\varepsilon E}.\tag{15}$$

The essential difference between (15) and (14) consists in the presence in (14) of off-diagonal matrix elements. Let us assume that at some moment of time $t=t_1$ the system, with temperature $T<T_{cr}$, is subjected to the action of the trains of coherent pulses schematically shown in Figs. 1 and 2. Then, depending on the type of exciting-pulse trains, the system at the appropriate moments of time generates free-induction and echo signals (including stimulated and secondary echoes) whose parameters are given in Tables I and II. (Table I presents the responses that can be excited by the coherent-pulse train shown in Fig. 1; Table II corresponds to the responses excited by the pulse train shown in Fig. 2.)

TABLE I. Types of spontaneous coherent responses excitable by the train of pulses shown in Fig. 1.

Type of response* at the transition $\|\alpha\rangle \leftrightarrow \|\beta\rangle$	$f_1(\theta_p, y_0)$	Wave vector of the response $\mathbf{k}_p+\xi\mathbf{k}_0;\ \psi(\mathbf{k}_p,\mathbf{k}_0)$	Response-formation time** $\varphi(\tau_1,\tau_2)$
IS $\|1\rangle \leftrightarrow \|2\rangle$	$S_1^2\cos^2\theta_1\sin^2\theta_1$	$\mathbf{k}_1$	IAP
IS $\|1\rangle \leftrightarrow \|3\rangle$	$\sin^2\theta_1(\varepsilon^2E^2/\|G_{\mathbf{k}_0}\|^2)y_0$	$\mathbf{k}_1+\mathbf{k}_0$	«
IS $\|1\rangle \leftrightarrow \|2\rangle$	$^1/_2\,S_1^2\cos^2\theta_1\sin^2 2\theta_2$	$\mathbf{k}_2$	«
ES $\|1\rangle \leftrightarrow \|2\rangle$	$^1/_2\,S_1^2\sin^2 2\theta_1\sin^4\theta_2$	$2\mathbf{k}_2-\mathbf{k}_1$	$t_1+2\tau_1$
IS $\|1\rangle \leftrightarrow \|2\rangle$	$^1/_2\,S_1^2\cos^2 2\theta_1\cos^2 2\theta_2\sin^2 2\theta_3$	$\mathbf{k}_3$	IAP
StES $\|1\rangle \leftrightarrow \|2\rangle$	$^1/_{16}\,S_1^2\sin^2 2\theta_1\sin^2 2\theta_2\sin^2 2\theta_3$	$-\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3$	$t_1+2\tau_1+\tau_2$
$SeES_1$ $\|1\rangle \leftrightarrow \|2\rangle$	$S_1^2\sin^4\theta_3\sin^4\theta_2\sin^2\theta_1\cos^2\theta_1$	$2\mathbf{k}_3-2\mathbf{k}_2+\mathbf{k}_3$	$t_1+2\tau_2$
$SeES_2$ $\|1\rangle \leftrightarrow \|2\rangle$	$S_1^2\sin^4\theta_3\cos^4\theta_2\sin^2\theta_1\cos^2 2\theta_1$	$2\mathbf{k}_3-\mathbf{k}_2$	$t_1+\tau_1+2\tau_2$
$SeES_3$ $\|1\rangle \leftrightarrow \|2\rangle$	$S_1^2\sin^4\theta_3\cos^4\theta_2\sin^2\theta_1\cos^2\theta_1$	$2\mathbf{k}_3-\mathbf{k}_1$	$t_1+2(\tau_1+\tau_2)$

*IS stands for induction signal; ES, StES, and SeES—normal, stimulated, and secondary echo signals, respectively.
**Time $t_1+\sum_{q=1}^{p}\tau_q$; IAP stands for: response is formed immediately after the pulse.

TABLE II. Types of spontaneous coherent responses excitable by the train of pulses shown in Fig. 2.

Type of response* at the transition $\|\alpha\rangle \leftrightarrow \|\beta\rangle$	$f_2(\theta_p, y_0)$	Wave vector of the response $\mathbf{k}_p+\xi\mathbf{k}_0;\ \psi(\mathbf{k}_p,\mathbf{k}_0)$	Response-formation time** $\varphi(\tau_1,\tau_2)$
IS $\|1\rangle \leftrightarrow \|2\rangle$	$\sin^2\theta_1(\varepsilon^2E^2/\|G_{\mathbf{k}_0}\|^2)y_0$	$\mathbf{k}_1-\mathbf{k}_0$	IAP**
IS $\|1\rangle \leftrightarrow \|3\rangle$	$^1/_2S_2^2\cos^2 2\theta_1\sin^2 2\theta_2$	$\mathbf{k}_1$	»
IS $\|1\rangle \leftrightarrow \|3\rangle$	$^1/_2\,S_2^2\sin^2 2\theta_1$	$\mathbf{k}_2$	»
ES $\|1\rangle \leftrightarrow \|2\rangle$	$\cos^2\theta_1\sin^2\theta_2\dfrac{\varepsilon^2E^2}{\|G_{\mathbf{k}_0}\|^2}y_0$	$\mathbf{k}_2-\mathbf{k}_0$	$t_1+(1+\varepsilon)\tau_1$
ES $\|1\rangle \leftrightarrow \|3\rangle$	$^1/_2\,S_2^2\sin^2 2\theta_1\sin^4\theta_2$	$2\mathbf{k}_2-\mathbf{k}_1$	$t_1+2\tau_1$
IS $\|1\rangle \leftrightarrow \|3\rangle$	$^1/_2\,S_2^2\cos^2 2\theta_1\cos^2 2\theta_2\sin^2 2\theta_3$	$\mathbf{k}_3$	IAP
StES $\|1\rangle \leftrightarrow \|3\rangle$	$^1/_{16}\,S_2^2\sin^2 2\theta_1\sin^2 2\theta_2\sin^2 2\theta_3$	$\mathbf{k}_3+\mathbf{k}_2-\mathbf{k}_1$	$t_1+2\tau_1+\tau_2$
$SeES_1$ $\|1\rangle \leftrightarrow \|3\rangle$	$S_2^2\sin^4\theta_3\sin^4\theta_2\sin^2\theta_1\cos^2\theta_1$	$2\mathbf{k}_3-2\mathbf{k}_2+\mathbf{k}_1$	$t_1+2\tau_2$
$SeES_2$ $\|1\rangle \leftrightarrow \|3\rangle$	$S_2^2\sin^4\theta_3\sin^2\theta_2\cos^2\theta_2\cos^2 2\theta_1$	$2\mathbf{k}_3-\mathbf{k}_2$	$t_1+\tau_1+2\tau_2$
$SeES_3$ $\|1\rangle \leftrightarrow \|3\rangle$	$S_2^2\sin^4\theta_3\cos^4\theta_2\sin^2\theta_1\cos^2\theta_1$	$2\mathbf{k}_3-\mathbf{k}_1$	$t_1+2(\tau_1+\tau_2)]$
ES $\|1\rangle \leftrightarrow \|2\rangle$	$\sin^2\theta_1\sin^2\theta_2\sin^2\theta_3\dfrac{\varepsilon^2E^2}{\|G_{\mathbf{k}_0}\|^2}y_0$	$\mathbf{k}_3-\mathbf{k}_2+\mathbf{k}_1-\mathbf{k}_0$	$t_1+(1+\varepsilon)\tau_2$
ES $\|1\rangle \leftrightarrow \|2\rangle$	$\cos^2\theta_1\cos^2\theta_2\sin^2\theta_3\dfrac{\varepsilon^2E^2}{\|G_{\mathbf{k}_0}\|^2}y_0$	$\mathbf{k}_3+\mathbf{k}_0$	$t_1+(1+\varepsilon)\times$ $\times(\tau_1+\tau_2)$

*IS stands for induction signal; ES, StES, and SeES—normal, stimulated, and secondary echo signals, respectively.
**Time $t_1+\sum_{q=1}^{p}\tau_q$; IAP stands for: response is formed immediately after the pulse.

The intensities of all the responses that can be generated were computed from the following formulas:

a) for the induction signals after the p-th exciting pulse ($p=1, 2, 3$):

$$I_{\alpha\beta}(\mathbf{k})=I_{\alpha\beta}^{(0)}(\mathbf{k})f_1(\theta_p, y_0)S_{1,2}^2\exp\left\{-\frac{1}{(T_{2,\alpha\beta}^*)^2}\left(t-t_1-\sum_{q=1}^{p}\tau_q\right)^2\right\}$$
$$\times\sum_{j\neq l}\exp\{i(\mathbf{k}_p+\xi\mathbf{k}_0-\mathbf{k})(\mathbf{r}_j-\mathbf{r}_l)\},\tag{16}$$

b) for the echo signals (including the stimulated and secondary ones) after the p-th exciting pulse:

$$I_{\alpha\beta}(\mathbf{k})=I_{\alpha\beta}^{(0)}(\mathbf{k})f_2(\theta_p, y_0)S_{1,2}^2\exp\left\{-\frac{(t-t_1-\varphi(\tau_1,\tau_2))^2}{(T_{2,\alpha\beta}^*)^2}\right\}$$
$$\times\sum_{j\neq l}^{N}\exp\{i[\psi(\mathbf{k}_p,\mathbf{k}_0)-\mathbf{k}](\mathbf{r}_j-\mathbf{r}_l)\},\tag{17}$$

where $f_1(\theta_p, y_0)$ and $f_2(\theta_p, y_0)$ are given in the tables, $\theta_p=\Delta t_p\hbar^{-1}|G_{\mathbf{k}_0}|\langle a_{\mathbf{k}_0}\rangle N^{-1/2}$, Δt_p is the width of the p-th pulse, S_1 and S_2 are factors applicable in the computations of the intensities of the responses excitable by the pulse trains shown in Figs. 1 and 2, respectively:

$$S_1=\begin{cases} Z^{-1}e^{-\beta E}+\dfrac{\varepsilon^2E^2}{2|G_{\mathbf{k}_0}|^2}-\dfrac{\sigma}{2}, & T\leqslant T_{cr}\\ Z_0^{-1}[e^{-\beta E}-1], & T>T_{cr}\end{cases},$$
$$S_2=\begin{cases} Z^{-1}e^{-\beta E}-\dfrac{\varepsilon^2E^2}{2|G_{\mathbf{k}_0}|^2}-\dfrac{\sigma}{2}, & T\leqslant T_{cr}\\ Z_0^{-1}[e^{-\beta E}-e^{\beta\varepsilon E}], & T>T_{cr}\end{cases},$$

τ_q is the interval of time between the q-th and ($q+1$)-th pulses; $\xi=0, 1$; $\varphi(\tau_1, \tau_2)$ and $\psi(\mathbf{k}_p, \mathbf{k}_0)$ are linear functions of the arguments (their form depends on the exciting-pulse train and the type of signal); $T_{2,\alpha\beta}^*$ is the reversible-phase-relaxation time with respect to the transition $|\alpha\rangle \longleftrightarrow |\beta\rangle$.

As can be seen from (16) and (17), the peaks of the induction- and echo-signal intensities are formed at the moments of time

$$t=t_1+\sum_{q=1}^{p}\tau_q, \quad t=t_1+\varphi(\tau_1,\tau_2)$$

and in the directions of the wave vectors $\mathbf{k}=\mathbf{k}_p+\xi\mathbf{k}_0$ and $\mathbf{k}=\psi(\mathbf{k}_p,\mathbf{k}_0)$ respectively. In the case of the train of exciting pulses that are resonant for the transition $|1\rangle\rightarrow|3\rangle$, a "richer" set of coherent responses is generated which includes signals generated in the $|1\rangle\rightarrow|2\rangle$ transition at the moments of time $t=t_1+(1+\epsilon)\tau_1, (1+\epsilon)\tau_2$, and $(1+\epsilon)(\tau_1+\tau_2)$, and due entirely to the presence of the SCS transition (at $T>T_{cr}$ the parameter $y_0=0$ and the intensity of these signals is equal to zero). Since this excitation scheme allows the time, spatial, and frequency separation of the exciting pulses and the phase-stimulated superradiant responses, it can primarily be recommended for the experimental detection of the SCS transtion. In view of the sufficiently wide generality of the employed model Hamiltonian, it is unimportant in which spectral regions (the optical, microwave, or radio-frequency regions) the transition frequencies lie, nor are the natures of the boson field $\mathbf{k}_0$, the exciting-generator fields, and the generated responses important.

Furthermore, similar results can be obtained also for semiconductors and semimetals if they are at temperatures below the critical temperature of the SCS transition with respect to the valence and first conduction bands. Using the results of Ref. 14, we can show that the intensity of the responses that can be generated in this case is given by the expressions given in the tables when $\epsilon^2E^2y_0/|G_{k_0}|^2$ is replaced by a quantity $\infty/\hbar^3NE_g(|A|^2Vmp)^{-1}y_0$, which can be derived with the use of the results of Ref. 14 for the SRPT in these systems (here A is the modulus of the matrix element of the interaction of the electrons with the electromagnetic field; E_g is the width of the first forbidden band; m the electron mass; V the crystal volume; and p the momentum at the boundary of the first Brillouin zone). With the same reservations, these results are also valid for exciton crystals. The replacement of the pulse generators by steady excitation also produces a steady spontaneous response both at the frequency of the exciting field and at frequencies different from this frequency. The signals generated in this case are phase-stimulated analogs of forced inductions, e.g., light,[15] phonon,[16] etc., inductions.

In conclusion, let us note that PSBSR is not only more direct and more convenient than any of the methods thus far proposed for the detection of the SCS transition, but also provides the simplest means of studying many-level systems with a nonequidistant spectrum.

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