absorption-emission of such photons by an electron is proportional to the parameter

$$\xi_1=\xi\left(\frac{mc^2}{\mathscr{E}_0}\right)\left(\frac{\hbar\omega}{\mathscr{E}_0}\right)\frac{\varepsilon_1}{\varepsilon_2}$$

which characterizes the field of the transformed wave ($E\rightarrow E\varepsilon_1/\varepsilon_2, \omega\rightarrow\mathscr{E}_0^2/mc^2\hbar$). Consequently, the probability of electron-positron pair production will be $\sim\xi_1^2$, in contrast to the case of pair production in a stationary plasma in the field of strong waves or radiation, where the probability of this process is $\sim\xi^{2s}(s\gtrsim 10^6$ for optical photons, and $\xi\ll 1)$. Thus, the principal small quantity in the probability of electron-positron pair production is eliminated in this case.

[1]V. L. Ginzburg, Radiofizika 16, 512 (1973).

[2]V. L. Ginzburg, and V. N. Tsytovich, Zh. Eksp. Teor. Fiz. 65, 132 (1973) [Sov. Phys. JETP 38, 65 (1974)].

[3]V. L. Ginzburg, Teoreticheskaya fizika i astrofizika (Theoretical Physics and Astrophysics), Nauka, 1975.

[4]V. L. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. 16, 15 (1946).

[5]V. M. Arutyunyan and G. K. Avetisyan, Kvantovaya Elektron. (Moscow) 7, 54 (1972) [Sov. J. Quantum Electron. 2, 39 (1972)]; Zh. Eksp. Teor. Fiz. 62, 1639 (1972) [Sov. Phys. JETP 35, 854 (1972)].

[6]H. K. Avetissian, Phys. Lett. A 58, 144 (1976); A 63, 7 (1977); A 63, 9 (1977).

[7]V. M. Arutyunyan and S. G. Oganesyan, Zh. Eksp. Teor. Fiz. 72, 466 (1977) [Sov. Phys. JETP 45, 244 (1977)].

Translated by J. G. Adashko

Radiative effects near cyclotron resonance

V. N. Baĭer and A. I. Mil'shteĭn

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences

(Submitted 26 January 1978)
Zh. Eksp. Teor. Fiz. 75, 390–401 (August 1978)

The mass operator is obtained for an electron moving in the field of an intense wave propagating along a magnetic field. An operator diagram technique is used for the analysis. The radiative shift of the levels and the electron radiation probability are obtained. The cross section is calculated of the Compton effect on an electron moving in a magnetic field. The region near cyclotron resonance is analyzed in detail.

PACS numbers: 41.70.+t

If a plane wave propagates along a magnetic field, a very interesting situation is realized: at the cyclotron-resonance point, where the wave frequency coincides with the frequency of the particle motion in the magnetic field (with allowance for the Doppler shift), resonant energy transfer from the particle to the wave and back is possible. This process (cyclotron resonance) can take place in a large number of physical phenomena, particularly in the formation of pulsar radiation, as well as in devices used to amplify electromagnetic waves or to accelerate particles by a laser wave.

In this connection, an analysis of radiative effects in a field of the indicated configuration, including the vicinity of the cyclotron resonance, is of undoubted interest. An approach to the analysis of this problem was formulated by us in an earlier paper,[1] where the case of particles with zero spin was considered, and where a brief bibliography concerning processes in this field is given. In the present paper we consider the case of fundamental physical interest, that of particles with spin 1/2. We used in our approach an operator diagram technique based on the operator representation of the Green's function of a charged particle in a given field with a subsequent specific transformation of the operator expressions. This technique was developed earlier for the analysis of radiative effects in the case of a homogeneous external field by Katkov, Strakhovenko and one of us,[2] and for the case of a plane electromagnetic wave by Katkov, Strakhovenko, and both of us.[3] The analysis of radiative effects in a field having the configuration considered in the present paper is a substantially more complicated problem, and the preceding papers were limited to an analysis of some particular cases. In the present paper we obtain a general expression for the mass operator of an electron in a given field, from which we deduce both the probability of the electron emission and the quasienergy level shift. We analyze some limiting cases and, in particular, obtain the cross section of Compton scattering in a magnetic field. Effects near cyclotron resonance are considered in detail.

We describe the electromagnetic field by a potential

$$\mathscr{A}_\mu=A_\mu(x_\parallel)+A_\mu(\varphi), \qquad (1)$$

where $\varphi=\varkappa x$ and $\varkappa x_\parallel=0$. Assume that the magnetic field is directed along the wave-propagation axis (the 3 axis); then

$$A^1(x_\parallel)=-x^2H, \quad A^\mu(\varphi)=n_1^\mu a_1(\varphi)+n_2^\mu a_2(\varphi); \qquad (2)$$

Here $\varphi=\varkappa x=x^0-x^3$; we have introduced the vectors $n_1^\mu=g_1^\mu$, $n_2^\mu=g_2^\mu$, $\varkappa^\mu=g_0^\mu+g_3^\mu$, where g_ν^μ are components of the metric tensor.

The electromagnetic field intensity is represented in the form

$$\mathcal{F}^{\mu\nu}=F^{\mu\nu}+\sum_{k=1}^{2} f_k^{\mu\nu}a_k'(\varphi), \quad f_k^{\mu\nu}=\varkappa^\mu n_k^\nu-\varkappa^\nu n_k^\mu, \tag{3}$$

where $F^{21}=H$, H is the magnetic field, and $a_k'(\varphi) \equiv da_k(\varphi)/d\varphi$.

The mass operator of an electron in an external field can be represented in the form [see Ref. 2, formula (3.1)]

$$M^{(1/2)}=-\frac{ie^2}{(2\pi)^4}\int\frac{d^4k}{k^2+i0}\gamma^\mu\frac{\hat{P}-\hat{k}+m}{(\hat{P}-\hat{k})^2-m^2+i0}\gamma_\mu, \tag{4}$$

where $P_\mu=i\partial_\mu-e\mathcal{A}_\mu(e>0)$. We carry out the standard parametrization of the integrand

$$\frac{1}{k^2+i0}\frac{1}{(\hat{P}-\hat{k})^2-m^2+i0}$$
$$=-\int_0^\infty s\,ds\int_0^1 du\exp(-isum^2)\exp\{i[su(\hat{P}-\hat{k})^2+s(1-u)k^2]\}. \tag{5}$$

Formula (4) can then be rewritten in the form

$$M^{(1/2)}=\frac{ie^2}{(2\pi)^4}\int_0^\infty s\,ds\int_0^1 du\,\tilde{M}^{(1/2)}; \tag{6}$$

Here

$$\tilde{M}^{(1/2)}=\int d^4k\gamma^\mu(\hat{P}-\hat{k}+m)\exp\{isu[(\hat{P}-\hat{k})^2-m^2]\}\gamma_\mu\exp\{is(1-u)k^2\}. \tag{7}$$

The evaluation of the integral with respect to k reduces primarily to calculation of the quantity

$$Q^{(1/2)}=\int d^4k\exp[isu(\hat{P}-\hat{k})^2]\exp[is(1-u)k^2]$$
$$=\int d^4k\,e^{-ikX}\exp(isu\hat{P}^2)e^{ikX}\exp[is(1-u)k^2]. \tag{8}$$

where the shift operator in momentum space is used; for a certain function $f(P)$ we have

$$e^{-ikX}f(P)e^{ikX}=f(P-k), \quad [P_\mu, X_\nu]=ig_{\mu\nu}.$$

To calculate (8) it is necessary to transform the operator expression $\exp(isu\hat{P}^2)$. This transformation (disentanglement), being one of the central points in the present method, is given in the Appendix. Substituting expression (A.14) in formula (8), we integrate with respect to k. The integration with respect to variables k^0 and k^3 can be carried out with the aid of a procedure used in Ref. 3. It is necessary in this case to change over to the variables

$$k_\varphi={}^1/_2(k^0+k^3), \quad k_\tau={}^1/_2(k^0-k^3), \tag{9}$$

and the integration with respect to k_φ yields $\delta(k_\tau-up_\tau)$, meaning that the integration with respect to k_τ reduces to the substitution $k_\tau\to up_\tau$. Integration with respect to the variables k^1 and k^2 can be carried out with the aid of the procedure adopted in Ref. 2. As a result we get

$$Q^{(1/2)}=-\frac{i\pi^2}{s^2}\frac{1}{D^{1/2}}e^{i\Phi}\exp\left\{i\left[(P-q)_\parallel^2\frac{\rho}{eH}\right]\right\}$$
$$\times(1+\hat{\varkappa}\hat{g})\exp\left(isu\frac{e\sigma\mathcal{F}}{2}\right)\exp(i\eta P_\perp^2), \tag{10}$$

where the following notation is used[1)] [see also formulas (A.4) and (A.7)]

$$q=x\int_0^1\Delta(\eta y)e^{-2xBy}dy(\operatorname{ctg}x+B), \quad \eta=su(1-u),$$
$$\rho(x)=x-a(x), \quad a(x)=\operatorname{arctg}\left[\frac{1-\cos 2x}{\sin 2x+2x(1-u)/u}\right],$$
$$\Phi=su\left[2su\int_0^1 dy_2\int_0^{y_2}\Delta(\eta y_1)e^{-2Bx(y_1-y_2)}eF\Delta(\eta y_2)\right.$$
$$\left.+\int_0^1\Delta^2(\eta y)dy-x\operatorname{ctg}x\left(\int_0^1\Delta(\eta y)e^{-2Bxy}dy\right)^2\right], \tag{11}$$
$$g=su\int_0^1\Delta'(\eta y)e^{-2eFsyu}dy,$$
$$D=\frac{u^2}{x^2}\left[\sin^2 x+x^2\left(\frac{1-u}{u}\right)^2+\frac{1-u}{u}x\sin 2x\right];$$

Here $x=sueH$ and the matrix $B^{\mu\nu}=F^{\mu\nu}/H$.

The expression for $\tilde{M}^{(1/2)}$(7) contains terms of two types, without and with k_μ in the factor preceding the exponential expression. We have calculated the terms of the first type, and it remains to find the terms of the second type. We use here the same method as in Ref. 3 (see formula (3.9) therein):

$$\int\hat{k}\exp[isu(\hat{P}-\hat{k})^2]\exp[is(1-u)k^2]d^4k=\frac{u}{2\eta}\gamma^\mu[X_\mu, Q^{(1/2)}]. \tag{12}$$

Calculating the commutators in this expression and substituting (11) and (12) in (7), we have for the mass operator in the given field

$$M^{(1/2)}=\frac{\alpha}{2\pi}\int_0^\infty\frac{ds}{s}\int_0^1\frac{du}{D^{1/2}}e^{-isum^2}\left\{\left[2m\cos x+E\hat{P}\right.\right.$$
$$\left.\left.+\frac{1}{2}\exp\left(-isu\frac{e\sigma\mathcal{F}}{2}\right)\hat{\varkappa}r_1+Er_2\right]Q_0-uEQ_0P_\perp\right\}, \tag{13}$$

where

$$E=-e^{-isue\sigma\mathcal{F}/2}+\hat{\varkappa}(\gamma e^{-Bx}g),$$
$$r_2=\frac{u}{2\eta}\left(\frac{e^{-2B\rho}-1}{eF}\right)(P-q)_\parallel, \quad r_1=\frac{\partial\Phi}{\partial p_\tau}\frac{u}{2\eta}+\frac{\partial q}{\partial p_\tau}r_2,$$
$$Q_0=e^{i\Phi}\exp\left[i(P-q)_\parallel^2\frac{\rho}{eH}\right]\exp(i\eta P_\perp^2); \tag{14}$$

for the remaining notations see (11) and (A.3).

The expression obtained for the mass operator should be renormalized in standard fashion [see Ref. 2, formula (3.15)]:

$$M_R^{(1/2)}=M^{(1/2)}-M^{(1/2)}(\hat{P}=m, \mathcal{A}_\mu=0)-(\hat{P}-m)\frac{\partial M^{(1/2)}}{\partial\hat{P}}(\hat{P}=m, \mathcal{A}_\mu=0), \tag{15}$$

i.e.,

$$M_R^{(1/2)}=M^{(1/2)}-\frac{\alpha}{2\pi}\int_0^\infty\frac{ds}{s}\int_0^1 due^{-isum^2}\{m(1+u)-(\hat{P}-m)(1-u)\cdot$$
$$\times[1-2ism^2u(1+u)]\}. \tag{16}$$

The obtained mass operator can be used in various applications. As shown by us earlier,[4] the change of the quasi-energy of the electron is determined by solving the secular equation (owing to the presence of the double degeneracy):

$$\left|(\varepsilon-\varepsilon_0)\delta_{ij}-\frac{\int d^4x\bar{\Psi}_i M^{(1/2)}\Psi_j}{\int d^4x\bar{\Psi}_i\gamma^0\Psi_i}\right|=0, \tag{17}$$

where Ψ_i is the solution of the Dirac equation in the given field ($i, j=1, 2$). Knowledge of the change of the quasienergy (at a given quasimomentum) makes it possible to obtain the quantities of interest, namely the radiative shift of the levels $\mathrm{Re}\,\Delta\varepsilon$ and the probability W of electron emission:

$$\Delta\varepsilon=\varepsilon-\varepsilon_0=\mathrm{Re}\,\Delta\varepsilon-{}^1/_2 iW^{1/2}. \tag{18}$$

A solution for the Dirac equation in the field of the considered consideration was obtained by Redmond.[5] We represent this solution in the form (a covariant normalization condition is used)

$$\Psi_{n\zeta}=\exp\left\{-i\left[\frac{\lambda}{2}\tau+\frac{m^2+b_n^{(1/2)}}{2\lambda}-\frac{1}{2}\int\limits_{-\infty}^{\varphi} eA(\varphi')\dot K(\varphi')d\varphi'+K\pi\right]\right\}\cdot$$
$$\times\exp\left(\frac{1}{2}\hat\varkappa\hat K(\varphi)\right)\psi_{n\zeta}(x_\parallel),\quad \zeta=1,2, \tag{19}$$

where

$$\pi_{\parallel\mu}=i\partial_\mu-eA_\mu(x_\parallel),\quad \tau=x^0+x^3\quad (p_\tau\Psi_n={}^1/_2\lambda\Psi_n),$$
$$b_n^{(1/2)}=2neH,\quad K(\varphi)=\exp\left(\frac{eF\varphi}{\lambda}\right)\int\limits_{-\infty}^{\varphi}\exp\left(-\frac{eF\varphi'}{\lambda}\right)eA(\varphi')\frac{d\varphi'}{\lambda},$$
$$\dot K(\varphi)=dK(\varphi)/d\varphi,$$
$$\psi_{n1}(x_\parallel)=\frac{1}{[2m(\varepsilon+m)]^{1/2}}\begin{pmatrix}(\varepsilon+m)v_n\\ 0\\ p^3v_n\\ (2neH)^{1/2}v_{n-1}\end{pmatrix},$$
$$\psi_{n2}(x_\parallel)=\frac{1}{[2m(\varepsilon+m)]^{1/2}}\begin{pmatrix}0\\ (\varepsilon+m)v_{n-1}\\ (2neH)^{1/2}v_n\\ -p^3v_{n-1}\end{pmatrix} \tag{20}$$
$$\varepsilon^2=m^2+(p^3)^2+2neH,$$
$$v_n=\left(\frac{\sqrt{eH}}{\pi^{1/2}\cdot 2^n n!}\right)^{1/2}e^{ip^1x^1}\exp\left[-\frac{eH}{2}\left(x^2+\frac{p^1}{eH}\right)^2\right]H_n\left(\sqrt{eH}\left(x^2+\frac{p^1}{eH}\right)\right).$$

Here H_n are Hermite polynomials.

Substituting in the secular equation (17) the mass operator (16) and the wave functions (19), we get a general solution of the problem for the analysis of radiative effects in the field (1), (2). Since the resultant expressions are very cumbersome, we confine ourselves hereafter to the case of circular polarization of the wave [see (2)]:

$$a_1=a\cos\varphi,\quad a_2=a\sin\varphi,$$

for which the result is relatively simple. We note that a substantial simplification of the formulas compared with the general case is a well known fact in the problem of radiative effects in the field of a circularly polarized wave.

Using the wave functions (19), we can obtain the quantity $\langle\gamma^0\rangle$ which enters in the secular equation (17):

$$\langle\gamma^0\rangle=\int d^4x\bar\Psi_i\gamma^0\Psi_i=\frac{\varepsilon}{m}+\frac{m\nu^2\xi^2}{2\lambda\delta^2},\quad \lambda=\varepsilon-p^3, \tag{21}$$

where $\nu=\omega\lambda/eH$ (the sign of ν depends on whether the wave is left- or right-polarized), $\delta=1+\nu$, and $\xi^2=a^2e^2/m^2$ is a parameter of the wave intensity.

After rather laborious calculations we obtain[2)] the following final formulas for the matrix element of the mass operator

$$\langle M^{(1/2)}\rangle_{ij}=\int d^4x\,\bar\Psi_i M^{(1/2)}\Psi_j$$

in the field of the considered configuration

$$\langle M^{(1/2)}\rangle_{ij}=\frac{\alpha}{2\pi}m\int\limits_0^\infty\frac{ds}{s}\int\limits_0^1 du\exp(-isu^2m^2)\left[\frac{Y_{ij}}{\sqrt{D}}\exp\left(i\Phi_{sp}-\frac{\vartheta}{2}\right)-(1+u)\delta_{ij}\right]. \tag{22}$$

Here

$$\Phi_{sp}=\Phi_0+\xi^2\Phi_1,\quad \vartheta=2\xi^2\frac{(\mathrm{Im}\,R)^2}{\sin^2\rho}\frac{H_0}{H},$$
$$\Phi_0=2n[x(1-u)-\rho],\quad \Phi_1=\frac{H_0}{H}\left[\left(\frac{x}{x+y}\right)^2\frac{\sin y}{\sin x}\sin(x+y)\right.$$
$$\left.-\frac{\sin 2y}{2(1+\nu)^2}-\frac{\nu xyu}{(x+y)(1+\nu)}-\mathrm{ctg}\,\rho|R|^2-\frac{2\,\mathrm{Im}\,R}{1+\nu}\sin y\right];$$
$$Y_{11}=Y_0+\xi^2Y_1,\quad Y_0=d_1L_n(\vartheta)+d_2L_{n-1}(\vartheta), \tag{23}$$
$$Y_1=b_1L_n(\vartheta)+b_2L_{n-1}(\vartheta)+b_3L_{n-1}^1(\vartheta)+b_4L_{n-2}^1(\vartheta)+b_5L_n^1(\vartheta),$$
$$Y_{12}=\xi^2\left[\frac{1}{\mu}(b_2L_{n-1}(\vartheta)+b_4L_{n-2}^1(\vartheta)+i\nu c_1^*(m\to-m)L_{n-1}^1(\vartheta))\right.$$
$$\left.-\mu(b_1L_n(\vartheta)+b_5L_n^1(\vartheta)+i\nu c_1L_{n-1}^1(\vartheta))\right],$$
$$Y_{22}=Y_{11}(m\to-m),\quad Y_{21}=Y_{12}(m\to-m).$$

We present below the explicity forms of the coefficients of (23)

$$d_1=\frac{1}{2}e^{-i(\rho-x)}\left\{\left(1+\frac{\varepsilon}{m}\right)(1+ue^{-2ix})\right.$$
$$\left.+2n\frac{H}{H_0}\left[2ie^{-ix}\sin x+ue^{i\rho}\left(e^{i\rho}-\frac{\sin\rho}{\eta H}\right)\right]\right\},$$
$$d_2=d_1^*(m\to-m),$$
$$b_1=\frac{\nu}{2}\left(1+\frac{m}{\varepsilon}\right)e^{-i\rho}\left\{2\sin y\frac{\nu}{1+\nu}z^*\right.$$
$$+\frac{ue^{-ix}}{\sin\rho}\mathrm{Re}\left[e^{i\rho}\left(\frac{x\sin y}{x+y}-R\right)\left(\frac{dR^*}{dy}+iR^*\right)\right]+\frac{u}{\eta H}\left(iRz^*-\frac{e^{-i(x+y)}\mathrm{Re}\,R}{1+\nu}\right)$$
$$\left.+ue^{-iy}\left[z^*\left(\frac{2\,\mathrm{Im}\,R\,e^{i(\rho+y)}}{\sin\rho}-\frac{i\nu}{1+\nu}\right)+\frac{e^{-ix}}{1+\nu}\left(-2\,\mathrm{Im}\,R+\frac{\nu\cos y}{1+\nu}\right)\right]\right\}$$
$$b_2=b_1^*(m\to-m),\quad b_3=i\nu[c_1+c_2+c_1^*(m\to-m)],$$
$$c_1=\left(R^*-\frac{\sin y}{1+\nu}\right)\left\{\left(1+\frac{m}{\varepsilon}\right)\left[\frac{u}{2}e^{-ix}\right.\right.$$
$$\left.\times\left(\frac{e^{-iy}\sin\rho}{\eta H(1+\nu)}-\frac{e^{-i(\rho+y)}}{1+\nu}+\left(\frac{dR}{dy}-iR\right)e^{-i\rho}\right)-iz^*e^{i\rho}\right]$$
$$\left.-i\frac{m}{\varepsilon}\frac{ue^{i\rho}}{1+\nu}\left(z+\frac{\sin x\,e^{iy}}{1+\nu}+\sin(y-x)\right)\right\},$$
$$c_2=2\,\mathrm{Re}\left\{\left(R^*-\frac{\sin y}{1+\nu}\right)e^{i\rho}\left[\frac{u}{\nu}e^{ix}\left(\frac{R}{\eta H}-\frac{e^{i\rho}(R-R^*)}{\sin\rho}\right)+2iz-\frac{ue^{i(x-y)}}{1+\nu}\right]\right\},$$
$$b_4=\frac{\nu}{2}\left(1-\frac{m}{\varepsilon}\right)\left(R^*-\frac{\sin y}{1+\nu}\right)e^{2i\rho}\left[2ze^{i\rho}+iue^{i(x-\rho)}\left(\frac{dR}{dy}-iR\right)\right.$$
$$\left.+2u\left(\frac{\sin\rho}{\eta H}-e^{i\rho}\right)\left(\frac{ie^{i(x+y)}}{2(1+\nu)}-z\right)\right],\quad b_5=-b_4^*(m\to-m),$$

where L_n are Laguerre polynomials,

$$R=e^{i(\rho+y)}\sin\rho\left(c^*+\frac{\nu}{1+\nu}\right),\quad c=\frac{x\sin(x+y)}{(x+y)\sin x}e^{iy}-1,$$
$$z=\frac{Re^{-i\rho}\sin x}{\sin\rho},\quad \mu=\left(\frac{\varepsilon-m}{\varepsilon+m}\right)^{1/2},\quad x=sueH,\quad y=\lambda\omega\eta. \tag{24}$$
$$\nu=\omega\lambda/eH,\quad H_0=m^2/e,\quad \varepsilon^2=m^2(1+2nH/H_0);$$

the remaining notation is given in (11).

The result (22) gives the general picture of the radiative effects[3)] (the emission probability, the level shift) when an electron moves in a field of the considered configuration. We analyze now the radiative effects in a number of limiting cases.

At $\xi^2\ll 1$ we have the description of the field in a magnetic field of any intensity in the presence of a weak plane wave. To obtain an explicit expression for $\langle M^{1/2}\rangle_{ij}$ we must expand in powers of ξ^2. The zeroth term of this expansion describes the radiative effects in a magnetic field, while the terms proportional to ξ^2, after division by $\lambda/\langle P_0\rangle$ and the substitution $\xi^2\to 4\pi\alpha/m^2\omega$ gives a complete cross section of the Compton effect on a polarized electron in a magnetic field of arbitrary intensity. For an electron in a state 1 [see (20)], the total cross section of the Compton effect

$$\sigma_1=-\frac{4\alpha^2}{m^2\Lambda}\operatorname{Im}\int_0^\infty\frac{ds}{s}\int_0^1 du\frac{e^{i\Phi_0}}{\sqrt{D}}\left[\frac{dY_0}{d\xi^2}+\left(i\Phi_1-\frac{\vartheta}{\xi^2}\right)Y_0+Y_1\right]_{\xi^2=0}, \tag{25}$$

where $\Lambda=\omega\lambda/m^2$ and the remaining quantities are defined in (23).

For the polarization of an electron in state 2 we have

$$\sigma_2=\sigma_1(m\to-m) \tag{26}$$

and for unpolarized electrons

$$\sigma_{\text{Comp}}={}^1/_2(\sigma_1+\sigma_2). \tag{27}$$

For any other choice of the polarized states of the electron, it is necessary to take the corresponding combination of ψ_1 and ψ_2 and of the matrix elements $\langle M^{1/2}\rangle_{ij}$.

The Compton effect on an electron in an external magnetic field was discussed earlier by Milton *et al.*[6] who succeeded in taking into account, within the framework of the Schwinger operator technique for an electron in a magnetic field in the lowest order of perturbation theory, the interaction with an electromagnetic monochromatic wave propagating along the field H. Our expression is substantially more compact.

In the case $H/H_0\ll 1$ and $\nu=\lambda\omega/eH\gg 1$ we can obtain an explicit expression for the cross section σ of the Compton effect. The contribution to the integral (25) is made in this case by the region of small x. Carrying out the expansion and integrating, we get

$$\sigma=\sigma_0+\sigma_H. \tag{28}$$

Here

$$\sigma_0=\frac{\pi\alpha^2}{m^2\Lambda}\left\{\left[1-\rho-\frac{\rho+2}{\Lambda}-\frac{2}{\Lambda^2}\right]\ln(1+2\Lambda)+\frac{1+4\rho}{2}+\frac{2}{\Lambda}+\frac{4\Lambda^2\rho-1}{2(1+2\Lambda)^2}\right\},$$

$$\begin{aligned}\sigma_H=-\frac{2\pi\alpha^2\xi_2}{m^2\Lambda\nu}\Bigg\{&\left(\frac{1}{2}+\frac{1}{\Lambda}+\frac{1}{\Lambda^2}\right)\ln(1+2\Lambda)+\frac{\Lambda}{2}-\frac{2}{\Lambda}-\frac{1}{2}+\frac{3}{2(1+2\Lambda)}\\&-\frac{5}{4}\frac{1}{(1+2\Lambda)^2}+\frac{1}{4(1+2\Lambda)^3}+\frac{\rho}{2}\Bigg[\left(\frac{1}{\Lambda}-1\right)\ln(1+2\Lambda)+\Lambda+1\\&-\frac{5}{1+2\Lambda}+\frac{5}{2(1+2\Lambda)^2}-\frac{1}{2(1+2\Lambda)^3}-\frac{\varepsilon^2}{m^2}\Bigg(\frac{1}{\Lambda}\ln(1+2\Lambda)\\&-\frac{3}{1+2\Lambda}+\frac{1}{(1+2\Lambda)^2}\Bigg)\Bigg]\Bigg\}\end{aligned} \tag{29}$$

where $\xi_2=\pm 1$ for right- and left-hand polarizations of the wave

$$\Lambda=\omega[\langle P_0\rangle-\langle P^3\rangle]/m^2,\quad \langle P_0\rangle^2=\langle P_3\rangle^2+m^2+2neH.$$

The expression for σ_0 agrees in form with the cross section of the Compton effect in the absence of a field if $\Lambda=kp/m^2$, k and p are the momenta of the photon and of the electron [see Ref. 3, formula (3.4)], and ρ is the spin correlation term. By virtue of the double degeneracy in spin, an electronic state with arbitrary polarization can be represented as a linear superposition of wave functions (20):

$$\psi=e^{-i\varphi/2}\cos\frac{\vartheta}{2}\psi_1+e^{+i\varphi/2}\sin\frac{\vartheta}{2}\psi_2; \tag{30}$$

Here ϑ and φ are certain angles. In this case, in a system where $p^3=0$, the spin correlation term in a circularly wave is

$$\xi_2\rho=\frac{m}{\varepsilon}\cos\vartheta-\frac{(2neH)^{1/2}}{\varepsilon}\sin\vartheta\cos\varphi.$$

The quantity σ_H is a correction to the cross section. For an unpolarized wave (and also in the case of linear polarization), all that remains in σ_H is the term proportional to ρ. This term agrees with that obtained by Milton *et al.*[6] The remaining contribution to σ_H is missing from Ref. 6, since an unpolarized wave was considered there.

As already noted, a resonant situation arises in the considered field configuration (cyclotron resonance, in which the wave frequency coincides with the frequency of particle motion in the magnetic field, $\nu=-1$). Let us consider this question in the case when

$$H/H_0\ll 1,\quad |\nu|\sim 1\quad (\Lambda=\lambda\omega/m^2\ll 1),\quad nH/H_0\ll 1.$$

In the case $\xi/\delta\sim 1$ the shift of the quasilevels is given by

$$\begin{aligned}\Delta\varepsilon_{1,2}=\frac{1}{1+\nu^2\xi^2/2\delta^2}\Bigg\{&-\frac{i\alpha}{2\pi}m\frac{\Lambda\xi^2\nu^2}{\delta^2}\int_0^\infty\frac{dy}{y^4}(\sin^2 y-y^2\cos 2y)\\&\times\left[1+\frac{\xi^2\nu^2}{\delta^2}\left(1-\frac{\sin^2 y}{y^2}\right)\right]^{-1}\pm\frac{\alpha}{2\pi}\frac{eH}{2m}\Bigg\}.\end{aligned} \tag{31}$$

In the first term, the main contribution to the integral with respect to u is made by the interval $u\sim H/H_0\ll 1$, while in the second term (the contribution anomalous magnetic moment) the main contribution is made by the region $x\sim H/H_0\ll 1$. The imaginary part of (31) determines the probability of radiation of an electron in the field of the given configuration in the classical limit (this result as obtained by us earlier). This term of the expansion of this probability in powers of ξ^2 [with account taken of the procedure used in the derivation of formula (25)] yields the cross section of the Thompson scattering or a circularly polarized wave by an electron in a magnetic field

$$\sigma_T=\frac{8\pi\alpha^2}{3m^2}\frac{\nu^2}{(1+\nu)^2}. \tag{32}$$

In the case when $\xi/\delta\gg 1$, the main contribution to the integral with respect to y is made by the integral $y\ll 1$. Carrying out the corresponding expansions in (22)–(24) (the terms with b_3, b_4, and b_5 make no contribution), we have for the quasi-energy shift

$$\begin{aligned}\Delta\varepsilon_{1,2}=\frac{\alpha m\delta^2}{\pi\xi^2}\Bigg\{&\frac{1}{3}\int_0^\infty\frac{dv(5+7v+5v^2)}{(1+v)^3}\left[-L_{2/3}\left(\frac{2v}{3\varkappa}\right)+\frac{i}{\sqrt{3}}K_{2/3}\left(\frac{2v}{3\varkappa}\right)\right]\\&\pm\frac{H}{H_0}\frac{1}{\varkappa}\int_0^\infty\frac{dv\,v}{(1+v)^3}\left[L_{1/3}\left(\frac{2v}{3\varkappa}\right)+\frac{i}{\sqrt{3}}K_{1/3}\left(\frac{2v}{3\varkappa}\right)\right]\Bigg\},\end{aligned} \tag{33}$$

where $\varkappa=\xi H/\delta H_0$, K_ν is a Bessel function of the imaginary argument (MacDonald function), while the function L_ν is defined in Ref. 7, p. 181. The last term in the curly brackets is the spin term.

We present here also asymptotic expansions for (33):

$$\begin{aligned}&\Delta\varepsilon_{1,2}=\frac{\alpha m\delta^2}{\pi\xi^2}\left[\frac{8\varkappa^2}{3}\ln\frac{1}{\varkappa}-i\frac{5}{2\sqrt{3}}\varkappa\pm\frac{H}{2H_0}\right]\quad(\varkappa\ll 1),\\&\Delta\varepsilon_{1,2}=\frac{2\alpha m\delta^2}{9\xi^2}\left[\frac{7}{3}\Gamma\left(\frac{2}{3}\right)(3\varkappa)^{2/3}\left(\frac{1}{\sqrt{3}}-i\right)\pm\frac{\Gamma(1/3)}{2(3\varkappa)^{2/3}}\frac{H}{H_0}\right],\quad(\varkappa\gg 1).\end{aligned} \tag{34}$$

Let us discuss the results. As resonance is approached, the energy of the electron increases [see (21)], so that the particle motion becomes quasiclassical. This is precisely why formula (33) agrees with the quasiclassical approximation of the radiative shift of the energy when a high-energy electron moves in a magnetic field [see Ref. 7, formula (12.7)]. It is easy to verify that in a system where $\langle p^{\sigma}\rangle = 0$ [see formula (21)], the parameter $\varkappa = H\xi/H_0\delta$ in (33) coincides with the characteristic parameter $\chi = H\langle P^0\rangle/H_0 m$, which enters in the formula that describes synchroton radiation.

The quasiclassical character of the motion of the electron makes it also possible to find other properties of the radiation, particularly the total intensity of the radiation of the electron resonance. Using the known results obtained in a magnetic field, we get

$$I=\frac{\alpha m^2}{3\pi\sqrt{3}}\int_0^\infty \frac{u(4+5u+4u^2)}{(1+u)^4}K_{2/3}\left(\frac{2u}{3\varkappa}\right)du.$$

We have left out of this expression the spin term, which makes a small contribution to the radiation intensity at $H/H_0 \ll 1$.

We present for the sake of completeness the asymptotic values of the expression for the density I:

$$I = {}^2/_3\alpha m^2\varkappa^2+\ldots,\quad \varkappa\ll 1;$$
$$I=({}^2/_3)^5\alpha m^2\Gamma({}^2/_3)(3\varkappa)^{2/3}+\ldots,\quad \varkappa\gg 1,$$

so that near resonance we have $I\propto\delta^{-2/3}$. We note also that the maximum of the spectral distribution of the intensity occurs at the frequency

$$\omega\sim m(H/H_0)(\xi/\delta)^2.$$

APPENDIX

Consider the operator

$$\exp(is\mathcal{P}^2)=\exp[is(P^2+e\sigma\mathcal{F}/2)],\quad \sigma\mathcal{F}=\sigma_{\mu\nu}\mathcal{F}_{\nu\mu}.$$

We represent this operator in the form

$$\exp(is\mathcal{P}^2)=L(s)e^{isb}e^{isa}, \tag{A.1}$$

where

$$a=P_0^2-P_3^2=P_\perp^2,\quad b=-(P_1^2+P_2^2+e\sigma\mathcal{F}/2)=-P_\parallel^2+e\sigma f(\varphi)/2+e\sigma F/2.$$

Differentiating (A.1) with respect to s and multiplying the result from the left by L^{-1} and from the right by $e^{-isa}e^{-isb}$, we obtain

$$iL^{-1}\frac{dL}{ds}=b(\varphi)-e^{isb(\varphi)}f(s)e^{-isb(\varphi)}, \tag{A.2}$$

where

$$f(s)=e^{isa}b(\varphi)e^{-isa}.$$

Using the variables

$$\varphi=x^0-x^3,\quad \tau=x^0+x^3,$$

we obtain

$$P_\perp^2=4p_\tau p_\varphi;\quad p_\tau=i\partial/\partial\tau,\quad p_\varphi=i\partial/\partial\varphi. \tag{A.3}$$

We see now that e^{isa} is the shift operator with respect to the variable φ, so that

$$f(s)=b(\phi_s),\quad \phi_s=\varphi-4p_\tau s. \tag{A.4}$$

We represent the operator $P_\parallel$ in the form

$$P_\parallel=i\partial_\parallel-eA(x_\parallel)-eA(\varphi)=\pi_\parallel-eA(\varphi).$$

Recognizing that

$$[\pi_{\parallel i},\pi_{\parallel k}]=-ieF_{ik},\quad \sigma f(\varphi)/2=-i\hat\varkappa A'(\varphi),\quad [\hat\varkappa,\sigma F]=0, \tag{A.5}$$

we can transform (A.2) into

$$iL^{-1}\frac{dL}{ds}=-\Delta^2+2\Delta e^{-2eFs}P_\parallel+i\hat\varkappa(\Delta' e^{-2eFs}\gamma), \tag{A.6}$$

where we use the matrix notation $(\Delta F\gamma)\equiv\Delta^\mu F_{\mu\nu}\gamma^\nu$

$$\Delta_\mu(s)=e[A_\mu(\phi_s)-A_\mu(\varphi)]. \tag{A.7}$$

In the derivation of (A.6) we used the relation

$$e^{ise\sigma F/2}\gamma_\mu e^{-ise\sigma F/2}=(e^{-2eFs}\gamma)_\mu, \tag{A.8}$$

which can be easily obtained by differentiating the left-hand side of the equation with respect to s and using the commutation relation between $\sigma_{\mu\nu}$ and γ_λ. The solution of (A.6) is

$$L=(1+\hat\varkappa g(s))\exp\left[is\int_0^1\Delta^2(ys)dy\right]T^{(-)}\exp\left[-2is\int_0^1\Delta(sy)e^{-2eFsy}P_\parallel dy\right], \tag{A.9}$$

where the symbol $T^{(-)}$ denotes the antichronological product in the "time" s,

$$g(s)=s\int_0^1\Delta'(sy)e^{-2eFsy}\gamma dy,\quad \Delta'=\partial\Delta/\partial\varphi.$$

Account was taken of the fact that $(\varkappa)^2=0$. The $T^{(-)}$ product contained in (A.9) can be calculated, since the commutator of the operators in the argument of the exponential is a c-number (Ref. 7, Sec. 6.3). It is easy to verify that

$$T^{(-)}\exp\left(\int_0^1 B(s)ds\right)=\exp\left[\int_0^1 B(s)ds\right]\exp\left\{\frac{1}{2}\int_0^1 ds_1\int_0^1 ds_2\cdot\right.$$
$$\left.\times\,\theta(s_2-s_1)[B(s_1),B(s_2)]\right\}. \tag{A.10}$$

Substituting (A.10) in (A.9), and then in (A.1), we get

$$e^{is\mathcal{P}^2}=(1+\hat\varkappa g(s))\exp\left[is\int_0^1\Delta^2(sy)dy\right]\cdot$$
$$\times\exp\left[-2is\int_0^1\Delta(sy)e^{-2eFsy}Pdy\right]\cdot$$
$$\times\exp\left[2is^2\int_0^1 dy_1\int_0^1 dy_2\theta(y_2-y_1)\Delta(sy_1)\exp[-2eFs(y_2-y_1)]\Delta(sy_2)\right]\cdot$$
$$\times\exp(ise\sigma\mathcal{F}/2)\exp(isP_\parallel^2)\exp(isP_\perp^2). \tag{A.11}$$

In what follows it is convenient to recast (A.11) in a form that does not contain terms linear in P. We use for this purpose the formula

$$\exp[is(P-q)_\parallel^2]=\exp\left\{i\left[q^2s+2s^2\int_0^1 dy_2\int_0^1 dy_1\theta(y_2-y_1)\cdot\right.\right.$$
$$\left.\left.\times q\exp[2eFs(y_2-y_1)]q\right]\right\}\exp\left[-2iqs\int_0^1 e^{-2eFsy}Pdy\right]\exp(isP_\parallel^2), \tag{A.12}$$

which can be easily obtained by the method described above. Putting

$$q=q_s=\int_0^1\Delta(sy)e^{-2eFsy}dy\left[\int_0^1 e^{-2eFsy}dy\right]^{-1}$$
$$=\int_0^1\Delta(sy)e^{-2eFsy}dy[eFs+eHs\operatorname{ctg}eHs], \tag{A.13}$$

we arrive at the following representation

$$\exp(is\mathcal{P}^2)=(1+\hat\varkappa g(s))\exp(i\Phi(s))\exp(ise\sigma\mathcal{F}/2)\cdot$$
$$\times\exp(is(P-q_s)_\parallel^2)\exp(isP_\perp^2), \tag{A.14}$$

where

$$\Phi(s)=2s^2\int_0^1 dy_2\int_0^{y_2} dy_1\Delta(sy_1)\exp[2eFs(y_2-y_1)]\Delta(sy_2)$$
$$+s\int_0^1\Delta^2(sy)dy-eHs^2\operatorname{ctg}eHs\left[\int_0^1\Delta(sy)e^{-2eFsy}dy\right]^2. \tag{A.15}$$

[1)]We use here and below extensively a matrix notation, e.g., $\Delta Bq = \Delta\mu B^{\mu\nu} q_\nu$.

[2)]Without loss of generality, we can change over to a system in which $p^2=0$. The operator transformations used in Sec. 3 of Ref. 1 are useful in this case.

[3)]Formal transition from a right- to a left-hand polarized wave is possible by making the substitution $\omega \to -\omega$.

[1]V. N. Baĭer and A. I. Milstein, J. Phys. A **11**, 297 (1978).

[2]V. N. Baĭer, V. M. Katkov, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **67**, 453 (1974) [Sov. Phys. JETP **40**, 225 (1975)].

[3]V. N. Baĭer, V. M. Katkov, A. I. Mil'shteĭn, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **69**, 783 (1975) [Sov. Phys. JETP **42**, 400 (1975)].

[4]V. N. Baĭer and A. I. Mil'shteĭn, Dokl. Akad. Nauk SSSR **231**, 1100 (1976) [Sov. Phys. Dokl. **21**, 734 (1976)].

[5]P. Redmond, J. Math. Phys. **6**, 1163 (1965).

[6]K. Milton, W. Tsai, L. De Raad, and N. Dass, Phys. Rev. D **10**, 1299 (1974).

[7]V. N. Baĭer, V. M. Katkov, and V. S. Fadin, Izluchenie relyativistskikh élektronov (Radiation of Relativistic Electrons), Atomizdat, 1973.

Translated by J. G. Adashko

Kinetics of saturation of a two-level system broadened inhomogeneously by the Doppler effect

L. D. Zusman

Institute of Inorganic Chemistry, Siberian Branch of the Academy of Sciences of the USSR, Novosibirsk

(Submitted 19 December 1977)
Zh. Eksp. Teor. **75**, 402–407 (August 1978)

Kinetics of saturation of a Doppler spectrum by a monochromatic field is determined. It is shown that the kinetics is exponential and the rate of saturation is found. At high incident-wave intensities, the rate of saturation of a Doppler profile is proportional to the collision frequency. The distribution of the population difference between the velocities is determined. It is demonstrated that the power absorbed per unit time is proportional to the rate of saturation of a Doppler spectrum.

PACS numbers: 31.30.Jv, 32.80.–t

Investigations of the nature of elastic collisions by nonlinear spectroscopy methods are now popular.[1-5] These methods are particularly interesting because the nature of the velocity-changing elastic collisions has practically no effect on the luminescence spectra.[6]

We shall consider the kinetics of saturation, by a monochromatic field, of a two-level system broadened inhomogeneously by the Doppler effect, and we shall also deal with the steady-state absorption of the field by such a system. Since in most nonlinear spectroscopic investigations of gases and in studies of gas lasers the experimental results are interpreted using the model of relaxation constants, which ignores the changes in the atomic velocities as a result of collisions, we shall allow for the influence of collisions on the kinetics of absorption or saturation of a two-level system, and also on the steady-state nonlinear absorption. Following Kol'chenko *et al.*[1] and Burshteĭn,[5] we shall use the model of weak collisions to show that the kinetics of saturation of a Doppler spectrum is exponential when the frequency of the incident field corresponds to the wings of the Doppler profile, and we shall find the rate of saturation of this profile. Moreover, we shall determine the steady-state power absorption. The rate of saturation of a Doppler profile carries information on the type of collisions and it is proportional to the effective collision frequency.

The saturation method is used widely in investigations of migration in magnetic resonance spectra[7] and in solid-state laser materials.[8] We shall also analyze the effects of diffusion in the velocity space. We shall show that if $\nu \gg 1/T$, where ν is the effective frequency of the velocity-changing in collisions and T is the longitudinal relaxation time, the distribution of the population difference between the velocities v has a dip, which is different from the well-known Lamb and Bennett dips, and is of diffusion origin. Kol'chenko *et al.*[1] also observed a dip of diffusion nature but it corresponds to the criterion $\nu \ll 1/T$ and is associated with the transient stage of diffusion, whereas the dip found in our investigation is associated with the quasisteady stage of diffusion.

The difference between the atomic populations $n(v,t) = \rho_{11}(v,t) - \rho_{22}(v,t)$, traveling at a velocity v, is described in the rate approximation by an equation which has the following form in the weak collision model[6]

$$\frac{\partial n(v,t)}{\partial t} = -2W(v,\omega-\omega_0)\,n(v,t) + \nu\left[1 + v\frac{\partial}{\partial v} + d\frac{\partial^2}{\partial v^2}\right] n(v,t) - \frac{1}{T}\left[n(v,t) - n_0\varphi(v)\right] \tag{1}$$

with the initial condition

$$n(v,0) = n_0\varphi(v) = \frac{n_0}{(2\pi d)^{1/2}}\exp\left(-\frac{v^2}{2d}\right). \tag{2}$$

Here, n_0 is the equilibrium difference between the populations if level 1 is the ground state, whereas if both levels are excited, then n_0/T represents pumping of level 1; T is the relaxation time of the population difference; ν is the frequency of the velocity-changing collisions. The probability of a transition $W(v, \omega - \omega_0)$ is described by