Crystallization of an electron-hole plasma in a strong magnetic field

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The conditions under which a lattice of heavy (compared to electrons) holes can be formed are determined. The thermodynamic quantities of the crystal are calculated. The crystals may be of three types: filaments stretched along the field, "point" particles, or systems of planes perpendicular to the magnetic field. The type depends on the hole parameters and on the magnetic field strength. The oscillation spectrum consists of a sound branch polarized along the magnetic field, and of a low-frequency branch with $\omega \sim \mathbf{k}^2$ as $\mathbf{k} \to 0$. The mean square displacement $\langle x^2 \rangle$ of a particle at a lattice site is calculated and found to be independent of the magnetic field intensity. The crystalline phase goes over discontinuously to the semiconductor phase at a finite value of the gap between the electron and hole bands.

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1. INTRODUCTION

It is well known that the spatially homogenous ground state of a system of charged particles becomes unstable at low density and low temperature.[1-4] The parameters and the number of the particles, and, consequently, the regions of instability, can be changed by an external pressure, by alloying, and by a magnetic field. The formation of an exciton phase has been studied in recent years,[5-7] and new phases of the electron-hole plasma in a strong magnetic field have been proposed.[8-10] In the works mentioned, the theory has been developed in the weak coupling (high density) approximation. The effective Coulomb interaction has been assumed to be small in comparison with the characteristic (Fermi) energy of the particles:

$$W_{\text{Coul}} \ll \varepsilon_F. \tag{1}$$

In the present work, we consider the case in which the masses of particles of one sign are much greater than those of the other (for example, the holes and electrons in Bi and other semimetals). The Fermi energies of the various particles differ widely and we can achieve fulfillment of the conditions

$$\varepsilon_h \ll W_{\text{Coul}} \ll \varepsilon_e \tag{2}$$

by changing the concentration.

The formation of a Wigner crystal is energetically advantageous here, when the holes are localized and form a discrete lattice against a background of an approximately uniform distribution of the electrons. This process is similar to the phase transition crystal–electron-proton liquid in matter at very high compressions.[11] The possibility of crystallization in a semiconducting plasma has been noted by Halperin and Rice.[12]

The possible types of hole crystals in a strong magnetic field under conditions of a quasi-one-dimensional spectrum of electrons and holes are studied in the current research.[1)] In that region of concentration in which condition (2) is satisfied, various structures are possible, as will be shown in Sec. 2. These structures depend on the values of the magnetic field H and the longitudinal and transverse masses of the holes $m_{\parallel}$ and $m_{\perp}$. This is, we have a crystal of filaments extended along the field, a "point" crystal in which the periods of the lattice along and transverse to the field are identical, and a system of planes perpendicular to the field. With decrease in the particle density (for example, upon approach to the metal-dielectric transition point) the Coulomb energy can become of the order of the Fermi energy of the electrons, which should now be taken into account in the determination of the parameters of the crystal (Sec. 3). A comparison is carried out of the energy of the crystalline phase with the energies of the exciton and liquid phases. Upon increase in the gap between the electron and hole bands, the crystalline structure disappears with a first order transition.

In Sec. 4 of the paper, the vibration spectrum of the crystal is considered; in particular, it is shown that there exists a branch with a quadratic dispersion law $\omega \sim \mathbf{k}^2$ as $\mathbf{k} \to 0$. The mean square of the particle displacement $\langle x^2 \rangle$ at the crystal site is calculated. The quadratic branch leads to a relatively large value of $\langle x^2 \rangle$; however, it is finite and at high temperatures it does not depend on the magnetic field. This result differs from the result of a number of researches, for example, that of Chaplik,[16] where it is maintained that the magnetic field destroys the long-range order.

2. STRUCTURE OF A WIGNER CRYSTAL

The Hamiltonian operator for electrons and holes is written in the form

$$H = \int d^3x \{\psi_h^+ \hat{H}_h \psi_h + \psi_e^+ (\hat{H}_e + G) \psi_e\} + \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \int d^3x \cdot \int d^3x' \psi_\alpha^+(\mathbf{x}') \psi_\beta^+(\mathbf{x}) \frac{e^2}{|\mathbf{x}_\alpha - \mathbf{x}_\beta'|} \psi_\alpha(\mathbf{x}') \psi_\beta(\mathbf{x}) = H_1 + H_2 + V_{11} + V_{12} + V_{22}. \tag{3}$$

The indices 1, 2, h, and e refer to holes and electrons, G is the distance between the extrema of the bands, and

$$H_\alpha = \frac{1}{2m_\alpha} \left(\mathbf{P} - \frac{e}{c} \mathbf{A} \right)^2$$

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is the Schrödinger operator for a particle with a quadratic spectrum in a magnetic field.

The case is considered of a strong magnetic field in which both types of particles fill only the lowest magnetic level (with one direction of spin); the system is multiply degenerate if no account is taken of the Coulomb interaction. If the local hole states give a smaller value of the energy H_1+V_{11} than the states distributed over the entire volume, crystallization occurs (the interaction with the homogeneous background of the opposite sign is taken into account in V_{11}). The parameters of the crystalline lattice are determined from the minimum condition. The potential V_{12} acting on the electrons is determined by the same token. The holes are characterized by the following parameters: the magnetic length λ, the Bohr radius r_h and the particle density n,

$$\lambda=\left(\frac{eH}{2\hbar c}\right)^{-1/2}, \quad r_h=\frac{\varepsilon_0\hbar^2}{m_\parallel e^2}, \quad n=\frac{1}{r_0^3};$$

ε_0 is the dielectric constant and in $\hbar=1$ what follows. The mass of the holes can be anisotropic, $m_\parallel$ and $m_\perp$ are the masses along and transverse to the magnetic field. The density n is expressed in terms of λ and the Fermi momentum p_h:

$$n^{-1}=r_0^3=\pi\lambda^2 d, \quad d=\pi\hbar/p_h. \tag{4}$$

The particles occupy only the lowest magnetic level if the condition $\varepsilon_h<eH/m_\perp c$ is satisfied, i.e.,

$$n^{-1}>{}^1/_2\pi^2(m_\perp/m_\parallel)^{1/2}\lambda^3. \tag{5}$$

The criterion for crystallization can be obtained from the condition that the kinetic energy of the motion along the magnetic field is smaller than the Coulomb energy, which, if we estimate the Coulomb energy at $e^2/\varepsilon_0 d$, yields

$$n^{-1}>{}^1/_2\pi^2\lambda^2 r_h. \tag{6}$$

The most advantageous structure of the crystal is determined by the relation among the parameters λ, r_h, and r_0. If $\lambda\ll r_0$, it is natural to try a structure extended along the field. Kaplan and Glasser[13] proposed a structure in the form of infinitely long filaments ordered in a triangular lattice. The particles move freely along the filament. However, this structure is unstable because of the collisions between particles in one filament. Since the collisions occur with an impact parameter of the order of λ, upon satisfaction of the condition

$$p^2/2m_\parallel\ll e^2/\varepsilon_0\lambda \tag{7}$$

the particles are mutually reflected and localized each in a bounded portion of the filament. Such a system is absolutely unstable: upon random displacements of the segments in a perpendicular direction, repelling forces are produced. A layered structure will be stable in which the filaments in neighboring layers are displaced by a half period (Fig. 1). The particle is retained in the layer by repulsion from the particles of the neighboring layers, for which satisfaction of the condition

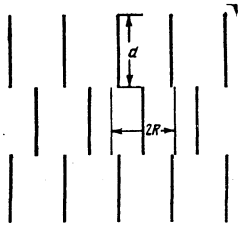

FIG. 1. Structure of a filamentary crystal.

$$p^2/2m_\parallel<e^2/\varepsilon_0 R \tag{8}$$

is necessary instead of (7). This crystal is the three-dimensional analog of a one-dimensional gas with short-range (δ-function) repulsion, considered by C. N. Yang and C. P. Yang.[17]

As in Ref. 17, we choose the wave function of the ground state in the form

$$\psi=\left\{\prod_{i,j}\varphi_{00}(\rho_{ij})\right\}\det\{\exp(ip_i z)\}, \quad z_{i-1}\leqslant z_i\leqslant z_{i+1}, \tag{9}$$

i and j are the number of the layer and the number of the particle in the layer. In the limit $d\gg R$, this function accurately describes the ground state of the system. In (9),

$$\varphi_{00}(\rho_{ij})=\frac{1}{(2\pi)^{1/2}\lambda}\exp\left\{-\frac{x_{ij}^2+y_{ij}^2}{2\lambda^2}\right\}$$

is a function with zero values of the magnetic number and of the projection of the momentum M. The states with $M\neq 0$ make a contribution to the vibrations of the crystal. In the ground state, the radius of the filament is of order λ.

The kinetic energy per particle is equal to 1/3 of $p^2/2m_\parallel$, i.e., $\pi^2/6m_\parallel d^2$. We can estimate the interaction energy by the Wigner-Seitz method, i.e., we can assume that the filament interacts only with a charge of opposite sign, distributed uniformly over a cylinder of radius R, which, upon neglect of the end effects, yields the principal term $(e^2/\varepsilon_0 d)\ln(d/R)$, i.e.,

$$W\approx\frac{1}{6}\frac{\pi^2}{m_\parallel d^2}-\frac{e^2}{\varepsilon_0 d}\ln\frac{d}{R}. \tag{10}$$

Minimization with account of constant density gives the following result for the equilibrium values, expressed in terms of r_h:

$$W=-\frac{3e^2}{8\pi r_h}\alpha^2, \quad R=\left(\frac{3}{2\pi^3}\frac{1}{r_h n}\right)^{1/2}\alpha^{1/2}, \quad d=\frac{2\pi^2}{3}r_h\alpha^{-1}, \tag{11}$$
$$\alpha\approx\ln[2.7(5r_h)^3 n].$$

These quantities do not depend on the magnetic field.

For an estimate of the contribution to the energy of interaction between layers, we must find the region δz in which the interaction between particles of different layers is of the order of the kinetic energy. This region $\delta z\sim 4d/\alpha$, i.e., the contribution to the energy is logarithmically small. As is seen from (10), the minimum energy is possible only at $d>R$. The filamentary crystal can thus exist at

$$\lambda\leqslant R\leqslant d,$$

which corresponds approximately to

$$\frac{2\pi^2 r_h\lambda^2}{3\ln(\pi r_h/\lambda)} \leqslant \frac{1}{n} \leqslant (5r_h)^3. \tag{12}$$

The left sign of the inequality makes more precise the condition (6). It follows from (12) that for the existence of the filamentary crystal, it is necessary that

$$\lambda \ll 5r_h. \tag{13}$$

Thus the filamentary crystal can exist at not too low densities. With decrease in the density, the ratio d/R decreases, and at $r_0 \sim 5r_h$ a transition takes place to a structure of a second type—a "point" crystal with distances between particles of the order of r_0, as at $H=0$. An estimate of the binding energy with the help of a Wigner-Seitz cell of spherical shape gives

$$W \approx -\frac{3}{2}\frac{e^2}{\varepsilon_0}\left(\frac{4}{3}\pi n\right)^{1/3}. \tag{14}$$

The structure and binding energy in this case do not differ from the crystal at $H=0$. The dimensions of a single particle are longitudinal $\sim(r_0^3/r_h)^{1/4}$ just as at $H=0$, and transverse λ.

If the condition (13) is satisfied, i.e., at not too strong magnetic fields, only the "point" structure is possible, and the filamentary form is not possible.

Finally, if $r_h \ll \lambda$, a crystal of third type is advantageous—a system of parallel planes perpendicular to the magnetic field. The layers do not overlap if

$$d \geqslant (r_h/16\pi n)^{1/4}. \tag{15}$$

Since a transition occurs at $r_0 \sim \lambda$ to a crystal of "points," the acceptable values of n are those for which

$$^1/_2\pi\lambda^3(r_h/\lambda)^{1/3} \leqslant n^{-1} \leqslant \pi\lambda^3. \tag{16}$$

With account of the condition of "one-dimensionality" (5), the requirements for the parameters take the form

$$m_\perp/m_\parallel < 4/\pi^2, \quad r_h^\parallel < 16\lambda < \pi r_h^\perp. \tag{17}$$

3. POLARIZATION OF THE ELECTRONS

The reaction of the electron system to the potential of a hole lattice is determined by a static dielectric function. For one-dimensional electrons this function is of the form

$$\varepsilon_e(k,0) = 1 + \frac{4\pi}{k^2}\Pi_e(k,0) = 1 + \frac{\exp(-\lambda^2 k_\perp^2)}{\pi^2\lambda^2 r_e k_\parallel (k_\parallel^2 + k_\perp^2)}\ln\left|\frac{2p_e + k_\parallel}{2p_e - k_\parallel}\right|. \tag{18}$$

Here r_e, p_e are the Bohr radius and the Fermi momentum.

It is not difficult to verify that for wave numbers of the order of the characteristic reciprocal distances $4\pi k^{-2}\Pi_e(k,0)$ is small for lattices of all types, i.e., the modulation of the electron density is not large and makes an insignificant contribution to the energy. For example, for a crystal of the "point" type, at $k_\perp \sim k_\parallel \sim n^{1/2} \gg p_e$ we have

$$\frac{4\pi}{k^2}\Pi_e(k,0) \sim \frac{1}{(\pi^3 r_e n^{1/3})^{1/3}} \ll 1. \tag{19}$$

Exceptions are the terms with $k_\parallel = 0$, where, for the lowest densities at which the existence of the crystal is possible (see below), the polarization is of the order of unity, i.e., the density depends significantly on the transverse coordinates. In view of the smallness of λ, the density can be calculated with the help of a two-dimensional equation of the type of the Thomas-Fermi equation. Because of the condition $p_e n^{1/3} \ll 1$, the inhomogeneities and the deformations of the lattice average the potential somewhat. The estimate (14) of the lattice energy is valid for all lattices that are nearly cubic. Arguments were advanced in the work of Ruderman[14] on behalf of a body-centered cubic structure; the electron "filaments," which contain chains of nuclei, shifted by a half period in neighboring filaments (we are speaking of the structure of the crust of a neutron star), are drawn together and coalesce into a crystal.

We shall assume the electron density to be constant and shall find the equilibrium parameters of the crystal, taking into account the possibility of a transition of the particles from the conduction band to the valence band. The expression for the energy per unit volume has the form

$$E = \left\{\frac{e^2}{\varepsilon_0}\frac{\pi^2}{6}(r_e\lambda^4)n^2 + G + W(n)\right\}n. \tag{20}$$

The first term is the kinetic energy of the electrons (exchange and correlation corrections are small, including the correlation energy $\sim n^{1/4}$ (see Ref. 8), estimates are given at the end of the section); $W(n)$ from (11) or (14) is the energy of the lattice. The condition

$$\left.\frac{\partial E}{\partial n}\right|_{n=n_0} = 0 \tag{21}$$

gives the equilibrium number of particles n_0. For values of G at which $E(n_0) < 0$, the existence of a crystalline phase is advantageous; at a value of the gap G_0, determined by the condition $E(n_0) = 0$, a first order transition to the semiconducting phase with a gap takes place

For a "point" lattice,

$$n_0 = \frac{1}{\pi}\left(\frac{9}{2}\right)^{1/5}(r_e\lambda^4)^{-3/5},$$
$$G_0 = \frac{e^2}{\varepsilon_0}2\left(\frac{9}{2}\right)^{1/5}\left[\left(\frac{1}{3}\right)^{1/3} - \frac{1}{8}\right](r_e\lambda^4)^{-1/5} \approx 1.6\frac{e^2}{\varepsilon_0}(r_e\lambda^4)^{-1/5}. \tag{22}$$

The transition to the semiconducting phase occurs immediately from the "filamentary" phase, bypassing the "point" crystal if the magnetic field is sufficiently large, $r_e\lambda^4 < r_h^5$. In this case,

$$n_0 \approx \left(\frac{9}{4\pi^3}\right)^{1/3}(r_h r_e\lambda^4)^{-1/2}\ln^{1/2}[r_h^3 r_e^{-1}\lambda^{-4}],$$
$$G_0 \approx \frac{3}{8\pi}\frac{e^2}{\varepsilon_0}\frac{1}{r_h}\ln^2[r_h^3 r_e^{-1}\lambda^{-4}]. \tag{23}$$

The pressure $p = -\partial E/\partial V$ is positive over the entire region of existence of the crystal and vanishes at the transition point. The electron-hole crystal thus always brings about small deformations of the host (atomic) lattice of the semimetal. At the transition point and at

$T=0$, the magnetic moment has a discontinuity:

$$\delta M=-n_0\frac{\partial G}{\partial H}+\frac{1}{H}\frac{e^2}{\varepsilon_0}\frac{\pi^2}{3}(r_e\lambda^4)n_0^3,$$
$$G<G_0,\quad \delta M=0,\quad G>G_0. \tag{24}$$

The obtained expressions are illustrated in Fig. 2., where the locations of the phases on the plane of r_0 and λ is shown (i.e., $n^{-1/2}$ and $H^{-1/2}$). The crystalline phase can exist both in strong fields and at $H=0$, and has a clearly expressed character at $m_h/m_e\gg 1$. The electron-hole interaction of order $n^{1/3}$ which determines it exceeds even at minimal density n_0 the maximum binding energy of the exciton phase

$$\frac{W_{exc}}{E}\sim\left(\frac{\lambda}{r_e}\right)^{4/5}\ln^2\left(\frac{r_e}{\lambda}\right)$$

and the electron-correlation energy $E\sim n^{1/4}$:

$$E_{cor}/E\sim(\lambda/r_e)^{4/35}. \tag{25}$$

The exchange energy is small in terms of the parameter $(\lambda^2 r_e n)^{-1}\lesssim(\lambda/r_e)^{2/5}$. At $m_h\sim m_e$, the crystal transforms into a charge-density wave, the energy $n^{1/3}$ is suppresed and an exciton phase or liquid phase with a charge wave can become favored.[9] As an example, we write down the parameters of the semimetal bismuth in a field ~100 kOe, oriented along the trigonal axis: $r_e=4\times10^{-5}$ cm, $r_h=6\times10^{-7}$ cm, $\lambda\sim10^{-6}$ cm. At these values of the parameters, $n_0\sim5\times10^{16}$ cm^{-3} and $G_0\sim9°$.

4. EXCITATION SPECTRUM

Crystallization of the holes leads to the appearance of a new branch of excitations of the electron-hole plasma–lattice vibrations. In a Wigner crystal with "rigid" (unpolarized) background at $H=0$, longitudinal vibrations have the spectrum of plasma waves, while the transverse vibrations have a linear spectrum with a small region of quadratic dependence at small momenta.[18] The external magnetic field and polarization of the electron background change the spectrum.

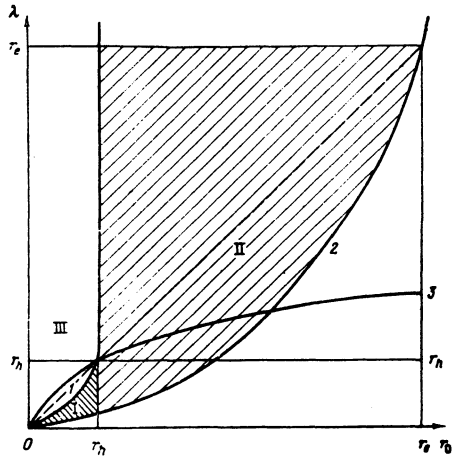

FIG. 2. Phase diagram of an electron-hole plasma. Region I—filamentary crystal; II—three-dimensional crystal. Region III—semimetal. On line 1), $r_h\lambda^2\sim r_0^3$, on line 2), $r_e\lambda^4\sim r_0^5$, on line 3), $\lambda^4\sim r_h r_0^3$. Line 3 limits the region in which the lattice vibrations depend on the magnetic field (see Sec. 4). The dashed line limits from above the region of a strong magnetic field, where the solitary magnetic level is occupied.

For the determination of the spectrum, it is necessary, as usual, to expand the potential in small displacements of the lattice. By a transformation to normal oscillations, the problem reduces to the problem of the motion of a single particle in a magnetic field and the field of a quadratic potential. If the longitudinal and transverse motions are separated, the transverse eigenfunctions are the same as in the magnetic field in cylindrical coordinates, while the energy levels are given by the expression[19,20]

$$\mathcal{E}(N,M)=(N+{}^1/_2)\Omega'+{}^1/_2M[\Omega'-\Omega],\quad N,M=0,1,2,\ldots, \tag{26}$$

where $\Omega'=(\Omega^2+4\omega^2)$, Ω is the Larmor frequency, and ω is the frequency of the oscillator.

The quadratic potential is anisotropic in the crystal and the variables are not separable at arbitrary k. The problem will be considered below in the limit of a strong field, when only the levels with $N=0$ are excited and $\Omega'-\Omega\approx 2\omega^2/\Omega$ and for lattice displacements greater than λ, i.e., $M\gg1$. In this case, the "magnetic" particle with a transverse dimension of the order of λ moves in the field of the lattice like a classical object.[2)] The equations of motion in the longitudinal and transverse directions have the form

$$\frac{e}{c}[\dot{\mathbf{x}}_i\times\mathbf{H}]_\alpha=(\mathbf{F}_i)_\alpha,\quad \alpha=1,2,$$
$$m_\parallel(\ddot{\mathbf{x}}_i)_\alpha=(\mathbf{F}_i)_\alpha,\quad \alpha=3. \tag{27}$$

Here $\mathbf{x}_i$ is the displacement of the particle in the i-th site of the crystal, $\mathbf{F}_i$ is the force of interaction with the particles of the crystal, $\dot{\mathbf{x}}_i$ in the first equation can be interpreted as the Hall velocity in the field $\mathbf{F}/e$. Thanks to the condition $\omega\ll\Omega$, the term $\ddot{\mathbf{x}}_i$ is omitted in the first equations.

The transition to the normal oscillations yields

$$i\omega\bar{\Omega}[\mathbf{q}^k\mathbf{n}]_\alpha=A_{\alpha\beta}q_\beta^k,\quad \alpha=1,2,\quad \bar{\Omega}=eH/m_\parallel c,$$
$$\omega^2 q_\alpha^k=A_{\alpha\beta}q_\beta^k,\quad \mathbf{n}=\mathbf{H}/|\mathbf{H}|, \tag{28}$$

$$A_{\alpha\beta}(\mathbf{k})=A_{\alpha\beta}^{(1)}+A_{\alpha\beta}^{(2)}=\frac{n}{m_\parallel}\Big[V_{\mathbf{k}}k_\alpha k_\beta$$
$$+\sum_{\mathbf{K}_n\neq0}\{V_{\mathbf{k}+\mathbf{K}_n}(k^\alpha+K_n^\alpha)(k^\beta+K_n^\beta)-V_{\mathbf{K}_n}K_n^\alpha K_n^\beta\}\Big]. \tag{29}$$

In a crystal with a rigid background, V_k are the Fourier components of the Coulomb potential. The summation is carried out over the reciprocal-lattice vectors $\mathbf{K}_n$ (see, for example, Ref. 22).

In order to take the polarization of the electron background into account, it is necessary, in the weak coupling approximation, to replace the Coulomb potential V_k by the "screened" potential

$$\tilde{V}_{\mathbf{k}}=V_{\mathbf{k}}/\varepsilon(\mathbf{k},0).$$

The second term in (29), which is connected with the structure of the crystal, is responsible for the transverse oscillations at $H=0$. The components of V_k with $|\mathbf{k}|\sim|\mathbf{K}_n|$ are not screened [see (18), (19)].

The dispersion relations

$$\Omega^2\omega^4-C_1\omega^2+C_2=0,$$
$$C_1=\Omega^2A_{zz}+A_{xx}A_{yy}-A_{xy}^2, \quad (30)$$
$$C_2=A_{xx}A_{yy}A_{zz}+2A_{xy}A_{yz}A_{zx}-A_{yy}A_{zx}^2-A_{xx}A_{yz}^2-A_{zz}A_{xy}^2$$

follow from (28).

There are two branches of the oscillations,

$$\omega_1^2\approx A_{zz}+C_3/\Omega^2A_{zz}, \quad \omega_2^2\approx C_2/A_{zz}\Omega^2; \quad (31)$$
$$C_3=2A_{xz}A_{zy}A_{yx}-A_{yy}A_{xz}^2-A_{xx}A_{yz}^2.$$

At arbitrary $\mathbf{k}$, the wave of the first branch is polarized along the magnetic field,

$$q_x\sim q_y\sim\omega^2q_z/\Omega^2\ll q_z. \quad (32)$$

In the second branch, all the q_α are generally of the same order, but the condition

$$A_{xx}q_\alpha\approx 0 \quad (33)$$

is satisfied.

The relative values of $A_{\alpha\beta}^{(1)}$ and $A_{\alpha\beta}^{(2)}$ depend on the value of the Debye radius $\varkappa^{-1}=(\pi^2\lambda^2p_er_e)^{1/2}$. If $\varkappa r_0\ll 1$, we have $A_{\alpha\beta}^{(1)}\gg A_{\alpha\beta}^{(2)}$ as $k\to 0$; at $\varkappa r_0\sim 1$, they are of the same order.

Using the expression (18) for $\varepsilon(\mathbf{k},0)$, we obtain $A_{\alpha\beta}^{(1)}=\omega_0^2k_\alpha k_\beta/\varkappa^2$ in the region $k_\parallel\ll\min\{\varkappa,(p_e\varkappa)^{1/2}\}$, $\omega_0^2=4\pi^2e^2n/m_\parallel$. At $k_\parallel\sim p_e$, $A_{\alpha\beta}^{(1)}$ has a minimum.

The maximum frequency of the first branch—anisotropic sound—is of the order of ω_0, that of the second branch is of the order of $\omega_0^2/\tilde{\Omega}$. The frequencies of the third branch of oscillations of the crystal, which are not contained in (28), are of order Ω. The condition $\omega_0\ll\tilde{\Omega}$ corresponds to

$$4\pi\lambda^3(\lambda/r_h)\ll r_0^3.$$

We also note that

$$\omega_0/W\sim(r_h/r_0)^{1/2}<1.$$

We estimate the mean square displacement of the particle at the crystal site. For this, it is convenient to use the formula (26). The quantity M is the projection of the angular momentum; therefore, the normal mode corresponds not to the oscillation but to the rotation of the particle about the equilibrium position. At $M\gg 1$, $\frac{1}{2}\langle q_k^2\rangle_\perp\approx\lambda^2M$, since this is the square of the radius of the orbit. Summation over M_k and over $\mathbf{k}$ yields

$$\langle x^2\rangle=\sum_{\mathbf{k}}\langle q_{\mathbf{k}}^2\rangle\sim\frac{1}{n}\int\frac{d^3k}{(2\pi^3)}\lambda^2\langle M_{\mathbf{k}}\rangle=\frac{1}{n}\int\frac{d^3k}{(2\pi)^3}\frac{\lambda^2}{\exp(\omega_{\mathbf{k}}/T)-1}. \quad (34)$$

The integrand converges at the lower limit. Let the temperature T be not too small:

$$\omega_D^2/\tilde{\Omega}\ll T\ll W. \quad (35)$$

The Debye frequency of the Coulomb lattice is equal to $\omega_0/3^{1/2}$. Since $m\omega_0^2r_0^2\sim W$,

$$\langle x^2\rangle\sim\frac{1}{n}\int\frac{d^3k}{(2\pi)^3}\frac{T\lambda^2}{\omega(\mathbf{k})}\sim\frac{T}{W}r_0^2\ll r_0^2. \quad (36)$$

The magnetic field does not enter into (36); the latter has the same form as at $H=0$, but enters at the low temperatures.

The same result can be obtained by use of classical statistics, averaging by means of the Gibbs distribution function:

$$\langle x^2\rangle=\sum_{\mathbf{k}}\left\{\left[\int d^3\dot{q}\,d^3q\,q^2\exp\left(-\frac{m}{2T}\dot{q}^2-\frac{m}{2T}A_{\alpha\beta}(\mathbf{k})q^\alpha q^\beta\right)\right]\right.$$
$$\left.\times\left[\int d^3\dot{q}\,d^3q\exp\left(-\frac{m}{2T}\dot{q}^2-\frac{m}{2T}A_{\alpha\beta}(\mathbf{k})q^\alpha q^\beta\right)\right]^{-1}\right\}=T\sum_{\mathbf{k}}\sum_{\gamma}\frac{1}{m\omega_\gamma^2}.$$

Here ω_γ^2 with $\gamma=1,2,3$ (or $\gamma=1,2$ for a two-dimensional lattice) are the eigenvalues of the matrix $A_{\alpha\beta}(\mathbf{k})$. At $H=0$, the quantities $\omega_\gamma(\mathbf{k})$ are identical with the frequencies of the normal oscillations; in a strong field, for the second mode $\omega_2(\mathbf{k})\ll\omega_\gamma(\mathbf{k})$ and $\omega_2(\mathbf{k})$ does not enter into the result.

Chaplik's conclusion that $\langle x^2\rangle$ diverges is based on the replacement in (34) of λ^2 by $(\hbar/m\omega_k)$, as is the case at $H=0$. Thus, the magnetic field does not disrupt the long-range order. Of course, for this, the inequality (36) should be satisfied with something to spare, although, because of the difficulty of transverse diffusion, the condition can be weaker than the usually applied criterion of Lindeman, $\langle x^2\rangle\lesssim(1/16)r_0^2$.

The vibrational spectrum of the filamentary crystal has the same character. Because of the smallness of the components A_{xz} and A_{yz} relative to the parameter α^{-1} from (11), the vibrations of the second branch are polarized transverse to the field. The lattice vibrations will make a contribution to the free energy. The contribution of the branch ω_2 at temperatures satisfying the inequality (35) has the form

$$\delta F_2=-nT\ln T+nT\ln(\omega_0^2/\tilde{\Omega}). \quad (37)$$

The quantity δF_1 can be calculated similarly.

CONCLUSION

Thus, the crystalline phase in a magnetic field at $m_h/m_e\gg 1$ is more advantageous and dynamically stable. The difference from the other phases can be determined by a measurement of the thermodynamic quantities, by finding the x-ray scattering by a hole lattice (with period ~100 Å) and also from the motion of the hole lattice in the electric field. Depending on the conditions of recombination of the electrons and holes on the boundaries of the sample, and also of the sticking to the impurities, the holes either make a periodic contribution to the conduction current and to the Hall current or make no contribution whatever. The intersections of the dispersion curves of the phonons of atomic and electric crystals lead to new regions of sound absorption.

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[1)]The structure of the crystal in a strong magnetic field in the envelope of a neutron star has been considered by Kaplan and Glasser[13] and by Ruderman.[14] Lozovik and Yudson,[15] and also Chaplik,[16] have considered a Wigner crystal in inversion films in a magnetic field.

[2)]In the work of Chaplik[21] the oscillation spectrum of a crystal in a film in a magnetic field is expressed in terms of the spectrum at $\mathbf{H}=0$.

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Inelastic scattering of electrons by dislocations in copper- and aluminum-base alloys

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The scattering of electrons in metals by quasilocal vibration modes of linear defects excited by increase in temperature results in a characteristic temperature dependence of the dislocation-induced electrical resistivity $\rho_d(T)$. This dependence $\rho_d(T)$ is in the form of a step, whose relative height depends on the density of dislocation-pinning points. If the free segment of a dislocation is limited in length to about 30 atomic distances by adding impurities to the base metal, the rise of $\rho_d(T)$ is suppressed.

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The dislocation-induced electrical resistivity ρ_d of a number of metals (Cu, Ag, Au, Al, Zn, and Mo) rises very rapidly with increase in temperature in a narrow range of the latter.[1-3] In some cases the cross section for the scattering of electrons by dislocations increases by a factor exceeding 2.[1,2]

If the dependence $\rho_d(T)$ is represented by the dimensionless curve $r(T)=[\rho_d(T)-\rho_d(0)]/\rho_d(0)$, it is found that pure fcc metals are characterized by $r_{max}\leq 2$ and the dependences $\rho_d(T)$ have a step whose height is a function of the density of the dislocation-pinning points. Such points can be dislocation line modes or impurity atoms. For example, in the case of copper samples of a given degree of purity and with the same dislocation density the step increases when dislocation lines have a certain preferred orientation.[2] The addition of dislocation-pinning impurities to dislocations increases very greatly the height of the step.[3]

The value of r_{max} can be altered by changing the number of pinning points. Foreign atoms present in a low concentration in a homogeneous alloy form a relatively regular sublattice with which dislocations interact. At a low dislocation density the length of a dislocation segment free of pinning points L_f is governed by the distance between the impurity atoms. However, at higher dislocation densities we can expect more frequent intersections (nodes).

Our aim was to determine the influence of dislocation pinning on the temperature dependence of the electrical resistivity ρ_d. Since it was of interest to find the conditions for complete suppression of the rise of $\rho_d(T)$, similar to that observed by the author earlier,[3] the impurity concentration in some of the alloys was selected to be $c\sim 0.1$ at.%. An important feature of the investigation was the determination of $\rho_d(T)$ for alloys with different ratios of the masses of the impurity and host (matrix) metal atoms.

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