not necessarily bring $F_0(\varepsilon)$ closer to a Maxwellian function: it was shown recently[18] that the $e-e$ collision integral can be equal to zero also for power-law distribution functions that describe, just as (12), distribution with flows of particles and of energy from a "source" ($\varepsilon = 0$) to a "sink" ($\varepsilon = \varepsilon_0$).

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Three-band Kane model and Auger recombination

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A derivation is given of a dispersion equation from which the spectrum of electrons, light holes, and heavy holes can be obtained in the three-band Kane model in which the interaction between the three bands is included rigorously but the interaction with the other bands is allowed for by the $\mathbf{k}\cdot\mathbf{p}$ approximation. Overlap integrals governing the Auger recombination rate are calculated. The overlap integral between the conduction and heavy-hole valence bands is zero for threshold values of the particle momenta if the interaction with the higher bands is ignored. Consequently, the preexponential function in the expression for the Auger recombination rate has a different temperature dependence from that obtained in the case of simple parabolic bands. This theoretical calculation is in good agreement with the experimental recombination time reported for InSb at 300°K.

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1. INTRODUCTION

The Kane model[1] allows rigorously for the interaction between the s and p bands, whereas the interaction with higher bands is included by the $\mathbf{k}\cdot\mathbf{p}$ approximation. However, in the case of narrow-gap semiconductors, such as InSb or $Hg_{1-x}Cd_xTe$, the spin–orbit splitting is large compared with the band gap E_g and the spectrum of electrons and holes can be determined using just the three-band approximation, i.e., by making rigorous allowance for the interaction with the conduction band with the light- and heavy-hole valence bands. The interaction with the spin–orbit–split band can also be included within the framework of the $\mathbf{k}\cdot\mathbf{p}$ approximation. This makes it possible to allow simultaneously for the band nonparabolicity and the corrugated nature of the constant-energy surfaces.

Allowance for the band nonparabolicity is essential in the calculation of the rate of the Auger recombination of electrons and holes[2] within the framework of the Kane model. The expression for this rate includes overlap

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integrals, whose initial estimates[2] have been refined repeatedly.[3-6] The overlap integrals are calculated for the threshold momenta of the particles participating in the recombination process. However, in the Kane model, the integrals of the overlap between the wave functions of electrons and heavy holes vanish for the threshold momenta of the particles if we ignore the interaction with higher bands. This is due to the fact that, in this model, the wave functions of heavy holes are orthogonal to the wave functions of electrons or light holes if the momenta of the latter particles are parallel to the momentum of a heavy hole, which is true at the threshold. The fact that the threshold overlap integrals vanish has led to the view that the Kane model is unsuitable for estimating these integrals.[4] We shall show that, in calculating the Auger recombination rate it is essential to know the behavior of the overlap integral near the threshold. This behavior modifies the temperature dependence of the recombination time, a quantity which is a product of the power function of the temperature T and an exponential function. In the case of semiconductors described satisfactorily by the Kane model, the power function is proportional to $T^{5/2}$ if $T/E_g \gg (m_c/m_h)^2$, where m_c is the mass of an electron at the bottom of the band and m_h is the mass of a heavy hole, whereas, in the case of purely parabolic bands, the power function is proportional to $T^{3/2}$ (Ref. 2). An estimate of the time constant of the auger recombination in InSb gives a value close to the experimental result.

2. ENERGY SPECTRUM OF ELECTRONS AND HOLES IN THE THREE-BAND KANE MODEL

In the three-band Kane model for a narrow-gap semiconductor, we shall include rigorously the matrix elements of the Hamiltonian H between the wave functions of the conduction band, which have the s symmetry at the point Γ ($u_1 = S\uparrow$, $u_2 = S\downarrow$), and the wave functions of heavy and light holes, which have the p symmetry:

$$u_3=2^{-1/2}(X+iY)\uparrow,\ u_4=i\cdot 6^{-1/2}[(X+iY)\downarrow-2Z\uparrow],$$
$$u_5=6^{-1/2}[(X-iY)\uparrow+2Z\downarrow],\ u_6=i\cdot 2^{-1/2}(X-iY)\downarrow.$$

The Hamiltonian is a 6×6 matrix whose nonvanishing elements are[7]:

$$\begin{aligned}
&H_{11}=H_{22}=E_g+\hbar^2k^2/2m_c',\ H_{13}=-iH_{26}^*=i\cdot 2^{-1/2}Pk_+,\\
&H_{14}=-iH_{25}=(^2/_3)^{1/2}Pk_z,\ H_{15}=-iH_{24}^*=i\cdot 6^{-1/2}Pk_-,\\
&H_{33}=H_{66}=\frac{\hbar^2}{2m_0}[3\gamma_2'k_x^2-(\gamma_1'+\gamma_2')k^2],\\
&H_{44}=H_{55}=-H_{33}-\frac{\hbar^2}{m_0}\gamma_1'k^2,\quad H_{34}=-H_{56}=\frac{i\hbar^2}{m_0}3^{1/2}\gamma_3'k_zk_-,\\
&H_{35}=H_{46}=\frac{\hbar^2}{m_0}3^{1/2}\left[i(\gamma_3'-\gamma_2')k_xk_y-\frac{1}{2}\gamma_2'k_-^2\right].
\end{aligned}\tag{1}$$

Here,

$$P=-\frac{i\hbar}{m_0}\langle S|\hat{p}_x|Z\rangle$$

is the matrix element of the momentum; $\mathbf{k}$ is the wave vector; $k_\pm = k_x \pm ik_y$. The constants m_c', γ_1', γ_2', and γ_3' differ from zero only if allowance is made for the interaction with higher bands.

The system (1) includes only the elements above the diagonal because the other elements can be found from the condition that the Hamiltonian is self-adjoint. The Hamiltonian is described by the system (1) in such a way as to show its relationship to the Luttinger Hamiltonian.[8] The Luttinger constants γ_1, γ_2, and γ_3 are related to γ_1', γ_2', and γ_3' by

$$\gamma_1=\gamma_1'+\frac{2m_0P^2}{3\hbar^2E_g},\quad \gamma_2=\gamma_2'+\frac{m_0P^2}{3\hbar^2E_g},\quad \gamma_3=\gamma_3'+\frac{m_0P^2}{3\hbar^2E_g}.\tag{2}$$

The mass of an electron at the bottom of the band is

$$m_c=(1/m_c'+4P^2/3\hbar^2E_g)^{-1}.$$

The eigenvalues of the Hamiltonian (1) can be found from the roots of the cubic equation

$$\left[E+\frac{\hbar^2k^2}{2m_0}(\gamma_1'-2\gamma_2')\right]\left\{\left(E-E_g-\frac{\hbar^2k^2}{2m_c'}\right)\left[E+\frac{\hbar^2k^2}{2m_0}(\gamma_1'+2\gamma_2')\right]-\frac{2}{3}P^2k^2\right\}+\frac{3\hbar^2}{m_0}k^4s(\gamma_2'-\gamma_3')\left[\frac{\hbar^2}{m_0}(\gamma_2'+\gamma_3')\left(E-E_g-\frac{\hbar^2k^2}{2m_c'}\right)-\frac{2}{3}P^2\right]=0,\tag{3}$$

where

$$s(\mathbf{k})=(k_x^2k_y^2+k_y^2k_z^2+k_z^2k_x^2)k^{-4}.$$

In the spherical approximation, when $\gamma_2'=\gamma_3'=\gamma'$, the roots of Eq. (3) can be found explicitly:

$$\begin{aligned}
&E_h(\mathbf{k})=-\varepsilon_h(\mathbf{k})=-k^2\hbar^2/2m_h,\ 1/m_h=\gamma_1'-2\gamma',\\
&E_{c,l}(\mathbf{k})=\frac{E_g}{2}+\frac{\hbar^2k^2}{4}\left(\frac{1}{m_c'}-\frac{\gamma_1'+2\gamma'}{m_0}\right)\\
&\pm\left\{\left[\frac{E_g}{2}+\frac{\hbar^2k^2}{4}\left(\frac{1}{m_c'}+\frac{\gamma_1'+2\gamma'}{m_0}\right)\right]^2+\frac{2}{3}P^2k^2\right\}^{1/2},
\end{aligned}\tag{4}$$

where E_h and E_l are the energies of heavy and light holes; E_c is the energy of electrons. Equation (3) can be used to obtain all the approximate formulas for the electron and hole energies in different parts of the dependence of E on $\mathbf{k}$ are (given—for example—in Madelung's book[9]), provided $(E, E_g) \ll \Delta$, where Δ is the spin–orbit splitting energy.

3. AUGER RECOMBINATION RATE

According to Beattie,[3] ten different transitions are possible between three bands under Auger recombination conditions. However, the recombination processes involving two electrons and a heavy hole or two heavy holes and an electron followed by the conversion of a heavy into a light hole have the lowest threshold energies. We shall consider the processes in which two electrons and a heavy hole participate. Processes of the second type mentioned above may be important in p-type semiconductors.

The overlap integrals occurring in the expression for the Auger recombination probability can be expressed conveniently in terms of the operators of projection onto the electron states $\Lambda^{(c)}(\mathbf{k})$ and onto the heavy-hole states $\Lambda^{(h)}(\mathbf{k})$:

$$\begin{aligned}
&\Lambda^{(c)}(\mathbf{k})=2[H(\mathbf{k})-E_l(\mathbf{k})][H(\mathbf{k})-E_h(\mathbf{k})]\\
&\times\{\mathrm{Sp}[H(\mathbf{k})-E_l(\mathbf{k})][H(\mathbf{k})-E_h(\mathbf{k})]\}^{-1},\\
&\Lambda^{(h)}(\mathbf{k})=2[H(\mathbf{k})-E_l(\mathbf{k})][H(\mathbf{k})-E_c(\mathbf{k})]\\
&\times\{\mathrm{Sp}[H(\mathbf{k})-E_l(\mathbf{k})][H(\mathbf{k})-E_c(\mathbf{k})]\}^{-1}.
\end{aligned}\tag{5}$$

The projection operators appear after summation over

two possible electron and hole states with a given wave vector $\mathbf{k}$.

The Auger recombination rate can be written in the form[2,5,10]

$$G=\frac{1}{(2\pi)^5\hbar}\left(\frac{4\pi e^2}{\varkappa}\right)^2\int d^3k\,d^3k_1\,d^3k_{12}f_e(\mathbf{k}_1)\delta\left[E_g+\varepsilon_e\left(\mathbf{k}+\frac{1}{2}\mathbf{k}_{12}\right)+\varepsilon_e(\mathbf{k}-{}^1/_2\mathbf{k}_{12})+\varepsilon_h(\mathbf{k}_1-2\mathbf{k})-\varepsilon_c(\mathbf{k}_1)\right]$$
$$\times\{|\mathbf{k}_1-\mathbf{k}-{}^1/_2\mathbf{k}_{12}|^{-4}B^{(cc)}(\mathbf{k}_1,\mathbf{k}+{}^1/_2\mathbf{k}_{12})B^{(eh)}(\mathbf{k}-{}^1/_2\mathbf{k}_{12},2\mathbf{k}-\mathbf{k}_1)$$
$$-|\mathbf{k}_1-\mathbf{k}-{}^1/_2\mathbf{k}_{12}|^{-2}|\mathbf{k}_1-\mathbf{k}+{}^1/_2\mathbf{k}_{12}|^{-2}D(\mathbf{k}-{}^1/_2\mathbf{k}_{12},\ \mathbf{k}_1,\ \mathbf{k}+{}^1/_2\mathbf{k}_{12};\ 2\mathbf{k}-\mathbf{k}_1)\}. \tag{6}$$

Here,

$$D(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3,\mathbf{q})=\mathrm{Sp}\,\Lambda^{(c)}(\mathbf{q}_1)\Lambda^{(c)}(\mathbf{q}_2)\Lambda^{(c)}(\mathbf{q}_3)\Lambda^{(h)}(\mathbf{q}),$$
$$B^{(ca)}(\mathbf{q}_1,\mathbf{q}_2)=\mathrm{Sp}\,\Lambda^{(c)}(\mathbf{q}_1)\Lambda^{(a)}(\mathbf{q}_2),\ \varepsilon_c=E_c-E_g,$$

$f_e=n\hbar^3(2\pi m_cT)^{-3/2}\exp(-\varepsilon_c/T)$ is the distribution function of the electrons present in a density n, $\varkappa$ is the permittivity, e is the electron charge. The law of conservation of momentum is allowed for in Eq. (6). The momenta of a recombining electron and a heavy hole are $\mathbf{k}-\frac{1}{2}\mathbf{k}_{12}$ and $\mathbf{k}_1-2\mathbf{k}$, whereas an electron whose momentum is $\mathbf{k}+\frac{1}{2}\mathbf{k}_{12}$ acquires an additional momentum $\mathbf{k}_1$ as a result of recombination. The second interference term appears because the wave function of electrons participating in the recombination process is, in accordance with the Pauli principle, antisymmetric in respect of the electron coordinates.

It follows from the law of conservation of energy that the threshold energies and momenta in the Auger recombination are

$$\varepsilon_{cT}=E_g+\varepsilon_h(\mathbf{k}_{1T}),\ k_{1T}^2=3E_g^2/P^2=4m_cE_g/\hbar^2,$$
$$k_{T,\alpha}=m_c\hbar^{-2}\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial k_{1\alpha}}\Big|_{\mathbf{k}_1=\mathbf{k}_{1T}},\quad \mathbf{k}_{12,T}=0, \tag{7}$$

where $\alpha=x,\ y,\ z$. The system (7) is derived ignoring the contribution of higher bands to the dispersion law for electrons and regarding the terms associated with the heavy holes as corrections. It follows from the system (7) that, after recombination, the electron energy is of the order of E_g, whereas before recombination the electron and heavy-hole energies are small compared with E_g. Therefore, in all the terms of the system (7) except for the exponential function in $f_e(\mathbf{k}_1)$, we shall utilize the smallness of the electron energy before recombination and the smallness of the heavy-hole energy (compared with the electron energy) after recombination. For example, in the expression

$$B^{(cc)}(\mathbf{k}_1,\mathbf{q}_1)=\{2[E_g^2+{}^4/_3P^2(\mathbf{k}_1\mathbf{q}_1)]-2E_g[E_l(\mathbf{k}_1)+E_l(\mathbf{q}_1)]+E_l(\mathbf{k}_1)E_l(\mathbf{q}_1)[{}^5/_2+{}^3/_2k_1^{-2}q_1^{-2}(\mathbf{k}_1\mathbf{q}_1)^2]\}\times(E_g^2+{}^8/_3P^2k_1^2)^{-1/2}(E_g^2+{}^8/_3P^2q_1^2)^{-1/2} \tag{8}$$

we can ignore the quantity $\mathbf{q}_1=\mathbf{k}+\frac{1}{2}\mathbf{k}_{12}$ compared with $\mathbf{k}_1$. Moreover, we find that $\varepsilon_c(\mathbf{q}_1)\ll E_g$. Therefore, in the first term inside the braces in Eq. (6), we can assume that

$$B^{(cc)}(\mathbf{k}_1,0)=\frac{2E_c(\mathbf{k}_1)}{(E_g^2+{}^8/_3P^2\mathbf{k}_1^2)^{1/2}}. \tag{9}$$

However, in the expression for the overlap integrals of the electron and heavy-hole bands

$$B^{(eh)}(\mathbf{q}_2,2\mathbf{k}-\mathbf{k}_1)=\frac{P^2[(\mathbf{k}_1-2\mathbf{k})\times\mathbf{q}_2]^2}{(\mathbf{k}_1-2\mathbf{k})^2E_c(\mathbf{q}_2)(E_g^2+{}^8/_3P^2q_2^2)^{1/2}}, \tag{10}$$

where $\mathbf{q}_2=\mathbf{k}-\frac{1}{2}\mathbf{k}_{12}$, we cannot assume that $\mathbf{q}_2=0$.

In the calculations of Beattie *et al.*,[2-5] all the overlap integrals are assumed to be constants which can be calculated for the threshold values of the particle momenta, whereas it follows from Eq. (10) that the integral representing the overlap of the electron and heavy-hole bands vanishes if the threshold values of the momenta are substituted in Eq. (10). This vanishing of the overlap integral at the threshold values of the momentum is due to the fact that the wave functions of a heavy hole in the Kane model are orthogonal, if the interaction with higher bands is ignored, to the wave function of an electron or a light hole provided the momenta of the latter particles are directed parallel to the heavy-hole momentum. This, in fact, is responsible for the modified temperature dependence of the preexponential factor in the expression for the Auger recombination rate. Similarly, we can show that the quantity D in Eq. (6) also vanishes for the threshold momenta. This follows from the fact that, in the Kane model without allowance for interaction with higher bands, the product of the relevant matrices is

$$\Lambda^{(c)}(\mathbf{q})\Lambda^{(h)}(\mathbf{k}_1)=0 \tag{11}$$

if the value of $\mathbf{q}\|\mathbf{k}_1$ is arbitrary. Estimates obtained by the sum rule[4,5] give incorrect results if the integrals of the overlap between the wave functions of heavy holes and higher bands are ignored. Beattie and Smith[11] give three different estimates of the overlap integrals in the Kane model. All of them proceed from the assumption that the integral of the overlap of the electron and heavy-hole wave functions is a constant quantity with small temperature-dependent corrections. The first estimate is based on the sum rule.[4,5] The second estimate relates the overlap integral to the absorption coefficient and the third is obtained by direct calculation of the matrix element of the momentum related, according to Eqs. (17) and (18) in the paper by Beattie and Smith,[11] to the overlap integral. However, although the formulas for the absorption coefficient of light and the Auger recombination rate can be written so that both contain the matrix element of the momentum, there are considerable differences between them. In calculating the absorption coefficient of unpolarized light, it is necessary to average over all the polarizations, whereas in the calculation of the Auger recombination rate the matrix element is evaluated for the momentum parallel to that of a heavy hole if we ignore the small electron momentum. This also follows from Eq. (18) in Ref. 11. On the other hand, a transition from the heavy-hole to the conduction band can be described by an oscillator localized in a plane perpendicular to the momentum.[12] Therefore, as pointed out above, the matrix element of the threshold momentum vanishes in the expression for the Auger recombination rate.

Since $q_2\ll k_1$, $k\ll k_1$, $Pq_2\ll E_g$, it follows that Eq. (10) can be expanded in terms of the small quantities

$$B^{(eh)}\left(\mathbf{k}-\frac{1}{2}\mathbf{k}_{12},2\mathbf{k}-\mathbf{k}_1\right)\approx\frac{P^2}{k_1^2E_g^2}\left([\mathbf{k}_1\times\mathbf{k}]-\frac{1}{2}[\mathbf{k}_1\times\mathbf{k}_{12}]\right)^2. \tag{12}$$

We shall expand the second term in the braces of Eq.

(6) in terms of the same small quantities. It follows from Eq. (11) that only terms of the second order in k and $\mathbf{k}_{12}$ should be retained in the expansion:

$$D(\mathbf{k}-{}^1/_2\mathbf{k}_{12},\ \mathbf{k}_1,\ \mathbf{k}+{}^1/_2\mathbf{k}_{12};\ 2\mathbf{k}-\mathbf{k}_1)\approx A_{\alpha\beta}(\mathbf{k}_1)\,(k_\alpha k_\beta-{}^1/_4 k_{12\alpha}k_{12\beta}), \quad (13)$$

where

$$A_{\alpha\beta}(\mathbf{k}_1)=\frac{1}{2}\frac{\partial^2}{\partial q_\alpha \partial q_\beta}D(\mathbf{q},\mathbf{k}_1,\mathbf{q};-\mathbf{k}_1)|_{\mathbf{q}=0}.$$

The expansion (13) does not include a term of the $k_\alpha k_{12\beta}$ type because the coefficient in front of it is

$$\mathrm{Sp}\left[\frac{\partial\Lambda^{(c)}(\mathbf{q})}{\partial q_\alpha}\Lambda^{(h)}(-\mathbf{k}_1)\frac{\partial\Lambda^{(c)}(\mathbf{q})}{\partial q_\beta}-\frac{\partial\Lambda^{(c)}(\mathbf{q})}{\partial q_\beta}\Lambda^{(h)}(-\mathbf{k}_1)\times\frac{\partial\Lambda^{(c)}(\mathbf{q})}{\partial q_\alpha}\right]\Lambda^{(c)}(\mathbf{k}_1)|_{q=0}=0.$$

The validity of the above expression follows from the fact that we cannot form an antisymmetric tensor from the component of the vector $\mathbf{k}_1$. It is clear from Eqs. (10) and (11) that $D(\mathbf{q},\ \mathbf{k}_1,\ \mathbf{q};\ -\mathbf{k}_1)=0$ if $\mathbf{q}\|\mathbf{k}_1$ and the value of q is arbitrary. Consequently, the tensor in question should be

$$A_{\alpha\beta}(\mathbf{k}_1)=F(k_1^2)\,(\delta_{\alpha\beta}-k_{1\alpha}k_{1\beta}/k_1^2). \quad (14)$$

We shall not give here the derivation of the expression for the function $F(k_1^2)$, which is

$$F(k_1^2)=\frac{P^2E_c(k_1)}{E_g^2(E_g^2+{}^8/_3P^2k_1^2)^{1/2}}, \quad (15)$$

because—as shown below—the term in Eq. (6) containing D vanishes in the approximation adopted here. It follows from Eqs. (13) and (14) that

$$D(\mathbf{k}-{}^1/_2\mathbf{k}_{12},\ \mathbf{k}_1,\ \mathbf{k}+{}^1/_2\mathbf{k}_{12};\ 2\mathbf{k}-\mathbf{k}_1)=F(k_1^2)k_1^{-2}\{[\mathbf{k}_1\times\mathbf{k}]^2-{}^1/_4[\mathbf{k}_1\times\mathbf{k}_{12}]^2\}. \quad (16)$$

We shall find the expression for the Auger recombination rate for the case when $E_g\gg T$. Therefore, we shall substitute in Eq. (6) all the quantities calculated for the threshold values of the parameters, with the exception of the exponential function in $f_e(\mathbf{k}_1)$, in which we shall include a correction associated with the mass ratio. In the δ function, we shall expand the energies in terms of the momenta and assume that the deviations from the threshold values are small:

$$G=\frac{4ne^4}{(2\pi)^{5/2}}(m_cT)^{-1/2}(\varkappa k_{1T}^2)^{-2}\int d^3k_1d^3kd^3k_{12}\times\delta\left[\frac{4P}{3^{1/2}}(k_1-k_{1T})-m_c^{-1}\left(k^2+\frac{1}{4}k_{12}^2\right)\hbar^2\right]\left\{\frac{4P^4}{3E_g^4}\left([\mathbf{k}_1\times\mathbf{k}]-\frac{1}{2}[\mathbf{k}_1\times\mathbf{k}_{12}]\right)^2-k_{1T}^{-2}F(k_{1T}^2)\left([\mathbf{k}_1\times\mathbf{k}]^2-\frac{1}{4}[\mathbf{k}_1\times\mathbf{k}_{12}]^2\right)\right\}\times\exp\left\{-\frac{E_g}{T}-\frac{4P}{3^{1/2}T}(k_1-k_{1T})-T^{-1}\varepsilon_h(k_{1T})\right\}. \quad (17)$$

After integration on the assumption of an isotropic parabolic dispersion law for heavy holes, we obtain

$$G=3\left(\frac{2}{\pi}\right)^{1/2}\frac{ne^4m_c}{\varkappa^2\hbar^3}\left(\frac{T}{E_g}\right)^{3/2}\exp\left[-\frac{E_g}{T}(1+2\mu)\right], \quad (18)$$

where $\mu=m_c/m_h$.

We can use the same approximation to calculate the Auger recombination rate for the spectrum of heavy holes, allowing for the corrugations of constant-energy surfaces. To include these corrugations, we have to find from Eq. (3) the value of ε_h corresponding to $k_{1T}=2\hbar^{-1}(m_cE_g)^{1/2}$, substitute it in the exponential function, and average over the angle. We finally obtain

$$G=\frac{6}{\pi}\left(\frac{2}{\pi}\right)^{1/2}\frac{ne^4m_c}{\varkappa^2\hbar^3}\left(\frac{T}{E_g}\right)^{1/2}\exp\left(-\frac{E_g}{T}\right)\int_0^{\pi/2}d\varphi\int_0^{\pi/2}d\theta\sin\theta\times\exp\left\{-\frac{2m_cE_g}{m_0T}\left[\gamma_1'-2\gamma_2'-6(\gamma_3'-\gamma_2')\sin^2\theta\left(\cos^2\theta+\frac{1}{4}\sin^2\theta\sin^2\varphi\right)\right]\right\} \quad (19)$$

We shall now compare the calculated value of G with the experimental data for InSb at $T=300$ °K (Ref. 13), when the Auger recombination process predominates. We shall assume the following parameters of InSb: $E_g=0.167$ eV (Ref. 9), $E_p=2m_0P^2\hbar^{-2}=21.2$ eV, $\gamma_1'=3.6$, $\gamma_2'=-0.47$, $\gamma_3'=0.70$ (Ref. 14), and $\varkappa=16.8$ (Refs. 9 and 15).

A calculation carried out using Eq. (18) in the approximation of a spherical heavy-hole band gives, for $\gamma'=\frac{1}{5}(2\gamma_2'+3\gamma_3')$ (Refs. 16 and 17), the recombination time $\tau_{Ai}=n/2G=10^{-8}$ sec. Equation (19) gives practically the same value of τ_{Ai}. The experimental value is $\tau_{Ai}=2\times10^{-8}$ sec (Ref. 13). Thus, Eq. (18) gives a recombination time close to the experimental values without the use of additional adjustable parameters.

4. INFLUENCE OF CORRECTIONS FOR THE INTERACTION WITH HIGHER BANDS ON THE AUGER RECOMBINATION RATE

The formulas for the overlap integrals in Sec. 3 are obtained ignoring the interaction with higher bands. The main reason for the modification of the temperature dependence of the preexponential function can be expressed in the form $\Lambda^{(c)}(0)\Lambda^{(h)}(-\mathbf{k}_1)=0$. The corrections due to higher bands may give a nonvanishing value of $\Lambda^{(c)}(0)\Lambda^{(h)}(-\mathbf{k}_1)$ and thus restore the original temperature dependence of the preexponential function.[2] However, in the spherical approximation, even the inclusion of corrections due to higher bands gives $B^{(ch)}(0,\ -\mathbf{k}_1)=0$. A finite value of $B^{(ch)}(0,\ -\mathbf{k}_1)$ is only obtained if allowance is made for the corrugations of constant-energy surfaces. If such corrugations are strong, it is found that $B^{(ch)}(0,\ -\mathbf{k}_1)\sim(m_c/m_h)^2$, and the original preexponential temperature dependence appears only for $(m_c/m_h)^2>T/E_g$. If the corrugations are weak, we find that $B^{(ch)}(0,\ -\mathbf{k}_1)\ll(m_c/m_h)^2$.

A calculation of $B^{(cc)}$, $B^{(ch)}$, and D for the threshold values of the particle momenta is given in the Appendix. Substituting these quantities in Eq. (6), we find that

$$G=\frac{3}{\pi}\left(\frac{2}{\pi}\right)^{1/2}\frac{ne^4m_c^3}{\hbar^3\varkappa^2m_0^2}(\gamma_3'-\gamma_2')^2\left(\frac{T}{E_g}\right)^{1/2}\exp\left(-\frac{E_g}{T}\right)\times\int_0^{2\pi}d\varphi\int_0^{\pi}d\theta\sin\theta\,[25s(\mathbf{k}_1)-96s^2(\mathbf{k}_1)+63k_1^{-6}(k_{1x}k_{1y}k_{1z})^2]\exp\left[-\frac{\varepsilon_h(k_{1T})}{T}\right]\approx\frac{432}{35}\left(\frac{2}{\pi}\right)^{1/2}\frac{ne^4m_c^3}{\hbar^3\varkappa^2m_0^2}(\gamma_3'-\gamma_2')^2\left(\frac{T}{E_g}\right)^{1/2}\exp\left[-\frac{E_g}{T}(1+2\mu)\right]. \quad (20)$$

This formula is valid for $(m_c/m_h)^2>T/E_g$, if the degree of corrugation of the constant-energy surfaces of the heavy holes is of the order of unity. If it is less than

unity, the range of validity of Eq. (20) is even narrower. An estimate obtained for InSb at $T = 300\,^{\circ}\mathrm{K}$ shows that the expression in Eq. (20) represents ~0.005 of G, calculated using Eq. (18).

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APPENDIX

To determine the overlap integrals corresponding to the threshold values of the particle momenta, we shall obtain relationships valid in the lowest nonvanishing order:

$$\Lambda^{(c)}(0)\Lambda^{(h)}(-\mathbf{k}_1)\Lambda^{(c)}(0)=\tfrac{1}{2}B^{(ch)}(0,\ -\mathbf{k}_1)\Lambda^{(c)}(0), \tag{A.1}$$

$$B^{(ch)}(0,-\mathbf{k}_1)=2\left\{\left[\varepsilon_h(\mathbf{k}_1)-\frac{\hbar^2}{2m_0}\gamma_1'k_1^2\right]^2-(\hbar^2k_1^2\gamma_2'm_0^{-1})^2+\frac{3\hbar^2}{m_0^2}(\gamma_2'^2-\gamma_3'^2)k_1^4s(\mathbf{k}_1)\right\}\{E_c(\mathbf{k}_1)E_l(\mathbf{k}_1)+2\varepsilon_h(\mathbf{k}_1)[\varepsilon_h(\mathbf{k}_1)+E_c(\mathbf{k}_1)+E_l(\mathbf{k}_1)]\}^{-1}, \tag{A.2}$$

$$\Lambda^{(c)}(0)\Lambda^{(c)}(\mathbf{k}_1)\Lambda^{(c)}(0)=E_c(\mathbf{k}_1)(E_g^2+\tfrac{8}{3}P^2k_1^2)^{-1/2}\Lambda^{(c)}(0); \tag{A.3}$$

Here, $\Lambda^{(c)}(0)$ represents a diagonal matrix in which the first two diagonal terms are equal to unity and the others vanish. Equation (A.2) is derived using the Vieta equalities relating the roots of Eq. (3):

$$E_c(\mathbf{k})+E_l(\mathbf{k})+E_h(\mathbf{k})=E_g+\hbar^2k^2/2m_c'-\hbar^2\gamma_1'k^2/m_0, \quad E_cE_l+E_lE_h+E_hE_c=3(k^2/m_0)^2s(\mathbf{k})(\gamma_2'^2-\gamma_3'^2)\hbar^4 \tag{A.4}$$

$$-\tfrac{2}{3}P^2k^2-\hbar^4(\gamma_2'^2+\tfrac{3}{4}\gamma_1'^2)k^4m_0^{-2}-\hbar^2\gamma_1'k^2m_0^{-1}(E_l+E_h+E_c). \tag{A.5}$$

It follows from Eq. (A.2) that only allowance for the corrugations of constant-energy surfaces gives a finite value of $B^{(ch)}(0,\ -\mathbf{k}_1)$. Since $E_g \gg k_1^2\hbar^2/m_h$, we can expand Eq. (A.2) in terms of small parameters γ_1', γ_2', and γ_3':

$$B^{(ch)}(0,-\mathbf{k}_1)=\frac{9\hbar^4k_1^2}{P^2m_0^2}(\gamma_3'-\gamma_2')^2s(\mathbf{k}_1)[1-3s(\mathbf{k}_1)]. \tag{A.6}$$

The relationships (A.1), (A.3), and (A.6) are then used to expand the expression in the braces of Eq. (6) in terms of a small quantity k_T:

$$B^{(ch)}(\mathbf{k}_T,2\mathbf{k}_T-\mathbf{k}_1)\approx B^{(ch)}(0,-\mathbf{k}_1)+m_c\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial k_{1\alpha}}\frac{\partial}{\partial q_\alpha}\times B^{(ch)}(\mathbf{q},-\mathbf{k}_1)|_{\mathbf{q}=0}+\left(\frac{m_cP}{E_gk_1\hbar^2}\right)^2\left[\mathbf{k}_1\times\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial\mathbf{k}_1}\right]^2, \tag{A.7}$$

$$D(\mathbf{k}_T,\mathbf{k}_1,\mathbf{k}_T;\ 2\mathbf{k}_T-\mathbf{k}_1)\approx D(0,\mathbf{k}_1;\ 0;\ -\mathbf{k}_1)+m_c\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial k_{1\alpha}}\frac{\partial}{\partial q_\alpha}D(\mathbf{q},\mathbf{k}_1,\mathbf{q};-\mathbf{k}_1)|_{\mathbf{q}=0}+\frac{m_c^2F(k_1^2)}{\hbar^4k_1^2}\left[\mathbf{k}_1\times\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial\mathbf{k}_1}\right]^2. \tag{A.8}$$

In the expansion, the second derivatives are calculated in accordance with Eqs. (10) and (16), ignoring higher bands. The results of calculations of the separate terms in Eqs. (A.7) and (A.8) are as follows:

$$\left[\mathbf{k}_1\times\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial\mathbf{k}_1}\right]^2=\frac{36\hbar^4}{m_0^2}(\gamma_3'-\gamma_2')^2k_1^4\{s(\mathbf{k}_1)[1-4s(\mathbf{k}_1)]+3k_1^{-6}k_{1x}^2k_{1y}^2k_{1z}^2\}, \tag{A.9}$$

$$D(0,\ \mathbf{k}_1,\ 0;\ -\mathbf{k}_1)=E_c(\mathbf{k}_1)(E_g^2+\tfrac{8}{3}P^2k_1^2)^{-1/2}B^{(ch)}(0,\ -\mathbf{k}_1), \tag{A.10}$$

$$\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial k_{1\alpha}}\frac{\partial}{\partial q_\alpha}B^{(ch)}(\mathbf{q},-\mathbf{k}_1)|_{\mathbf{q}=0}=(k_1^2E_g\hbar^2)^{-1}\left[\mathbf{k}_1\times\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial\mathbf{k}_1}\right]^2, \tag{A.11}$$

$$\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial k_{1\alpha}}\frac{\partial}{\partial q_\alpha}D(\mathbf{q},\mathbf{k}_1,\mathbf{q};-\mathbf{k}_1)|_{\mathbf{q}=0}=(\hbar^2k_1^2E_g)^{-1}E_c(\mathbf{k}_1)\left(E_g^2+\frac{8}{3}P^2k_1^2\right)^{-1/2}\left[\mathbf{k}_1\times\frac{\partial\varepsilon_h(\mathbf{k}_1)}{\partial\mathbf{k}_1}\right]^2. \tag{A.12}$$

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