into account, are smaller than the Néel temperature (16), i.e., our analysis is valid at concentrations not too close to the percolation limit.

[1)]It can be shown that in the infinite cluster there is short-range magnetic order over lengths

$$r \leqslant r(T) = a\left(\frac{V_0}{T}\right)^{\nu/(1-\nu)},$$

where a is the lattice constant.

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Topological instability of singularities at small distances in nematics

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At short distances, the order parameter in a nematic is degenerate on the sphere S^4; this leads to topological removability of the singularities. As a result, the disclinations may have a nonsingular core.

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According to experimental data on measurement of the heat of transition, critical scattering, etc.,[1,2] the transition from isotropic liquid to nematic is a weak transition of the first kind, nearly of the second. Furthermore, the temperature range of existence of the nematic phase is much smaller than the transition temperature. Therefore Landau's theory may be a reasonable approximation for description of a nematic over its whole range of existence.

The order parameter in a nematic is a traceless, symmetric, real second-rank tensor $\hat{Q}=Q_{\alpha\beta}$. In general it has five independent components. This tensor can be represented in the following form:

$$Q_{\alpha\beta}=\sqrt{2}Q_0\{\sin(\varphi+{}^1/_3\pi)(n_\alpha n_\beta-{}^1/_3\delta_{\alpha\beta})+\sin\varphi(l_\alpha l_\beta-{}^1/_3\delta_{\alpha\beta})\}, \tag{1}$$

where Q_0 is the modulus of the order parameter, $\mathrm{Sp}\,\hat{Q}^2=Q_0^2$, and **n** and **l** are mutually perpendicular unit vectors. The angle φ describes the degree of biaxiality of the tensor $Q_{\alpha\beta}$. When $\varphi=0$, $Q_{\alpha\beta}$ is uniaxial, and **n** is the director. The tensor $Q_{\alpha\beta}$ must not change sign upon change of sign of **n** or **l**. Therefore $Q_{\alpha\beta}$ contains no terms of the form $n_\alpha l_\beta$.

There are only two independent invariants of the rotation group constructed from the components of $Q_{\alpha\beta}$, for example $\mathrm{Sp}\hat{Q}^2$ and $\mathrm{Sp}\hat{Q}^3$. Therefore the Landau expansion in powers of Q_0 can be represented in the form[1)]

$$F={}^1/_2A\,\mathrm{Sp}\,Q^2-{}^1/_3\sqrt{{}^3/_2}B\,\mathrm{Sp}\,Q^3+{}^1/_4C(\mathrm{Sp}\,Q^2)^2, \tag{2}$$

where $A=a(T-T^*)$; T^* is a fictitious Curie temperature. On substituting (1) in (2), we get

$$F={}^1/_2AQ_0^2-{}^1/_3B\cos 3\varphi Q_0^3+{}^1/_4CQ_0^4. \tag{3}$$

Minimization of (3) with respect to φ gives $\varphi=0$, provided $B\neq 0$. This means that when $B\neq 0$, only the uniaxial state is stable. If $B=0$, the free energy (2) is a function only of $\mathrm{Sp}\hat{Q}^2$, and the symmetry of the order parameter is higher than in the uniaxial case. The only constraint is

$$\mathrm{Sp}\,Q^2=Q_0^2, \tag{4}$$

where Q_0 can be found by minimization of the free energy (3) with $B=0$. It is easy to show that (4) is the equation of a four-dimensional sphere S^4 in the five-dimensional space of the components of the matrix $Q_{\alpha\beta}$.

Now let B be nonzero but small. The biaxial perturbation corresponds to certain motions on the sphere S^4. Such motions have an energy gap $\Delta\sim BQ_0^3$. The corresponding correlation radius is $R_B\sim kB^{-1}Q_0^{-3}$, where k is a quantity of the order of the Frank constants. The criterion for smallness of B is the condition $R_B\gg R_0$, where R_0 is the correlation radius of fluctuations of the modulus Q_0 of the order parameter. In the distance range $R_0<R<R_B$, the order parameter is degenerate on the sphere S^4. When $R>R_B$, the degeneracy parameter will be the ordinary nematic director **n**. Its domain of variation is the sphere S^2, on which diametrically opposite points are equivalent (since **n** and $-$**n** are

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indistinguishable). Thus for small B, the symmetry of the degeneracy parameter will be different at small and at large distances. A situation with different symmetries of the degeneracy parameter at small and at large distances was first considered by Volovik and Mineev[4] for the case of the superfluid phase of He^3.

We shall consider the problem of singularities at small distances $R_0 < R < R_B$, using the method of homotopic topology. It has been shown[4,5] that the possibility of existence of stable singularities in degenerate systems is determined by the topological properties of the domain of variation of the degeneracy parameter. We assume that when $R < R_B$, there is a singular line. We surround it with a closed contour. Its mapping on the sphere S^4 is a closed contour on this sphere. Such a contour can always be deformed to a point. Consequently the singular line in real space can also be removed by continuous transformation of the order parameter. This means that the topology of the sphere S^4 does not permit the existence of stable line singularities when $R_0 < R < R_B$. In other words, the fundamental group $\pi_1(S^4)$ is trivial: $\pi_1(S^4) = 0$. The homotypic groups $\pi_2(S^4)$ and $\pi_3(S^4)$ are also trivial. Therefore point singularities and particle-like solitons[6] also do not exist for $R_0 < R < R_B$.

At large distances ($R > R_B$), the symmetry permits the existence of singular lines—disclinations. Upon passage around a closed contour about such a line, the director changes from **n** to −**n**.[2] In the space of the degeneracy parameter, such a contour corresponds to a line joining diametrically opposite points of the sphere S^2. Such a line can no longer be deformed to a point.[4] Since there are no singularities for $R_0 < R < R_B$, a singularity on a disclination line can be removed in this range of distances. Therefore for small B a disclination will have a large nonsingular core with radius $R \sim R_0$.

We shall consider an example of removal of a singularity on a disclination line in the range $R < R_B$. Let there be in the range $R > R_B$ a disclination with Frank index $\frac{1}{2}$, with director **n** lying in the XY plane, and with singularity line along Z. We consider a continuous change of the angle φ in the range $R < R_B$. We direct the vector **1** along Z. As is evident from the expression (1), it is possible, by changing φ from 0 to $\frac{2}{3}\pi$, to change the direction of uniaxial ordering from **n** to **1**. The field of the vector **1** is nonsingular in the case under consideration. Thus the singularity on the disclination line can be removed. This process is depicted in Fig. 1. The order parameter is represented geometrically by the semiaxes of ellipses. The axes of these ellipses are directed along the vectors **n** and **1**.

This spreading and disappearance of singularities with decrease and vanishing of the constant B should be observable on approach to a point on the phase diagram at which B vanishes. We shall estimate the values of R_B and R_0 for the typical nematic MBBA. The ratio between R_B and R_0 can be easily estimated from the following considerations. At the transition point, R_B and R_0 are of the same order. Away from it R_B is almost constant, since the order parameter varies only slightly. The value of R_0 behaves like $(T^* - T)^{1/2}$, where T is the temperature of the observation. Therefore

$$R_B/R_0 \sim (T^*-T)^{1/2}(T_c-T^*)^{-1/2}.$$

For MBBA we have $T_c - T^* \sim 0.2$ K[7]; choosing $T^* - T \sim 20$ K, we get $R_B/R_0 \sim 10$. The value of B was calculated by Ostrovskiĭ *et al.*,[7] who obtained $B \sim 6\cdot10^5$ erg/cm^3. Supposing that $k \sim 10^{-6}$ dyn and $Q_0 \sim 0.6$, we find $R_B \sim 2\cdot10^{-6}$ cm. Thus even in ordinary nematics there may occur a situation with "broadening" of the disclination core.

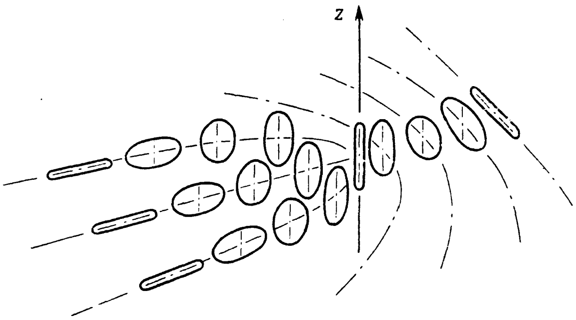

FIG. 1. Example of removal of a singularity on a disclination line.

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[1)]These considerations on the thermodynamics of a nematic are due to Kats and Pokrovskiĭ.[3]

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