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Collective excitations of electrons and holes in a strong electromagnetic field and absorption of light in a semiconductor under parametric resonance conditions

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The Bogolyubov–Mitropol'skiĭ averaging method is used to solve the two-band equation in the Kane model of a semiconductor and to investigate the quasienergy spectrum of electron–hole excitations in the field of a strong circularly polarized electromagnetic wave whose frequency ω is closed to the band gap ε_g of the semiconductor. The main feature of the spectrum is that the electromagnetic wave lifts the spin degeneracy of the levels due to free electrons and holes. One of the split dispersion curves is shifted only slightly compared with the curve for a free particle and the other exhibits—under certain conditions—a discontinuity of the order of $\rho\varepsilon_g$ (ρ is a parameter proportional to the electric field of wave). The gap in the spectrum appears for $0<\omega-\varepsilon_g<\rho\varepsilon_g$; the overlap of the quasienergy bands closes the gap for $\omega-\epsilon_g>\rho\varepsilon_g$ but discontinuities of the dispersion curves remain. The absorption coefficient of a weak electromagnetic wave of frequency ω_1 is calculated. It is shown that creation of electron–hole excitations at levels without a discontinuity in a strong field results in considerable absorption near the frequency $\omega_1=\omega$.

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1. INTRODUCTION

A strong electromagnetic field acting on a semiconductor not only heats the carriers but can also cause more fundamental dynamic changes in the electron–hole subsystem, deforming and modifying its energy spectrum.[1-3] Distortions of the spectrum are particularly large in various resonance situations, for example, in the case of cyclotron resonance,[4,5] when a semiconductor is subjected to electromagnetic radiation of frequency close to the width of the band gap (forbidden band)[6] or of the gap between two conduction bands.[7] It is important to note that under resonance conditions even a weak interaction may alter radically the energy spectrum of the system (see, for example, Oleĭnik's paper).[4] Physically, a change in the energy spectrum of a particle under the action of an electromagnetic wave implies the appearance of collective excitations of the "particle (electron or hole)+photons" type, which are states of electrons and holes strongly coupled to the electromagnetic wave field. In electromagnetic fields causing radical changes in the energy spectrum and, consequently, altering the very nature of the electron and hole motion, the behavior of the system could be described entirely in terms of these collective excitations.

We shall use the Bogolyubov–Mitropol'skiĭ averaging method[8] to solve the two-band equation in the Kane model of a semiconductor (Sec. 2) and we shall investigate the quasienergy spectrum of electron–hole excitations in the field of a strong circularly polarized electromagnetic wave described by the potential[1)]

$$\mathbf{A}=a(\cos kx, \sin kx, 0), \tag{1}$$

where $a=\text{const}>0$ is the amplitude of the potential; $k=(k_0/c, 0, 0, k_z)$ is the wave four vector of a photon; $k_z=c^{-1}\varepsilon^{1/2}k_0$; $\varepsilon=\text{const}$ is the permittivity of the meddium. We shall consider the case of a parametric resonance when the frequency of an electromagnetic wave $\omega\equiv k_0$ is related to the band gap ε_g of a semiconductor by

$$\omega=\varepsilon_g+\Delta, \quad |\Delta|\ll\varepsilon_g. \tag{2}$$

We shall define a strong electromagnetic field by the inequality

$$\Omega\gg\omega_{\text{ст}}, \tag{3}$$

where Ω is the frequency of transitions between the

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electron and hole (conduction and valence) bands under the action of an external field; ω_{coll} is the frequency of carrier collisions with one another, and also with phonons and impurity atoms. The condition (3) means that in a time between two consecutive relaxation events an electron can undergo many transitions between the bands. Naturally, in this case the main interaction governing the behavior of the system is the interaction of the electron–hole subsystem with the electromagnetic field, which should be included even in the zeroth approximation.[6,9] Ignoring the momentum of the electromagnetic field, we find that the following relationships are obeyed in the problem under consideration (they are given in the standard units):

$$\Omega \approx \rho\varepsilon_g, \quad \rho = 2eEs/\hbar\omega^2, \quad s = (\varepsilon_g/2m)^{1/2}, \tag{4}$$

where m is the effective mass of an electron or a hole; E is the intensity of the electric field of the electromagnetic wave. For typical values of the parameters of a semiconductor ($\varepsilon_g \sim 1$ eV, $m = 0.1m_e$) and for $E \sim 10^5$ V/cm the value of ρ reaches 10^{-2}.

The results obtained (Sec. 3) indicate that the quasienergy spectrum of the electron–hole subsystem has the following characteristics. The electromagnetic field lifts the spin degeneracy of the energy levels of free electrons and holes,[10] and one of the dispersion curves ($\sigma = 1$, where σ is the spin variable) is only slightly displaced (by an amount of the order of $\rho^2\varepsilon_g$) and the second ($\sigma = -1$) may, under certain conditions, have a discontinuity of the order of $\rho\varepsilon_g$. Consequently, a quasienergy band formed by the $\sigma = -1$ levels splits into two subbands (denoted by A and B in Fig. 1). For

$$0 < \Delta < \rho\varepsilon_g \equiv \Delta_0 \tag{5}$$

an energy gap forms between the subband B and the minimum of the $\sigma = 1$ band; this gap is

$$\tilde{\Delta} = {}^1/_2(\Delta_0 - \Delta). \tag{6}$$

If $\Delta > \Delta_0$, the $\sigma = 1$ band partly overlaps the B subband.

The appearance and properties of this energy gap are considered elsewhere[6,10] on the basis of a different, non-Kane model. This gives results[6,10] which differ from ours, based on the Kane model, in the following respect: 1) in our case the discontinuity of the dispersion curves and the gap width are independent of the angle between the particle quasimomentum $\mathbf{p}$ and the electric field $\mathbf{E}$; 2) the gap vanishes for $\Delta > \Delta_0$ but the discontinuities of the dispersion curves remain; 3) for fixed values of the transverse momentum $p_\perp = (p_x^2 + p_y^2)^{1/2}$ the discontinuities appear only in the $\sigma = -1$ dispersion curves; the $\sigma = 1$ dispersion curves differ from the corresponding results for a free particle only by a slight displacement and weak distortion.

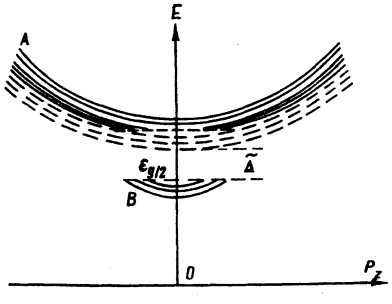

FIG. 1. Electron part of the quasienergy spectrum of electron—hole excitations in a semiconductor interacting with a circularly polarized electromagnetic wave. Here, E and p_z are the quasienergy and the z component of the quasimomentum. The continuous and dashed curves represent, respectively, the $\sigma = -1$ and $\sigma = 1$ dispersion curves for various values of p_x and p_y.

The absorption coefficient $K = K(\omega_1)$ of a weak electromagnetic wave of frequency ω_1 indident on a semiconductor subjected to a strong electromagnetic wave of frequency ω is discussed in Sec. 4. It is shown there that the appearance of a gap in the spectrum modifies the dependence of K on ω_1 in such a way that maxima appear at $\omega_1 \approx \rho\omega$ and $\omega_1 \approx (1+\rho)\omega$ and a minimum (negative absorption) is observed at $\omega_1 \approx (1-\rho)\omega$. Creation of electron–hole excitations at the $\sigma = 1$ levels makes the absorption coefficient large also near the frequency $\omega_1 = \omega$. It is interesting to note that a resonance amplification peak of electromagnetic radiation appears at a frequency $\omega_1 \approx \varepsilon_g - \frac{1}{2}\rho\omega$, which lies within the forbidden band of the semiconductor in the absence of a strong electromagnetic field.

2. SOLUTION OF THE TWO-BAND EQUATION

The solution of the equation

$$\left[i\gamma_0 \frac{\partial}{\partial t} + s\gamma\left(i\frac{\partial}{\partial \mathbf{r}} - \frac{e_0\mathbf{A}}{c} \right) - ms^2 \right] \Psi = 0, \tag{7}$$

where γ_μ are the Dirac matrices describing the behavior of a semiconductor in an external field $\mathbf{A}$ in Eq. (1), can be represented in the following form in the Kane model:

$$\Psi = e^{-i\varepsilon r}\Lambda\Phi(\zeta), \tag{8}$$

where

$$q = \text{const}, \quad \zeta = {}^1/_2 kx, \quad \Lambda = \exp(r\gamma_0\gamma_x), \quad r = {}^1/_2 \operatorname{Arch}[(1-\varepsilon s^2/c^2)^{-1/2}].$$

We shall now assume that $1 - \varepsilon s^2/c^2 > 0$. The function $\Phi(\zeta)$ satisfies an equation which is identical in form with the secular Schrödinger equation in which time is represented by the quantity ζ:

$$i\frac{d}{d\zeta}\Phi(\zeta) = (H_0 + V)\Phi(\zeta), \tag{9}$$

$$H_0 = -2k\tilde{q}/\tilde{k}^2 + \gamma_0 f_0 - \alpha_x f_1 + (\alpha_x \cos\varphi_q + \alpha_y \sin\varphi_q) f_2,$$

$$V = (\alpha_x \cos 2\zeta + \alpha_y \sin 2\zeta)\rho, \quad \alpha_i = \gamma_0\gamma_i \quad (i = x, y, z),$$

$$(f_0, f_1, f_2, \rho) = 2\frac{s}{c}(\tilde{k}^2)^{-1/2}\left(ms, \frac{\varkappa q}{(\tilde{k}^2)^{1/2}}, q_\perp, \frac{e_0 a}{c} \right),$$

$$q_x = q_\perp \cos\varphi_q, \quad q_y = q_\perp \sin\varphi_q.$$

We have introduced above the following four-vectors

$$\tilde{q} = (q_0/c, sq/c), \quad \varkappa = (k_z, 0, 0, k_0/c).$$

The function $\Phi(\zeta)$ can be expanded in terms of the complete set of bispinors u_n which diagonalize the matrix H_0:

$$\Phi(\zeta)=\exp\left(2i\frac{k\tilde{q}}{k^2}\zeta\right)\sum_{m=1}^{4}\varphi_m(\zeta)u_m. \tag{10}$$

The bispinors u_m are defined by

$$u_1=e^{-2i\zeta}u_1^{(+)},\quad u_2=e^{2i\zeta}u_{-1}^{(+)},\quad u_3=u_1^{(-)},\quad u_4=u_{-1}^{(-)};$$
$$u_1^{(\pm)}=c_\pm\begin{pmatrix} f_0\pm\alpha_0^{1/2}\\ 0\\ -f_1\\ f_2\exp(i\varphi_q)\end{pmatrix},\quad u_{-1}^{(\pm)}=c_\pm\begin{pmatrix} 0\\ f_0\pm\alpha_0^{1/2}\\ f_2\exp(-i\varphi_q)\\ f_1\end{pmatrix}, \tag{11}$$
$$c_\pm=(2\alpha_0^{1/2})^{-1/2}(\alpha_0^{1/2}\pm f_0)^{-1/2},\quad \alpha_0=f_0^2+f_1^2+f_2^2.$$

Substituting the expansion (10) into Eq. (9) and forming a bispinor φ from the quantities $\varphi_m\equiv\varphi_m(\zeta)$, we obtain

$$i\frac{d}{d\zeta}\varphi+\nu\varphi=(V_1+V_2)\varphi, \tag{12}$$

where

$$\nu=-\gamma_0(\alpha_0)^{1/2}+2\begin{pmatrix}\sigma_z & 0\\ 0 & 0\end{pmatrix},$$

σ_z is the Pauli matrix,

$$V_1=\rho[i\gamma_1 A_1+(\gamma_3\cos\varphi_q+i\gamma_2\sin\varphi_q)A_2],$$

$$V_2=\frac{\beta}{2\alpha_0^{1/2}}[\gamma_0-B(\gamma_3\cos 2\zeta+i\gamma_2\sin 2\zeta)]\cos(2\zeta-\varphi_q),$$

$$A_n=f_n(f_1^2+f_2^2)^{-1/2}\quad(n=1,2),\quad B=(\alpha_0^{1/2}-f_0)(f_1^2+f_2^2)^{-1/2},$$

$$\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3,\quad \beta=2\rho f_2.$$

We shall be interested in the following values of the parameters f_i and ρ:

$$|f_0|\approx 1,\quad |f_1|,\ |f_2|,\ \rho\ll 1. \tag{13}$$

In this case the right-hand side of Eq. (12) is proportional to a small parameter and, consequently, this equation can be solved by the Bogolyubov–Mitropol'skiĭ method.[8] However, before applying this averaging method, we shall carry out an additional transformation of Eq. (12) which allows us to include the main part of the interaction described by the vector V_1 ($|V_1|\gg|V_2|$) even in the zeroth approximation. Representing the solution of Eq. (12) in the form

$$\varphi(\zeta)=\sum_{n=1}^{4}g_n\varphi_0^{(n)}\exp(-i\nu_n\zeta), \tag{14}$$

where $\varphi_0^{(n)}\exp(-i\nu_n\zeta)$ are the solutions of Eq. (12) for $V_2=0$, we obtain the following system of equations:

$$i\frac{d}{d\zeta}g_k=\sum_{n=1}^{4}V_{kn}(\zeta)g_n\exp[i(\nu_k-\nu_n)\zeta],$$
$$g_n=g_n(\zeta),\quad V_{kn}(\zeta)=\varphi_0^{+(k)}V_2\varphi_0^{(n)}. \tag{15}$$

We shall now give the explicit expressions for ν_n and $\varphi_0^{(n)}$:

$$(\nu_1,\nu_2,\nu_3,\nu_4)=(\nu_1^{(+)},\nu_{-1}^{(+)},\nu_1^{(-)},\nu_{-1}^{(-)}),$$
$$\nu_\sigma^{(\pm)}=\sigma\pm(1+\alpha+2\sigma\sqrt{\alpha-\rho^2})^{1/2},\quad \sigma=\pm 1,\quad \alpha=\alpha_0+\rho^2, \tag{16}$$
$$\varphi_0^{(n)}=c_n\begin{pmatrix}0\\ p\\ -A_1(\nu_n-2-\alpha_0^{1/2})\\ A_2(\nu_n-2-\alpha_0^{1/2})\exp(i\varphi_q)\end{pmatrix},\quad c_n=[\rho^2+(\nu_n-2-\alpha_0^{1/2})^2]^{-1/2},$$
$$n=1,3,$$
$$\varphi_0^{(n)}=c_n\begin{pmatrix}\rho\\ 0\\ A_2(\nu_n+2-\alpha_0^{1/2})\exp(-i\varphi_q)\\ A_1(\nu_n+2-\alpha_0^{1/2})\end{pmatrix},\quad c_n=[\rho^2+(\nu_n+2-\alpha_0^{1/2})^2]^{-1/2},$$
$$n=2,4.$$

It follows from the inequalities (13) that the quantities α and ν_k can be described by the following expressions:

$$\alpha=1+\delta,\quad |\delta|\ll 1,$$
$$\nu_k=n_k+\Delta_k,\quad |\Delta_k|\ll 1\quad(k=1,2,3,4), \tag{17}$$

where n_k is an integer. If we assume that

$$g_k(\zeta)=\exp\left\{i(\Delta_k+\Delta_k^{(1)})\zeta-i\int_0^\zeta d\zeta' V_{kk}(\zeta')\right\}\tilde{g}_k(\zeta),$$
$$\Delta_k^{(1)}=\pi^{-1}\int_0^\pi d\zeta\, V_{kk}(\zeta), \tag{18}$$

we find from Eq. (15) that

$$i\frac{d}{d\zeta}\tilde{g}_k(\zeta)-(\Delta_k+\Delta_k^{(1)})\tilde{g}_k(\zeta)=\sum_{\substack{k'=1\\(k'\neq k)}}^{4}\bar{V}_{kk'}(\zeta)\exp(2iN_{kk'}\zeta)\tilde{g}_{k'}(\zeta),$$
$$\bar{V}_{kn}(\zeta)=\exp i\left\{\int_0^\zeta d\zeta'(U_k(\zeta')-U_n(\zeta'))\right\}V_{kn}(\zeta), \tag{19}$$
$$N_{kk'}={}^1/_2(n_k-n_{k'}),\quad U_k(\zeta)=V_{kk}(\zeta)-\Delta_k^{(1)}.$$

According to the Bogolyubov–Mitropol'skiĭ averaging method, the solution of the system (19) has the following form in the first approximation:

$$\tilde{g}_k(\zeta)=g_k^{(0)}\exp(-i\lambda\zeta), \tag{20}$$

where the quantities $g_k^{(0)}$ satisfy the following system of the algebraic equations

$$(\lambda-\Delta_k-\Delta_k^{(1)})g_k^{(0)}=\sum_{\substack{k'=1\\(k'\neq k)}}^{4}g_{k'}^{(0)}f_{kk'}^{(-N_{kk'})}, \tag{21}$$

λ are the roots of the determinant of this system, and $f_{kk'}^{(s)}$ are the coefficients of the Fourier expansion

$$\bar{V}_{kn}(\zeta)=\sum_{s=-\infty}^{+\infty}f_{kn}^{(s)}e^{2is\zeta}.$$

Using λ_n ($n=1,2,3,4$) to denote the roots of the determinant of the system (21) and $g_{kn}^{(0)}$ for the corresponding values of $g_k^{(0)}$, we find from the system (21) (always retaining only the principal terms)

$$\lambda_n=\Delta_n+\Delta_n^{(1)}+x_n,\quad x_n=\sum_{\substack{k=1\\(k\neq n)}}^{4}\frac{|f_{nk}|^2}{\Delta_n-\Delta_k},$$
$$g_{kn}^{(0)}=\begin{cases}1 & \text{for } k=n\\ f_{nk'}/(\Delta_n-\Delta_k) & \text{for } n\neq k\end{cases} \tag{22}$$

Here, for simplicity, we have used $f_{kk'}$ to denote $f_{kk'}^{(-N_{kk'})}$ ($f_{kk'}^*=f_{k'k}$).

The expressions (22) for x_{π} and $g_{kn}^{(0)}$ with $n=1$ or 3 are valid only if $|\Delta_1| \gg |f_{13}|$. We can easily obtain the corresponding expressions also for $|\Delta_1| \leq |f_{13}|$ but for reasons of space we shall not give them here.

Combining Eqs. (8), (10), (14), (18), and (20) and introducing the notation

$$\mu_1=\nu_1+x_1+\Delta_1^{(1)}-2, \quad \mu_3=\nu_3+x_3+\Delta_3^{(1)}, \qquad (23)$$
$$\mu_2=\nu_2+x_2+\Delta_2^{(1)}+2\theta(\alpha-1), \quad \mu_4=\nu_4+x_4+\Delta_4^{(1)}+2\theta(1-\alpha)$$

and the quasienergy-quasimomentum four-vector $p \equiv p^{(n)}$,

$$p^{(n)}=q-(k\tilde{q}/k^2-{}^1/_2\mu_n)k, \qquad (24)$$

we obtain the solutions of the two-band equation (7) in the form (c_{pn} is the normalization constant)

$$\Psi_{pn}(\mathbf{r},t)=c_{pn}e^{-ip^{(n)}x}\Lambda\sum_{s=1}^{4}g_{sn}^{(0)}\exp\left\{-i\int_0^t d\zeta' U_s(\zeta')-i(n_s+\tau_n)\zeta\right\}\sum_{m=0}^{4}\varphi_{0m}^{(s)}u_m, \qquad (25)$$

where the quantity τ_n is defined as follows:

for $\alpha>1$,

$$\tau_n=\begin{cases}-1 & \text{for } n=1,2\\ 1 & \text{for } n=3,4\end{cases};$$

for $\alpha<1$,

$$\tau_n=\begin{cases}-1 & \text{for } n=1,4\\ 1 & \text{for } n=2,3\end{cases}.$$

The quasienergy spectrum, i.e., the dependence of the quasienergy $p_0^{(n)}$ on the quasimomentum $\mathbf{p}$, is given by

$$\mu_n \equiv \mu_n(p)=2\tilde{k}\tilde{p}/k^2. \qquad (26)$$

The wave functions corresponding to the electron $\Psi_{p\sigma}^{(+)}(\mathbf{r},t)$ and hole $\Psi_{p\sigma}^{(-)}(\mathbf{r},t)$ states are found by applying the boundary condition

$$\Psi_{p\sigma}^{(\pm)}(\mathbf{r},t) \to \psi_{p\sigma}^{(\pm)}(\mathbf{r},t) \quad \text{for} \quad a \to 0,$$

where $\Psi_{p\sigma}^{(\pm)}(\mathbf{r},t)$ are the electron and hole wave functions in the absence of the external field $\mathbf{A}$ and $\sigma=\pm1$ is the spin index. Using the above boundary condition, we obtain the following expressions:

$$\Psi_{p,1}^{(+)}(\mathbf{r},t)=\Psi_{p1}(\mathbf{r},t), \quad \Psi_{p,1}^{(-)}(\mathbf{r},t)=\Psi_{p3}(\mathbf{r},t),$$
$$\Psi_{p,-1}^{(+)}(\mathbf{r},t)=\theta(\alpha-1)\Psi_{p2}(\mathbf{r},t)+\theta(1-\alpha)\Psi_{p4}(\mathbf{r},t), \qquad (27)$$
$$\Psi_{p,-1}^{(-)}(\mathbf{r},t)=\theta(\alpha-1)\Psi_{p4}(\mathbf{r},t)+\theta(1-\alpha)\Psi_{p2}(\mathbf{r},t).$$

The normalization constants in c_{pn} in Eq. (25) are found using the identity

$$\Psi_{p'n'}^{+}\Psi_{pn}=-\frac{i}{c}\frac{k_z\partial/\partial t+k_0\partial/\partial z}{\varkappa p'-\varkappa p}\Psi_{p'n'}^{+}\Psi_{pn}, \quad \varkappa p' \neq \varkappa p,$$

and the equation of continuity for the electric current density

$$\frac{\partial}{\partial t}\rho'+c\,\mathrm{div}\,\mathbf{j}=0, \quad \rho'\equiv\Psi_{p'n'}^{+}\Psi_{pn}, \quad \mathbf{j}=\frac{s}{c}\Psi_{p'n'}^{+}\alpha\Psi_{pn}.$$

These relationships lead to the equality

$$\int dr\,\Psi_{p'n'}^{+}\Psi_{pn}=\frac{-i}{\varkappa p'-\varkappa p}\int dr\frac{d}{dz}\Psi_{p'n'}\hat{k}\Psi_{pn},$$

where

$$\hat{k}=\gamma_0\frac{k_0}{c}-\frac{s}{c}\gamma_z k_z,$$

which can be used to calculate quite easily the normalization constant. If the wave functions are normalized by the condition

$$\int dr\,\Psi_{p'n'}^{+}(\mathbf{r},t)\Psi_{pn}(\mathbf{r},t)=(2\pi)^3\delta(\mathbf{p}'-\mathbf{p})\delta_{n'n},$$

it is found that

$$c_{pn}=\left(\sum_{s=1}^{4}|g_{sn}^{(0)}|^2\right)^{-1/2}\left(1-\frac{\varepsilon^{1/2}}{c}\frac{\partial p_0^{(n)}}{\partial p_z}\right)^{1/2}\left(1-\varepsilon\frac{s^2}{c^2}\right)^{-1/4}. \qquad (28)$$

3. QUASIENERGY SPECTRUM

It follows from the preceding section that the dispersion curves describing the dependence of the quasienergy p_0 on the quasimomentum $\mathbf{p}$ are given by the following equations for electron and hole states[5,11]

$$\mu_\sigma^{(\pm)}=2\tilde{k}\tilde{p}/k^2, \quad \sigma=\pm1, \qquad (29)$$

where

$$\mu_1^{(+)}=-\mu_1^{(-)}=\nu^{(+)}-1+x_1+\Delta_1^{(1)}, \quad \mu_{-1}^{(+)}=-\mu_{-1}^{(-)}=1$$
$$+(\nu^{(-)}+x_2+\Delta_2^{(1)})\,\mathrm{sign}(\alpha-1),$$
$$\nu^{(\pm)}=(1+\alpha\pm2\sqrt{\alpha-\rho^2})^{1/2}.$$

The equality $\mu_\sigma^{(+)}+\mu_\sigma^{(-)}=0$ implies symmetry of the electron and hole branches of the spectrum relative to the origin of the coordinate system. The application of the expressions in Eq. (22) shows readily that in the range of the parameters described by Eq. (13) the dispersion curves are deformed only slightly by the inclusion of the quantities x_n and $\Delta_n^{(1)}$ ($n=1$ or 2) and, therefore, these quantities can be ignored in investigations of the spectra.

For simplicity, we shall neglect the momentum of the incident electromagnetic wave and assume that $\varepsilon=0$. Introducing $E_\sigma^{(\pm)}(\mathbf{p})$ for the quasienergy of a particle in a state $\Psi_{p\sigma}^{(\pm)}$ and assuming that the condition (2) is satisfied, we obtain the following dispersion equations from Eq. (29):

$$E_1^{(+)}(\mathbf{p})={}^1/_2\varepsilon_g(1+\mathbf{p}^2/m\varepsilon_g+{}^1/_4\rho^2),$$
$$E_{-1}^{(+)}(\mathbf{p})={}^1/_2\omega[1+(\rho^2+({}^1/_2\delta)^2)^{1/2}(1-{}^1/_4\delta+{}^1/_8\rho^2)\,\mathrm{sign}\,\delta], \qquad (30)$$
$$\delta=\alpha-1=-2\Delta/\varepsilon_g+2\mathbf{p}^2/m\varepsilon_g+\rho^2.$$

According to Eq. (30), the curve $\sigma=1$ is a parabola displaced by an amount $\frac{1}{8}\rho^2\varepsilon_g$ relative to the dispersion curve for a free electron. The behavior of the $\sigma=-1$ dispersion curve is more complex. Generally speaking, this curve has three extrema: one at the point $p_z=0$ and two at the points where the equality $\delta=\rho^2$ is satisfied. This equality gives

$$p_z=\pm(m\varepsilon_g)^{1/2}\left(\frac{\Delta}{\varepsilon_g}-\frac{p_\perp^2}{m\varepsilon_g}\right)^{1/2}=\pm p_{z1}, \quad p_\perp=(p_x^2+p_y^2)^{1/2}.$$

Since

$$\frac{d^2}{dp_z^2}E_{-1}^{(+)}|_{p_z=0}=c\,\mathrm{sign}[\delta_0(\delta_0-\rho^2)],$$

where $c>0$, $\delta_0=\delta|_{p_z=0}$, we find that there are three ranges of the value of Δ:

a) $\Delta/\varepsilon_g<p_\perp^2/m\varepsilon_g$—in this range the dispersion curve has a single minimum at the point $p_z=0$;

b) $p_\perp^2/m\varepsilon_g<\Delta/\varepsilon_g<p_\perp^2/m\varepsilon_g+\frac{1}{2}\rho^2$—we now have a maximum at $p_z=0$ and two minima at $p_z=\pm p_{z1}$ separated by a barrier whose maximum height is $\frac{1}{16}\varepsilon_g\rho^3$;

c) $\Delta/\varepsilon_g>p_\perp^2/m\varepsilon_g+\frac{1}{2}\rho^2$—in this range there are three minima located at the points $p_z=0$ and $\pm p_{z1}$, the corresponding energies being

$$E_{-1}^{(+)}|_{p_z=0}=\frac{\omega}{2}\left[1-\left(\rho^2+\frac{1}{4}\delta_0^2\right)^{1/2}\left(1-\frac{\delta_0}{4}+\frac{1}{8}\rho^2\right)\right],$$

$$E_{-1}^{(+)}|_{p_z=\pm p_{z1}}=\frac{\omega}{2}(1+\rho).$$

The dispersion curve has a discontinuity at $\delta=0$, i.e., at

$$p_z=\pm(m\varepsilon_g)^{1/2}(\Delta/\varepsilon_g-p_\perp^2/m\varepsilon_g-{}^1/_2\rho^2)^{1/2}=\pm p_{z2}.$$

Then,

$$E_{-1}^{(+)}|_{p_z=p_{z2}}=\frac{\omega}{2}\left[1\pm\rho\left(1+\frac{1}{8}\rho^2\right)\right]=E_0^{(\pm)}. \tag{31}$$

It follows from the above relationships that the discontinuity of the dispersion curve in respect of the quasienergy is $\rho\varepsilon_g$ and the depth of the minima lying above the $\sigma=1$ curve is $\frac{1}{16}\varepsilon_g\rho^3$. If $\Delta/\varepsilon_g<p_\perp^2/m\varepsilon_g+\frac{1}{2}\rho^2$, the $\sigma=-1$ dispersion curve lies above the $\sigma=1$ curve (at a fixed value of $p_\perp$). If $\Delta/\varepsilon_g>p_\perp^2/m\varepsilon_g+\frac{1}{2}\rho^2$, the $\sigma=-1$ curve has a discontinuity and part of this curve corresponding to $\delta>0$ lies above $\sigma=1$, whereas the remainder is below this curve. For different values of $p_\perp$ the σ dispersion curves form quasienergy bands, which we shall call the σ bands. The $\sigma=-1$ band splits into two parts, one of which (subband A) lies above the $\sigma=1$ curve corresponding to $p_\perp=0$ and the minimum of the other (subband B) lies below the $\sigma=1$ band. If the condition (5) is satisfied, the $\sigma=1$ band is separated by a gap $\tilde{\Delta}$ from the B subband [see Eq. (6)]. When the opposite condition is satisfied, there is a partial overlap between the B subband and the $\sigma=1$ band. The quantity $E_0^{(-)}$, which is the upper limit of the B subband, is independent of the transverse momentum $p_\perp$. The dimensions (Δp_g) and (Δp_0) of the B subband (in the $|\Delta|\gg\rho^2\varepsilon_g$) case are:

$$(\Delta p_z)=\rho[2m\varepsilon_g(x-1)]^{1/2},\quad (\Delta p_0)={}^1/_2\{[\rho^2+(\Delta/\varepsilon_g)^2]^{1/2}-\rho\}\varepsilon_g, \tag{32}$$

where the value of x $(x>1)$ is given by

$$\Delta/\varepsilon_g=p_\perp^2/m\varepsilon_g+{}^1/_2\rho^2x.$$

The quasienergy spectrum is analyzed above ignoring the momentum k of the incident electromagnetic wave. Using the dispersion equation (29), which is valid for $\mathbf{k}\neq 0$, we can see how important are the distortions of the dispersion curves resulting from neglect of the wave momentum. We shall not give the full analysis but mention that the deformation of the $\sigma=1$ dispersion curves due to the wave momentum is negligible. The displacement of the $\sigma=-1$ dispersion curves is much greater but even in the most interesting range $\Delta<\rho\varepsilon_g$, where the energy gap is formed and the width of the split-off subband B is of the same order of magnitude as the gap, the corrections to the dispersion curves due to the electromagnetic wave momentum are always small and can be ignored.

4. ABSORPTION COEFFICIENT

We shall consider the absorption of photons from a weak electromagnetic wave of frequency ω_1 by an electron–hole system interacting with a strong electromagnetic wave described by Eq. (1) and we shall assume that the condition (3) is satisfied by the strong wave. As usual, we shall define the absorption coefficient K by

$$K=-\varepsilon^{1/2}\Gamma_1/c,$$

where Γ_1 is the damping of the weak-wave phonons absorbed in the semiconductor. If we use the results of the preceding sections and calculate Γ_1 to within terms of the second order in respect of the interaction with photons of frequency ω_1, we obtain

$$K(\omega_1)=\sum_{\sigma,\,\sigma_1=\pm1}\sum_{r=-\infty}^{+\infty}[K_{\sigma\sigma_1}^{(+)}(r)-K_{\sigma\sigma_1}^{(-)}(r)], \tag{33}$$

$$K_{\sigma\sigma_1}^{(\pm)}(r)=\alpha'\frac{s^2}{\varepsilon^{1/2}\omega_1}\int_0^\infty dp\,p^2F_{\sigma\sigma_1}(p)L_{\sigma\sigma_1}(r)\delta(E_\sigma^{(+)}(\mathbf{p})-E_{\sigma_1}^{(-)}(\mathbf{p})+r\omega\mp\omega_1),$$

where [we shall give only the principal coefficients $L_{\sigma\sigma1}(r)$]

$$F_{\sigma\sigma_1}=1-f_{p\sigma}^{(+)}-f_{p\sigma_1}^{(-)},\quad f_{p\sigma}^{(\pm)}=[\exp\{\beta(E_\sigma^{(+)}(\mathbf{p})\mp\mu)\}+1]^{-1},$$
$$\beta=(kT)^{-1},\quad \alpha'=e^2/4\pi c,$$
$$L_{11}(0)=e_x^2+e_y^2,\quad L_{1,-1}(0)=L_{-1,1}(0)=e_z^2(\Delta_2+{}^1/_2|\delta|)/2\Delta_2,$$
$$L_{-1,-1}(0)=(e_x^2+e_y^2)[(\Delta_2+{}^1/_2|\delta|)/2\Delta_2]^2,\quad L_{-1,-1}(-1)={}^1/_3(\rho/\Delta_2)^2\xi,$$
$$L_{-1,-1}(-2)=(e_x^2+e_y^2)[(\Delta_2-{}^1/_2|\delta|)/2\Delta_2]^2,\quad \Delta_2=(\rho^2+{}^1/_4\delta^2)^{1/2},$$
$$L_{11}(-1)=\frac{1}{2}\rho^2\xi\left\{\frac{1}{3}\left(1-\frac{4}{5}\frac{\xi}{\delta}\right)(e_x^2+e_y^2)+\frac{4}{35}\left(\frac{\xi}{\delta}\right)^2(e_x^2+e_y^2+\frac{1}{3}e_z^2)\right\}, \tag{34}$$
$$L_{1,-1}(-1)=L_{-1,1}(-1)={}^1/_6\rho^2\xi[(\Delta_2+{}^1/_2|\delta|)/2\Delta_2]^2e_z^2,\quad \xi=4(s/\omega)^2\mathbf{p}^2,$$

$\mathbf{e}$ is the polarization vector of the absorbed photons, and μ is the chemical potential. In the derivation of the first expression in the system (33) we have ignored the photon momentum of the strong field, which deforms the energy spectrum of electrons and holes, and of the weak field, so that the absorption coefficient is entirely due to interband transitions. The quantities $K_{\sigma\sigma1}^{(+)}(r)$ and $K_{\sigma\sigma1}^{(+)}(r)$ describe, respectively, the absorption and emission of a photon of frequency ω_1 accompanied by the creation of an electron–hole pair in such a way that the created electron and hole excitations are in the σ and σ_1 bands, respectively. The terms in the sum over r in Eq. (33) for $r<0$ $(r>0)$ correspond to the absorption (emission) of $|r|$ of strong-field photons.

We shall confine our attention to the absorption of photons of frequency ω_1 lying within the following two intervals:

$$\text{i) } |\omega_1-\varepsilon_g|\ll\varepsilon_g, \quad \text{ii) } \rho^2\varepsilon_g\ll\omega_1\ll\varepsilon_g. \tag{35}$$

In this case, the sum over r in Eq. (33) consists only of contributions of the terms with $r=0$, -2 (interval i) and with $r=-1$ (interval ii). Introducing

$$\mp\omega_1=-(r+1)\omega\mp\tilde{\omega}, \quad |\tilde{\omega}|<\omega,$$

we shall represent the law of conservation of quasienergy, corresponding to the absorption (upper sign) or emission (lower sign) of a photon of frequency ω_1, in the form

$$E_\sigma^{(+)}(\mathbf{p})-E_{\sigma_1}^{(-)}(\mathbf{p})-\omega\mp\tilde{\omega}=0,$$

where, in accordance with the inequalities of Eq. (35), we have $\rho^2\varepsilon_g\ll|\tilde{\omega}|\ll\varepsilon_g$.

Applying the above expressions we can easily show that the quantities $K_{11}^{(\pm)}$, $K_{-1,1}^{(\pm)}$, and $K_{1,-1}^{(\pm)}$ vary smoothly with ω_1, whereas the dependence of $K_{-1-1}^{(\pm)}$ on ω_1 is of resonance nature. In fact, it follows from the relationship

$$\delta(E_{-1}^{(+)}(\mathbf{p})-E_{-1}^{(-)}(\mathbf{p})-\omega\mp\tilde{\omega})=\sum_{i=+1,-1}\frac{m}{2p_i}\frac{(\delta_i^2+4\rho^2)^{1/2}}{|\delta_i-\rho^2|}\delta(p-p_i), \tag{36}$$

where

$$p_i=(m\varepsilon_g)^{1/2}(\delta_i/2+\Delta/\varepsilon_g)^{1/2}, \quad \delta_i=2(\tilde{\omega}^2/\omega^2-\rho^2)^{1/2}i,$$

that the coefficient $K_{-1,-1}^{(\pm)}$ has a singularity at $\delta_i=\rho^2$. Therefore, the quantity $K_{-1,-1}^{(+)}(0)-K_{-1,-1}^{(-)}(-2)$ has a maximum at $\omega_1\approx(1+\rho)\omega$ and a minimum (negative absorption) at $\omega_1\approx(1-\rho)\omega$. The limit $K\to\pm\infty$ is reached at frequencies $\omega_1\approx(1\pm\rho)\omega$ because we have ignored dissipative processes, for example, the interaction between the electron–hole subsystem with phonons. Allowance for this interaction imparts a finite half-width Γ to the electron and hole states. As a result, the quantity $|\delta_i-\rho^2|$, in the resonance range, which is in the denominator of Eq. (36), assumes a finite value γ proportional to the half-width Γ. It should be noted that in the interval i of the frequencies ω_1 the value of $K_{11}^{(\pm)}(r)$ differs from zero, whereas

$$K_{-1,-1}^{(+)}(0)-K_{-1,-1}^{(-)}(-2)=0 \quad \text{for} \quad (1-\rho)\omega<\omega_1<(1+\rho)\omega,$$

$$K_{-1,1}^{(\pm)}(0)=K_{1,-1}^{(\pm)}(0)=0 \quad \text{for} \quad (1-{}^1/_2\rho)\omega<\omega_1<(1+{}^1/_2\rho)\omega.$$

The absorption coefficient can be estimated for $\Delta=\frac{1}{2}\rho\varepsilon_g$. In this case the energy gap between the $\sigma=1$ band and B subband is $\frac{1}{4}\rho\varepsilon_g$. The calculations will be carried out for the following parameters: $\varepsilon_g=1$ eV, $m=0.1m_e$, $\varepsilon=16$, and $E=10^5$ V/cm. For these values of the parameters, we have $\rho\approx10^{-2}$. Assuming additionally that $F_{\sigma\sigma_1}(p)\approx1$ and $\gamma\approx10^{-4}$ (the latter corresponds to the case when the half-width Γ due to the electron–phonon interaction is of the order of 10^{11} sec^{-1}), we find that

$$K|_{\omega_1\approx(1+\rho)\omega}=-K|_{\omega_1\approx(1-\rho)\omega}\sim10^4(e_x^2+e_y^2)\text{ cm}^{-1}.$$

Far from the resonance region and for $|\omega_1-\omega|\ll\varepsilon_g$, we have

$$K\sim10^3\text{cm}^{-1}.$$

A sharp peak of the absorption coefficient appears also at $\omega_1\approx\rho\omega$, where $K|_{\omega_1\approx\rho\omega}\sim10^4$ cm^{-1}. It is interesting to note that the absorption coefficient in the $\omega_1\ll\omega$ range is dominated by the processes creating electrons and holes in the $\sigma=-1$ band. In fact, calculations indicate that $K_{11}^{(\pm)}(-1)\sim K_{1,-1}^{(\pm)}(-1)\sim K_{-1,1}^{(\pm)}(-1)\sim10^{-2}$ cm^{-1}, whereas $K_{-1,-1}^{(\pm)}(-1)\sim10^2-10^3$ cm^{-1} (far from the resonance region). In the range $|\omega_1-\omega|\ll\omega$ and far from the resonance peak all the quantities $K_{\sigma\sigma_1}(0)$ $(\sigma,\sigma_1=\pm1)$ are of the same order of magnitude ($\sim10^2-10^3$ cm^{-1}).

According to our estimates, the quantity $K_{-1,-1}^{(+)}(0)-K_{-1,-1}^{(-)}(-2)$ considered as a function of ω_1 behaves near $\omega_1=\omega$ in the same way as the absorption coefficient for the case of an isotropic gap.[12] However, if the spectrum includes the $\sigma=1$ quasienergy levels, which do not have a discontinuity but are shifted slightly by the strong field, the behavior of the total absorption coefficient is not affected in any basic manner: the coefficient is fairly high also in the $\varepsilon_g<\omega_1<(1+\rho)\omega$ range because of creation of pairs at the $\sigma=\sigma_1=1$ levels (compare with the results of Krokhin[13] and Klimontovich and Pogorelova).[14]

The following characteristic feature should also be noted: electromagnetic waves traveling along the z axis are absorbed and amplified mainly in the range $\omega_1\approx(1\pm\rho)\omega$, whereas in the range $\omega_1\approx\rho\omega$ there is no preferred direction along which photons are absorbed more strongly. It is worth noting also the circumstance that the resonance amplification peak appears at $\omega_1=\varepsilon_g-\frac{1}{2}\rho\omega$, i.e., at a frequency within the forbidden gap of the semiconductor not subjected to a strong electromagnetic field.

[1)]The four-dimensional notation is used here: $qx=q_0t-\mathbf{q}\cdot\mathbf{r}$ is the scalar product of the four-vectors $q=(q_0/c,\mathbf{q})$ and $x=(ct,\mathbf{r})$. The system of units in which $\hbar=1$ is employed.

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Features of antiferromagnetic ordering in the garnet $Mn_3Cr_2Ge_3O_{12}$

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The specific heat and the magnetic properties are investigated in the garnet $Mn_3Cr_2Ge_3O_{12}$, in which the weakly interacting *a* and *c* sublattices are completely filled by magnetic ions. Independent antiferromagnetic ordering of the manganese and chromium sublattices is observed at temperatures 3.0 and 5.1 K, respectively. It is shown that the magnetic symmetry of this garnet is responsible for the peculiarities of its magnetic properties, in particular the nonlinear field dependence of the magnetization below T_N. A calculation carried out in the molecular-field approximation, with use of the exchange-interaction parameters of the corresponding "single-sublattice" garnets, gives good agreement with experimental data.

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1. INTRODUCTION

In compounds with the garnet structure, the magnetic ions may occupy three types of crystallographic sites (sublattices): dodecahedral $\{c\}$, octahedral $[a]$, and tetrahedral (d). The character of the magnetic ordering of garnets is determined to a significant degree by the relation between the magnitudes of the antiferromagnetic intra- and intersublattice exchange interactions: in iron garnets, the strong intersublattice a–d interaction suppresses the considerably weaker intrasublattice interactions and causes ferrimagnetism of these compounds below $T_c \approx 550$ K; by diamagnetic substitution, it is possible to weaken or even completely eliminate the a–d interaction, and then there is observed in the garnet, at sufficiently low temperatures ($T_N \sim 10$ K), antiferromagnetic ordering of the remaining magnetic sublattice. The properties of such antiferromagnetic or "single-sublattice" garnets, in which the magnetic $3d$ or $4f$ ions completely occupy one of the three types of crystallographic sites, have been investigated quite thoroughly (see, for example, the review by Belov and Sokolov[1]).

In order to obtain information about the complex mechanism of indirect exchange interactions, which occur in garnets through one, two, or even three intermediate links (two oxygen ions and a nonmagnetic cation), it is of interest to study magnetic phase transitions in garnets in which the intra- and intersublattice exchange interactions have approximately equal magnitudes. Such a situation occurs, for example, in yttrium iron garnets with a sufficiently high concentration of diamagnetic ions in the a and d sublattices. In a paper of Plakhtiĭ *et al.*[2] it was shown that in such substituted garnets, the transition from ferri- to antiferromagnetism occurs over a quite appreciable range of concentrations with a cluster type of magnetic ordering.

Another variant of systems that have comparable values of the intra- and intersublattice exchange interactions is garnets in which the weakly interacting c and a sublattices are *completely* filled by magnetic ions. The features of the magnetic ordering of one of these compouds, $\{Mn_3\}[Fe_2]Ge_3O_{12}$, have already been discussed.[3,4] The present paper reports the results of measurements of the specific heat and magnetic properties of the garnet $\{Mn_3\}[Cr_2]Ge_3O_{12}$ (MnCrG), in which the antiferromagnetic ordering has an unusual character.[5,6] We shall also consider a simple model, which enables us to give a qualitative interpretation of the

TABLE I. Lattice parameters (a_0), Néel temperatures (T_N), paramagnetic Curie points (Θ_P), values of the susceptibility (χ) at $T \leq T_N$, spin-flip fields (H_E), and exchange-interaction integrals for first and second nearest neighbors (J_1, J_2), for single-sublattice chromium and manganese garnets.

Garnet	a_0, Å	T_N, K	$-\Theta_P$, K	χ, cgs emu/mol	$H_E(T)$, kOe	$-J_1$, K	$-J_2$, K
CdCrG	12.205	12.55	21.5	0.11	260 (4.2)	0.81	0.28
MnAlG	11.894	6.65	28	0.38	210 (3.0)	0.58	0.12

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