Stimulated interaction of charged particles with electromagnetic radiation in a medium with nonstationary properties

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Classical and quantum theory are used to investigate stimulated interaction of charged particles with intense electromagnetic radiation in a medium with a nonstationary dielectric constant. As a result of the real energy exchange with the external field, the particles are accelerated (inverse transition effect) or decelerated (and give up energy to the wave in the form of stimulated transition radiation), depending on the initial phase. Simultaneous absorption-emission leads to inelastic diffraction scattering of the particles. Quantum modulation of a beam of charged particles at the frequency of the external wave and its harmonics is also obtained. The modulation depth becomes of the order of unity even in weak laser fields.

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INTRODUCTION

If a charge moves uniformly in a medium with nonstationary properties (specifically, in which the dielectric constant varies with time), radiation is produced[1-3] similar to transition radiation on the interface between different dielectrics.[4] In the presence of an external electromagnetic wave, this radiation acquires a stimulated character and enhancement of weak electromagnetic radiation at the expense of the energy of the charged particles is possible. The inverse process, stimulated absorption of quanta from the external field and their acceleration, also takes place (the inverse transition effect). Real multiphoton exchange between particles in the wave leads to the quantum effect of inelastic diffraction scattering of particles in the field of the electromagnetic wave. An important consequence of the direct and inverse stimulated transition effects is also the quantum modulation of a beam of charged particles at the frequency of the external wave and its harmonics.

Analogous phenomena in a homogeneous stationary medium (stimulated Cerenkov effect) were investigated in detail in Refs. 5 and 6, and effects in a spatially inhomogeneous medium were investigated in Ref. 7.

In the present paper we examine stimulated interaction of charged particles with a plane electomagnetic wave in a spatially homogeneous medium whose properties vary strongly with time.

In Sec. 1 we present the classical theory of the direct and inverse stimulated nonstationary transition effects. From the classical equations of motion we obtain the changes of the momentum and of the energy of the particle after the interaction. Depending on the initial phase, the particle is either accelerated (inverse transition effect) or decelerated and gives up its energy to the wave in the form of stimulated emission.

In Sec. 2 is presented the quantum theory of this effect, which has a multiphoton character. On the basis of the Klein-Gordon equation, we obtain the probability of the multiphoton absorption and emission, corresponding to inelastic diffraction scattering of electrons. It is shown that, depending on the energy spreads of the real beams, the diffraction scattering can lead to an energy broadening of the beam by a value of the order of the initial width.

In Sec. 3 we present the theory of quantum modulation of the beam of charged particles at the frequency of the wave and its harmonics. Owing to the nonstationary character of the medium, the beam is modulated also in time, and the modulation period depends strongly on the change of the dielectric constant of the medium. At sufficiently large variation of the latter, hard quanta (on the order of the electron energy) appear in the spectrum of the wave and the probability amplitude of their absorption is proportional to the intensity of the transformed wave (in the one-photon case). The interaction of the electron with such quanta leads also to formation of electron-positron pairs even in first order in the field, in contrast to the case of a stationary plasma, where this is possible in an incomparably higher order in the field.

1. CLASSICAL THEORY OF THE INTERACTION

Let a charged particle (electron) with constant initial velocity v_0 move in a spatially homogeneous medium whose dielectric constant ε changes abruptly at the instant of time $t=0$ from a value ε_1 $(t<0)$ to ε_2 $(t>0)$, and let a strong electromagnetic wave propagate in this medium. Such a jumplike change of the dielectric constant can be realized by abruptly changing the density (pressure) of the medium. Particular interest attaches to the case when the nonstationary character of the dielectric constant of the medium is automatically produced by the wave intensity (nonlinear polarization of the medium, formation of a laser plasma in solid films, etc.).

We consider first the question of how the change of the properties of the medium affects the external monochromatic wave. If a plane monochromatic wave of frequency ω and electric intensity amplitude E propagates in such a medium ($\varepsilon=\varepsilon_1$) at $t<0$, then at $t>0(\varepsilon=\varepsilon_2)$ there are two waves—incident and reflected. Since the medium is assumed to be spatially homogeneous, the wave vectors are $\mathbf{k}=\mathbf{k}_1=\mathbf{k}_2=\text{const}$, and the nonstationarity of the medium leads to a change of frequency. Using the boundary conditions for the electric induction of the waves $D=D_1+D_2$ and for the magnetic field intensity $H=H_1+H_2$ at $t=0$ we obtain, in the case of linear polarization of the wave, for the amplitudes of the electric field of the transmitted and reflected waves, respectively,

$$E_1=\frac{\varepsilon_1^{1/2}(\varepsilon_1^{1/2}+\varepsilon_2^{1/2})}{2\varepsilon_2}E,\quad E_2=\frac{\varepsilon_1^{1/2}(\varepsilon_1^{1/2}-\varepsilon_2^{1/2})}{2\varepsilon_2}E. \tag{1}$$

If ε changes abruptly, the initial monochromatic wave is transformed into a continuous wave spectrum. Taking (1) into account, we obtain for the spectral amplitude of the field

$$E(\Omega)=i\frac{E}{2\pi}\frac{\varepsilon_2-\varepsilon_1}{\varepsilon_2}\frac{\Omega^2}{(\Omega-\omega)(\Omega^2-\omega^2\varepsilon_1/\varepsilon_2)}. \tag{2}$$

Actually the Ω spectrum depends on the time during which the properties of the medium change. For an abrupt change of ε it is necessary that this time be $\tau\ll 2\pi/\Omega$, and then the monochromatic wave is transformed into the spectrum (2).

The problem of the interaction of the electron with the electromagnetic radiation in a medium with nonstationary properties reduces now to the interaction of an electron with the field (2). We consider the dynamics of such an interaction with the aid of classical theory.

Let the wave propagate along the x axis and let the electric field intensity be directed along the y axis. Then the relativistic equations of motion of the electron in the field (2) take the form

$$\begin{gathered}\frac{dp_x}{dt}=-i\frac{e}{c}k\int_{-\infty}^{+\infty}v_yF(\Omega,x,t)\,d\Omega,\\ \frac{dp_y}{dt}=i\frac{e}{c}\int_{-\infty}^{+\infty}(kv_x-\Omega)F(\Omega,x,t)\,d\Omega,\quad \frac{dp_z}{dt}=0,\end{gathered} \tag{3}$$

where

$$F(\Omega,x,t)=A(\Omega)e^{i\Omega t-ikx}-A^*(\Omega)e^{-i\Omega t+ikx},$$

and $A(\Omega)$ is the spectral amplitude of the vector potential of the field (2). The electron motion along the z axis remains free, we can choose the electron velocity in the xy plane:

We shall solve the system (3) in a perturbation-theory approximation with respect to the field. The parameter of the perturbation theory is $\xi=eA/mc^2$ ($\xi^2=e^2A^2/m^2c^4$ is a relativistically invariant parameter of the wave intensity), which is much less than unity even for strong laser fields, $\xi\ll 1$. Integrating (3) with respect to time from $-\infty$ to $+\infty$, we obtain in first-order approximation in ξ the following expressions for the changes of the momentum and energy after the interaction:

$$\begin{gathered}\Delta p_y=\Delta p_z=0,\quad \Delta p_x=\Delta\mathscr{E}/v_0\cos\theta,\\ \Delta\mathscr{E}=2mc^2\xi\frac{v_0^3}{c^3}(\varepsilon_1-\varepsilon_2)\frac{\sin\theta\cos^2\theta}{(1-\varepsilon_1^{1/2}v_0c^{-1}\cos\theta)(1-\varepsilon_2v_0^2c^{-2}\cos^2\theta)}\\ \times\sin\left(\omega t_0\varepsilon_1^{1/2}\frac{v_0}{c}\cos\theta\right),\end{gathered} \tag{4}$$

where t_0 is the instant of time corresponding to the initial phase of the electron in the wave, which has become transformed into a spectrum at $t=0$. As seen from (4), depending on this phase, the electron is either accelerated after the interaction or is decelerated and gives up energy to the wave. This real energy exchange is due to the direct and inverse stimulated (nonstationary) transition effect. In the case of an electron beam, different electrons in different initial phases $\Phi_0=\omega t_0\varepsilon_1^{1/2}c^{-1}v_0\cos\theta$, acquire or lose different energies at the interaction. As a result, the initial monoenergetic beam will broaden, provided that the temporal relative retardation of the electrons in the beam exceeds the value

$$T=2\pi/\omega\varepsilon_1^{1/2}\frac{v_0}{c}\cos\theta,$$

(this is always satisfied for real beams). The beam acquires in this case width $\gamma=2\Delta\mathscr{E}_{\max}$.

Let us estimate the energy acquired or lost by the electron after the interaction. Since we are interested only in the stimulated transition effect due to the change of ε, we must exclude the Čerenkov effect in the medium with ε_1 and ε_2, i.e., we put

$$v_0<\frac{c}{\cos\theta}\frac{1}{\varepsilon_{1,2}^{1/2}}.$$

Then an electron with initial energy $\mathscr{E}_0\sim 10$ MeV in a plasma with $\varepsilon_1\sim\varepsilon_2\sim 1$ at $\xi\sim 10^{-2}$ (which corresponds, for example, to a CO_2-laser intensity $E\sim 10^7$ V/cm) will acquire or lose an energy $\Delta\mathscr{E}\sim 1$ MeV ($\Delta\mathscr{E}\ll\mathscr{E}_0$, which agrees with the perturbation-theory approximation). In the case of an electron beam this can be observed in the form of an energy broadening of the beam by an amount $\gamma\sim 2$ MeV.

2. QUANTUM THEORY OF STIMULATED MULTIPHOTON PRODUCTION

The classical change of the energy and momentum of the electron (4) as a result of stimulated interaction with electromagnetic radiation in a nonstationary medium shows that such a real energy exchange with the field (2) corresponds to stimulated absorption and emission of a large number of photons. We elucidate the multiphoton character of the stimulated nonstationary transition effect on the basis of quantum theory. To this end, we consider the quantum motion of a relativistic electron in a field (2). The role of the spin is inessential here, and we therefore solve the Klein-Gordon equation. Assuming the same geometry as in Sec. 1, this equation takes the form

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2}=[-c^2\hbar^2\nabla^2+2iec\hbar\nabla_\nu A_\nu+e^2A^2+m^2c^4]\psi. \tag{5}$$

We shall solve (5) in the impulse approximation, when the initial plane wave of the electron is slowly distorted in this field, i.e., we take the solution in the form

$$\psi(\mathbf{r},t)=f(x,t)\exp\left[\frac{i}{\hbar}(\mathbf{p}_0\mathbf{r}-\mathscr{E}_0t)\right], \tag{6}$$

where $f(x,t)$ is a function, slower than exponential, of the coordinates and of the time. This corresponds to a small change of the momentum and energy of the electron in the field compared with the initial values: $\Delta p \ll p_0, \Delta\mathscr{E} \ll \mathscr{E}_0$, i.e., this is perturbation theory in terms of the energy-momentum and corresponds to the classical case. Substituting (6) in (5) taking into account the slow variation of the function $f(\partial f/\partial t \ll f\mathscr{E}_0/\hbar, \partial f/\partial x \ll fp_{0x}/\hbar)$, and changing to the Fourier transform of the field, we obtain for an electron beam with initial density $N_0=\text{const}$ after the interaction $(t\to+\infty)$, taking (2) into account,

$$\psi(\mathbf{r},t)=\sqrt{N_0}\exp\left(\frac{i}{\hbar}p_{0y}y\right)\sum_{s=-\infty}^{+\infty}i^sJ_s(\alpha)$$
$$\times\exp\left[-\frac{i}{\hbar}\left(\mathscr{E}_0-s\hbar\omega\varepsilon_1^{1/2}\frac{v_0}{c}\cos\theta\right)t+\frac{i}{\hbar}\left(p_{0x}-s\hbar\omega\frac{\varepsilon_1^{1/2}}{c}\right)x\right], \tag{7}$$

where the argument of the Bessel function is

$$\alpha=2\xi\frac{mc^2}{\hbar\omega}\frac{v_0^2}{c^2}\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_1^{1/2}}\frac{\sin\theta\cos\theta}{(1-\varepsilon_1^{1/2}v_0c^{-1}\cos\theta)(1-\varepsilon_2v_0^2c^{-2}\cos^2\theta)}. \tag{8}$$

As seen from (7), after stimulated interaction with the wave the electron actually absorbs and emits s photons, as a result of which the momentum and energy after the interaction are altered:

$$\Delta p_x=s\hbar\frac{\omega}{c}\varepsilon_1^{1/2},\quad \Delta p_y=0,\quad \Delta\mathscr{E}=s\hbar\omega\varepsilon_1^{1/2}\frac{v_0}{c}\cos\theta. \tag{9}$$

The probability of this process is

$$W_s=J_s^2(\alpha), \tag{10}$$

and corresponds to inelastic diffraction scattering of the electrons. The initial plane wave of the electron is spread out by the diffraction into a packet of waves with arbitrary number of absorbed and emitted photons (since $J_s^2(\alpha)=J_{-s}^2(\alpha)$, and this spreading is equally probable: all electrons have the same probability of absorbing and emitting s quanta). Comparison of the expression for α with a classical change of the energy-momentum of the the electron (4) shows that

$$\alpha=(\Delta p_x)_{max}/\hbar k. \tag{11}$$

Since the classical change Δp_s corresponds to $\alpha \gg 1$, the main contribution to the scattering is made by photons $s\sim\alpha$ (as seen from (10), photons $s\sim\alpha\gg 1$ are absorbed and emitted with maximum probability, in full agreement with the classical change of the electron momentum). For the numerical values of the parameters given in Sec. 1, we have $s_{max}\sim 10^6$. The probability of emission and absorption of this number of photons is $W_{s\,max}\sim 10^{-4}$. For a monochromatic electron beam this means that $\approx 0.01\%$ of the beam has classical probability of being accelerated after scattering (conversely, the same number of electrons is slowed down). For real particle beams having energy spreads, it is necessary to average the scattering probability (9) over these spreads.

Let $\rho_0(\mathscr{E})$ be the initial distribution function of the electron beam with energy width $\gamma_0=\mathscr{E}-\bar{\mathscr{E}}$. In the final distribution function of the electrons in the beam, after the scattering, is

$$\rho_f(\mathscr{E}')=\int_{-\infty}^{+\infty}\rho_0(\mathscr{E})J_s^2(\alpha)d\mathscr{E}\bigg/\int_{-\infty}^{+\infty}\rho_0(\mathscr{E})d\mathscr{E}. \tag{12}$$

Since the main contribution to the integral is made by photons $s\sim\alpha$ (principal diffraction maxima), and since on the other hand

$$s=\Delta\mathscr{E}/\hbar\omega\varepsilon_1^{1/2}v_0c^{-1}\cos\theta,$$

the final distribution function $\rho_f(\mathscr{E}')$ depends on the ratio $\gamma_0/\Delta\mathscr{E}$.

If $\gamma_0\ll\Delta\mathscr{E}$, then $\rho_f(\mathscr{E}')\to W_s=J_s^2(\bar\alpha)$, where $\bar\alpha$ corresponds to the average energy $\bar{\mathscr{E}}$ of the electrons of the initial beam, and diffractive spreading of the electron energy will be observed (a pure quantum effect). If $\gamma_0\sim\Delta\mathscr{E}$, then we get from (12)

$$\rho_f(\mathscr{E}')={}^1/_2[\rho_0(\mathscr{E}_0+\Delta\mathscr{E})+\rho_0(\mathscr{E}_0-\Delta\mathscr{E})]. \tag{13}$$

We see therefore that a finite width of the beam $\gamma=\gamma_0+2|\Delta\mathscr{E}|$, i.e., multiphoton absorption or emission of electrons, leads in this case to an energy broadening of the beam by a value on the order of the initial widths. This is in full agreement with classical theory and it appears that it is precisely this case which can be observed in experiment, namely, beam broadening which is a consequence of the direct and inverse stimulated nonstationary transition effect. In the case $\gamma_0\gg\Delta\mathscr{E}$ we $\rho_f(\mathscr{E}')\to\rho_0(\mathscr{E})$ and it is clear that such an energy exchange $(\Delta\mathscr{E}\ll\gamma_0)$ cannot lead to a change in the energy width of the beam.

3. QUANTUM MODULATION OF ELECTRON BEAM

In the preceding section, when solving the quantum equation of motion, we did not take into account the quantum recoil of the electron (at a slow variation of ψ we neglected the second derivatives of ψ compared with the first derivatives). If account is also taken of the quantum recoil obtained by the electron in stimulated absorption or emission of field photons, then this leads to quantum modulation of the initially homogeneous electron beam. We solve for this purpose the Klein-Gordon equation (5) with the aid of perturbation theory in the field. In first-order approximation (single-photon absorption-emission) we obtain for the wave function of the electron from (5) the equation

$$\frac{\partial^2\psi_1}{\partial x^2}-\frac{1}{c^2}\frac{\partial^2\psi_1}{\partial t^2}-\frac{1}{\hbar^2c^2}(m^2c^4+c^2p_{0y}^2)\psi_1=-2\frac{ep_{0y}}{c\hbar^2}[A_y(t)e^{-ikx}+A_y^*(t)e^{ikx}]\psi_0, \quad (14)$$

where

$$\psi_1(\mathbf{r},t)=[\Phi_1(t)e^{-ikx}+\Phi_2(t)e^{ikx}]\exp\left[\frac{i}{\hbar}(\mathbf{p}_0\mathbf{r}-\mathscr{E}_0t)\right]. \quad (15)$$

describes the initial beam with density $N_0=\text{const}$. The solution of (14) is sought in the form

$$\psi_0=\sqrt{N_0}\exp\left[\frac{i}{\hbar}(\mathbf{p}_0\mathbf{r}-\mathscr{E}_0t)\right]$$

Substituting (15) in (14) and changing over from $A_y(t)$ to the Fourier component of the field we obtain for $\Phi_1(t)$ and $\Phi_2(t)$ after the interaction ($t\to+\infty$)

$$\Phi_1(t)=-4i\sqrt{N_0}\frac{\pi ecp_{0y}}{\hbar^2(\Omega_1-\Omega_2)}[A(\Omega_1)e^{i\Omega_1t}-A(\Omega_2)e^{i\Omega_2t}],$$

$$\Phi_2(t)=-4i\sqrt{N_0}\frac{\pi ecp_{0y}}{\hbar^2(\Omega_1'-\Omega_2')}[A^*(-\Omega_1')\exp(i\Omega_1't)-A^*(-\Omega_2')\exp(i\Omega_2't)], \quad (16)$$

where

$$\Omega_{1,2}=\frac{\mathscr{E}_0}{\hbar}\mp\left[\left(\frac{\mathscr{E}_0}{\hbar}-\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}\right)^2+\varepsilon_1\omega^2\left(1-\frac{v_{0x}^2}{c^2}\right)\right]^{1/2},$$
$$\Omega_{1,2}'=\frac{\mathscr{E}_0}{\hbar}\mp\left[\left(\frac{\mathscr{E}_0}{\hbar}+\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}\right)^2+\varepsilon_1\omega^2\left(1-\frac{v_{0x}^2}{c^2}\right)\right]^{1/2}. \quad (17)$$

Formulas (17) correspond to the energy-momentum conservation laws for the electron: in this process the electron can emit only photons with frequencies $\Omega_{1,2}$ and absorb photons with frequencies $\Omega_{1,2}'$. Inasmuch as $\mathscr{E}_0/\hbar\gg\varepsilon_1^{1/2}\omega v_{0x}/c$, at laser frequencies we expand the square roots in (17) in a series, and retain only the small terms of first order. We then obtain

$$\Omega_1\approx\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}-\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right),$$
$$\Omega_2\approx2\frac{\mathscr{E}_0}{\hbar}-\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}+\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right),$$
$$\Omega_1'\approx-\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}-\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right),$$
$$\Omega_2'\approx2\frac{\mathscr{E}_0}{\hbar}+\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}+\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right). \quad (18)$$

These expressions show that emission of a photon with frequency Ω_2 and absorption with frequency Ω_2' has a clearly quantum character, and its probability, as seen from (2), depends on the change of the dielectric constant of the medium $\varepsilon_1-\varepsilon_2$. We consider therefore two cases: $\varepsilon_1/\varepsilon_2\lesssim1$ and $\varepsilon_1/\varepsilon_2\gg1$.

If $\varepsilon_1/\varepsilon_2\lesssim1$ (this corresponds in fact to real situations), we get from (2)

$$A(\Omega_2)\approx A\left(2\frac{\mathscr{E}_0}{\hbar}\right)\ll A(\Omega_1)\approx A\left(\varepsilon_1^{1/2}\omega\frac{v_{0x}}{c}\right),$$

so that in this case we can neglect in (16) the pure quantum process of emission and absorption of excessively hard photons $\Omega_2\simeq2\mathscr{E}_0/\hbar$. In this case we obtain for the density of the beam ($|\psi_0+\psi_1|^2$) after the interaction

$$N=N_0\left\{1+2\alpha\sin\left[\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right)t\right]\cos\left[\varepsilon_1^{1/2}\frac{v_{0x}}{c}\omega\left(t-\frac{x}{v_{0x}}\right)\right]\right\} \quad (19)$$

where

$$\alpha=\frac{(\Delta p_x)_{max}}{\hbar\varepsilon_1^{1/2}\omega/c}$$

coincides with the argument of the Bessel function (8). As seen from (19), stimulated absorption and emission of the photons Ω_1 causes the beam to be modulated at a frequency $\Omega_1=\varepsilon_1^{1/2}v_{0x}\omega/c$ with a depth $\Gamma_1=2\alpha$. The period of the time modulation is

$$T_1=\frac{4\pi\mathscr{E}_0}{\varepsilon_1\hbar\omega^2(1-v_{0x}^2/c^2)}.$$

Formula (19) corresponds to modulation of a beam at the frequency of the wave (fundamental frequency). If we obtain the wave function of the electron in the next higher orders of perturbation theory, then we obtain the beam modulation at higher harmonics of the wave frequency. The modulation depth at the s-th harmonic is $\Gamma_s\sim\Gamma_1^s$.

Let us estimate the depth of modulation. For relativistic beams at $\sin\theta\sim\cos\theta\sim1$ ($v_0<c/\varepsilon_{1,2}^{1/2}\cos\theta$ to exclude the Čerenkov effect), as seen from (8),

$$\Gamma_1\sim4\xi mc^2/\hbar\omega\sim10^6\xi$$

and $\Gamma_1\sim10\%$ already at $\xi\sim10^{-7}$, which corresponds to an Nd:YAG – laser($\lambda=1.06\ \mu m$) intensity $E\sim10^3\ V/cm$ or a CO_2 – laser ($\lambda=10.6\ \mu m$) intensity $E\sim10^2\ V/cm$. The period of the time modulation (for $\mathscr{E}_0\sim1$ MeV) is $T_1\sim10$ nsec. These estimates show that in practice it is possible to obtain beam modulation also at higher harmonics of the laser frequency.

In the case when $\varepsilon_1/\varepsilon_2\gg1$, it is necessary to take also $A(\Omega_2)$ into account in (16). This leads to the following expression for the beam density:

$$N=N_0\left\{1+\Gamma_1\sin\left[\varepsilon_1\frac{\hbar\omega^2}{2\mathscr{E}_0}\left(1-\frac{v_{0x}^2}{c^2}\right)t\right]\cos\left[\varepsilon_1^{1/2}\frac{v_{0x}}{c}\omega\left(t-\frac{x}{v_{0x}}\right)\right]\right.$$
$$\left.+\Gamma_2\sin\left(2\frac{\mathscr{E}_0}{\hbar}t\right)\cos\left[\varepsilon_1^{1/2}\frac{v_{0x}}{c}\omega\left(t+\frac{x}{v_{0x}}\right)\right]\right\}, \quad (20)$$

where the modulation depth due to the absorption-emission of photons Ω_2 is

$$\Gamma_2=\xi\frac{mc^2}{\mathscr{E}_0}\frac{\hbar\omega}{\mathscr{E}_0}\frac{v_0}{c}\sin\theta\frac{\varepsilon_1/\varepsilon_2-1}{1-(\varepsilon_1/\varepsilon_2)(\hbar\omega/2\mathscr{E}_0)^2}. \quad (21)$$

The period of the time modulation is $T_2=\pi\hbar/\mathscr{E}_0$.

Comparison of the expressions for Γ_2 with Γ_1 and for T_2 with T_1 shows that $\Gamma_2\ll\Gamma_1$ and $T_2\ll T_1$. At the same values of the parameters $T_2\sim10^{-21}$ sec, and $\Gamma_2\sim10^{-5}\xi\varepsilon_1/\varepsilon_2$. In practice it is extremely difficult to realize such a large change of ε ($\varepsilon_1/\varepsilon_2\gg1$). This case, however, can be of interest in astrophysics, where the radiating matter is in a strongly nonstationary state. Since the modulated particle beam emits coherently, this mechanism can be used to explain the pulsar radiation, which has high intensity.

Finally, we note that the formation of hard photons $\sim\mathscr{E}_0$ as a result of the abrupt change in the properties of the medium leads to production of electron-positron pairs. As seen from (12), the probability amplitude of absorption-emission of such photons by an electron is proportional to the parameter

$$\xi_1=\xi\left(\frac{mc^2}{\mathscr{E}_0}\right)\left(\frac{\hbar\omega}{\mathscr{E}_0}\right)\frac{\varepsilon_1}{\varepsilon_2}$$

which characterizes the field of the transformed wave ($E\to E\varepsilon_1/\varepsilon_2$, $\omega\to\mathscr{E}_0^2/mc^2\hbar$). Consequently, the probability of electron-positron pair production will be $\sim\xi_1^2$, in contrast to the case of pair production in a stationary plasma in the field of strong waves or radiation, where the probability of this process is $\sim\xi^{2s}$ ($s\gtrsim 10^6$ for optical photons, and $\xi\ll 1$). Thus, the principal small quantity in the probability of electron-positron pair production is eliminated in this case.

[1] V. L. Ginzburg, Radiofizika 16, 512 (1973).
[2] V. L. Ginzburg, and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. 65, 132 (1973) [Sov. Phys. JETP 38, 65 (1974)].
[3] V. L. Ginzburg, Teoreticheskaya fizika i astrofizika (Theoretical Physics and Astrophysics), Nauka, 1975.
[4] V. L. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. 16, 15 (1946).
[5] V. M. Arutyunyan and G. K. Avetisyan, Kvantovaya Elektron. (Moscow) 7, 54 (1972) [Sov. J. Quantum Electron. 2, 39 (1972)]; Zh. Eksp. Teor. Fiz. 62, 1639 (1972) [Sov. Phys. JETP 35, 854 (1972)].
[6] H. K. Avetissian, Phys. Lett. A 58, 144 (1976); A 63, 7 (1977); A 63, 9 (1977).
[7] V. M. Arutyunyan and S. G. Oganesyan, Zh. Eksp. Teor. Fiz. 72, 466 (1977) [Sov. Phys. JETP 45, 244 (1977)].

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Radiative effects near cyclotron resonance

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The mass operator is obtained for an electron moving in the field of an intense wave propagating along a magnetic field. An operator diagram technique is used for the analysis. The radiative shift of the levels and the electron radiation probability are obtained. The cross section is calculated of the Compton effect on an electron moving in a magnetic field. The region near cyclotron resonance is analyzed in detail.

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If a plane wave propagates along a magnetic field, a very interesting situation is realized: at the cyclotron-resonance point, where the wave frequency coincides with the frequency of the particle motion in the magnetic field (with allowance for the Doppler shift), resonant energy transfer from the particle to the wave and back is possible. This process (cyclotron resonance) can take place in a large number of physical phenomena, particularly in the formation of pulsar radiation, as well as in devices used to amplify electromagnetic waves or to accelerate particles by a laser wave.

In this connection, an analysis of radiative effects in a field of the indicated configuration, including the vicinity of the cyclotron resonance, is of undoubted interest. An approach to the analysis of this problem was formulated by us in an earlier paper,[1] where the case of particles with zero spin was considered, and where a brief bibliography concerning processes in this field is given. In the present paper we consider the case of fundamental physical interest, that of particles with spin 1/2. We used in our approach an operator diagram technique based on the operator representation of the Green's function of a charged particle in a given field with a subsequent specific transformation of the operator expressions. This technique was developed earlier for the analysis of radiative effects in the case of a homogeneous external field by Katkov, Strakhovenko and one of us,[2] and for the case of a plane electromagnetic wave by Katkov, Strakhovenko, and both of us.[3] The analysis of radiative effects in a field having the configuration considered in the present paper is a substantially more complicated problem, and the preceding papers were limited to an analysis of some particular cases. In the present paper we obtain a general expression for the mass operator of an electron in a given field, from which we deduce both the probability of the electron emission and the quasienergy level shift. We analyze some limiting cases and, in particular, obtain the cross section of Compton scattering in a magnetic field. Effects near cyclotron resonance are considered in detail.

We describe the electromagnetic field by a potential

$$\mathscr{A}_\mu=A_\mu(x_\parallel)+A_\mu(\varphi), \tag{1}$$

where $\varphi=\varkappa x$ and $\varkappa x_\parallel=0$. Assume that the magnetic field is directed along the wave-propagation axis (the 3 axis); then

$$A^1(x_\parallel)=-x^2H,\quad A^\mu(\varphi)=n_1^\mu a_1(\varphi)+n_2^\mu a_2(\varphi); \tag{2}$$

Here $\varphi=\varkappa x=x^0-x^3$; we have introduced the vectors $n_1^\mu=g_1^\mu$, $n_2^\mu=g_2^\mu$, $\varkappa^\mu=g_0^\mu+g_3^\mu$, where g_ν^μ are components of

0038-5646/78/080196-06$02.40