Radiation pressure on charged particles in stimulated inverse bremsstrahlung

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An investigation is made of the distribution of the photon momentum between subsystems of colliding particles in the course of stimulated inverse bremsstrahlung of light in an isotropic plasma. The general case of an arbitrary ratio of the charges and masses of particles is considered. The mass ratio corresponding to the complete absorption of the momentum by the lighter particles is found. It is shown that the pressure forces exerted on electrons and ions in an electron–ion plasma are, respectively, parallel and antiparallel to the direction of wave propagation. The density of the electric current representing the photon drag of charged particles is calculated.

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1. When an electron beam penetrates a plasma, the electron motion becomes isotropic. The directional momentum is transferred to the plasma electrons and ions. The average rate of change of the momentum in an electron beam due to collisions with ions is given by the well-known expressions

$$\frac{d\mathbf{p}}{dt}=-\nu_{ei}\mathbf{p}=-\frac{4\pi n_i Z^2 e^4}{m^2 v^3}\mathbf{p}\ln\frac{r_{max}}{r_{min}}, \qquad (1)$$

where $\mathbf{p}$ is momentum, m is the mass, and $\mathbf{v}=\mathbf{p}/m$ is the velocity of electrons; ν_{ei} is the effective collision frequency; n_i is the concentration of ions; e and $-Ze$ are the charges of an electron and an ion, respectively; r_{max} and r_{min} are the usual (for a plasma) cutoff factors in the Coulomb logarithm.[1] Electrons with an isotropic distribution of the velocities $\mathbf{v}$ bombard ions at the same rate from all sides. In this case, collisions with ions do not alter the momenta of the electron and ion subsystems. Averaging of Eq. (1) over the directions of $\mathbf{p}$ gives zero. In the absence of external fields the isotropic electron component of a plasma does not shift relative to the ion component (there is no current).

If the scattering of electrons by ions occurs in the presence of an external electromagnetic wave, we can expect not only elastic scattering but also stimulated direct and inverse bremsstrahlung of the wave photons.[1] This process is accompanied, respectively, by a reduction or increase in the momentum of the system. In the case of directional motion of particles a change in their momentum due to stimulated bremsstrahlung processes is a small correction to Eq. (1). In the case of an isotropic distribution the Coulomb exchange of the momenta makes zero contribution and the average rate of change of the momentum of the particles is determined entirely by the stimulated inverse bremsstrahlung. Clearly then the total change in the plasma momentum (or the total volume density of a force acting on the medium $\mathbf{f}_{tot}$) is equal to the product of the photon momentum $\mathbf{k}$ and the number of the absorbed photons:

$$\mathbf{f}_{tot}=\mathbf{k}\frac{d}{dt}n_{ph}=\mathbf{n}\frac{\alpha\mathscr{E}_0^2}{4\pi}=\mathbf{n}\frac{8\pi}{3}\frac{n_i n_e Z^2 e^6\mathscr{E}_0^2}{p^3\omega^2} \qquad (2)$$

where n_{ph} and n_e are the photon and electron concentrations; $\mathscr{E}_0$ is the amplitude of the electric field of the wave given by $\mathscr{E}_0=(4\pi\omega n_{ph})^{1/2}$; ω is the frequency of this wave; α is the absorption coefficient; $\mathbf{n}=\mathbf{k}/\omega$ is a unit vector in the direction of propagation of the wave. Here and later, we shall use the system of units in which $\hbar=c=1$. For simplicity, we shall also postulate that the incident wave is unpolarized and that the coefficients in Eq. (2) imply summation over the polarizations.

The absorption coefficient of light α thus represents the total rate of change of the plasma momentum $\sim\mathbf{f}_{tot}$. However, this information is insufficient to resolve a more detailed problem of how the absorbed photon momentum $\mathbf{k}$ is distributed between the two types of particle (between electrons and ions in a plasma). To the best of our knowledge, this problem has not yet been considered and we shall deal with it below. The distribution of the absorbed momentum between colliding particles is of interest not only in the case of electron–ion but also semiconductor plasmas. In the latter case the dominant scattering mechanism may be the Coulomb scattering by impurities or the electron–hole interaction (we shall ignore the electron–phonon scattering). Therefore, we shall consider the general problem of the determination of the rate of change of the momenta of particles with arbitrary masses and charges being scattered in the process of stimulated inverse bremsstrahlung of photons. The initial velocity distribution of the particles is assumed to be isotropic. Other approximations made in the course of calculations are as follows:

1) the nonrelativistic approximation $v\ll 1, \omega\ll m$ is used;

2) the Coulomb interaction between particles is considered in the first Born approximation $|e_1e_2|/v\ll 1$;

3) the wave field is assumed to be sufficiently weak and is considered only in the lowest order of the perturbation theory; the condition of validity of this approximation is the smallness of the velocity of electron oscillations $v_E=e\mathscr{E}_0/m\omega$ compared with the velocity of the translational motion of electrons.[1]

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In the theory of stimulated bremsstrahlung it is usual to employ also the dipole approximation in which the dependence of the field intensity on the spatial coordinates is ignored completely.[1] This approximation is insufficient to deal with the problem formulated above because the rate of change of the particle momenta is governed by the photon momentum **k**, i.e., by the wave vector which is assumed to be zero in the dipole approximation. We shall obtain general formulas for the rate of change of the particle momenta without invoking the dipole approximation.

The validity of the dipole approximation in the usual theory of stimulated bremsstrahlung[1] is related to the smallness of the photon momentum $k=\omega$ compared with the minimum momentum transferred to a nucleus $\sim\omega/v$, i.e., it is governed by the nonrelativistic criterion $v \ll 1$. Allowance for the nondipole aspects means that some relativistic corrections are included. We then face the question whether the allowance for the nondipole aspects does not represent a redundant precision in combination with the nonrelativistic approximation. In fact, there is no redundant precision if we confine ourselves to the first two terms in the expansion of the cross sections in powers of **k**. The corrections to the cross sections proportional to **k** are characterized by a parameter v and are considerably greater than the true relativistic corrections $\sim v^2$ so that we have $(\omega/m) \ll v$ (Ref. 2). We shall show later that the zeroth order with respect to **k** makes no contribution to the rate of change of the momentum of the relative motion of the particles (when their distribution is isotropic). The linear terms in the expansion of the cross sections in powers of **k** give the volume densities of the forces acting on the particles and these naturally are of the same order as the total volume density of the force (2).

Calculations carried out using the perturbation theory and the nonrelativistic approximation are quite simple in the fundamental sense. Therefore, one might only give the final result without considering the details. However, the results of such a calculation (given below) are somewhat unexpected. Moreover, the question of the distribution of the absorbed momentum is essentially quantitative so that we have to consider certain basic aspects of the calculation procedure.

2. We shall discuss the processes of stimulated direct and inverse bremsstrahlung in the case of scattering of two particles of charges and masses e_1, m_1 and e_2, m_2, respectively. The interaction between these particles is described by the potential energy $V=e_1e_2/r$ (r is the distance between the particles). The field of an external electromagnetic wave is described by a vector potential whose value at the point of location of a particle i ($\mathbf{r}_i$) is

$$\mathbf{A}(\mathbf{r}_i, t)=\mathbf{A}_0\cos(\mathbf{kr}-\omega t), \quad i=1, 2.$$

Selecting independent variables in the form of the coordinates of the relative position of the particles $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ and of the center of mass

$$\mathbf{R}=(m_1\mathbf{r}_1+m_2\mathbf{r}_2)/M, \quad M=m_1+m_2,$$

we shall represent the Hamiltonian of the system in the form

$$H=-\frac{1}{2\mu}\frac{\partial^2}{\partial\mathbf{r}^2}-\frac{1}{2M}\frac{\partial^2}{\partial\mathbf{R}^2}+V(r)+W, \tag{3}$$

where $\mu=m_1m_2/M$ is the reduced mass and W is the operator of the interaction with the wave field:

$$\begin{aligned}W=&\frac{i}{m_1}\cos\left[\mathbf{k}\left(\mathbf{R}+\frac{m_2}{M}\mathbf{r}\right)-\omega t\right]\mathbf{A}_0\left(\frac{\partial}{\partial\mathbf{r}}+\frac{m_1}{M}\frac{\partial}{\partial\mathbf{R}}\right)\\&-\frac{i}{m_2}\cos\left[\mathbf{k}\left(\mathbf{R}-\frac{m_1}{M}\mathbf{r}\right)-\omega t\right]\mathbf{A}_0\left(\frac{\partial}{\partial\mathbf{r}}-\frac{m_2}{M}\frac{\partial}{\partial\mathbf{R}}\right).\end{aligned} \tag{4}$$

In view of the assumptions made above, we shall regard the potential V and the operator W as small perturbations. We shall determine in the usual way the probability of scattering in the second order of the perturbation theory, retaining only the cross terms $\sim VW$ (and ignoring V^2 and W^2) in the composite matrix elements. Let us assume that in the initial state the momenta of the relative motion and of the center-of-mass motion are, respectively, **p** and **P** and that in the final state they are **p′** and **P′**. The differential cross sections of the scattering accompanied by the emission ($d\sigma_e$) or absorption ($d\sigma_a$) of quantum ω are given by the following perturbation theory expressions:

$$\begin{aligned}d\sigma_{e,a}=&\frac{(2\mu e_1e_2)^2}{v}d\mathbf{p}'\,d\mathbf{P}'\delta(\mathbf{P}'-\mathbf{P}\pm\mathbf{k})\delta\left(\frac{p'^2-p^2}{2\mu}+\frac{P'^2-P^2}{2M}\pm\omega\right)\\&\times\left|\frac{e_1}{m_1}\left(\mathbf{p}'\pm\frac{m_2}{M}\mathbf{k}-\mathbf{p}\right)^{-2}\left\{\mathbf{A}_0\left(\mathbf{p}'+\frac{m_1}{M}\mathbf{P}\right)\left(\left(p'\pm\frac{m_2}{M}\mathbf{k}\right)^2-\mathbf{p}^2\right)^{-1}\right.\right.\\&\left.-\mathbf{A}_0\left(\mathbf{p}+\frac{m_1}{M}\mathbf{P}\right)\left(\mathbf{p}'^2-\left(\mathbf{p}\mp\frac{m_2}{M}\mathbf{k}\right)^2\right)^{-1}\right\}\\&-\frac{e_2}{m_2}\left(\mathbf{p}'\mp\frac{m_1}{M}\mathbf{k}-\mathbf{p}\right)^{-2}\left\{\mathbf{A}_0\left(\mathbf{p}'-\frac{m_2}{M}\mathbf{P}\right)\left(\left(\mathbf{p}'-\frac{m_1}{M}\mathbf{k}\right)^2-\mathbf{p}^2\right)^{-1}\right.\\&\left.\left.-\mathbf{A}_0\left(\mathbf{p}-\frac{m_2}{M}\mathbf{P}\right)\left(\mathbf{p}'^2-\left(\mathbf{p}\pm\frac{m_1}{M}\mathbf{k}\right)^2\right)^{-1}\right\}\right|^2.\end{aligned} \tag{5}$$

If the mass of one of the particles is large ($m_2 \approx M \gg m_1 \approx \mu$), Eq. (5) simplifies greatly to

$$\begin{aligned}d\sigma_{e,a}=&\frac{e_1^4e_2^2}{v\omega^2}\frac{d\mathbf{p}'d\mathbf{P}'}{|\mathbf{p}'-\mathbf{p}\pm\mathbf{k}|^4}\delta(\mathbf{P}'-\mathbf{P}\pm\mathbf{k})\delta\left(\frac{p'^2-p^2}{2m_1}\pm\omega\right)\\&\times\left|\frac{\mathbf{A}_0\mathbf{p}'}{m_1-\mathbf{p}'\mathbf{n}}-\frac{\mathbf{A}_0\mathbf{p}}{m_1-\mathbf{pn}}\right|^2.\end{aligned} \tag{6}$$

The above expressions for the cross sections $d\sigma_{e,a}$ show explicitly that allowance for the nondipole aspects is equivalent to inclusion of certain relativistic corrections which appear in the dependences of the energy denominators of Eq. (6) on the momenta **p′** and **p**. Equation (6) can be obtained from the familiar formulas for the cross section of the spontaneous bremsstrahlung emission from an electron.[2] The obvious changes have to be made in these formulas because we are considering a stimulated rather than a spontaneous process; moreover, it is necessary to ignore the relativistic dependence of the mass on the velocity (corresponding to the corrections $\sim v^2$) and small correction terms $\sim\omega/m_1$.

In general, the cross sections of Eq. (5) represent a change of the momentum of the relative motion of the particles and of the momentum of the whole system per unit time in a unit volume:

$$\begin{aligned}\frac{1}{V}\frac{d\mathbf{p}}{dt}&=n_1n_2v\int(\mathbf{p}'-\mathbf{p})(d\sigma_e+d\sigma_a),\\\frac{1}{V}\frac{d\mathbf{P}}{dt}&=n_1n_2v\int(\mathbf{P}'-\mathbf{P})(d\sigma_e+d\sigma_a),\end{aligned} \tag{7}$$

where n_1 and n_2 are the concentrations of the particles of type 1 and 2.

The occurrence in Eq. (5) of the delta function $\delta(\mathbf{P}' - \mathbf{P} \pm \mathbf{k})$ reflects the self-evident fact that the total momentum of the system changes by $\mp\mathbf{k}$ as a result of the emission or absorption of a photon. This allows us to represent the rate of change of the momentum of the center of mass in the form

$$\frac{1}{V}\frac{d\mathbf{P}}{dt} = -\mathbf{k}\sigma_T n_1 n_2 v, \tag{8}$$

where σ_T is the total cross section for the emission of a photon:

$$\sigma_T = \int (d\sigma_e - d\sigma_a). \tag{9}$$

We can now calculate σ_T applying the dipole approximation because the photon momentum $\mathbf{k}$ occurs in Eq. (8) as a factor. In this approximation, the averaging of σ_T over the directions of the momenta $\mathbf{p}$ and $\mathbf{P}$ and summation over the polarizations of light on the basis of Eqs. (5), (8), and (9) gives

$$\frac{1}{V}\frac{d\mathbf{P}}{dt} = \mathbf{n}\frac{\alpha \mathscr{E}_0^2}{4\pi} = \mathbf{n}\frac{8\pi}{3}\frac{n_1 n_2 e_1^2 e_2^2 \mathscr{E}_0^2}{\mu v^3 \omega^2}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)^2, \tag{10}$$

where α still represents the absorption coefficient. For $e_1 = e, e_2 = -Ze, m_1 = \mu = m$, and $m_2 = M = \infty$, Eq. (10) reduces to Eq. (2).

In contrast to the total momentum of the system P, the momentum of the relative motion p does not satisfy any laws of conservation and it can be calculated only by rigorous integration of Eqs. (5) and (7) allowing for the dependences of the cross sections $d\sigma_{e,a}$ on $\mathbf{k}$. Expanding $d\sigma_{e,a}$ in powers of the wave vector $\mathbf{k}$, averaging $V^{-1}d\mathbf{p}/dt$ over the directions of the vectors $\mathbf{p}$ and P, and summing over the wave polarizations $\mathbf{A}_0$, we can easily demonstrate that - as stated above - the contribution of the dipole term ($\sim\mathbf{k}^0$) vanishes. In the first order in respect of the wave vector $\mathbf{k}$, the rate of change of the momentum $\mathbf{p}$ is

$$\frac{1}{V}\frac{d\mathbf{p}}{dt} = \frac{4e_1^2 e_2^2 n_1 n_2}{\omega^2 M}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)\left(\frac{e_1 m_2}{m_1} + \frac{e_2 m_1}{m_2}\right)$$
$$\times \int d\mathbf{p}' \left\langle \frac{\mathbf{p}'-\mathbf{p}}{|\mathbf{p}'-\mathbf{p}|^4}(\mathbf{A}_0, \mathbf{p}'-\mathbf{p}) \sum_{\pm} \delta\left(\frac{\mathbf{p}'^2 - \mathbf{p}^2}{2\mu} \pm \omega\right)\right.$$
$$\left. \times \left\{ \mp \frac{(\mathbf{A}_0, \mathbf{p}'-\mathbf{p})(\mathbf{k}, \mathbf{p}'-\mathbf{p})}{|\mathbf{p}'-\mathbf{p}|^2} + \frac{(\mathbf{A}_0\mathbf{p}')(\mathbf{k}\mathbf{p}') - (\mathbf{A}_0\mathbf{p})(\mathbf{k}\mathbf{p})}{2\mu\omega}\right\}\right\rangle, \tag{11}$$

where the angular brackets represent averaging over the directions of the momentum $\mathbf{p}$ and summing over the polarizations $\mathbf{A}_0$.

The dependence on the mass of the particles in Eq. (11) is mainly concentrated in the factor in front of the integral sign. The integrand in Eq. (11) depends only on the reduced mass μ. Therefore, in fact, the whole structure of the integrals (11) follows from Eq. (6) in the first order of its expansion in powers of $\mathbf{k}$, i.e., it can be obtained in the approximation of an infinite mass of one of the particles. The form of the integrand in Eq. (11) can also be found from familiar formulas for spontaneous bremsstrahlung[2] if we go over to the nonrelativistic limit.

Summation over the polarizations $\mathbf{A}_0$ allows us to express the rate of change of the momentum $\mathbf{p}$ in the form

$$\frac{1}{V}\frac{d\mathbf{p}}{dt} = \frac{e_1^2 e_2^2 n_1 n_2 \mathscr{E}_0^2}{\pi\omega^2 M}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)\left(\frac{e_1 m_2}{m_1} + \frac{e_2 m_1}{m_2}\right)$$
$$\times \int dp'\, d\Omega_{\mathbf{p}} \frac{\mathbf{p}'-\mathbf{p}}{|\mathbf{p}'-\mathbf{p}|^4} \sum_{\pm} \delta\left(\frac{p'^2 - p^2}{2\mu} \pm \omega\right)$$
$$\times \left\{ \mp \frac{[\mathbf{p}'-\mathbf{p}, \mathbf{n}]^2 (\mathbf{p}'-\mathbf{p}, \mathbf{n})}{|\mathbf{p}'-\mathbf{p}|^2} + \frac{[\mathbf{p}'-\mathbf{p}, \mathbf{n}]((\mathbf{p}'\mathbf{n})[\mathbf{p}'\mathbf{n}] - (\mathbf{p}\mathbf{n})[\mathbf{p}\mathbf{n}])}{2\mu\omega}\right\}, \tag{12}$$

where $d\Omega_{\mathbf{p}}$ is an element of the solid angle in the direction of the vector $\mathbf{p}$. Further integration can be carried out conveniently by replacing $\mathbf{p}'$ with a new integration variable $\mathbf{q} = \mathbf{p}' - \mathbf{p}$. After integration with respect to $d\Omega_{\mathbf{p}}$, Eq. (12) becomes

$$\frac{1}{V}\frac{d\mathbf{p}}{dt} = \frac{4e_1^2 e_2^2 n_1 n_2 \mathscr{E}_0^2}{\omega^2 p}\frac{\mu}{M}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right) \cdot$$
$$\times \left(\frac{e_1 m_2}{m_1} + \frac{e_2 m_1}{m_2}\right) \sum_{\pm} \int \frac{\mathbf{q}\, dq (\mathbf{q}\mathbf{n})[\mathbf{q}\mathbf{n}]^2}{q^7}, \tag{13}$$

where the domain of integration with respect to $\mathbf{q}$ is limited by the conditions

$$q_{min} < q < q_{max},$$
$$q_{min} = |p - \sqrt{p^2 \mp 2\mu\omega}|, \quad q_{max} = p + \sqrt{p^2 \mp 2\mu\omega}. \tag{14}$$

Clearly, the integral with respect to $\mathbf{q}$ in Eq. (13) is

$$\mathbf{n}\frac{8}{15}\ln\frac{q_{max}}{q_{min}} = \mathbf{n}\frac{8}{15}\ln\frac{p^2 \mp \mu\omega + p(p^2 \mp 2\mu\omega)^{1/2}}{\mu\omega}. \tag{15}$$

The logarithm is governed by the dimensionless paramater $\mu\omega/p^2$ which is usually very small (in the optical frequency range and under the conditions of validity of the Born approximation, we have $\mu\omega/p^2 \sim 10^{-5}$). As usual,[1] this smallness results in substantial compensation of the contributions due to stimulated emission and absorption. Expanding the logarithm in Eq. (15) in terms of this small parameter, we finally obtain from Eq. (13) the following expression for the rate of change of the momentum of the relative motion of the investigated system of particles:

$$\frac{1}{V}\frac{d\mathbf{p}}{dt} = \mathbf{n}\frac{64\pi}{15}\frac{n_1 n_2 e_1^2 e_2^2 \mathscr{E}_0^2}{p^3\omega^2}\frac{\mu^2}{M}\left(\frac{e_2}{m_1} - \frac{e_2}{m_2}\right)\left(\frac{e_1 m_2}{m_1} + \frac{e_2 m_1}{m_2}\right). \tag{16}$$

3. Equations (10) and (16) describe completely the distribution of the momentum of the absorbed photons between the particles. In fact, the rates of change of the particle momenta $d\mathbf{p}_i/dt$ are clearly related to $d\mathbf{p}/dt$ and $d\mathbf{P}/dt$ in the same way as the momenta $\mathbf{p}_i$ to $\mathbf{p}$ and $\mathbf{P}$. Hence, it follows that the volume densities of the forces acting on the particle subsystems 1 and 2 are given by

$$\mathbf{f}_1 = \frac{1}{V}\frac{d\mathbf{p}_1}{dt} = \frac{1}{V}\frac{d\mathbf{p}}{dt} + \frac{m_1}{M}\frac{1}{V}\frac{d\mathbf{P}}{dt}$$
$$= \mathbf{n}\frac{8\pi}{3}\frac{n_1 n_2 e_1^2 e_2^2 \mathscr{E}_0^2 \mu^2}{p^3\omega^2 M}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)\left[e_1\left(1 + \frac{8}{5}\frac{m_2}{m_1}\right) + \frac{3e_2 m_1}{5m_2}\right], \tag{17}$$

$$\mathbf{f}_2 = \frac{1}{V}\frac{d\mathbf{p}_2}{dt} = \frac{1}{V}\frac{d\mathbf{p}}{dt} - \frac{m_2}{M}\frac{1}{V}\frac{d\mathbf{P}}{dt}$$
$$= \mathbf{n}\frac{8\pi}{3}\frac{n_1 n_2 e_1^2 e_2^2 \mathscr{E}_0^2 \mu^2}{p^3\omega^2 M}\left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)\left[-\frac{3e_1 m_2}{5m_1} - e_2\left(1 + \frac{8}{5}\frac{m_1}{m_2}\right)\right]. \tag{18}$$

The above formulas can be written conveniently in the form

$$\mathbf{f}_i=\mathbf{k}_{ef}^{(i)}\frac{dn_{ph}}{dt}=\varphi_i\mathbf{k}\frac{dn_{ph}}{dt}=\varphi_i\mathbf{n}\frac{\alpha\mathscr{E}_0^2}{4\pi}, \tag{19}$$

where

$$\begin{gathered}\mathbf{k}_{ef}^{(i)}=\varphi_i\mathbf{k},\\ \varphi_1=\frac{\mu}{e_1m_2-e_2m_1}\left[e_1\left(1+\frac{8}{5}\frac{m_2}{m_1}\right)+\frac{3}{5}\frac{e_2m_1}{m_2}\right],\\ \varphi_2=-\frac{\mu}{e_1m_2-e_2m_1}\left[\frac{3}{5}\frac{e_1m_2}{m_1}+e_2\left(1+\frac{8}{5}\frac{m_1}{m_2}\right)\right]\end{gathered} \tag{20}$$

and the absorption coefficient α is given by Eq. (10).

The quantities $\varphi_i=k_{ef}^{(i)}/\omega$ can be regarded as the "fraction" of the photon momentum absorbed separately by the subsystems of particles of the first and second kind. In general, $\varphi_1+\varphi_2=1$, i.e., $\mathbf{k}_{ef}^{(1)}+\mathbf{k}_{ef}^{(2)}=\mathbf{k}$ and $\mathbf{f}_1+\mathbf{f}_2=\mathbf{f}_{tot}$, where the total force is given by the first two equalities in Eq. (2).

We shall consider the two most interesting special cases which follow from Eqs. (19) and (20).

1) Let us assume that $m_2\approx M\gg m_1\approx\mu$, $e_1=e$, and $e_2=-Ze$ (this corresponds to the scattering of electrons by ions in a plasma or by impurity centers in a semiconductor). We then have

$$\mathbf{k}_{ef}^{(e)}={}^8/_5\mathbf{k},\quad \mathbf{k}_{ef}^{(i)}=-{}^3/_5\mathbf{k}. \tag{21}$$

The result is somewhat unexpected because $|\mathbf{k}_{ef}^{(e)}|>|\mathbf{k}|$, i.e., the "fraction" of the photon momentum absorbed by the electron subsystem exceeds unity. This excess momentum undoubtedly arises from the fact that stimulated inverse bremsstrahlung is accompanied by a much greater exchange of the momentum because of the Coulomb scattering. The inequality $|\mathbf{k}_{ef}^{(e)}|>|\mathbf{k}|$ means that in the presence of an external field the compensation in the Coulomb exchange of the momenta is incomplete even when the distribution of the electron translational velocities $\mathbf{v}$ is isotropic. This may be due to the fact that an external field disturbs the isotropy of the system. The momentum $\mathbf{k}_{ef}^{(e)}$ absorbed by the electrons consists of the true fraction of the photon momentum (which can hardly be determined in its pure form) and the uncompensated residue of the Coulomb exchange of the momentum between electrons and ions.

It follows from Eqs. (19) and (21) that the force $\mathbf{f}_e$ acting on electrons is directed along the wave vector of light and represents eight-fifths of the total radiation pressure force on the medium $\mathbf{f}_{tot}$. The force $\mathbf{f}_i$ acting on ions represents three-fifths of $\mathbf{f}_{tot}$ and its direction is opposite to that of the wave propagation.

2) Let us now assume that $e_1=-e_2=e$ and that the ratio of the masses m_1 and m_2 is arbitrary (this corresponds to the scattering of electrons by holes in a semiconductor). Then, the expressions in Eq. (20) become

$$\varphi_1=\varphi(x)=\frac{8x^2+5x-3}{5(1+x)^2},\quad \varphi_2=\varphi\left(\frac{1}{x}\right),\quad x=\frac{m_2}{m_1}. \tag{22}$$

As in the preceding case, we find that $\varphi_1=8/5$ and $\varphi_2=-3/5$ for $m_2\gg m_1$ ($x\to\infty$). Conversely, in the limit $x\to 0$ ($m_2\ll m_1$), we have $\varphi_1=-3/5$ and $\varphi_2=8/5$. For equal masses ($m_1=m_2$), we find that $\varphi_1=\varphi_2=\frac{1}{2}$, i.e., $\mathbf{f}_1=\mathbf{f}_2=\frac{1}{2}\mathbf{f}_{tot}$. Consequently, if the masses of the particles are equal and their charges are equal and opposite in sign, we find - as expected - that each of the particles obtains exactly one-half of the photon momentum.

It follows from the expressions in Eq. (22) and also from Fig. 1 that the momentum absorbed by the heavier particles is zero if the mass of the heavier particles represents eight-thirds of the mass of the light particle.

The density of the electric current resulting from stimulated inverse bremsstrahlung[3] can generally be described by

$$\mathbf{j}=e_1\mathbf{v}_1+e_2\mathbf{v}_2=\frac{e_1\mathbf{f}_1\tau_1}{m_1n_1}+\frac{e_2\mathbf{f}_2\tau_2}{m_2n_2}, \tag{23}$$

where τ_1 and τ_2 are the characteristic collision times of particles 1 and 2 with one another and with other objects in the medium. In the case when $e_1=-e_2=e$, $n_1=n_2=n_0$, and $\tau_1=\tau_2=\tau$, we find from Eqs. (19), (22), and (23) that

$$\mathbf{j}=\frac{8}{5}\frac{e\tau}{n_0}\mathbf{n}\frac{\alpha\mathscr{E}_0^2}{4\pi}\left(\frac{1}{m_1}-\frac{1}{m_2}\right). \tag{24}$$

If $m_1=m_2$, it follows from Eq. (24) that the current is $\mathbf{j}=0$. The optical drag of particles of equal mass and with equal (but opposite in sign) charges does not produce an electric field because the momenta and velocities acquired by the particles are equal (in view of the assumption that $\tau_1=\tau_2$, these particles also have equal mobilities; if this is not true, the drag current may differ from zero even for equal electron and hole masses).

In the case of real systems we have to average not only the directions of velocities of particles in the expressions for the force density $\mathbf{f}_{1,2}$ and current $\mathbf{j}$ but also the energy distributions (i.e., for example, averaging over the Maxwellian distribution is needed in the case of an electron–ion plasma). Clearly, such averaging does not affect the conclusions formulated above on the distribution of the absorbed momentum between the particle subsystems.

An estimate of the value of j in the case of a laser plasma is given in a recent paper,[3] where it is shown that currents of very high densities may be generated by the optical pressure on electrons. It is assumed that the momentum absorbed by electrons is equal to

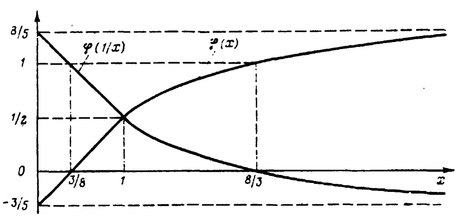

FIG. 1. Relative momenta acquired by particles of different masses (m_1 and m_2) in stimulated bremsstrahlung absorption of photons; $x=m_2/m_1$.

the photon momentum $k_{ef}^{(e)} = k$. In fact, as shown above, the value of $k_{ef}^{(e)}$ has an additional numerical factor of 8/5. Clearly, this factor (~1) does not affect the qualitative estimate given in Ref. 3. On the other hand, an accurate quantitative determination of the distribution of the photon momentum between electrons and ions in a laser plasma is hardly possible because of many secondary phenomena. It is likely that the optical pressure forces exerted on particles in the stimulated inverse bremsstrahlung can be determined more easily in the case of a semiconductor plasma, in which case the current j can be measured with a high accuracy. The drag of particles in the process of stimulated inverse bremsstrahlung may also be manifested in astrophysical phenomena.

In conclusion, we should point out that in addition to any practical applications of the above results, the distribution of the photon momentum between charged particles is undoubtedly of intrinsic interest because it is related to such classical phenomena as stimulated direct and inverse bremsstrahlung of light.

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Nonequilibrium distribution of $N_2(C_3\Pi)$ molecules over the rotational levels in a gas discharge

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The distributions of the intensities in the rotational structure of the 2^+ bands of the N_2 system in a dc discharge and in strong-current discharge were investigated. The causes of the appearance of a "hot" group of $N_2(C\,^3\Pi_u)$ molecules with a high rotational excitation level are discussed. It is shown that this group is the result of impact de-excitation of the $N_2(E\,^3\Sigma_g^+)$ molecules. The effect of alternation of the intensities in the emission spectrum of the hot group is considered and found to be connected with the existence of selection rules for the transfer of electron excitations in collisions of heavy particles.

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1. INTRODUCTION

We have previously established[1] that under conditions of a low temperature glow-discharge plasma at reduced pressure, when the lifetime of $N_2(C^3\Pi)$ is shorter than the time between the gaskinetic collisions, the rotational-level distribution of the $N_2(C^3\Pi)$ molecules deviates from Boltzmann's law. The deviations consist in the fact that a so-called "hot" group of molecules is produced, with a high (~2100 K) population temperature. If we subtract the hot part from the total distribution, then the remaining "cold" group has a Boltzmann distribution described by the temperature of the translational motion of the neutral gas. This must be taken into account when attempts are made to determine the gas temperature by measuring the spectrum of the 2^+ system of N_2. The errors that can appear in the determined temperature because of failure to take the hot group into account depends both on the experimental conditions and on the spectral interval in which the measurements are made. These deviations from equilibrium appear in the electron-vibrational-rotational spectrum in discharges in pure nitrogen as well as in mixtures with other atomic and molecular gases.

We note that similar anomalous rotational distributions in discharges in N_2-Ar mixtures were pointed out earlier in a number of papers,[2-5] although no clearcut separation into cold and hot groups was made. It was established in those molecules that N_2^* molecules with fast rotation are produced in the reaction

$$N_2(X^1\Sigma^+)+Ar(3p^54s)\to N_2^*(C^3\Pi_u)+Ar(3p^6\,^1S_0). \qquad (1)$$

Many details, however, remained unexplained in this frequently described case, particularly the unique alternation of the intensities in the rotational structure of the spectrum of the 2^+ system of the N_2.[1)]

We have therefore verified experimentally[1] the possibility of the appearance of hot N_2 molecules when no argon was specially added to the initial gases, but residual impurities could be present. Measurements of the densities of the $Ar(2p^54S)$ metastable states have shown that the observed anomalies cannot be ascribed to the influence of the impurities.

In many reports of the study of the mechanisms of

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