Bremsstrahlung from a vector meson on a nucleus

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A meson theory in which a vector particle has a scalar partner is considered. The equations of motion remain causal and describe wave propagation for any type of interaction with the electromagnetic field. The bremsstrahlung cross section in the nuclear field is calculated in second-order perturbation theory in e.

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INTRODUCTION

There is now an extensive literature[1–8] on different variants on the theory of vector particles. Most of these treatments are found to be equivalent to the usual Proca theory[9] both in the free case and in the presence of electromagnetic interaction. On the other hand, the Proca theory is known to suffer from a number of important defects: (1) there are no stable solutions in the Kepler problem when the anomalous magnetic moment (AMM) is zero, (2) renormalization cannot be achieved by the usual methods, (3) when the AMM is introduced, the S-matrix loses its relativistic covariance, and (4) the quadrupole interaction leads to the propagation of the particle in an electric field with velocity greater than that of light, and to the absence of propagation when the magnetic field is strong enough. There is also one further important point. In the free case, the Proca equations

$$\partial_\mu \Psi_{\mu\nu} - m^2 \Psi_\nu = 0 \tag{1}$$

lead to the condition

$$\partial_\mu \Psi_\mu = 0, \tag{2}$$

which enables us to exclude zero-spin quanta. When the electromagnetic field is turned on,

$$D_\mu \Psi_{\mu\nu} - m^2 \Psi_\nu = 0, \quad D_\mu = \partial_\mu + ieA_\mu, \tag{3}$$

and (2) is replaced by

$$D_\mu \Psi_\mu = \frac{ie}{2m^2} F_{\mu\nu} \Psi_{\mu\nu}, \tag{4}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

It then no longer follows from (4) that $\Psi_0 = 0$. Thus, the absence of scalar quanta in the initial and final states is ensured by (2), but the S-matrix elements contain contributions due to Ψ_0.[5] In our view, this exclusion is not logically satisfactory and is fundamentally different from the situation in quantum electrodynamics and in the current-conserving theory of interactions of neutral vector mesons in which conditions analogous to (2) are satisfied whether the interaction is present or not. We therefore feel justified in searching for other formulations of the theory of the vector field.

In this paper, we propose a variant of the meson theory in which the vector particle can have a scalar partner. The arguments in favor of objects with such properties have already been mentioned. Thus, for example, the presence of daughter trajectories in dual models[10,11] the existence of "abnormal" solutions of the Bethe-Salpeter equations,[12] and the expressions for the propagators introduced in connection with $O(4)$ symmetry[13] predict for particles with higher spins ($s \geqslant 1$) the existence of partners with lower spins. There is also the weak interaction model proposed by Lee,[14] in which the intermediate vector boson and its scalar partner have different masses. This theory does not lead to a discrepancy between the experimental and theoretical values of the $K_L \to \mu^+\mu^-$ decay rate and the mass difference between K_L and K_s if it is assumed that the intermediate boson masses are ~10 GeV.[15] Finally, there are papers outlining possible experimental searches for such particles. In this connection, we note the paper by Jouge[16] on the detection of the scalar partner O_s of the hypothetic vector particle O_ν in decays of the form

$$\Psi\ (3.1\ \text{GeV}) \to O_s O_\nu \to \text{hadrons}.$$

In this paper, we begin, in Sec. 1, with the procedure for the quantization of the meson field. In Sec. 2, we consider the bremsstrahlung process on a nucleus. The Appendix treats problems connected with the motion of the meson in external fields with a view to establishing the connection between spin-1 and spin-0 states. The system of units in which $\hbar = c = 1$ is used throughout.

1. QUANTIZATION

We start with the Lagrangian density

$$\mathscr{L} = {}^1/_2 \Psi_{\lambda\sigma}^* ({}^1/_2 \Psi_{\lambda\sigma} - \partial_{[\lambda} \Psi_{\sigma]}) + \Psi^* ({}^1/_2 \Psi - \partial_\lambda \Psi_\lambda) + {}^1/_2 \Psi_{\lambda\sigma} ({}^1/_2 \Psi_{\lambda\sigma}^* - \partial_{[\lambda} \Psi_{\sigma]}^*) + \Psi ({}^1/_2 \Psi^* - \partial_\lambda \Psi_\lambda^*) - m^2 \Psi_\lambda^* \Psi_\lambda, \tag{1.1}$$

from which variation with respect to Ψ, Ψ_μ, and $\Psi_{\lambda\sigma}$ yields the following equations of motion:

$$\partial_\lambda \Psi_{\lambda\sigma} - m^2 \Psi_\sigma + \partial_\sigma \Psi = 0, \tag{1.2}$$

$$\Psi_{\lambda\sigma} = \partial_\lambda \Psi_\sigma - \partial_\sigma \Psi_\lambda, \tag{1.3}$$

$$\Psi = \partial_\lambda \Psi_\lambda \tag{1.4}$$

(+conj). Standard methods can then be used to obtain the following expressions for the dynamic variables: for the energy-momentum tensor

$$T_{\mu\nu}^{metr} = T_{\mu\nu}^{can} + \partial_\lambda f_{[\mu\lambda]\nu}, \tag{1.5}$$

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where

$$T_{\mu\nu}^{can}=\Psi_{\mu\lambda}{}^{*}\partial_\nu\Psi_\lambda+\Psi^{*}\partial_\nu\Psi_\mu+\text{conj}-\mathscr{L}\delta_{\mu\nu}, \quad (1.6)$$

$$f_{[\mu\lambda]\nu}=\Psi_{\lambda\mu}{}^{*}\Psi_\nu-\delta_{\lambda\nu}\Psi^{*}\Psi_\mu+\delta_{\mu\nu}\Psi^{*}\Psi_\lambda+\text{conj}, \quad (1.7)$$

and for the total angular momentum density tensor

$$M_{\mu[\rho\sigma]}=M^{0}_{\mu[\rho\sigma]}+S_{\mu[\rho\sigma]}, \quad (1.8)$$

where $M^{0}_{\mu[\rho\sigma]}$ and $S_{\mu[\rho\sigma]}$ are the orbital and spin angular momentum density tensors

$$S_{\mu[\rho\sigma]}=\Psi_{\mu\sigma}{}^{*}\Psi_\rho+\delta_{\mu\sigma}\Psi^{*}\Psi_\rho-\Psi_{\mu\rho}{}^{*}\Psi_\sigma-\delta_{\rho\mu}\Psi^{*}\Psi_\sigma+\text{conj}. \quad (1.9)$$

The following expression for the intrinsic spin of the field follows from (1.9):

$$\mathbf{s}=\int[\vec{\mathscr{E}}^{*}\boldsymbol{\Psi}]d^3x, \quad (1.10)$$

where $\mathscr{E}_i=i\Psi_{4i}$.

The formula given by (1.10) describes a spin-1 particle and is identical with the analogous formula in the Proca theory.

The equal-time commutation relations have the form

$$[\Psi_\mu(\mathbf{r},t),\Pi_\nu(\mathbf{r}',t)]=-i\delta_{\mu\nu}\delta(\mathbf{r}-\mathbf{r}'), \quad (1.11)$$

where

$$\Pi_i=i\Psi_{4i}{}^{*},\quad \Pi_4=i\Psi^{*}. \quad (1.12)$$

The fact that the time component of Ψ_μ contains the canonically conjugate momentum suggests that Ψ_0 and Ψ_k are present in the theory with equal validity, i.e., the quantization process extends not only to the vector field but to the scalar field as well. We recall that $\Pi_4 \equiv 0$ in the Proca theory, which is a source of difficulty connected with the divergence and noncovariance of the elements of the scattering matrix.

The plane-wave expansion for Ψ_μ has the form

$$\Psi_\mu=(2\pi)^{-3/2}\sum_{r=1}^{4}\int\frac{d^3p}{(4E)^{1/2}}(\xi_\mu{}^r a_r e^{ipx}+\xi_\mu{}^r a_r{}^{*}e^{-ipx}), \quad (1.13)$$

where a_r^* and a_r are the particle creation and annihilation operators, and ξ_μ^r are the polarization vectors.

If the direction of the x_3 axis is parallel to the direction of propagation, then, by using the explicit form of the projection operators onto the spin 1 and spin 0 states

$$\Lambda_{\mu\nu}^{(1)}=\delta_{\mu\nu}+\frac{p_\mu p_\nu}{m^2},\quad \Lambda_{\mu\nu}^{(0)}=-\frac{p_\mu p_\nu}{m^2},$$

we can show that the choice of polarization vectors in the form

$$\xi_\mu{}^1=(1,0,0,0),\ \xi_\mu{}^2=(0,1,0,0),\ \xi_\mu{}^3=\left(0,0,\frac{E}{m},i\frac{p}{m}\right)$$
$$\xi_\mu{}^4=\left(0,0,\frac{n}{m},i\frac{E}{m}\right) \quad (1.14)$$

is unambiguous in the helicity basis.

For the Hamiltonian density, we have

$$H=\int\frac{d^3p}{2}E(a_k{}^{*}a_k-a_0{}^{*}a_0). \quad (1.15)$$

The creation and annihilation operators satisfy the commutation relations

$$[a_l,a_k{}^{*}]=\delta_{kl}, \quad (1.16)$$

$$[a_0,a_0{}^{*}]=-1. \quad (1.17)$$

It follows from (1.15)–(1.17) that the vector and scalar particles have opposite metric in precisely the same way as in the Lee model.[14] The associated difficulties can be removed by introducing an indefinite metric. We shall not reproduce all the intermediate steps involved in the redefinition of the various quantities in Hilbert space with indefinite metric, or develop the quantization procedure. This is done in a general form for high spin fields ($s \geqslant 1$) by Barut and Mullen.[17] However, this leaves as an open question the unitarity of the S-matrix. Within the framework of our assumption, i.e., the simultaneous existance of the vector and scalar particles, this can be resolved by showing that the Hamiltonian for the system is positive-definite, i.e., that E^2 is always greater than zero. Thus, we must show that, for any type of interaction with the electromagnetic field, the set of equations of motion describes causal propagation of the particles. Let us examine our equations of motion by the method proposed by Velo and Zwanzinger.[18] Wave propagation is usually associated with a hyperbolic set of partial differential equations. Their solutions have wave fronts that propagate in space time along the characteristic surfaces. These surfaces form the characteristic cone which divides space time into the past, the present, and the future. The characteristic cone is not necessarily identical with the light cone. If causality is not violated, the characteristic cone lies inside the light cone, whereas, in the case of signal propagation with velocity greater than the speed of light, it lies outside the light cone. To verify whether a particular set of equations is hyperbolic and casual, we must evaluate the characteristic determinant

$$D(n)=|A^{\mu_1\ldots\mu_n}n_{\mu_1}\ldots n_{\mu_n}|,$$

where $A^{\mu_1\cdots\mu_n}$ is the term in the equation of motion with highest-order derivative and n_μ is the normal to the characteristic surface.

If for any $\mathbf{n}$ the solution n_0 of the equation

$$D(n)=0$$

is real, then (1.2)–(1.4) are hyperbolic equations. The maximum velocity of signal propagation is then

$$n_0/|\mathbf{n}|.$$

When the electromagnetic field is present, (1.2)–(1.4) assume the form

$$[(D_\lambda{}^2-m^2)\delta_{\mu\nu}-ieF_{\mu\nu}]\Psi_\mu=0. \quad (1.18)$$

The characteristic determinant corresponding to (1.18) is

$$D(n)=|n^2\delta_{\mu\nu}|=(n^2)^4. \quad (1.19)$$

It follows from (1.19) that

$$n_0^2 = \mathbf{n}^2.$$

Thus, the system given by (1.18) is hyperbolic and causal ($n_0/|\mathbf{n}|=1$). Introduction of the dipole magnetic and quadrupole interactions lead to the equations

$$[(D_\lambda^2-m^2)\delta_{\mu\nu}-ie(1+k)F_{\mu\nu}]\Psi_\mu=0, \qquad (1.20)$$

$$[(D_\lambda^2-m^2)\delta_{\mu\nu}-ieF_{\mu\nu}]\Psi_\mu-iq[\partial_\nu F_{\mu\lambda}D_\mu\Psi_\lambda+D_\mu(\partial_\lambda F_{\mu\nu})\Psi_\lambda]=0, \qquad (1.21)$$

where k is the AMM in units of $e/2m$ and q is the quadrupole interaction in units of e/m^2.

The determinant of (1.20) and (1.21) is identical with (1.19). Consequently, the S-matrix is unitary for any type of interaction with the electromagnetic field.

We note that, if the masses of the vector particle and its partner are not equal, which is the case in the Lee model, the Lagrangian (1.1) must be replaced by

$$\mathscr{L}=\Psi_{\lambda\sigma}^*\left(\frac{1}{2}\Psi_{\lambda\sigma}-\partial_{[\lambda}\Psi_{\sigma]}\right)+\Psi^*\left(\frac{1}{2}\frac{m_0^2}{m_1^2}\Psi-\partial_\lambda\Psi_\lambda\right)+\text{conj}-m_1^2\Psi_\lambda^*\Psi_\lambda, \qquad (1.22)$$

from which we have the following equations for the field potential:

$$(\partial_\lambda^2-m_1^2)\Psi_\mu-\eta\partial_\mu\partial_\lambda\Psi_\lambda=0, \qquad (1.23)$$

where $\eta=(m_0^2-m_1^2)/m_0^2$, and m_1 and m_0 are the masses of the vector and scalar particles, respectively.

Thus, even in the free case, there is an interaction between the vector and scalar parts of the field, which is characterized by the parameter η. It is easily seen that this interaction will mix the longitudinal and scalar parts of the field, whereas the transverse part will remain free. It is known that the Lagrangian given by (1.1) is invariant under the gauge transformation

$$\Psi_\mu\to\Psi_\mu+\partial_\mu\Lambda, \quad \Psi\to\Psi+m^2\Lambda \qquad (1.24)$$

provided the arbitrary scalar function $\Lambda(x)$ satisfies the equation

$$(\partial_\mu^2-m^2)\Lambda(x)=0.$$

If we choose this function so that

$$(\partial_\mu^2-m_0^2)\Lambda(x)=\frac{m_0^2-m_0'^2}{m_1^2}\Psi,$$

the transformation given by (1.24) ($m=m_1$) will induce a change in the mass of the scalar particle from m_0 to m_0' in (1.22).

We shall now show that there is a class of interactions between bosons and the electromagnetic field which admits of separation of the spin states. Thus, let us introduce the proper vector in an arbitrary Lorentz reference frame, n_μ, for which

$$n_\mu^2=-1.$$

It gives the origin for the the two projection operators $\Lambda_{\mu\nu}^{(s)}$:

$$\Lambda_{\mu\nu}^{(1)}=\delta_{\mu\nu}+n_\mu n_\nu, \quad \Lambda_{\mu\nu}^{(0)}=-n_\mu n_\nu, \qquad (1.25)$$

the first of which segregates from Ψ_μ the part connected with spin 1 and the second the part with spin 0. The canonical energy-momentum tensor (1.6) can be written in the form (AMM $\neq 0$)

$$T_{\mu\nu}^{can}=T_{\mu\nu}^{(0)}+T_{\mu\nu}^{(1)}+T_{\mu\nu}^{interf}, \qquad (1.26)$$

where

$$T_{\mu\nu}^{(s)}=\frac{1}{2}\Lambda_{\lambda\sigma}^{(s)}\Lambda_{\sigma\tau}^{(s)}[D_\mu^+\Psi_\lambda^*\partial_\nu\Psi_\tau+D_\mu\Psi_\lambda\partial_\nu\Psi_\tau^*]$$
$$+\frac{ies(1+k)}{2}F_{\mu\sigma}\Lambda_{\sigma\tau}^{(s)}\Lambda_{\nu\varkappa}^{(s)}\Psi_\tau^*\Psi_\varkappa, \quad s=0,1, \qquad (1.27)$$

$$T_{\mu\nu}^{interf}=-\delta_{\mu\nu}\partial_\lambda^2(\Psi_\rho^*\Psi_\rho)-ie(1+k)F_{\mu\sigma}\Lambda_{\nu\rho}^{(1)}\Lambda_{\sigma\tau}^{(0)}\Psi_\tau^*\Psi_\rho. \qquad (1.28)$$

It is readily seen that, by virtue of the Gauss theorem, $T_{\mu\nu}^{inter}$ will not provide a contribution to the energy-momentum vector field

$$P_\mu=\int T_{\mu\nu}^{(1)}d\sigma_\nu+\int T_{\mu\nu}^{(0)}d\sigma_\nu=P_\mu^{(1)}+P_\mu^{(0)}. \qquad (1.29)$$

Similarly, the current is given by

$$J_\mu=J_\mu^{(1)}+J_\mu^{(0)}+J_\mu^{interf},$$

where

$$J_\mu^{(s)}=\frac{ie}{2}\Lambda_{\rho\lambda}^{(s)}\Lambda_{\rho\sigma}^{(s)}(D_\mu^+\Psi_\sigma^*\Psi_\lambda-D_\mu\Psi_\sigma\Psi_\lambda^*)$$
$$+\frac{ies(1+k)}{2}\Lambda_{\mu\sigma}^{(s)}\Lambda_{\nu\tau}^{(s)}\partial_\nu(\Psi_\sigma^*\Psi_\tau), \qquad (1.30)$$

$$J_\mu^{interf}=-\frac{ie(1+k)}{2}\Lambda_{\mu\rho}^{(0)}\Lambda_{\nu\sigma}^{(1)}\partial_\nu(\Psi_\rho^*\Psi_\sigma-\Psi_\rho\Psi_\sigma^*). \qquad (1.31)$$

The total charge of the field is given by

$$Q=\int J_\mu^{(1)}d\sigma_\mu+\int J_\mu^{(0)}d\sigma_\mu=Q^{(1)}+Q^{(0)}. \qquad (1.32)$$

It follows from (1.29) and (1.32) that, if the initial and final states lie on spatially similar hypersurfaces, whose unit normals n_μ are equal, the spin-1 and spin-0 states will not interfere. An example of this interaction is the motion of a particle in the field of an arbitrarily polarized plane wave, the potential for which is nonzero only in a bounded region of space (see the Appendix). It is readily seen that interference between the vector and scalar particles will also be absent in the free case. In all other cases, the electromagnetic interaction will mix the spin-1 and spin-0 states.

The S-matrix is given by

$$S=1+\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int_{-\infty}^{\infty}P\left(\prod_{k=1}^{n}H^{int}[x^k]d^4x^k\right), \qquad (1.33)$$

where

$$H^{int}=ieB_{\mu\nu,\lambda\sigma}(\Psi_\mu A_\lambda\partial_\sigma\Psi_\nu^*-\Psi_\nu^*A_\sigma\partial_\lambda\Psi_\mu)-e^2B_{\mu\nu,\lambda\sigma}\Psi_\mu^*\Psi_\nu A_\lambda A_\sigma,$$
$$B_{\mu\nu,\lambda\sigma}=\delta_{\mu\nu}\delta_{\lambda\sigma}+(1+k)(\delta_{\mu\lambda}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\lambda}). \qquad (1.34)$$

Relativistic covariance of the S-matrix was demonstrated by Boyarkin[19] for nonzero AMM. The expressions for the one- and two-photon vertices in momentum space have the form

$$V_{\mu\nu\sigma}=ie(B_{\mu\nu,\lambda\sigma}p_{1\lambda}+B_{\mu\nu,\sigma\lambda}p_{2\lambda}), \qquad (1.35)$$

$$V_{\mu\nu\lambda\sigma}=-2e^2\delta_{\mu\nu}\delta_{\lambda\sigma}, \qquad (1.36)$$

where p_1 and p_2 are the momenta of the incident and emitted mesons, μ and ν are the coordinate indices of their polarization vectors, the indices σ are given in (1.35), and σ and λ in (1.36) are connected with the photon polarizations.

The meson propagator is

$$P_{\mu\nu}=\frac{\delta_{\mu\nu}}{p^2+m^2}.$$

It follows from (1.13) that a particle in the initial (final) state corresponds to the following factor in the matrix element:

$$\xi_\mu{}^r a_r/2(E)^{1/2},\ (\xi_\mu{}^r a_r^*/2(E)^{1/2}).$$

The meson field can therefore assume four polarization states, two of which are purely vector states and correspond to transversely polarized particles ($\mu=1,2$) and the other two are mixtures of longitudinal and scalar states ($\mu=3,4$) because of the interaction.

2. BREMSSTRAHLUNG

We now consider the bremsstrahlung process for a meson on a nucleus in second-order perturbation theory in e. We shall assume that the particles are unpolarized in the initial and final states. The diagrams providing nonzero contributions to the matrix elements are shown in the figure.

The matrix elements are given by the following expressions:

$$M^a=\frac{ie^2(2\pi)^4}{2(2E_1E_2k_0)^{1/2}}a_\mu(p_1)\Big\{A_\sigma^e(q)\,[B_{\mu\nu,\rho\sigma}p_{1\rho}+B_{\mu\nu,\sigma\rho}(p_2+k)_\rho]$$
$$\times\frac{\delta_{\nu n}}{(p_2+k)^2+m^2}e_\lambda[B_{nm,\varkappa\lambda}(p_2+k)_\varkappa+B_{nm,\lambda\varkappa}p_{2\varkappa}]\Big\}a_m^*(p_2)\delta(E_1-E_2-k_0),\quad (2.1)$$

$$M^b=\frac{ie^2(2\pi)^4}{2(2E_1E_2k_0)^{1/2}}a_\mu(p_1)\Big\{e_\lambda[B_{\mu\nu,\rho\lambda}p_{1\rho}+B_{\mu\nu,\lambda\rho}(p_1-k)_\rho]$$
$$\times\frac{M^b\,\delta_{\nu n}}{(p_1-k)^2+m^2}A_\sigma^e(q)\,[B_{nm,\varkappa\sigma}(p_1-k)_\varkappa+B_{nm,\sigma\varkappa}p_{2\varkappa}]\Big\}a_m^*(p_2)\delta(E_1-E_2-k_0),\quad (2.2)$$

where e_λ is the photon polarization vector, A_σ^e is the potential for the nuclear field, and $\mathbf{q}=\mathbf{p}_2+\mathbf{k}-\mathbf{p}_1$.

The differential cross section is

$$d\sigma=\frac{Z^2\alpha^3}{(2\pi)^2}\frac{p_2}{p_1}\frac{dk_0}{k_0}\frac{d\Omega_2\,d\Omega}{q^4}\Big\{\frac{p_1^2\sin^2\theta_1}{\Delta_1^2}\Big[4E_2^2-\frac{1}{2}(1+k)^2q^2\Big]$$
$$+\frac{p_2^2\sin^2\theta_2}{\Delta_2^2}\Big[4E_1^2-\frac{1}{2}(1+k)^2q^2\Big]+(1+k)^2k_0^2\frac{p_1^2\sin^2\theta_1+p_2^2\sin^2\theta_2}{\Delta_1\Delta_2}$$
$$-2\frac{p_1p_2\sin\theta_1\sin\theta_2\cos\varphi}{\Delta_1\Delta_2}\Big[4E_1E_2-\frac{1}{2}(1+k)^2q^2+(1+k)^2k_0^2\Big]$$
$$+\frac{(1+k)^4}{8}k_0^2(p_1^2\sin^2\theta_1+p_2^2\sin^2\theta_2-2p_1p_2\sin\theta_1\sin\theta_2\cos\varphi)\Big[\frac{1}{\Delta_1}-\frac{1}{\Delta_2}\Big]^2\Big\},\quad (2.3)$$

where $\Delta_{1,2}=E_{1,2}-p_{1,2}\cos\theta_{1,2}$; θ_1 and θ_2 are the angles between the vectors $\mathbf{k}$, $\mathbf{p}_1$ and $\mathbf{k}$, $\mathbf{p}_2$, respectively; φ is the angle between the $(\mathbf{k},\mathbf{p}_1)$ and $(\mathbf{k},\mathbf{p}_2)$ planes; $d\Omega_2$ and $d\Omega$ are solid-angle elements containing the vectors $\mathbf{p}_2$ and $\mathbf{k}$; and $p_{1,2}=|\mathbf{p}_{1,2}|$.

We must now integrate with respect to the directions of the secondary meson. The integrals in (2.3) can be evaluated exactly with the aid of the results reported by Glückstern and Hull[20]:

$$d\sigma=\frac{Z^2\alpha^3}{8\pi}\frac{p_2}{p_1}\frac{dk_0}{k_0}d\Omega\Big\{\frac{L}{p_2p_1}\Big[\frac{4m^2E_1\sin^2\theta_1}{p_1^2\Delta_1^4}(3k_0m^2-E_2p_1^2)$$
$$+\frac{8E_1^3E_2-8E_1^2m^2}{p_1^2\Delta_1^2}+(1+k)^2\Big(\frac{k_0(E_1^2+E_1E_2-m^2}{p_1^2\Delta_1}$$
$$+\frac{2E_1^2k_0^2-m^2(3E_1^2+E_2^2-3E_1E_2-m^2)}{p_1^2\Delta_1^2}\Big)+\frac{(1+k)^4}{8}k_0^2$$
$$\times\Big(\frac{6k_0E_1^2}{p_1^4\Delta_1^2}+\frac{3E_1E_2-2E_1^2-m^2}{p_1^4}+\frac{E_1^2(2E_1E_2-4E_1^2)+m^2(2E_1^2+E_1E_2)-m^4}{p_1^4\Delta_1^2}\Big)\Big]$$
$$+(1+k)^2\frac{\varepsilon^T}{p_2T}\Big[\frac{4m^2}{\Delta_1^2}-\frac{3k_0}{\Delta_1}-\frac{k_0(p_1^2-k_0^2)}{T^2\Delta_1}+\frac{(1+k)^2}{8}\Big(\frac{k_0}{\Delta_1}$$
$$-\frac{k_0(p_1^4+2p_2^2k_0E_2+k_0^4-2p_1^2k_0^2}{T^4\Delta_1}\Big)+\frac{3k_0^2}{\Delta_1^2}+\frac{m^2k_0^2}{T^2\Delta_1^2}$$
$$-\frac{7k_0^2}{T^2}+\frac{p_2^2k_0^2}{T^4}+\frac{6k_0^2E_2}{T^2\Delta_1}+\frac{k_0^2E_2^2p_2^2}{T^4\Delta_1^2}\Big)\Big]$$
$$-\frac{2(1+k)^2}{p_2\Delta_1}\ln\Big|\frac{E_2+p_2}{E_2-p_2}\Big|+\frac{8m^2\sin^2\theta_1}{p_1^2\Delta_1^2}(2E_1^2+m^2)$$
$$-\frac{16E_1^2-(1+k)^2(3E_1^2-3m^2-2E_1E_2)}{p_1^2\Delta_1^2}-(1+k)^2\Big(\frac{p_1^2-k_0^2}{T^2\Delta_1^2}-\frac{2E_2}{p_1^2\Delta_1}\Big)$$
$$+\frac{(1+k)^4}{8}\Big[\frac{E_1^2(32E_1E_2-27E_1^2-12E_2^2)+m^2(22E_1^2-8E_1E_2-7m^2)}{2p_1^4\Delta_1^2}$$
$$+\frac{k_0E_1^2(4E_1^2-4E_1E_2)+k_0m^2(12E_1^2-8E_1E_2-4m^2)}{p_1^4m^2\Delta_1}$$
$$-\frac{k_0^2(4m^2+2E_1^2)}{p_1^4m^2}+\frac{10p_1^2-6k_0^2}{2T^2\Delta_1^2}-\frac{3(p_1^2-k_0^2)^2}{2T^4\Delta_1^2}\Big]\Big\},\quad (2.4)$$

where

$$L=\ln\Big|\frac{E_1E_2-m^2+p_1p_2}{E_1E_2-m^2-p_1p_2}\Big|\qquad \varepsilon^T=\ln\Big|\frac{T+p_2}{T-p_2}\Big|$$
$$T=|\mathbf{p}_1-\mathbf{k}|.$$

Integration of (2.4) with respect to Ω leads to the following expression for the differential cross section for emission in the frequency band k_0, k_0+dk_0:

$$d\sigma_\gamma=\frac{Z^2\alpha^3}{m^2}\frac{p_2}{p_1}\frac{dk_0}{k_0}\Big\{\frac{L}{p_2p_1}\Big[\frac{8E_1E_2}{3}+\frac{k_0^2(1+k)^2}{4}$$
$$\times\frac{(2E_1^2E_2^2+2m^2E_1E_2+2p_1^2p_2^2)}{p_1^2p_2^2}+\frac{k_0m^2(1+k)^2}{4}$$
$$\times\Big(\frac{p_1^2+E_1E_2}{p_1^3}l_1-\frac{p_2^2+E_1E_2}{p_2^3}l_2\Big)+\frac{k_0^3m^2(1+k)^4}{16}$$
$$\times\Big(3\frac{E_1^2}{p_1^5}l_1-3\frac{E_2^2}{p_2^5}l_2+6\frac{E_2}{p_2^4}-6\frac{E_1}{p_1^4}+2\frac{E_2}{p_2^2m^2}$$
$$-2\frac{E_1}{p_1^2m^2}\Big)\Big]-\frac{8}{3}+\frac{(1+k)^2}{2}\Big[4-2E_1E_2\frac{p_1^2+p_2^2}{p_1^2p_2^2}$$
$$+m^2\Big(\frac{E_1}{p_2^3}l_2+\frac{E_2}{p_1^3}l_1-\frac{l_1l_2}{p_1p_2}\Big)\Big]+\frac{(1+k)^4}{8}m^2\Big[\Big(\frac{3k_0^2E_1}{p_1^5}+\frac{k_0^2E_1+k_0m^2}{p_1^3m^2}\Big)l_1$$
$$+\Big(\frac{3k_0^2E_2}{p_2^5}+\frac{k_0^2E_2-k_0m^2}{p_2^3m^2}\Big)l_2-\frac{(1+k)^4k_0^2}{4p_1^2p_2^2}\Big[\frac{3m^2(p_1^4+p_2^4)}{p_1^2p_2^2}-$$
$$-E_1E_2-5m^2+2E_1^2+2E_2^2\Big]\Big\}.\quad (2.5)$$

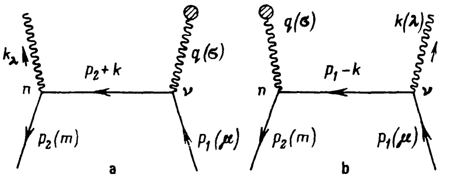
where

$$l_{1,2}=\ln\Big|\frac{E_{1,2}+p_{1,2}}{E_{1,2}-p_{1,2}}\Big|.$$

It follows from (2.5) that the emission intensity $k_0 d\sigma_\gamma$ diverges logarithmically as $k_0\to\infty$.

In the ultrarelativistic case ($E_1, E_2\gg m$), the spectral distribution of the radiation is given by

$$d\sigma_\gamma=\frac{Z^2\alpha^3}{m^2}\frac{E_2}{E_1}\frac{dk_0}{k_0}\Big\{\Big[2\ln\frac{2E_1E_2}{mk_0}-1\Big]\Big[\frac{8}{3}+\frac{(1+k)^2k_0^2}{E_1E_2}$$
$$+\frac{(1+k)^4k_0^4}{16E_1E_2}\Big]+\frac{(1+k)^4k_0^2}{8E_1^2}\Big[2\ln\frac{2E_1}{m}-3\Big]\quad (2.6)$$
$$+\frac{(1+k)^4k_0^2}{8E_2^2}\Big[2\ln\frac{2E_2}{m}-3\Big]\Big\}.$$



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This expression can be used to determine the energy lost by a meson traveling through matter by radiation. The energy loss will be defined as follows:

$$\sigma = E_1^{-1} \int\limits_0^{E_1 - m} k_0 \, d\sigma_\gamma \, dk_0.$$

We then have

$$\sigma = \frac{Z^2\alpha^3}{m^2} \left\{ \left[\frac{8}{3} + \frac{2}{3}(1+k)^2 \right] \ln \frac{2E_1}{m} + \frac{4}{3} - \frac{4}{3}(1+k)^2 + \frac{(1+k)^4}{8} \left[2\left(\ln \frac{2E_1}{m} \right)^2 - c_1 \ln \frac{2E_1}{m} + c_2 \right] \right\},$$
$$c_1 \approx 12.386, \quad c_2 \approx 15.422. \qquad (2.7)$$

Thus, in the ultrarelativistic case, σ increases as the square of the logarithm of the energy. This result is valid only if we neglect the screening effect which becomes important for energies $E_1 \geq 137Z^{-1/3}m$. The energy loss σ tends to a constant in the case of complete screening. The bremsstrahlung cross section within the framework of the Proca theory was determined by Christy and Kusaka[21] and Bludman and Young[22] using the Weizsäcker-Williams. It was found that the cross section was a linear function of the energy of the initial meson. It is well known that a more rapid variation with energy in the Proca theory is due to transitions with boson spin flip.[22] Since the evaluation of the cross sections for polarized vector particles is very laborious, and the resulting formulas are very unwieldy, the situation is usually illustrated by the scattering of particle by the Coulomb field of the nucleus (first-order perturbation theory). We shall also use this example to illustrate the behavior of the cross sections for polarized particles. The differential cross section for scattering by the Coulomb field of the nucleus is

$$\begin{aligned} d\sigma = (\sigma_R/4E^2)\{&4E^2(\xi_1\xi_2)^2 + (A^e\xi_2)^2[(p_1\xi_1)^2 \\ &+ (p_2\xi_1)^2 - 2(p_1\xi_1)(p_2\xi_1)] + (A^e\xi_1)^2[(p_2\xi_2)^2 \\ &+ (p_1\xi_2)^2 - 2(p_1\xi_2)(p_2\xi_2)] - 4E(\xi_2\xi_1)(A^e\xi_2) \\ &\times[(p_1\xi_1) - (p_2\xi_1)] + 4E(\xi_2\xi_1)(A^e\xi_1)[(p_1\xi_2) \\ &- (p_2\xi_2)] - 2(A^e\xi_2)(A^e\xi_1)[(p_1\xi_1)(p_1\xi_2) - (p_1\xi_1) \\ &\times(p_2\xi_2) - (p_2\xi_1)(p_1\xi_2) + (p_2\xi_1)(p_2\xi_2)]\}, \end{aligned} \qquad (2.8)$$

where $\sigma_R = [Ze^2/2pv \sin^2(\theta/2)]^2$; p and v are the momentum and velocity of the meson, respectively; ξ_i is the polarization index of the i-th meson; and θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_2$.

Summation over the polarizations is performed with the aid of the formula

$$\sum_{r=1}^{4} \xi_\mu{}^r \xi_\nu{}^r = \delta_{\mu\nu}, \quad \xi_\mu{}^4 \xi_\nu{}^4 = -\frac{p_\mu p_\nu}{m^2},$$
$$\sum_{r=1}^{2} \xi_i{}^r \xi_j{}^r = \delta_{ij} - \frac{p_i p_j}{p^2}, \quad i, j = 1, 2, 3. \qquad (2.9)$$

If the initial and final mesons are transversely polarized, then

$$\frac{d\sigma^{t\to t}}{d\Omega} = \sigma_R(1 + \cos^2\theta). \qquad (2.10)$$

If, prior to and after scattering, we have a mixture of longitudinal and scalar states, then

$$\frac{d\sigma^{l+0\to 0+l}}{d\Omega} = \sigma_R\left[2 + 2\sin^4\frac{\theta}{2} + 2\frac{m^2}{E^2}\sin^4\frac{\theta}{2} - 4\sin^2\frac{\theta}{2}\right]. \qquad (2.11)$$

If, in the initial state, we have a transversely polarized meson and, in the final, a mixture of longitudinal and scalar states, then

$$\frac{d\sigma^{t\to l+0}}{d\Omega} = \sigma_R\left[\sin^2\theta - \frac{v^2}{4}\sin^2\theta\right]. \qquad (2.12)$$

The expressions given by (2.10)–(2.12) can be used to obtain the cross section for unpolarized particles:

$$\frac{d\sigma}{d\Omega} = \sigma_R\left[1 - \frac{v^2}{2}\sin^2\frac{\theta}{2}\right]. \qquad (2.13)$$

Thus, in this particular variant of the meson theory, transitions with spin flip ($t \to l + 0$) do not result in a rise in the cross section with increasing energy. This conclusion can also be reached in other ways. In the Proca theory, the degree of divergence of the expressions corresponding to the Feynman diagrams is given by[23]

$$D \leq 4 - 2E_s - E_i + 2(N_3 + N_4), \qquad (2.14)$$

where E_s is the number of external meson lines, E_i is the number of external photon lines, and N_3 and N_4 are the numbers of one- and two-photon vertices, respectively. The dependence of D on N_3 and N_4 leads to an infinite number of primitive divergences, i.e., to a nonrenormalizable theory. It turns out that, if the longitudinal component of the meson field is absent, the number of primitive divergences becomes finite since

$$D \leq 4 - E_s - E_i, \qquad (2.15)$$

as in scalar electrodynamics,[24] and the theory is renormalizable. In our case, the convergence condition is also determined by (2.15). Consequently, the presence of longitudinal quanta is unrelated to the presence of any additional divergences.

It is also important to recall the most successful variant of the Proca theory, namely, the theory of ξ-limiting formalism of Lee and Yang.[5] This formalism can be used to construct a relativistically covariant S-matrix for nonzero AMM, to achieve renormalizability, and to ensure that the S-matrix is unitary. In fact, unitarity and renormalizability are achieved for different values of ξ.[7] The quantity ξ is allowed to tend to zero in the final stage of the calculation, and the resulting scattering and reaction cross sections turn out to be exactly the same as in the Proca theory.[25] Thus, the cross sections exhibit a persistent increase with the energy of the initial particles, which is due to the contribution of the longitudinal quanta.

APPENDIX

1. Particle in the field of an arbitrarily polarized plane wave

The wave potential will be taken in the form

$$A_\mu(\theta) = a_\mu{}^m A_m(\theta),$$

where $\theta = x_\lambda n_\lambda$, $n_\lambda = (0, 0, 1, i)$, $m = 1, 2$, $n_\lambda a_\lambda^m = n_\lambda^2 = 0$.

If we substitute

$$\Psi_\mu=\exp\left\{-ipx+\frac{ie}{2p_\lambda x_\lambda}\int_{-\infty}^{\theta}d\theta'[-2p_\sigma A_\sigma+eA_\sigma^2]\right\}\chi_\nu(\theta),$$

we find that the equations of motion (1.18) assume the form

$$\frac{d}{d\theta}\chi_\mu+\frac{e}{2p_\lambda n_\lambda}A_m'(\theta)f_{\mu\nu}{}^m\chi_\nu=0, \tag{A.1}$$

where p is the free-particle four-momentum and

$$A_m'(\theta)=\frac{dA_m}{d\theta},\quad f_{\mu\nu}{}^m=n_\mu a_\nu{}^m-n_\nu a_\mu{}^m.$$

The solution of (A.1) is

$$\chi_\mu(\theta)=-\frac{e}{2p_\lambda n_\lambda}\left[n_\mu\left(\frac{eA_m{}^2(\theta)b_\nu n_\nu}{4p_\sigma n_\sigma}-A_m b_\nu a_\nu{}^m\right)+a_\mu{}^m A_m(\theta)b_\nu n_\nu\right]+b_\mu. \tag{A.2}$$

The constants b_μ are determined by the initial conditions. The existence of the constant of motion

$$\chi_\mu n_\mu=\text{const} \tag{A.3}$$

shows that the particle has only three independent polarization states in the wave field.

We now assume that the wave potential satisfies the condition $A_m|_{\pm\infty}=0$. We take the initial state of the particle to be the spin 1 state, i.e.,

$$\Psi_\mu=N\left(\delta_{\mu\nu}+\frac{p_\mu p_\nu}{m^2}\right)k_\nu\exp\{-ipx\},$$

where N is the normalization constant and k_ν is an arbitrary four-dimensional constant vector.

We now substitute $k_\nu=n_\nu$ and obtain the following expression:

$$D_\mu\Psi_\mu=\frac{ie^2A_\lambda^2(\theta)}{8m}\exp\left\{-ipx+\frac{ie}{2p_\lambda n_\lambda}\int_{-\infty}^{\theta}d\theta'[-2p_\sigma A_\sigma+eA_\sigma^2]\right\}. \tag{A.4}$$

In the final state with $A_m=0$, the wave function is then again transverse, i.e., scalar particles do not appear in the scattering process. If we take a scalar initial state, then, after scattering, we obtain only a spin-0 particle. In this situation, therefore, there are no transitions between different spin states. However, if, prior to scattering, we have a vector particle with its scalar partner, then, as can readily be noted from (A.2), we will have both spin-1 and spin-0 particles after the scattering event. Thus, the plane wave cannot be used to separate the scalar partner from the vector particle.

2. Magnetic field

Consider a magnetic field parallel to the z axis, which is uniform in space and constant in time. The potentials can then be taken in the form

$$A_x=-\frac{yH}{2},\quad A_y=\frac{xH}{2},\quad A_z=A_t=0.$$

The equations given by (1.20) then assume the form

$$(D_\lambda^2-m^2)\Psi_z=(D_\lambda^2-m^2)\Psi_0=0,$$
$$(D_\lambda^2-m^2)\Psi_x-ie(1+k)H\Psi_y=0, \tag{A.5}$$
$$(D_\lambda^2-m^2)\Psi_y+ie(1+k)H\Psi_x=0. \tag{A.6}$$

The solution of (A.5) is known to be[26]

$$\Psi_0=\exp\left\{i[p_zz-Et+l\varphi]-\frac{eHr^2}{4}\right\}(2\pi L)^{-1/2}\times\left(\frac{eHr^2}{2}\right)^{l/2}\frac{l!}{(l+s)!}L_s{}^l\left(\frac{eHr^2}{2}\right),$$

where l is the azimuthal quantum number, s is the radial quantum number, L is the edge of the cube containing the particle, p_z is the eigenvalue of the momentum projection operator on the z axis, L_s^l are the Laguerre polynomials, and E is the particle energy.

The functions $\Psi_\pm=(\Psi_x\pm\Psi_y)/\sqrt{2}$ satisfy the equations

$$[(D_\lambda^2-m^2)\mp e(1+k)H]\Psi_\pm=0. \tag{A.7}$$

The solution of this is

$$\Psi_\pm=\exp\left\{i[p_zz-Et+(l\pm1)\varphi]-\frac{eHr^2}{4}\right\}(2\pi L)^{-1/2}\times\left(\frac{eHr^2}{2}\right)^{(l\pm1)/2}\frac{(l\pm1)!}{(l\pm1+s)}L_s{}^{l\pm1}\left(\frac{eHr^2}{2}\right). \tag{A.8}$$

We note that the eigenfunctions of the total angular momentum projection operator in the direction of the magnetic field are Ψ_+ and Ψ_- and not Ψ_x and Ψ_y.

The energy levels are then given by

$$E^2=m^2+p_z^2+2eH[n+{}^1/_2+q-{}^1/_2q(1+k)], \tag{A.9}$$

where $q=\pm1$ for transverse (relative to the field) states and $q=0$ for both longitudinal and scalar states; n is the principal quantum number.

The above results are valid for

$$H<H_{cr},\ H_{cr}=m^2/e,$$

since otherwise $E^2<0$. To remove this formal defect (classical field theory becomes invalid well before H_{cr} is reached), we can introduce terms representing the magnetic polarizability of the particles into the equations of motion in the same way as in the Proca theory.[27]

The evaluation of $D_\mu\Psi_\mu$ leads to a nonzero result which enables us to conclude that there is interference between spin-1 and spin-0 states. Thus, the magnetic field can be used to separate transverse quanta, but the behavior of the longitudinal and scalar quanta is identical.

3. Electric field

Let $\mathbf{E}=(0,0,E)$. The potential can then be taken in the form

$$A_0=-Ez.$$

The equations of motion (1.18) then take the form

$$(D_\lambda^2-m^2)\Psi_x=(D_\lambda^2-m^2)\Psi_y=0, \tag{A.10}$$
$$(D_\lambda^2-m^2)\Psi_0-ieE\Psi_z=0, \tag{A.11}$$
$$(D_\lambda^2-m^2)\Psi_z-ieE\Psi_0=0. \tag{A.12}$$

The solution of (A.10) is known to be[28]

$$g_\pm=(\Psi_0\pm\Psi_3)/\sqrt{2},$$

where $\xi=-ieE(z+\mathcal{E}/eE)^2$, $\mathcal{E}$ is the particle energy, p_x and p_y are, respectively, the momentum components along the x and y axes, $a=(aE+iM^2)/4eE$, F is the confluent hypergeometric function, and $M^2=p_x^2+p_y^2+m^2$.

Let

$$\Psi_x=\exp\{i[\mathcal{E}t-p_xx-p_yy]-{}^1/_2\xi\}\{c_1F(a, {}^1/_2, \xi)+c_2\xi^{1/2}F(a+{}^1/_2, {}^3/_2, \xi)\},$$

From (A.11) and (A.12), we find that the equations for $g_\pm$ are

$$(D_z^2+ieE)g_+=0, \quad (D_z^2-ieE)g_-=0,$$
$$D_z^2=\frac{d^2}{dz^2}+e^2E^2\left(z+\frac{\mathcal{E}}{eE}\right)^2-M^2. \tag{A.13}$$

The solution of this is

$$g_\pm=\exp\{i[\mathcal{E}t-p_xx-p_yy]-{}^1/_2\xi\}\{c_3F(a_\pm, {}^1/_2, \xi)+c_4\xi^{1/2}F(a_\pm+{}^1/_2, {}^3/_2, \xi)\},$$

where

$$a_\pm=(eE+iM^2\mp eE)/4eE.$$

The constants c_1, c_2, c_3, and c_4 can be determined from the initial conditions and from the normalization condition. Direct evaluation leads to $D_\mu\Psi_\mu\neq 0$, i.e., interference between vector and scalar states also takes place. It is easily seen from (A.13) that the longitudinal and scalar parts of the field are coupled.

Borgardt and Karpendko[29] have demonstrated the existence of stable solutions of the Kepler problem for the charged vector boson. The Lagrangian density in the form given by (1.1) was taken as the starting point. The condition $D_\mu\Psi_\mu=0$ was imposed on the asymptotic behavior at long distances from the force center. This fixed the constants in the solution of the equations of motion, so that the scalar part of the field cancelled by the longitudinal part. The result was that the vector meson could be only in transversely polarized states.

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The Aharonov-Bohm effect in a toroidal solenoid

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We consider the Aharonov-Bohm effect in the case in which the magnetic field is concentrated in a finite region of space (a closed solenoid). Formulas are obtained for the scattering of charged particles in a toroidal solenoid in the eikonal and Born approximations.

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1. INTRODUCTION

Aharonov and Bohm called attention to the fact that a magnetic field affects the interference of coherent beams of charged particles propagating outside the region of localization of the field.[1] They also discussed the scattering of charges in an infinitely long solenoid and showed that the scattering is due to the change in phase of the wave function in the region in which there is no magnetic field but in which the vector potential is not zero; here the total cross section for scattering turns out to be infinite. Subsequently a large number of articles have appeared on the interpretation of this effect and on discussion of locality in quantum mechanics (see for example Refs. 2–5[1)]).

The analysis contained in the studies cited has a definite methodological deficiency due to the infinite

