Experimental test of the equivalence principle

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A quantum superconducting interferometer was used to test whether the changes in the magnetization of a ferrite to which an electric field is applied depend on the orientation in space. Such a dependence could be expected in the framework of the general theory of relativity if the Earth moves along a nongeodesic path in the gravitational field of the cosmic bodies. A systematic effect correlated to the diurnal rotation of the Earth was not found.

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INTRODUCTION

The present paper describes an attempt to detect magnetic induction in a volume in which there is a constant electric field. One could, for example, expect the occurrence of such an effect to be the result of the influence of a gravitational field on electromagnetic processes in accordance with the conclusions of the general theory of relativity. In this sense, the experiment can be regarded as an experimental test of general relativity.

The experimental verification of general relativity hitherto achieved is rather one sided. The well-known effects such as the red shift, deflection of light rays by the Sun, and precession of Mercury's perihelion, as well as the time delay of a radar echo in the field of the Sun are due to the deviation from unity of the diagonal components of the metric tensor, so that only these components are tested experimentally. The fundamental tensorial nature of Einstein's theory of gravitation is not tested. It would be tested in experiments on the relativistic precession of a gyroscope, but these have not yet been performed.

Another way to test general relativity could be made by measuring the effect of gravitation on electromagnetism as predicted by general relativity. In a gravitational field, Maxwell's equations take on a somewhat modified form and one has noncustomary relationships between the electric displacement **D** and the magnetic induction **B** on the one hand and the fields **E** and **H** in the vacuum on the other[1]:

$$\mathbf{B}=\mathbf{H}/\sqrt{g_{00}}+[\mathbf{g}\times\mathbf{E}], \tag{1}$$

$$\mathbf{D}=\mathbf{E}/\sqrt{g_{00}}+[\mathbf{H}\times\mathbf{g}]. \tag{2}$$

Here, g is a vector whose projections onto the coordinate axes are equal to the crossed components $g_{0\alpha}$ of the metric tensor. The presence of the first terms on the right-hand sides of these equations can be interpreted as the appearance of a permittivity and magnetic permeability in vacuum in the presence of gravitation that are not equal to unity, this leading to a refractive index which is not equal to unity and depends on the distance to the gravitating body and therefore describes the experimentally observed deflection and delay of light rays and radio waves by the field of the Sun. However, from our point of view, the second terms in Eqs. (1) and (2) are of greater interest since they are determined by the nondiagonal components of the metric tensor and can in principle provide the basis for a new test of general relativity. The vector g can be represented in the form (see, for example, Ref. 2)

$$\mathbf{g}=\frac{2\gamma}{c^3R^2}[\mathbf{m}\times\mathbf{n}] . \tag{3}$$

Here, m is the angular momentum of the rotating gravitating body, n is a unit vector in the direction of the observer at distance R from the rotating gravitating body, and γ is the gravitational constant.

It is readily seen that by rotating a body of laboratory scale one cannot obtain any experimentally measurable values of the vector g. Motion relative to the Earth in, for example, an aircraft (under noninertial conditions) makes it possible to obtain $|g|\approx10^{-15}$. In the region of the Earth's orbit, much larger values of g are produced by the rotation of the Sun and the Galaxy in a frame attached to the fixed stars.

If it is assumed, in agreement with the astrophysical data, that the stars which populate the spherical component of the Galaxy and are concentrated in its central part and nucleus have randomly oriented velocities of their rotation and do not produce an appreciable field g and that the stars of the flat component, which are concentrated on the periphery and form a relatively thin disk, rotate around the center in accordance with Kepler's law in one direction, then the field of the vector g associated with this rotation can be obtained in the frame of the fixed stars in the region of the Earth's orbit by integration over the flat component. According to the astrophysical data,[3] the mass of the flat component of the Galaxy is $8\cdot10^{22}$ g and its radius $5\cdot10^{22}$ cm while the Earth is at a distance $3\cdot10^{22}$ cm from the center of the Galaxy and rotates in a Kepler orbit with velocity 250 km/sec; hence, in the region of the Earth

$$|g|\approx2.5\cdot10^{-10}. \tag{4}$$

If it is assumed that the Earth moves with velocity 250 km/sec in the static gravitational field of the Galaxy rectilinearly and uniformly, then in a Lorentz frame attached to the Earth we obtain

$$|g|\approx2.3\cdot10^{-9}. \tag{5}$$

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With current measurement technology, these effects could be readily measured, but because of the equivalence principle they must be inobservable in a freely falling laboratory. Indeed, in accordance with the equivalence principle, "an observer in a closed box could in no way establish whether the box is at rest in a (homogeneous) gravitational field or is in a region of space free of gravitational fields but is moving with acceleration due to forces applied to the box".[4] Therefore, the results of all local experiments made in a system falling with natural acceleration characteristic of the existing gravitational field will be identical to the results of experiments in an inertial frame. In other words, it follows from this principle that a freely falling laboratory (the Earth) selects a trajectory which is such that inside the laboratory the local gravitational effects proportional to the gravitational field (but not its gradient) are absent. In connection with these limitations, it is to be expected that under terrestrial conditions the effects described by the second terms in Eqs. (1) and (2) and due to the galactic gravitational field could exist either because of failure of the equivalence principle for the components $g_{0\alpha}$ or if the Earth does not fall freely in the galactic field, i.e., falls with an acceleration that is determined by not only the gravitational field.

The effects associated with the rotation of the Sun and the motion of the Earth around it have the same nature, but since the calculations show that they are several orders of magnitude smaller we shall not consider them. Noninertial motion of a laboratory with respect to the Earth should give a nonvanishing effect whose observation would be a test of general relativity and not the equivalence principle, but with the existing technological possibilities we are still short of several orders of magnitude in the sensitivity for its observation.

DESCRIPTION OF EXPERIMENT

The test of the equivalence principle in the approach described above can be reduced to a measurement of the magnetic induction in a volume in which there is an electric field or, more precisely, to a measurement of the change in the magnetic induction accompanying a change in the orientation of the electric field. The greatest sensitivity in the measurement of small magnetic effects is currently achieved by using superconducting quantum interferometers.[5,6] Their operation is based on the principle that the magnetic flux through a superconducting circuit does not change when there are changes in the magnetic field produced by sources either outside or inside the loop. If the latter is represented in the form of two superconducting rings connected by superconducting leads (Fig. 1), the condition that the magnetic flux remains unaltered leads to the appearance in the complete circuit of an undamped current when the magnetic field which passes through one of the rings changes. This current can be measured (in the second ring) with extremely high accuracy because of the quantization of the magnetic flux and the Josephson effect.[6,7] An electric field can be conveniently produced in the investigated region by means of a charged capacitor.

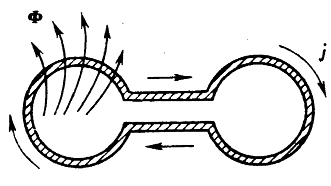

FIG. 1.

Let us consider this question: Does the placing of a capacitor within such a superconducting ring lead to measurable effects? Suppose that the electric field in a planar capacitor, the vector g, and the axis of the superconducting ring are mutually perpendicular to one another (Fig. 2), i.e., such that the magnetic flux which arises in the capacitor can give rise in accordance with Eq. (1) to a current in the superconducting ring. For simplicity, we can assume that one of the plates of the capacitor and the ring are at zero potential. If the second plate is charged, then in such a system an electric field will exist in the capacitor gap and in the gap between the second plate and the ring. It is not difficult to see that the magnetic fluxes that could arise in these gaps because of the considered effect exactly compensate one another.

One can prove a theorem: For any configuration of the ring and plates and any number of them, one cannot obtain a nonzero total magnetic flux with this arrangement. Indeed, if there were a nonzero magnetic flux through the ring this would mean in the considered case that the integral of the electric field strength in the plane of the ring over the area of the ring would be nonzero:

$$\Phi=\iint ds\, B=g_{0y}\int dy \int dx\, E_x. \qquad (6)$$

Since the contour of integration in our case is restricted to the conductor (superconductor), between any two points of which there is, by definition, no potential difference, the integral vanishes identically:

$$\int_A^B dx\, E_x=U_{AB}=0. \qquad (7)$$

If the total magnetic flux through such a ring is to be nonzero, one of the gaps must be filled with a ferromagnetic medium with magentic permeability $\mu\gg 1$. Because an electric field must be applied to this filler, our choice is directed to ferrites with low conductivity and satisfactory electronic strength. Note that in accordance with Eq. (2) the presence of a magnetic field in the field of the vector g results in an increment of

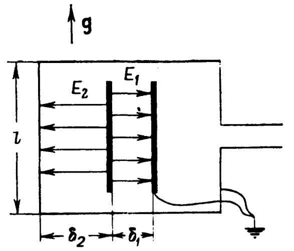

FIG. 2.

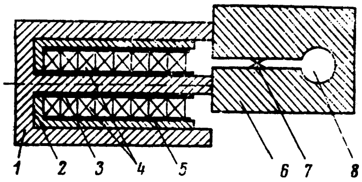

FIG. 3. Sensor of interferometer with flux transformer: *1)* superconducting flux transformer–the niobium beaker with rod, *2)* insulator, *3)* inner plate of capacitor and copper foil, *4)* ferrite rings, *5)* outer plate of capacitor, *6)* frame of sensor, *7)* point junction, *8)* opening of sensor.

the electric displacement and not the electric field strength, so that this feedback effect will not influence the orientation of the magnetic moments in the ferrite.

The main technical and constructional characteristics of our superconducting quantum interferometer have been described earlier in detail in Refs. 6 and 8. In the present experiment, a single-inductance squid was used as sensor of the interferometer. It consisted of a block of niobium ***6*** (see Fig. 3) with one 2-mm diameter opening ***8***, which plays the role of the working inductance of the squid. A thin slit joined the outer surface of the block to the inner opening, making the complete block simply connected. A point Josephson junction ***7*** was placed within the slit. The niobium beaker *1* with toroidal topology, within which the ferrite *4* was placed, played the role of the primary coil of the superconducting transformer, the working inductance of the squid serving simultaneously as the second coil of the transformer, within which the coil of the oscillator circuit of the interferometer was introduced. The previously achieved optimization of the squid and circuit parameters[6,9] and well chosen electronics made it possible to obtain a sensitivity at the level $10^{-4}\Phi_0$ (Φ_0 is the magnetic flux quantum) at a time constant 1 sec of the instrument without introducing the ferrites into the transformer and applying a voltage.

Measurements made earlier had shown that for all the investigated ferrites the reversible permeability at helium temperature is strongly reduced, and for nickel-zinc ferrite as well as magnetic insulators of the type $CdCrO_4$ and EuO it is close to 10. The experiment described here was performed with the nickel-zinc ferrite because it has simultaneoulsy low electrical conductivity, appreciable electrical strength (up to 3 kV/cm)[1)] and satisfactory magnetic properties.

In contrast to the previous experiment,[11] in which a ferrite slab was placed directly inside the squid, in the present experiment a higher sensitivity was achieved by applying a voltage to a column made of ferrite rings with inner and outer diameters 2 mm and 3 mm. To obtain a reliable electrical contact between the ferrite and the conducting electrodes, the inner and outer surface of the ferrite rings were silvered. The ferrite rings *4* were placed inside a niobium beaker with a central niobium rod *1*. The electric field was applied radially to the ferrite rings. The electrodes were the niobium beaker with the rod and a special cylinder of copper foil *5* wound over the ferrite cylinder. This copper cylinder was carefully insulated from the niobium one in order to eliminate loss and breakdown. To ensure a reliable contact between the inner silvered surface of the ferrite rings and the central niobium rod, and also to compensate for the difference between the coefficients of thermal expansion of the ferrite and the niobium, we also placed between them a cylinder ***3*** of 20-micron copper foil, which played the role of a spring lead.

The electric field applied to the ferrite ring in the radial direction should give rise in accordance with Eq. (1) in the presence of a vector g along the axis of a ring to a circular magnetization of the ferrite rings and, as a result of the conservation of the magnetic flux, to a dc current in the superconducting circuit formed by the niobium beaker and rod and the squid.

RESULTS OF THE EXPERIMENT

Thus, the existence of the field of the vector g should lead to a magnetic flux in the ferrite when the electric field **E** is switched on:

$$\Phi_f=\mu S_f E|\mathbf{g}|. \tag{8}$$

Here, S_f is the cross section of the ferrite rings, and μ is the permeability of the ferrite. The total magnetic flux in the toroidal transformer (with allowance for the effect of the second part of the cylindrical capacitor, which does not contain ferrite) is

$$\Phi_t=(\mu-1)S_f E|\mathbf{g}|. \tag{9}$$

In this case, the squid can detect the flux

$$\Phi_s=\frac{L_s}{L_s+L_t}\Phi_t. \tag{10}$$

(L_s and L_t are the inductances of the squid and the toroidal transformer with the ferrite), whence

$$|\mathbf{g}|=\frac{3\Phi_s(L_s+L_t)\delta}{(\varepsilon+2)US_fL_s(\mu-1)}. \tag{11}$$

Here we have used the fact that the electric field in the ferrite with cubic structure is

$$E=U(\varepsilon+2)/3\delta; \tag{12}$$

U is the voltage applied to the ferrite; ϵ is the permittivity of the ferrite; and δ is the thickness of the wall of the ferrite ring.

Preliminary measurements showed that $L_s=(2.88\pm0.04)\cdot10^{-10}$ H, $L_t=(4.7\pm0.1)\cdot10^{-10}$ H, $\mu=11\pm2$, $\varepsilon=2.3\pm0.1$, $\delta=0.05$ cm, $S_f=6.5\cdot10^{-2}$ cm². The voltage applied to the ferrites was in the form of rectangular pulses from a generator, the amplitude swinging from +100 to −100 V at a frequency of about 30 Hz.

The detection circuit was arranged in such a way that the signal, i.e., the voltage at the interferometer output coded by means of an ADC, was detected by a special conversion instrument in the form of a difference between the indications of the interferometer during the positive and negative half-periods of the voltage applied to the ferrite.[12] This synchronous detection made it possible to obtain a time constant of the instrument equal to tens of minutes and more and reduce the noise appreciably.

To avoid interference due to the instability of the external magnetic field, the squid with the toroidal transformer containing the ferrite were placed in a superconducting magnetic screen made of lead in the form of somewhat flattened bottle. The screening coefficient was better than 10^8, so that effects of external fields on the interferometer output were not noted.

Within the cryostat, the axis of the toroidal transformer (i.e., the axis of the ferrite rings) was oriented horizontally. The setup could therefore detect only the projection of the vector g onto the horizontal plane. To have the possibility of changing the angle between the electric field and the direction of g, the complete cryostat together with the primary electronics was placed on a rotating platform and could be rotated around the vertical axis. Special measures were taken to achieve antivibrational suspension of the rotating platform.

The experiments were made in two series of measurements during a day. The first series of measurements was made when the star Deneb (α Cyg), toward which the solar system is moving in the Galaxy (according to the data of the astronomers), so that the vector g also points in its direction, was near the line of the horizon. The axis of the ferrite rings together with the cryostat was oriented along the direction to Deneb, and the digital synchronous detector measured the readings of the interferometer for about 3 min. The cryostat was then rotated through 180° about the vertical axis and the readings determined for the new orientation. The results obtained in this way were subtracted and the difference taken as the desired effect. The procedure was then repeated. A further series of measurements was made about 12 hours later, when the constellation of Cygnus was at the zenith and for any orientation of the cryostat the axis of the ferrite rings was orthogonal to the direction toward Deneb. In this control series of measurements, for any orientation of the cryostat, even if the equivalent principle is violated, there should be no effect of the galactic field g.

In the first series of measurements, which was made during about one hour, the interferometer measured the following change in the flux:

$$\Delta\Phi=(4.1\pm10.1)\cdot10^{-5}\Phi_0, \tag{13}$$

and in the second

$$\Delta\Phi=(1.2\pm11.2)\cdot10^{-5}\Phi_0. \tag{14}$$

Thus, using Eq. (11), we find that in the first series

$$|g|=(0.92\pm4.20)\cdot10^{-12} \tag{15}$$

and in the second

$$|g|=(0.27\pm3.67)\cdot10^{-12}. \tag{16}$$

On the basis of this, we can assert that there was no effect in either the first or the second series of measurements.

CONCLUSIONS

This experiment shows that under terrestrial conditions effects due to the crossed components of the galactic gravitational field g are absent, in complete agreement with the principle of equivalence, whereas the motion of the Earth in the galactic field can be determined to an accuracy not worse than 10^{-3} by purely gravitational forces. The negative result of the experiment cannot be regarded as contradicting the prediction of relativistic precession of a gyroscope on an Earth satellite[13] even though in both cases the effects are described in terms of the effect of a gravitational field with components $g_{0\alpha}$ and the measurements are made in a freely falling laboratory. The precession angle can be measured relative to the fixed stars, and therefore such an experiment is performed in this sense in a noninertial system. But the experiment described here can be regarded as performed in a locally inertial frame of reference.

It should be noted that the results of our experiment cannot, strictly speaking, be unambiguously interpreted since at the present time there are neither experimental data not theoretical predictions on the behavior of the magnetic moments in a ferrite in the presence of electric and gravitational fields.

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Three-boson stimulated scattering

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A quantum-mechanical justification is obtained for the three-boson stimulated scattering (TBSS) hypothesis advanced by Skorobatov and Dzevitskiĭ [Sov. Phys. Dokl. **18**, 668 (1974)] in an analysis of the Boltzmann quantum equation. The resonant properties of TBSS are investigated, i.e., the increased probability of scattering of one of the two colliding bosons with the momentum belonging to the third boson located at a distance comparable with the de Broglie wavelength of the fastest of the three particles (in their mass center). Some singularities of "cumulative" production of hadrons on a nucleus are explained with TBSS taken into account.

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1. INTRODUCTION

Assume that only two- and three-particle exchanges of a certain substance take place in a spatially-homogeneous system of identical particles with the number of particles conserved and let the role of the "substance" be assumed by the momentum. Then the time evolution of the system in the N/V limit[1] is described by the balance equation[2,3]:

$$\frac{\partial f(\mathbf{p}_1,t)}{\partial t}=\int_{-\infty}^{\infty}[\varkappa(\mathbf{p}_1',\mathbf{p}_2';\mathbf{p}_1,\mathbf{p}_2)f(\mathbf{p}_1',t)f(\mathbf{p}_2',t)-\varkappa(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_1',\mathbf{p}_2')f(\mathbf{p}_1,t)f(\mathbf{p}_2,t)]d^3\mathbf{p}_2\,d^3\mathbf{p}_1'\,d^3\mathbf{p}_2'$$
$$+\int_{-\infty}^{\infty}[\varkappa(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3';\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)f(\mathbf{p}_1',t)f(\mathbf{p}_2',t)f(\mathbf{p}_3',t)-\varkappa(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3;\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')f(\mathbf{p}_1,t)f(\mathbf{p}_2,t)f(\mathbf{p}_3,t)]d^3\mathbf{p}_2\,d^3\mathbf{p}_3\,d^3\mathbf{p}_1'\,d^3\mathbf{p}_2'\,d^3\mathbf{p}_3', \quad (1)$$

where $f(\mathbf{p},t)\geqslant 0$ is the coarse-structure distribution function, and $\varkappa(\mathbf{p}_1,\mathbf{p}_2;\ \mathbf{p}_1',\mathbf{p}_2')$, $\varkappa(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3;\ \mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')\geqslant 0$ are the probabilities of the elementary acts $\mathbf{p}_1,\mathbf{p}_2\to\mathbf{p}_1'$, $\mathbf{p}_2'$ and $\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3\to\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3'$.

It was recently proved that the Uhlenbeck-Uehling equation (the Boltzmann quantum equation) for bosons

$$\frac{\partial f(\mathbf{p}_1,t)}{\partial t}\times\int\delta(\mathbf{p}_1'^2+\mathbf{p}_2'^2-\mathbf{p}_1^2-\mathbf{p}_2^2)\delta(\mathbf{p}_1'+\mathbf{p}_2'-\mathbf{p}_1-\mathbf{p}_2)\Lambda(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_1'/|\mathbf{p}_1'|)\times\{f(\mathbf{p}_1',t)f(\mathbf{p}_2',t)[1+sf(\mathbf{p}_1,t)][1+sf(\mathbf{p}_2,t)]-f(\mathbf{p}_1,t)f(\mathbf{p}_2,t)[1+sf(\mathbf{p}_1',t)][1+sf(\mathbf{p}_2',t)]\}d^3\mathbf{p}_2\,d^3\mathbf{p}_1'\,d^3\mathbf{p}_2', \quad (2)$$

where $\Lambda\geqslant 0$, $s=(2\pi\hbar)^3(2S+1)^{-1}$, and S is the particle spin, can be written in the canonical form (1) if we put

$$\varkappa(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_1',\mathbf{p}_2')=\delta(\mathbf{p}_1'^2+\mathbf{p}_2'^2-\mathbf{p}_1^2-\mathbf{p}_2^2)\delta(\mathbf{p}_1'+\mathbf{p}_2'-\mathbf{p}_1-\mathbf{p}_2)\times\Lambda(\mathbf{p}_1,\mathbf{p}_2;\mathbf{p}_1'/|\mathbf{p}_1'|), \quad (3)$$

$$\varkappa(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3;\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')=\frac{s}{3}\sum_{\{i,j,k\}}\sum_{\{l,m,n\}}\delta(\mathbf{p}_l'-\mathbf{p}_i)\delta(\mathbf{p}_m'-\mathbf{p}_i)\varkappa(\mathbf{p}_j,\mathbf{p}_k;p_i,\mathbf{p}_n'), \quad (4)$$

where $\{i,j,k\}$, $\{l,m,n\}$ are cyclic permutations of $\{1,2,3\}$. The effect of increased probability of scattering of one of the two colliding bosons with the momentum of the present "provocator" boson [see (4)] was named in Ref. 2 "three-boson simulated scattering (TBSS)." There was no complete assurance, however, of the existence of TBSS as a true three-particle process, where one could not exclude the possibility that when the Uhlenbeck-Uehling-Boltzmann Eq. (2) was derived from the Liouville-Neumann equation[1,4,5] the collective effects were not implicitly taken into account. In fact, in Refs. 1, 4, and 5 Eq. (2) is derived only in the N/V limit, and the method of the derivation consists of taking into account the successive terms of the correlation functions[4,5] or terms of the perturbation series[1] until characteristic quantum-mechanical expressions appear in the Boltzmann collision integral. The terms in the diagrams of higher order are then discarded without rigorous justification. Thus, it might turn out that the TBSS does not exist as an elementary three-particle act, and occurs only in a medium with a sufficiently large assembly of identical bosons.

Nor was it proved in Ref. 2 that, besides (3) and (4), there exist no other functions $\varkappa(\mathbf{p}_1,\mathbf{p}_2;\ \mathbf{p}_1',\mathbf{p}_2')$, $\varkappa(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3;\ \mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_3')$, that yield (2) when substituted in (1). To resolve these problems we derive below formula (4) from quantum-mechanical considerations. In addition, we deduce from the conservation laws resonant properties of the TBSS, which consist in the fact that the TBSS takes place only when the momentum of a provocator lies on the so-called "resonance shell" an equation for which is given below.

By the same method used in the present article to investigate TBSS, it is possible to obtain known results on simulated emission. The analogy between TBSS and

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