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Translated by J. G. Adashko

Reversal of the wave front of light in the case of depolarized pumping

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(Submitted 12 January 1978)
Zh. Eksp. Teor. Fiz. **75**, 428–438 (August 1978)

Some specific features of the reversal (or reproduction) of the wave front during stimulated scattering of light, which are due to the inhomogeneity of the pump polarization state, are considered. It is found that there are four linearly independent solutions for the scattered field with a structure correlated with the structure of the pump field. The growth rates are determined and the form of the solutions is investigated in detail. It is shown that the polarization inhomogeneity is favorable for reproduction of the wave front in the case of forward stimulated scattering. For backward stimulated scattering, the pumping depolarization impairs the quality of the wave front reversal (the reaction of the scattered wave on the pump is neglected).

PACS numbers: 42.65.Cq

1. INTRODUCTION

The reversal of the wave front in backward stimulated scattering (SS) of light in pump beams $E_L(\mathbf{r},z)$ with a highly developed transverse structure[1] consists of the fact that the preferred gain is possessed by the configuration of the scattered field $E_s(\mathbf{r},z)$ that is the complex conjugate of the pump field, i.e.,

$$E_s(\mathbf{r},z)=\text{const}\cdot E_L^*(\mathbf{r},z). \quad (1a)$$

Similarly, in forward scattering a configuration of the form

$$E_s(\mathbf{r},z)=\text{const}\cdot E_L(\mathbf{r},z) \quad (2a)$$

should possess a large gain. These effects were investigated later in a number of researches (see, for example, Refs. 2–17 and papers cited therein); however, the theory and the experiments pertained to radiation with a definite state of polarization that is the same at all points $(\mathbf{r},z)$ of the scattering medium. On the other hand, it is well known that the inhomogeneity of the pump polarization leads to a number of specific features of the SS process—see for example, Refs. 18–20. In the present work, we examine how the appearance of reversal (in backward SS) or reproduction (in forward SS) of the wavefront takes place when the pump polarization is modulated randomly over the cross section.

The following results are obtained in the present work for the most interesting case of scattering of the scalar type. For forward SS, the phenomenon of reproduction with account of depolarization differs in general little from the case of homogeneous polarization of the pumping, and the relation (2) is essentially generalized to

 0038-5646/78/080214-06$02.40

$$\mathbf{E}_s(\mathbf{r},z)=\text{const}\cdot\mathbf{E}_L(\mathbf{r},z). \quad (2b)$$

In contrast with this, for backward SS the relation

$$\mathbf{E}_s(\mathbf{r},z)=\text{const}\cdot\mathbf{E}_L^*(\mathbf{r},z) \quad (1b)$$

is generally not satisfied. In fact, the relation (1b) denotes a transition from some unit vector of elliptical polarization of the pump $\mathbf{e}_L$, for example $\mathbf{e}_L=\cos\alpha\,\mathbf{e}_x+i\sin\alpha\,\mathbf{e}_y$, to the unit vector of the scattered wave $\mathbf{e}_s=\cos\alpha\,\mathbf{e}_x-i\sin\alpha\,\mathbf{e}_y$. In the special case of circular polarization ($\alpha=\pi/4$), the unit vector $\mathbf{e}_s$ turns out to be orthogonal to $\mathbf{e}_L$ and then there is no interaction of the scalar type at all. For this reason, the generalization of the relation (1a) to the vector case is not trivial and requires the solution of the set of corresponding equations. Just this program is carried through in the present work.

2. THE SYSTEM OF BASIC EQUATIONS

We shall write the equations for the pump wave $E_L(\mathbf{r},z)$ and the Stokes wave $E_S(\mathbf{r},z)$ in the usual parabolic approximation. For simplicity in writing, we introduce symbols for the two-dimensional vectors: $\mathbf{E}_L=\mathbf{E}$ and $\mathbf{E}_s=\mathbf{B}$; these vectors have components in the (x,y) plane that is perpendicular to the direction of propagation z. Here we shall assume that the scattered wave $\mathbf{B}$ always propagates in the $+z$ direction, while the pump field propagates in the $+z$ or $-z$ direction depending on whether forward or back scattering is considered.

Designating the Cartesian components of the vectors of the wave field by the symbols E_i and B_i $(i=1,2)$, we obtain the following relations in the parabolic approximation and neglecting the reaction of the Stokes wave on the pump:

$$\mp\frac{\partial E_i}{\partial z}+\frac{i}{2k}\Delta_\perp E_i=0,$$
$$-\frac{\partial B_i}{\partial z}+\frac{i}{2k}(1+\alpha)\Delta_\perp B_i+\frac{1}{2}G_{iklm}E_kE_l^*B_m=0. \quad (3)$$

Here $\Delta_\perp=(\partial^2/\partial x^2+\partial^2/\partial y^2)$, $k=k_L=\omega_L\sqrt{\varepsilon(\omega_L)}/c$, $\alpha=(k_L-k_s)/k_s$ and summation over repeated indices is assumed. The minus sign in (3) refers to the case of forward scattering, the plus sign to back scattering.

The matrix with four subscripts G_{iklm} characterizes the polarization dependence of the scattering process (both stimulated and the corresponding spontaneous). In liquids or gases, this matrix is characterized by three independent constants: G_{sc}, G_s and G_a, corresponding to scattering of scalar, symmetric traceless, and antisymmetric type:

$$G_{iklm}=G_{SC}\delta_{ik}\delta_{lm}+{}^3/_4G_s(\delta_{il}\delta_{km}+\delta_{im}\delta_{kl}-{}^2/_3\delta_{ik}\delta_{lm})+G_a(\delta_{il}\delta_{km}-\delta_{im}\delta_{kl}). \quad (4)$$

Normalization of the quantities G_{sc}, G_s and G_a in (5) is obtained in the following way. If scattering of only any one type is "turned on," then in a linearly polarized pump wave of constant amplitude, the greater (of the two possible, see Ref. 19) gain (in the intensity) g (in cm^{-1}) is equal to

$$g_\beta=G_\beta|\mathbf{E}|^2,$$

where the index runs through the values $\beta=sc$, s, and a.

We shall solve the problem of stimulated light scattering in a rectangular light pipe, assuming periodicity boundary conditions of the Born-von Karman type. A discussion of this assumption may be found in Refs. 11–13. Here it is convenient to use the expansion in a Fourier series in the transverse coordinates (see Refs. 6, 7, 11–13):

$$\mathbf{E}(\mathbf{r},z)=\sum_{\mathbf{q}}\mathbf{C}(\mathbf{q},z)\exp(i\mathbf{qr}),$$
$$\mathbf{B}(\mathbf{r},z)=\sum_{\mathbf{q}}\mathbf{S}(\mathbf{q},z)\exp(i\mathbf{qr}). \quad (5)$$

The solution of the equation for the pump (3) in the Fourier representation is trivial:

$$C_i(\mathbf{q},z)=C_i(\mathbf{q})\exp(\mp i\mathbf{q}^2z/2k). \quad (6)$$

In the following, we shall assume that $C_i(\mathbf{q})$ are a set of Gaussian complex random quantities and their correlator is equal to

$$\langle C_i^*(\mathbf{q}_1)C_j(\mathbf{q}_2)\rangle=T_{ij}(\mathbf{q}_1)\delta(\mathbf{q}_1-\mathbf{q}_2), \quad (7)$$

where $\delta(\mathbf{q}_1-\mathbf{q}_2)$ is the discrete Kronecker delta: $\delta(\mathbf{q}\neq0)=0$, $\delta(0)=1$. The relation (7), together with the boundary conditions of the Born-von Karman type, corresponds to an assumption that the properties of the pump are the same (in a statistical sense) at all points in the volume of the light pipe. The Hermitian matrix $T_{ij}(\mathbf{q})=T_{ji}^*(\mathbf{q})$ as a function of its indices i and j, characterizes the polarization properties of the angular component of the pump with given $\mathbf{q}$ (see, for example, Ref. 21, Sec. 50).

Equation (3) for the Stokes wave takes the form

$$\frac{\partial S_i(\mathbf{q},z)}{\partial z}+i\frac{\mathbf{q}^2}{2k}(1+\alpha)S_i(\mathbf{q},z)=$$
$$=\frac{G_{iklm}}{2}\sum_{\mathbf{q}_1}\sum_{\mathbf{q}_2}\sum_{\mathbf{q}_3}C_k(\mathbf{q}_1,z)C_l^*(\mathbf{q}_2,z)S_m(\mathbf{q}_3,z)\delta(\mathbf{q}_1-\mathbf{q}_2+\mathbf{q}_3-\mathbf{q}) \quad (8)$$

in the Fourier representation.

The reversal (or reproduction) of the wavefront in Raman scattering with a significant relative frequency shift α, where $\alpha=(k_L-k_s)/k_s$, was discussed by us earlier.[11] Therefore, aiming to discuss here the specifics of the process that are connected with the polarizations, we limit ourselves below to the case of scattering with a small frequency shift and set $\alpha=0$.

3. SOLUTIONS UNCORRELATED WITH THE PUMP

In this section we consider such solutions of Eq. (8) which correspond to the Stokes waves $S_i(q,z)$, uncorrelated with the pumping. Then the right side of Eq. (8) can be averaged over the ensemble of fluctuations of the pump, and the equation transforms to

$$\left[\frac{\partial}{\partial z}+i\frac{\mathbf{q}^2}{2k}\right]S_i(\mathbf{q},z)=\frac{1}{2}g_{im}S_m(\mathbf{q},z), \quad (9)$$

where the Hermitian gain matrix g_{im} (dimensionality cm^{-1}) is equal to

$$g_{im}=G_{iklm}I_{kl},\quad I_{kl}=\sum_{\mathbf{q}}T_{kl}(\mathbf{q}). \quad (10)$$

The Hermitian matrix $I_{kl}=I_{lk}^*$ characterizes the pump polarization properties averaged over the cross section of the light pipe (or, what amounts to the same thing, over all q). The trace of this matrix, $I=I_{11}+I_{22}$, is equal to the total pump power density summed over the polarizations $I=\langle E_L^*(\mathbf{r},z)\,E_L(\mathbf{r},z)\rangle$.

It is convenient to represent the matrix $\hat{I}$ itself in the form of an expansion in Pauli matrices:

$$\hat{1}=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad \hat{\sigma}_1=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \hat{\sigma}_2=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \hat{\sigma}_3=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{11}$$

This expansion has the form

$$\hat{I}={}^1/_2 I(\hat{1}+\boldsymbol{\xi}\hat{\boldsymbol{\sigma}}), \tag{12}$$

where the real vector $\boldsymbol{\xi}=(\xi_1,\xi_2,\xi_3)$ is the so-called reduced Stokes vector, which characterizes the mean state of polarization of the radiation. The value $p=|\boldsymbol{\xi}|$ characterizes the degree of polarization, so that $p=1$ corresponds to completely polarized radiation and the direction of the $\boldsymbol{\xi}$ characterizes the mean orientation (ξ_1,ξ_3) and the degree of ellipticity (ξ_2) of the polarization (see, for example, Ref. 21).

We note that for waves that are uncorrelated with the pump, we obtained Eq. (9), without indicating beforehand which case of scattering is considered—backward or forward; therefore, for such waves, the two cases of scattering do not differ in the approximation considered.

The general solution of Eq. (9) has the form

$$S(\mathbf{q},z)=\exp(-i\mathbf{q}^2z/2k)\exp({}^1/_2\hat{g}z)S(\mathbf{q},z=0) \tag{13}$$

In (13) we introduced a symbol for the exponential function of the matrix $1/2\hat{g}z$. If we represent the arbitrary matrix $\hat{A}$ in the form $\hat{A}=A_0\cdot\hat{1}+\mathbf{A}\cdot\hat{\boldsymbol{\sigma}}$, then the following relation holds:

$$\exp\hat{A}=\frac{1}{2}\left(\hat{1}+\frac{\mathbf{A}\hat{\boldsymbol{\sigma}}}{(\mathbf{AA})^{1/2}}\right)\exp[A_0+(\mathbf{AA})^{1/2}] + \frac{1}{2}\left(\hat{1}-\frac{\mathbf{A}\hat{\boldsymbol{\sigma}}}{(\mathbf{AA})^{1/2}}\right)\exp[A_0-(\mathbf{AA})^{1/2}]. \tag{14}$$

Representing $\hat{A}$ in the form of the matrix $1/2\hat{g}z$ with $\hat{g}$ from (4) and (10), we can obtain an explicit solution for the Stokes waves that are uncorrelated with the pump. In our case $1/2\hat{g}z$ is a Hermitian matrix, and the vector $\boldsymbol{\rho}=/\mathbf{A}(\mathbf{A}\cdot\mathbf{A})^{-1/2}$ in (14) is purely real and equal with modulus unity: $|\boldsymbol{\rho}|=1$. Its value is expressed in terms of $\boldsymbol{\xi}$ and the gains G_{sc}, G_s and G_a. The matrices $1/2(1\pm\boldsymbol{\rho}\cdot\boldsymbol{\sigma})$ are projection operators along the mutually orthogonal directions of polarization, characterized by the Stokes vectors $\pm\boldsymbol{\rho}$. These polarization directions correspond to the natural waves of the Stokes field, amplified (at a given state of the pump polarization of the Stokes vector $\boldsymbol{\xi}$—and at given G_{sc}, G_s and G_a) with a definite amplitude increment $\mu_{1,2}$ and with preservation of the orientation of the polarization.

Referring to our Refs. 19 and 20 for details of the general case of all three forms of scattering, we discuss here the most interesting case of scattering of a purely scalar type ($G_s=G_a=0$) and set $G_{sc}=G$. Then

$$\boldsymbol{\rho}=\boldsymbol{\xi}/|\boldsymbol{\xi}|,\quad \mu_{1,2}=GI(1\pm|\boldsymbol{\xi}|)/4. \tag{15}$$

The completely polarized Stokes wave, with direction of polarization closest to the polarization of the pump, is amplified with a gain (in intensity) $2\mu_1=GI(1+|\boldsymbol{\xi}|)/2$ and the Stokes-wave polarization that is orthogonal to it has a gain $2\mu_2=GI(1-|\boldsymbol{\xi}|)/2$. In particular, in the case of arbitrary elliptical (but complete!) polarization of the pump, i.e., at $|\boldsymbol{\xi}|=1$, we have $2\mu=GI$ and $2\mu_2=0$. On the other hand, in the case of completely unpolarized pumping, i.e., at $|\boldsymbol{\xi}|=0$, we have $2\mu_1=2\mu_2=GI/2$.

4. SOLUTIONS CORRELATED WITH THE PUMP FOR FORWARD SS

Similar to what was done in Refs. 11–14 without account of the polarizations, we now consider the solutions for a Stokes wave correlated with the pump field. For the case of forward SS we shall seek such a solution in the form

$$S_i(\mathbf{q},z)=f_{ik}(\mathbf{q})C_k(\mathbf{q},z)e^{\mu z}, \tag{16a}$$

where μ is the complex amplitude increment. The matrix $f_{ik}(\mathbf{q})$ characterizes the coupling of the i-th component of the scattered field with the k-th component of the pump. As will be seen from the solution obtained below, at $(k_L-k_s)/k_s=\alpha=0$ this matrix turns out to be independent of $\mathbf{q}$.

A field of the form (16a) does not satisfy Eq. (8) exactly. A search for an approximate solution in the form (16a) means that we are seeking the so-called "modes" of a medium with a spatially inhomogeneous dielectric constant $\delta\varepsilon(\mathbf{r},z)\propto i|\mathbf{E}_L(\mathbf{r},z)|^2$. In the coordinate representation, (16a) corresponds to writing

$$\mathbf{S}(\mathbf{r},z)=\mathbf{M}(\mathbf{r},z)e^{\mu z},\quad \frac{\partial}{\partial z}\mathbf{M}(\mathbf{r},z)-\frac{i}{2k}\Delta_\perp\mathbf{M}(\mathbf{r},z)=0. \tag{16b}$$

In other words, the spatial structure of the mode $\mathbf{M}(\mathbf{r},z)$ is described by the law of propagation in the case of free diffraction, while the total complicated effect of multiple re-scattering by the inhomogeneities $\delta\varepsilon(\mathbf{r},z)$ reduces to the appearance of the exponential factor $\exp(\mu z)$.

We shall assume that the condition of existence of the phenomenon of reproduction, discussed in Refs. 6 and 13, for completely polarized radiation, is satisfied, i.e., we shall assume that $GI\lesssim k\theta^2$. Here θ is the angular spread of the pump radiation. Then, similar to the method of Refs. 11–14, we substitute the expression (16a) in Eq. (8), multiply the right and left sides of the equation by $C_r^*(\mathbf{q}_4)$ and average over the ensemble of pump fields. As a result, we obtain the following equation:

$$\mu f_{ip}(\mathbf{q})T_{rp}(\mathbf{q})=\frac{G_{iklm}}{2}\left\{I_{lk}f_{mn}(\mathbf{q})T_{rn}(\mathbf{q})+T_{rk}(\mathbf{q})\sum_{\mathbf{q}_1}f_{mn}(\mathbf{q}_1)T_{ln}(\mathbf{q}_1)\right\}. \tag{17a}$$

This procedure corresponds to the discarding, in the right side of Eq. (8), of a number of terms corresponding to re-scattering of the wave $S(q)$ by "foreign" lattices of the form

$$\delta\varepsilon\propto C(\mathbf{q}_1)C^*(\mathbf{q}_2)\exp\left\{i(\mathbf{q}_1-\mathbf{q}_2)\mathbf{r}-i\frac{\mathbf{q}_1^2-\mathbf{q}_2^2}{2k}z\right\}$$

with $\mathbf{q}\neq\mathbf{q}_1$ and $\mathbf{q}_1\neq\mathbf{q}_2$. Actually, the interference of the amplitudes re-scattered by the "foreign" lattices, under the assumption of δ correlation of (7), gives a mean value of zero over the ensemble, while the fluctuations around the zero mean value are small upon satisfaction of the condition $GI\lesssim k\theta^2$. What is more, the overwhelming part of the discarded terms oscillates rapidly with change of the z coordinate, i.e., it does not satisfy the Bragg condition.

Assuming the matrix $T_{ik}(\mathbf{q})$ to be nondegenerate, we can multiply the right and left sides by T^{-1} and obtain

$$\left(\mu\delta_{im}-\frac{g_{im}}{2}\right)f_{ms}(\mathbf{q})=\frac{G_{islm}}{2}\sum_{\mathbf{q}_1}f_{mn}(\mathbf{q}_1)T_{ln}(\mathbf{q}_1),\qquad(17b)$$

where g_{im} is determined by formula (10). It is seen first of all from this equation that $f(\mathbf{q})$ actually does not depend on $\mathbf{q}$, and therefore in what follows, the argument $\mathbf{q}$ will be omitted in the matrix f. With account of this circumstance, Eq. (17b) can be rewritten in the form

$$(\mu\delta_{im}\delta_{sn}-{}^1/_2 g_{im}\delta_{sn}-{}^1/_2 G_{islm}I_{ln})f_{mn}=0.\qquad(17c)$$

The system (17c) contains four ($i=1,2$; $s=1,2$) linear homogeneous algebraic equations for four ($m=1,2$; $n=1,2$) unknowns f_{mn}. The condition of compatibility of this system—the vanishing of its determinant—gives an equation for the eigenvalues of the increment μ. This equation is of fourth degree and in the general case it has four different roots: μ_1, μ_2, μ_3, μ_4.

We now explain in detail the reason for the appearance of such a number of eigenvalues $\mu_i (i=1,2,3,4)$ and the correlated solutions $\hat{f}^{(i)}\mathbf{C}(\mathbf{q},z)$ corresponding to them for the Stokes wave. The pump field is characterized by two functions $\mathbf{E}(\mathbf{r},z)=[E_x(\mathbf{r},z), E_y(r,z)]$, which are independent in the general case. From these two functions we can construct four linearly independent fields of the scattered wave $\mathbf{B}$, correlated with the pump:

$$\mathbf{B}_1(\mathbf{r})=[E_x(\mathbf{r}),0],\ \mathbf{B}_2(\mathbf{r})=[0,E_y(\mathbf{r})],\qquad(18)$$
$$\mathbf{B}_3(\mathbf{r})=[E_y(\mathbf{r}),0],\quad \mathbf{B}_4(\mathbf{r})=[0,E_x(\mathbf{r})]$$

The eigensolutions determined from (17a) are in the general case linear combinations of the fields $\mathbf{B}_1$ to $\mathbf{B}_4$ from (18); however, the number of linearly independent fields (four) found in (18) does not change in the transition to such combinations.

For the more interesting case of scattering of the scalar type, it is convenient to represent Eq. (17c) in matrix form:

$$\mu f={}^1/_2 G[\hat{I}\tilde{f}+\mathrm{Sp}(\hat{I}\tilde{f})\cdot\hat{1}],\qquad(19)$$

where $\tilde{I}_{ij}=I_{ji}$ denotes the transposed matrix and the index of G_{sc} is omitted for brevity. Writing $\hat{I}$ and $\hat{f}$ in the form of an expansion in the Pauli matrices: $\hat{f}=f_0\cdot\hat{1}+\mathbf{f}\cdot\hat{\sigma}$ and for $\hat{I}$ the formula (12), we transform (19) to the form

$$\mu f_0={}^1/_4 GI(f_0+\tilde{\boldsymbol{\xi}}\mathbf{f}),$$
$$\mu\mathbf{f}={}^1/_4 GI(f_0\tilde{\boldsymbol{\xi}}+\mathbf{f}+i[\tilde{\boldsymbol{\xi}}\times\mathbf{f}]).\qquad(20)$$

Here $(\tilde{\xi}_1,\tilde{\xi}_2,\tilde{\xi}_3)=(\xi_1,-\xi_2,\xi_3)$ and the square brackets denote the vector product. The eigenvalues μ_i corresponding to the vanishing of the determinant of the system (20) are then

$$\mu_{1,2}={}^1/_4 GI(1\pm|\boldsymbol{\xi}|),$$
$$\mu_{3,4}={}^1/_4 GI(2\pm(1+3|\boldsymbol{\xi}|^2)^{1/2}).\qquad(21)$$

Graphs of the dependence of μ_i on $|\xi|$ are given in Fig. 1. Here the inequalities $\mu_1<\mu_2<\mu_3<\mu_4$ are satisfied for all $|\xi|$ in the range $0<|\xi|<1$.

In order to make clear the meaning of the obtained solutions, it is convenient to represent the complex vector $\mathbf{E}(\mathbf{r},z)$ of the pump field in the form of a resolution into two orthogonal polarizations $\mathbf{e}_1$ and $\mathbf{e}_2$, such that the Stokes vectors for these two unit vectors are respectively $\boldsymbol{\xi}_1=\boldsymbol{\xi}/(\boldsymbol{\xi})$ and $\boldsymbol{\xi}_2=-\boldsymbol{\xi}/|\boldsymbol{\xi}|$. Here $\boldsymbol{\xi}$ is the

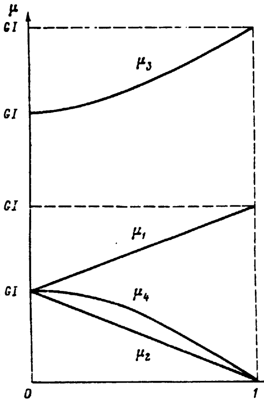

FIG. 1. Dependence of the amplitude increments $\mu_i (i=1,2,3,4)$, for solutions correlated with the pump, on the degree of polarization of the pump $|\xi|$ at fixed total intensity of the pump for forward SS.

Stokes vector, which characterizes the state of polarization (generally speaking, partial) of the pump field. Also,

$$\mathbf{E}(\mathbf{r},z)=R_1(\mathbf{r},z)\mathbf{e}_1+R_2(\mathbf{r},z)\mathbf{e}_2\qquad(22)$$

and the functions $R_1(\mathbf{r},z)$ and $R_2(\mathbf{r},z)$ are orthogonal and statistically independent. The degree of polarization $|\xi|$ and the total intensity I of the pump field are given by the expressions

$$I=\langle|R_1|^2\rangle+\langle|R_2|^2\rangle,\quad |\boldsymbol{\xi}|=(\langle|R_1|^2\rangle-\langle|R_2|^2\rangle)/I\qquad(23)$$

(it is certain here that $\langle|R_1|^2\rangle\geq\langle|R_2|^2\rangle$.

The study of the characteristic matrices $\hat{f}^{(i)}(i=1,2,3,4)$, corresponding to the characteristic increments found above, shows that the Stokes field here has the following form for each of the solutions:

$$\mathbf{B}_1(\mathbf{r},z)=\mathrm{const}\cdot\exp(\mu_1 z)R_2(\mathbf{r},z)\mathbf{e}_1,$$
$$\mathbf{B}_2(\mathbf{r},z)=\mathrm{const}\cdot\exp(\mu_2 z)R_1(\mathbf{r},z)\mathbf{e}_2,$$
$$\mathbf{B}_{3,4}(\mathbf{r},z)=\mathrm{const}\cdot\exp(\mu_{3,4}z)\left\{\left(1+\frac{|\boldsymbol{\xi}|}{1\pm(1+3|\boldsymbol{\xi}|^2)^{1/2}}\right)R_1\mathbf{e}_1\right.\qquad(24)$$
$$\left.+\left(1-\frac{|\boldsymbol{\xi}|}{1\pm(1+3|\boldsymbol{\xi}|^2)^{1/2}}\right)R_2\mathbf{e}_2\right\}.$$

The solution $\mathbf{B}_1(\mathbf{r},z)$ is completely polarized in the direction of the unit vector $\mathbf{e}_1$, while its spatial dependence is given by the function $R_2(\mathbf{r},z)$. By virtue of the scalar character of the interaction, the role of the pump for this solution is played by the field $R_1(\mathbf{r},z)$, and because of the statistical independence of the fields $R_1(\mathbf{r},z)$ and $R_2(\mathbf{r},z)$ the solution $\mathbf{B}_1$ corresponds to the amplification of the uncorrelated wave. The same arguments apply to the solution $\mathbf{B}_2(\mathbf{r},z)$ with replacement $1\rightleftarrows 2$ of the indices. As expected, the increments $\mu_{1,2}$ are described by the formula (15) obtained in Sec. 3 for the uncorrelated solutions.

The solution $B_3(\mathbf{r},z)$ has the largest increment μ_3. It duplicates mainly the spatial and polarization structure of the pump field, slightly accentuating the field component $R_1\mathbf{e}_1$ which carries a large fraction of the pump energy. In the case of a completely unpolarized pump, $|\xi|\to 0$, the duplication is complete and the amplitude increment μ_3 is equal to $(3/4)GI$. This quantity is three times the increment $\mu_3=1/4GI$ for the uncorrelated solutions—formula (15) as $\xi|\to 0$. The correspondence is complete also in the case of a completely polarized pumping $|\xi| \to 1$. Here the increment is equal to $\mu_3 = GI$; it is twice the increment for the uncorrelated solutions; this circumstance is well known at the present time.

The solution $\mathbf{B}_4(\mathbf{r}, z)$ in the case of completely unpolarized pumping, $|\xi| \to 0$ corresponds to separately duplicates the $R_1\mathbf{e}_1$ and $R_2\mathbf{e}_2$ components of the pump field; however, these components enter into the field with opposite signs (in comparison with the pump): $\mathbf{B}_4(|\xi| \to 0) \to R_1\mathbf{e}_1 - R_2\mathbf{e}_2$. Here $\mu_4 \to 1/4GI$, i.e., it coincides with the increment of the uncorrelated solutions. In the other limiting case, for a completely polarized pump, $\mathbf{B}_4(|\xi| \to 1) \to R_2\mathbf{e}_2$, i.e., this field duplicates the "weak" polarization component, and its increment tends to zero.

We note that the obtained expressions (24) for the field allow us to determine the Stokes vectors $\xi^{(i)}$ characterizing the states of polarization of the scattered wave:

$$\xi^{(1,2)} = \pm \frac{\xi}{|\xi|}, \quad \xi^{(3,4)} = \pm \frac{2}{(1+3|\xi|^2)^{1/2}} \xi. \qquad (25)$$

5. SOLUTIONS CORRELATED WITH THE PUMP FOR BACKWARD SS

For backward SS, we seek solutions correlated with the pump, in a form similar to the expression (22) for the forward SS case, with this difference, that the Stokes waves here basically duplicates the complex-conjugate wave of the pump. We shall seek the solution $S_i(\mathbf{q}, z)$ in the Fourier representation, in the form

$$S_i(\mathbf{q}, z) = f_{ik}(\mathbf{q}) C_k^*(-\mathbf{q}, z) e^{\mu z}. \qquad (26)$$

Limiting ourselves, as above in Sec. 4, to the more interesting case of scattering of the scalar type and, furthermore, with a small frequency shift ($\alpha \to 0$), and carrying out similar averaging procedures, we arrive at the following equation for the increments μ and the eigen matrices $\hat{f}$ (which are independent of $\mathbf{q}$):

$$\mu \hat{f} = {}^1/_2 G \hat{I} (\hat{f} + \tilde{\hat{f}}). \qquad (27)$$

The four eigenvalues $\mu_{1,2,3,4}$ of this equation are equal to

$$\mu_1 = 0, \quad \mu_2 = GI/2, \quad \mu_{3,4} = {}^1/_2 GI(1 \pm |\xi|). \qquad (28)$$

Graphs of the dependence of the increments $\mu_{1,2,3,4}$ on the degree of polarization $|\xi|$ at a fixed total intensity of pumping I are shown in Fig. 2. Using the same representation (22) for the pump field, we can write the solution for the Stokes wave in the form

$$\begin{aligned} \mathbf{B}_1(\mathbf{r}, z) &= \text{const} \cdot e^{0 \cdot z} \{R_1^*(\mathbf{r}, z)\mathbf{e}_2 - R_2^*(\mathbf{r}, z)\mathbf{e}_1\} = \text{const} \cdot [\mathbf{E}^*(\mathbf{r}, z) \times \mathbf{e}_z], \\ \mathbf{B}_2(\mathbf{r}, z) &= \text{const} \cdot e^{\mu_2 z} \{R_1^*(1 - |\xi|)\mathbf{e}_2 + R_2^*(1 + |\xi|)\mathbf{e}_1\}, \\ \mathbf{B}_3(\mathbf{r}, z) &= \text{const} \cdot e^{\mu_3 z} R_1^*(\mathbf{r}, z)\mathbf{e}_1, \\ \mathbf{B}_4(\mathbf{r}, z) &= \text{const} \cdot e^{\mu_4 z} R_2^*(\mathbf{r}, z)\mathbf{e}_2. \end{aligned} \qquad (29)$$

We note a characteristic feature of the expression (29) that is specific to the problem of stimulated forward scattering of the light. The vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ in (29) are generally complex unit vectors, corresponding to elliptically polarized waves. The structure of the solutions of (29) that we have found is such that the spatial dependence of the components of the Stokes wave corresponds to the complex-conjugate components of the pump field, i.e., a reversal of the wave front takes place for the individual terms R_1 and R_2. At the same time,

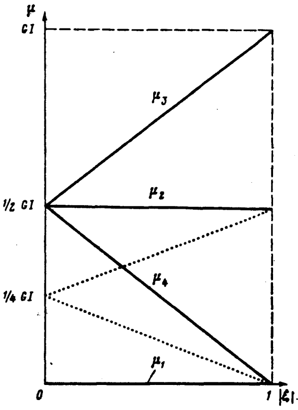

FIG. 2. The same as in Fig. 1, but for backward SS. The points show the dependence of the amplitude increments $\mu_{1,2}$ for solutions that are uncorrelated with the pumping in spatial structure; the properties of these solutions are identical for forward and backward scattering.

the polarization unit vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are not subject to complex conjugation, but simply coincide with the unit vectors of the pump.

The solution $\mathbf{B}_1$ is the solution of the free wave equation, i.e., without amplification. The absence of amplification ($\mu_1 = 0$) for this solution is connected with the fact that the field $B_1(\mathbf{r}, z)$ is strictly orthogonal to the field of the pump at each point in space, i.e., $\mathbf{B}_1(\mathbf{r}, z) \cdot \mathbf{E}^*(\mathbf{r}, z) = 0$; we recall that we are discussing scattering of scalar type, the interaction in which is determined by the scalar product just written down. It is interesting to observe that in the case of forward scattering, for a pump not fully transversely polarized over its cross section, ($|\xi| < 1$), it is not possible to find for the free wave equation a solution that would be strictly orthogonal to the pump at all points of three-dimensional space $(\mathbf{r}, z)$. For this reason, there are no zero values of the increment at $|\xi| < 1$ for the problem of forward scattering, for either the correlated or uncorrelated (with the pump) solutions.

The solution $\mathbf{B}_2$ possesses the interesting feature that its increment does not generally depend on the degree of polarization of the pump and is equal to the value of the increment in the case of interaction of strictly polarized plane waves of the same polarization. We can say that for this solution the deterioration of the amplification because of the inexact coincidence of the polarizations is compensated for by improvement of the polarization because of the coincidence of the maxima of the intensity of the pump and the scattered wave.

The solution $\mathbf{B}_3$ corresponds to reversal of the front of the stronger component of the pump wave, while the solution $\mathbf{B}_4$ corresponds to the same for the weaker pump component. The form of the solution $\mathbf{B}_3$ and the increment μ_3 are the same as if the usual reversal of the wave front took place in the field of a strictly polarized pump of the form $\mathbf{E} = R_1\mathbf{e}_1$, i.e., without the weak component in the pump, with a corresponding decrease in the total intensity of the pump. A similar statement can be made for the solution $\mathbf{B}_4$ with only the weak component remaining in the pump. In correspondence with this, as $|\xi| \to 1$, the behavior of $\mu_{3,4}$ is the following $\mu_3 \to GI$, $\mu_4 \to 0$. On the other hand, in the case of completely unpolarized pumping, all three increments $\mu_{2,3,4}$ tend to the same value $GI/2$.

We also note the following circumstance. The introduction of a polarization of the pumping beam that is random over the cross section leads to different changes in the amplification coefficients for forward SS and backward SS. To be precise, in the case of a completely unpolarized pump, the maximum increment of the forward SS exceeds the maximum increment of the backward SS by a factor of 1.5. This can be of interest in problems in which the amplification coefficient in Mandel'shtam-Brillouin SS (SMBSS in only the backward direction) exceeds the amplification coefficient on any line of the Raman scattering by only a small amount. In this case, the above mentioned difference of the forward and backward increments in the case of a depolarized pumping can serve as a method for suppression of SMBS and separating of the Raman scattering which is not excited under other conditions against the SMBS background.

6. CONCLUSIONS

The different "modes" of the amplifying medium with spatially inhomogeneous pumping were found above and the values of the increments corresponding to them were determined. Since amplification in the regime above the threshold of stimulated scattering corresponds to values of $\exp(2\mathrm{Re}\,\mu L) \sim e^{25}$, then the component from the spontaneous noise is more sharply isolated that corresponds to the solution with the larger increment. Just this component corresponds to the reversal (duplication) of the wave front in the case of back (forward) scattering of completely polarized pumping. The results obtained in the present work show that the spatially inhomogeneous depolarization of the pumping wave impairs the regime of reversal of the wave front in the case of backward SS in comparison with the case of completely homogeneous polarization. In contrast with this regime, the duplication of the wave front for forward SS is improved upon introduction of a strong, spatially inhomogeneous depolarization, since the increment of the duplicating solution in this case is three times the increments of all the remaining solutions. From our viewpoint, experiments on reversal and duplication of the wave front under conditions of depolarized beams of pumping is of undoubted interest.

It should also be noted that for both cases, the forward and backward scattering, the process of stimulated scattering of the scalar type in the field of a spatially inhomogeneous, incompletely polarized pump leads to an increase in the degree of polarization of the scattered radiation in comparison with the degree of polarization of the pump. This refers not only to the pump-correlated field that, possesses the largest increment, but also to the scattered fields that are uncorrelated with the pump. The increase in the degree of polarization of the radiation at the shifted Stokes frequency can lead to instability relative to the polarization of the process of stimulated scattering of the radiation with a broad spectrum (broader than the value of the Stokes shift), which is evidently of interest in astrophysical problems of stimulated Compton scattering by the electrons of the cosmic plasma.[22]

The authors express their deep gratitude to V. N. Blashchuk, B. F. Pilipetskiĭ and V. V. Ragul'skiĭ for valued discussions.

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Translated by R. T. Beyer