Laser detector of gravitational waves

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The interaction of gravitational waves with laser fields under situation conditions is considered. It is shown that the influence of gravitational radiation the absorption or emission spectra contain not only the ordinary nonlinear spectral structures but also nonlinear laser–gravitation resonances at the points $\omega' = \omega \pm \xi$, where ω is the central frequency of the ordinary nonlinear resonance and ξ is the frequency of the gravitational wave. The widths and amplitudes of the laser–gravitation resonances depend on the characteristics of the laser systems and the parameters of the gravitational radiation. The experimental realization of a laser detector of gravitational waves is discussed and its sensitivity estimated.

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1. INTRODUCTION

Gravitational radiation is so hard to detect because it interacts so extremely weakly with other forms of motion, giving up virtually no energy.[1-3] This is why detector antennas have a very low sensitivity under usual conditions. The force produced by a gravitational wave of extraterrestrial or laboratory origin is so weak that its effect is comparable with the effect of the fluctuation force of the Brownian motion in a gravitational antenna. It is therefore necessary to cool the detecting systems to extremely low temperatures.

In this paper, we consider a laser detector of gravitational waves that makes it possible to look for and receive gravitational radiation at frequencies $10-10^5$ Hz, a rich range from the astrophysical point of view in which one can expect prolonged and powerful gravitational pulses.[1,4] The proposed scheme differs from the ordinary electromagnetic variant of a gravitational antenna[5,6] based on direct interaction between the gravitational wave and electromagnetic radiation in that here the interaction takes place through a resonantly absorbing gas medium, and the gravitational waves are detected through the spectra of nonlinear absorption. The procedure for tuning such an antenna reduces to scanning the frequency of the laser field.

The effectiveness of the laser variant of the detector derives from the production of a nonequilibrium velocity distribution of excited particles in the field of the strong electromagnetic wave. The lower the velocities of the particles in resonance with the light field, the greater the influence on the spectra of the gravitation-induced acceleration.[7] Under saturation conditions, the only particles that interact with the laser field are those that have low velocities of the order of the width of the Bennett[8] dip (peak):

$$v \sim \Gamma/k \ll \bar{v}, \tag{1.1}$$

where Γ is the homogeneous width of the absorption line, k is the wave vector, and $\bar{v}$ is the mean thermal velocity. Therefore, the substitution of the saturation-induced Bennett velocity distribution for the Maxwellian strongly enhances the intensity of the laser-gravitation resonances.[9]

2. KINETIC EQUATION

We shall assume that two-level particles which interact resonantly with the light field E execute thermal motion and, in addition, are subject to the acceleration a due to gravitation:

$$a_k = g_k - c^2 \sum_l R_{k0l0} x_l, \tag{2.1}$$

where x_l are the coordinates of the particles in the absorbing cavity. The time dependence of the acceleration is given by the components R_{k0l0} of the curvature tensor.[10] Because the perturbations of the metric produced by the gravitational wave are small, the covariant differentiation can be replaced by ordinary differentiation. The kinetic equation for the density matrix describing the dynamical evolution of the system then reduces to a result which is well known in the spectroscopy of accelerated particles (Wigner distribution)[7,11]:

$$(\partial/\partial t + \mathbf{v}\nabla_r + \mathbf{a}\nabla_v + \Gamma_j)\rho_j = q_j \mp 2\,\mathrm{Re}(iV^*\rho), \quad j = m, n, \tag{2.2}$$

$$(\partial/\partial t + \mathbf{v}\nabla_r + \mathbf{a}\nabla_v + \Gamma)\rho = iV(\rho_n - \rho_m). \tag{2.3}$$

The diagonal elements of the density matrix ρ_j characterize the populations of the working states m and n; the nondiagonal element ρ describes the properties of the spectral line. The interaction with the light field is taken into account by means of the operator V; Γ_j are the decay constants of the levels; we assume that the state excitation functions q_j ($j = m, n$) are Maxwellian:

$$q_j = Q_j F_M(\mathbf{v}), \quad F_M(\mathbf{v}) = (\pi^{1/2}\bar{v})^{-3} \exp(-v^2/\bar{v}^2). \tag{2.4}$$

The profile of the spectral line is determined by the magnitude of the acceleration and the mutual orientation of the vectors **a** and **k**. If the wave vectors of the electromagnetic and gravitational fields are collinear, the gravitational radiation does not affect the spectra. Suppose a linearly polarized gravitational wave with frequency ξ and amplitude h propagates in such a way that the constant component of the acceleration a is at right angles to the wave vector **k** of a strong travelling wave

$$\mathbf{E} = \mathrm{Re}[\mathbf{E}_0 e^{-i(\omega t - \mathbf{k}\mathbf{r})}] \tag{2.5}$$

that acts on the $m-n$ transition. In this case, the influence of the constant component of the acceleration on the spectrum, which was considered earlier in Ref. 7, is eliminated. The projection of the acceleration onto **k** becomes an oscillating function of the time:

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$$a_{\parallel}=h\xi^2 x\cos\xi t. \tag{2.6}$$

The coordinate x is measured in the plane of the gravitational wave front along the axis of the nonlinearly absorbing cavity, reflecting in this way the quadrupole nature of the antenna. The coordinate origin is at the end of the cavity.[1)]

The gravitational radiation modulates the populations of the excited states with frequency ξ and excites in the medium a polarization at the combination frequencies $\omega\pm\xi$. We therefore seek a solution of Eqs. (2.2) and (2.3) in the form

$$\rho_j=R_j+2\mathrm{Re}(r_j e^{i\xi t}),$$
$$\rho=Re^{-i(\Omega t-kx)}+re^{-i(\Omega t-\xi t-kx)}+r'e^{-i(\Omega t+\xi t-kx)}, \tag{2.7}$$

where $\Omega=\omega-\omega_{mn}$ is the amount by which the frequency of the light wave deviates from the frequency ω_{mn} of the working transition.

The variables R_j and R, which describe the interaction with the electromagnetic field in the absence of gravitational waves, satisfy the equations

$$\Gamma_j R_j=q_j\mp 2\mathrm{Re}(iG^*R), \tag{2.8}$$

$$(\Gamma-i\Omega+ikv_x)R=iG(R_n-R_m), \tag{2.9}$$

$$V=Ge^{-i(\Omega t-kx)},\quad G=\mathbf{E}_0\mathbf{d}_{mn}/2\hbar. \tag{2.10}$$

The weak effect of the gravitational radiation is taken into account perturbatively by the equations for r_j and r:

$$(i\xi+\Gamma_j)r_j+\frac{h\xi^2 x}{2}\frac{\partial R_j}{\partial v_x}=\mp i(G^*r-Gr'^*),\quad j=m,n, \tag{2.11}$$

$$(\Gamma-i\Omega+i\xi+ikv_x)r+\frac{h\xi^2 x}{2}\frac{\partial R}{\partial v_x}=iG(r_n-r_m). \tag{2.12}$$

We obtain the equations for r' from (2.12) by the substitutions $r\to r'$, $r_j\to r_j^*$, $\xi\to-\xi$. Knowing the solution of Eqs. (2.8)–(2.12), we can follow the frequency dependence of the coefficient of resonance absorption, which is the result of averaging over the ensemble.

3. SPECTRAL RESONANCES

We consider the profile of resonance absorption of a weak light field in the form of a travelling wave which acts on the same transition as the strong wave. In this case, we must add to the interaction operator (see Eq. (2.10)) the term

$$V_\mu=G_\mu\exp[-i(\Omega_\mu t-k_\mu x)],\quad G_\mu=\mathbf{E}_\mu\mathbf{d}_{mn}/2\hbar,\quad \Omega_\mu=\omega_\mu-\omega_{mn}, \tag{3.1}$$

where ω_μ is the frequency of the weak signal and $\mathbf{E}_\mu$ is its amplitude. Corresponding terms also appear in the expressions for ρ and ρ_j. In the approximation $\Gamma\ll k\bar{v}$ and to the first corrections that are nonlinear in the intensity, the coefficient of absorption of the weak electromagnetic wave has the form ($\mathbf{k}_\mu\uparrow\uparrow\mathbf{k}$)

$$\alpha_\mu=\alpha_\mu'+\alpha_g, \tag{3.2}$$

$$\alpha_\mu'\propto(N_n-N_m)\exp\left[-\frac{\Omega_\mu^2}{(k\bar{v})^2}\right]$$
$$\times\left\{1-2|G|^2\mathrm{Re}\sum_{j=m,n}\left[\left(\frac{1}{\Gamma_j}+\frac{1}{\Gamma_j-i\varepsilon}\right)\frac{1}{2\Gamma-i\varepsilon}\right]\right\}, \tag{3.3}$$

$$\varepsilon=\omega_\mu-\omega,\quad N_j=Q_j/\Gamma_j,$$
$$\alpha_g\propto a_0\exp\left[-\frac{\Omega_\mu^2}{(k\bar{v})^2}\right]\sum_{q=\pm1}\delta_{\omega_\mu,\omega-q\xi}\mathrm{Re}\left\{\left(\frac{N_n}{\Gamma_n+iq\xi}-\frac{N_m}{\Gamma_m+iq\xi}\right)\right.$$
$$\times\frac{2GG_\mu^*}{i\pi^{1/2}\bar{v}|G_\mu|^2}+(N_n-N_m)GG_\mu^*\frac{|G|^2}{|G_\mu|^2}ik\sum_{j=m,n}\left[\left(\frac{2}{\Gamma_j(\Gamma_j+iq\xi)}\right.\right.$$
$$\left.\left.\left.+\frac{1}{\Gamma\Gamma_j}+\frac{1}{(\Gamma_j+iq\xi)(\Gamma+iq\xi)}\right)\right]\frac{1}{(2\Gamma+iq\xi)^2}\right\},$$
$$a_0=h\xi^2 L, \tag{3.4}$$

where L is the length of the nonlinearly absorbing cavity.

The absorption coefficient decomposes into two terms. In the absence of gravitational waves, the absorption line profile has a Doppler background on which there are superimposed narrow spectral structures with centers at the frequency $\omega_\mu=\omega$, the profile being described by the term α_μ'[12,13] α_g takes into account the effect of the gravitational radiation. The oscillations of the level populations due to the gravitational wave lead to the appearance of parametric laser-gravitation resonances. They arise near the ordinary nonlinear interference peak at the frequencies $\omega_\mu=\omega\pm\xi$. The widths and amplitudes of the laser-gravitation resonances depend on the characteristics of the resonantly absorbing medium and the parameters of the gravitational wave.

For linear spectra, the relative intensity of the laser-gravitation resonances can be estimated at

$$\alpha_g/\alpha_\mu'\sim h\xi^2 L|G|/\bar{v}\xi|G_\mu|. \tag{3.5}$$

Substituting here the values $\xi/2\pi\sim0.5\cdot10^4$ Hz, $h\sim10^{-18}$, $L\sim10^3$ cm, $\Gamma_j/2\pi\sim10^2$ Hz, $\bar{v}\sim10^5$ cm/sec, we obtain the small value 10^{-13} for the ratio α_c/α_μ'.

The sensitivity of the detector increases significantly if there is saturation, when particles from the "Bennett peak"[8] begin to play a decisive role in the formation of the laser-gravitation resonances. It can be seen from (3.4) that in this case the ratio of the amplitude α_g of the laser-gravitation resonance to $\alpha_g^{(0)}$ satisfies

$$\frac{\alpha_g}{\alpha_g^{(0)}}\sim\frac{|G|^2 k\bar{v}}{\xi\Gamma_j\Gamma}\gg1. \tag{3.6}$$

Comparing the mean thermal velocity $\bar{v}$ with the magnitude of the Bennett dip Γ/k (for the optical frequency range $\Gamma/2\pi\sim10^2$ Hz and $|G|\lesssim\xi$) we find that the gain in the sensitivity when the nonlinear effects are used is $\alpha_g/\alpha_g^{(0)}\sim10^9$. The ratio of the amplitude of the nonlinear laser-gravitation resonance $\Delta\alpha_g\approx\alpha_g$ to the amplitude of the nonlinear interference correction $\Delta\alpha_\mu'$ is then

$$\Delta\alpha_g/\Delta\alpha_\mu'\sim h\xi^2 Lk|G|/|G_\mu|\Gamma_j\Gamma\sim10^{-4}. \tag{3.7}$$

A structure of this kind is detectable by the means of modern nonlinear spectroscopy.

Uncertainty is introduced into the estimate (3.7) by the hypothetical value of h. If we take h equal to $4\cdot10^{-20}-4\cdot10^{-22}$, then for the previous values of the remaining parameters the ratio (3.7) varies in the range $10^{-6}-10^{-8}$. The frequency stability requirements on the laser radiation are determined by the widths of the nonlinear resonances 2Γ and Γ_j, i.e., the frequency variations $\delta\omega$ must be appreciably smaller than 2Γ and Γ_j (for example, $\delta\omega \sim 10$ Hz). To detect gravitational pulses of duration $\tau_{gr} \sim 10^{-3}$ sec it is sufficient to have a power 10 mW of the light radiation.

Following,[5,6] let us estimate the effects due to the direct influence of the gravitational radiation on laser fields. If a laser wave propagates in a medium which is not bounded by resonator walls, the signal accumulation time is the lifetime of a molecule in the excitated state, $\tau_j = \Gamma_j^{-1}$, and the change in the energy of the laser wave is given by

$$\Delta\mathscr{E}/\mathscr{E} \sim h\xi/\Gamma_j \sim 10^{-16}. \tag{3.8}$$

If the laser radiation propagates in a resonator, the corresponding correction to the energy due to the curving of space by the gravitational wave is equal in order of magnitude to

$$\Delta\mathscr{E}/\mathscr{E} \sim (h\omega/8\sigma)^2 \sim 10^{-16} - 10^{-14}, \tag{3.9}$$

where $\sigma \sim 10^5 - 10^6$ sec^{-1} is the effective conductivity of the resonator, $\omega \sim 10^{15}$ sec^{-1}. Therefore, under these conditions the effect of the change in the energy of the light fields under the direct influence of the gravitational wave is much weaker than the effect described above.

Let us now consider possible ways in which the effectiveness of a laser detector could be further increased.

First, the sensitivity is improved if the width of the Bennett dip is reduced, and this can be achieved by narrowing the nonlinear resonance and decreasing the wavelength of the coherent radiation. For example, a reduction in the width of the resonance to 10 Hz and transition to the vacuum ultraviolet region increase the relative intensity of the nonlinear laser-gravitation resonance by three orders of magnitude. This resonance then becomes comparable with the ordinary nonlinear structures. The complication of the spectral picture by the recoil effect[14] and time-of-flight effects[15] does not reduce the sensitivity of the detector.

Second, one can examine the spectrum of the weak field at a transition adjacent to the one at which the strong standing wave acts (Raman scattering scheme).[16,17] Suppose the weak wave acts on the transition $m-l$; then the additional factor d_{mn}/d_{ml} appears in (3.7). If the adjacent transition has a smaller oscillator strength, the gain in the sensitivity will come about because of the increase of this factor.

Finally, the detection of gravitational radiation can be significantly facilitated by comparing the signals of differently orientated detectors. This last method also makes it possible to detect gravitational pulses that are not strictly monochromatic. The gravitational flux which can be detected by the laser detector is $1-0.1$ erg·cm^{-2}·sec^{-1}.

To detect gravitational waves, one can also use two-component gas media consisting of amplifying and absorbing particles of different species with coincident resonance lines. In this case, the spectral profile of the weak signal contains not only peaks corresponding to resonance absorption but also dips that are responsible for the resonance amplification and have large widths. This last circumstance is the reason why the laser-gravitation resonances formed near the center of the amplification line have much less contrast than the laser-gravitation absorption resonances considered above. Thus, the use of two-component media does not lead to a radical qualitative change in the results obtained here.

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[1)]Estimates using the above values for the parameters of the gravitational wave and the absorbing cavity show that the influence of the gravitational wave on the frequency of the laser radiation is unimportant.

[1]W. H. Press and K. S. Thorne, Ann. Rev. Astron. Astrophys. **10** (1972) [Russian translation in Usp. Fiz. Nauk. **110**, 589 (1973)].

[2]V. B. Braginskiĭ and A. B. Manukin, Izmeremie Malykh Sil v Fizicheskikh Éksperimentakh (Measurement of Small Forces in Physics Experiments), Nauka (1974).

[3]L. P. Grishchuk, Usp. Fiz. Nauk **121**, 629 (1977) [Sov. Phys. Usp. **20**, 319 (1977)].

[4]V. B. Braginskiĭ, Ya. B. Zel'dovich, and V. N. Rudenko, Pis'ma Zh. Eksp. Teor. Fiz. **10**, 437 (1969) [JETP Lett. **10**, 280 (1969)].

[5]V. B. Braginskiĭ and M. B. Menskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **13**, 585 (1971) [JETP Lett. **13**, 417 (1971)].

[6]V. B. Braginskiĭ, L. P. Grishchuk, A. G. Doroshkevich, Ya. B. Zel'dovich, N. D. Novikov, and M. V. Sazhin, Zh. Eksp. Teor. Fiz. **65**, 1729 (1973) [Sov. Phys. JETP **38**, 865 (1974)].

[7]A. P. Kol'chenko and G. I. Smirnov, Zh. Eksp. Teor. Fiz. **71**, 925 (1976) [Sov. Phys. JETP **44**, 486 (1976)].

[8]W. R. Bennett, Jr., Phys. Rev. **126**, 580 (1962).

[9]Yu. E. Nesterikhin, S. G. Rautian, and G. I. Smirnov, Talk at Fifth Vavilov Conference on Nonlinear Optics, Novosibirsk, 1977.

[10]L. D. Landau and E. M. Lifshitz, Teoriya Polya, Nauka (1967) (English translation: The Classical Theory of Fields, Pergamon Press, Oxford (1975)).

[11]A. P. Kol'chenko, S. G. Rautian, and G. I. Smirnov, Preprint No. 42 [In Russian], Institute of Automation and Electrometry, Siberian Branch, Academy of Sciences of the USSR (1977).

[12]S. G. Rautian, Tr. Fiz. Inst. Akad. Nauk SSSR **43**, 3 (1968).

[13]S. G. Rautian, G. I. Smirnov, and A. M. Shalagin, Zh. Eksp. Teor. Fiz. **62**, 2097 (1972) [Sov. Phys. JETP **35**, 1095 (1972)].

[14]A. P. Kol'chenko, S. G. Rautian, and R. I. Sokolovskiĭ, Zh. Eksp. Teor. Fiz. **55**, 1864 (1968) [Sov. Phys. JETP **28**, 986 (1969)].

[15]S. G. Rautian and A. M. Shalagin, Pis'ma Zh. Eksp. Teor. Fiz. **9**, 686 (1969) [JETP Lett. **9**, 427 (1969)].

[16]S. G. Rautian, Mod. Opt. Polytechn. Inst. of Brooklyn, New York, March (1967), p. 353; H. K. Holt, Phys. Rev. Lett. **20**, 410 (1968).

[17]A. I. Burshteĭn, É. G. Saprykin, and G. I. Smirnov, Zh. Eksp. Teor. Fiz. **66**, 1570 (1974) [Sov. Phys. JETP **39**, 769 (1974)].

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