Self-localized states of the magnetic moment in a rotating magnetic field

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It is shown that the Landau–Lifshitz equations admit the existence of self-localized solutions of the magnetic solution type if allowance is made for the dissipation and internal magnetic fields in the presence of a magnetizing field parallel to anisotropy axis and an external magnetic field rotating in a plane orthogonal to this axis.

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1. The existence of self-localized states of the magnetic moment in uniaxial ferromagnets has been demonstrated in several papers.[1-3] These states are characterized by the localization of the magnetic moment in respect of the polar angle θ and precession in respect of the azimuthal angle φ. The problem is essentially similar to that of a domain structure in an effective external magnetic field directed along the anisotropy axis and governed by the precession frequency of the magnetic moment ω. In more thorough studies of the properties of such situations, it is necessary to solve the problem of excitation of self-localized states of the magnetic moment.

We shall show that, in a spatially one-dimensional case, the Landau–Lifshitz equations admit the existence of self-localized solutions if allowance is made for internal magnetic fields, dissipation, and an external magnetic field rotating in a plane orthogonal to the anisotropy axis.

If the dissipation is ignored, the problem of self-localized (along the easy magnetization axis) states of the magnetic moment admits exact solution if allowance is made for internal magnetic fields: the problem then reduces essentially to one of a domain structure in an effective magnetic field whose component along the anisotropy axis is governed by the pump field frequency ω and the component orthogonal to this axis is determined by the amplitude of the pump field $h_\perp$.

In the $(\omega, h_\perp)$ plane, the range of existence of self-localized states of the magnetic moment is bounded by the curve

$$\omega^{2/3}+h_\perp^{2/3}=1 \tag{1.1}$$

(the pump field frequency is in units of the characteristic frequency of precession in the anisotropy field and the pump field amplitude is expressed in terms of the anisotropy field). It should be noted that a similar curve bounds the range of existence of a domain structure in an inclined external magnetic field.[4] Small-amplitude localized states of the magnetic moment appear in the vicinity of the boundary curve (1.1) shown in Fig. 1. Well inside the region bounded by the curve (1.1), the amplitude of the self-localized solution increases and the characteristic size of the spatial localization region decreases. It is important to note that the characteristic size of the spatial localization of the magnetic moment in the case of small-amplitude solutions is governed by the frequency and amplitude of the pump field. The exact solutions of the Landau–Lifshitz equations corresponds to self-localization in respect of the polar angle θ and absence of a phase shift between the homogeneous precession of the magnetic moment and rotation of the magnetic vector of the pump field.

If allowance is made for the damping, self-localized solutions are characterized by a phase shift between the azimuthal angle φ, which determines the spatially inhomogeneous precession of the magnetic moment, and the angle of rotation of the magnetic vector of the pump field. There is a threshold of the amplitude of the pump field (Fig. 1). In the absence of damping, the Landau–Lifshitz equations have the first integral. If allowance is made for the damping and pump field, the derivative of the "first integral" has a variable sign. It is this circumstance that is responsible for the retention of self-localized distributions of the magnetic moment.

2. The Landau–Lifshitz equations for a uniaxial ferromagnet in an external magnetic field rotating in a plane orthogonal to the anisotropy axis have the following form if allowance is made for the spatial inhomogeneity of the magnetic moment along this axis:

$$-\sin\theta\frac{\partial\varphi}{\partial t}+\alpha\frac{\partial\theta}{\partial t}=\frac{\partial^2\theta}{\partial z^2}-\left[1+\left(\frac{\partial\varphi}{\partial z}\right)^2\right]\sin\theta\cos\theta$$
$$-\varepsilon(\cos\theta-\cos\theta_*)\sin\theta+h_\perp\cos(\varphi-\omega t)\cos\theta,$$

$$\sin\theta\frac{\partial\theta}{\partial t}+\alpha\sin^2\theta\frac{\partial\varphi}{\partial t}=\frac{\partial}{\partial z}\left(\sin^2\theta\frac{\partial\varphi}{\partial z}\right)-h_\perp\sin(\varphi-\omega t)\sin\theta. \tag{2.1}$$

Here, α is the damping parameter; ω and $h_\perp$ are the frequency and amplitude of the magnetic pump field;

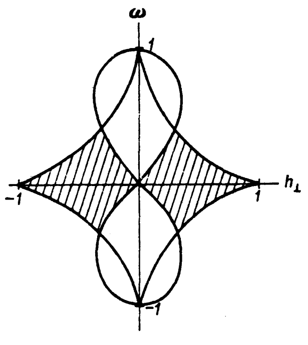

FIG. 1.

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$\varepsilon = 2\pi M_s^2/K_1$ is the parameter of the magnetic medium (M_s is the saturation magnetization and K_1 is the uniaxial anisotropy energy constant); the spatial variable z is expressed in terms of the characteristic size of a Bloch–Landau domain boundary which is $(A/K_1)^{1/2}$ (A is the exchange energy constant); the temporal variable t is expressed in terms of the reciprocal precession frequency in the anisotropy field. The Landau–Lifshitz equations (2.1) are derived on the assumption that the internal magnetic fields in the adopted geometry of the problem are given by

$$h_z^{(i)} = -\varepsilon(\cos\theta - \cos\theta_s), \tag{2.2}$$

where θ_s is the integration constant defined below. The transformation

$$\varphi \Rightarrow \Phi = \varphi - \omega t \tag{2.3}$$

reduces the system (2.1) to

$$\begin{aligned} -\sin\theta\frac{\partial\Phi}{\partial t} + \alpha\frac{\partial\theta}{\partial t} &= \frac{\partial^2\theta}{\partial z^2} - \left[1 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right]\sin\theta\cos\theta \\ &+ \omega\sin\theta + h_\perp\cos\Phi\cos\theta - \varepsilon\sin\theta(\cos\theta - \cos\theta_s), \\ \sin\theta\frac{\partial\theta}{\partial t} + \alpha\sin^2\theta\frac{\partial\Phi}{\partial t} &= \frac{\partial}{\partial z}\left(\sin^2\theta\frac{\partial\Phi}{\partial z}\right) - h_\perp\sin\Phi\sin\theta + \alpha\omega\sin^2\theta. \end{aligned} \tag{2.4}$$

One of the features of the system (2.4) is the absence of an explicit time dependence, which allows us to identify the exact solutions

$$\frac{\partial\theta}{\partial t} = 0, \quad \frac{\partial\Phi}{\partial t} = 0 \tag{2.5}$$

or

$$m_z = \cos\theta, \quad m_x \pm i m_y = \exp[\pm i(\omega t + \Phi)]\sin\theta.$$

In this case, the spatial distribution (θ, Φ) is given by the system of equations

$$\begin{aligned} &\frac{\partial^2\theta}{\partial z^2} - \left[1 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right]\sin\theta\cos\theta + \omega\sin\theta + h_\perp\cos\Phi\cos\theta \\ &\quad - \varepsilon(\cos\theta - \cos\theta_s)\sin\theta = 0, \\ &\frac{\partial}{\partial z}\left(\sin^2\theta\frac{\partial\Phi}{\partial z}\right) - h_\perp\sin\Phi\sin\theta + \alpha\omega\sin^2\theta = 0. \end{aligned} \tag{2.6}$$

A consequence of the system (2.6) is the relationship

$$\frac{d\mathscr{H}}{dz} = 2\alpha\omega\sin^2\theta\frac{d\Phi}{dz}, \tag{2.7}$$

in which

$$\begin{aligned} \mathscr{H} = \left(\frac{d\theta}{dz}\right)^2 + \left(\frac{d\Phi}{dz}\right)^2\sin^2\theta - \sin^2\theta + \varepsilon(\cos\theta - \cos\theta_s)^2 \\ - 2\omega\cos\theta + 2h_\perp\cos\Phi\sin\theta. \end{aligned} \tag{2.8}$$

In the absence of damping ($\alpha = 0$), Eq. (2.8) gives the first integral of the system (2.6):

$$\mathscr{H}\left(\theta, \Phi, \frac{d\theta}{dz}, \frac{d\Phi}{dz}\right) = 0.$$

If we identify the integration constant θ_s with the polar angle governing the orientation of the magnetic moment in the spatially homogeneous case, we find that, for spatially homogeneous states, the polar angle θ_s and the phase shift between the rotating pump field and the azimuthal angle φ_s of the magnetic moment are given by solutions of the system

$$\begin{aligned} \cos\theta_s &= \frac{\omega\sin\theta_s}{\sin\theta_s - h_\perp\cos\Phi_s}, \\ \sin\Phi_s &= -\frac{\alpha\omega}{h_\perp}\sin\theta_s. \end{aligned} \tag{2.9}$$

Eliminating Φ_s, we obtain

$$\cos\theta_s = \frac{\omega\sin\theta_s}{\sin\theta_s - [h_\perp^2 - (\alpha\omega)^2\sin^2\theta_s]^{1/2}}. \tag{2.10}$$

Equation (2.10) correponds to the algebraic equation of the fourth degree

$$\begin{aligned} &[1 + (\alpha\omega)^2]\cos^4\theta_s - 2\omega\cos^3\theta_s \\ &+ [\omega^2 + h_\perp^2 - 1 - (\alpha\omega)^2]\cos^2\theta_s + 2\omega\cos\theta_s - \omega^2 = 1. \end{aligned} \tag{2.11}$$

It should be noted that the degree of this equation remains the same in the $\alpha = 0$ case.

In the absence of damping, we have $\alpha = 0$ and the phase shift Φ_s vanishes. In this case, an analysis of Eq. (2.10) demonstrates the existence in the $(\omega, h_\perp)$ plane of the curve (1.1) bounding the permissible values of the pump field parameters for which self-localized states of the magnetic moment can exist. In other words, the boundary curve (1.1) encloses the region of existence of the separatrix solutions (magnetic solitons). The situation is then fully analogous to the problem of a domain structure in a field of arbitrary orientation, described by Landau and Lifshitz.[4] If, in addition to a rotating pump field, there is also a static magnetic field h_z along the anisotropy axis, the equation for the boundary curve becomes

$$(\omega - h_z)^{2/3} + h_\perp^{2/3} = 1. \tag{2.12}$$

Thus, in the absence of damping, the problem of self-localized (along the anisotropy axis) states of the magnetic moment reduces to an analysis of singular phase trajectories (separatrices) for the first integral

$$\mathscr{H} = \left(\frac{d\theta}{dz}\right)^2 - \sin^2\theta + \varepsilon(\cos\theta - \cos\theta_s)^2 - 2\omega\cos\theta + 2h_\perp\sin\theta = \text{const}, \tag{2.13}$$

in which θ_s satisfies

$$\cos\theta_s = \frac{\omega\sin\theta_s}{\sin\theta_s - h_\perp}. \tag{2.14}$$

The merging of a pair of roots of Eq. (2.14) in the phase plane corresponds to merging singularities of the saddle and center type. This occurs on the boundary curve (1.1) or (2.12) if the static magnetization field is not zero.

If there is damping, the phase shift Φ_s no longer vanishes and an analysis of Eq. (2.10) demonstrates the existence of a pump threshold. If $h_\perp/\alpha\omega \gg 1$, which corresponds to a considerable excess above the threshold, the boundary curve is always close to that given by Eq. (1.1), which describes the range of existence of self-localized states of the magnetic moment. However, at a given pump field frequency, $h_\perp$, the amplitude of the pump field cannot be less than the value given by

$$\omega^2 + (h_\perp/\alpha\omega)^2 = 1. \tag{2.15}$$

In the $(\omega, h_\perp)$ plane, Eq. (2.15) represents a figure-of-eight curve, shown in Fig. 1. For $h_\perp/\alpha\omega \gg 1$, the range of existence of self-localized states is, in the first approximation, represented by the area under the curve

(1.1) except for the area bounded by Eq. (2.15). The permissible range of values $h_\perp$ and ω is shown shaded in Fig. 1. Thus, at a given pump field frequency ω, the amplitude of the pump field has upper and lower bounds.

In the presence of damping, an important feature ensuring the retention of self-localized distributions of the magnetic moment (of the soliton type) is the possibility of a change of the sign of the derivative of the first integral (2.8), which is associated—according to Eq. (2.7)—with a change in the sign of the spatial derivative of the phase shift. The establishment of a constant phase shift in the homogeneous magnetization region, namely, the condition

$$\lim_{|z|\to\infty}\left(\frac{d\Phi}{dz}\right)=0$$

corresponds to

$$\lim_{|z|\to\infty}\mathscr{H}=-\sin^2\theta_\infty-2\omega\cos\theta_\infty+2h_\perp\cos\Phi_\infty\sin\theta_\infty.$$

Consequently, a self-localized state of the magnetic moment should satisfy the condition

$$\int_{-\infty}^{\infty}dz\sin^2\theta\frac{d\Phi}{dz}=0.$$

It should be noted that the above type of self-localized solution corresponds to the conservation of the projection of the total moment along the anisotropy axis:

$$\int dz(\cos\theta-\cos\theta_\infty)=\text{const}.$$

We shall conclude by mentioning that the possibility of excitation of magnetic solitons by a traveling external magnetic field was considered by Akhiezer and Borovik.[5]

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Optical properties and structural ordering of the planar texture of a cholesteric liquid crystal

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The experimental dependences of the circular dichroism are used to calculate the optical-anisotropy parameter and the number of turns of the helix of the planar texture of cholesteric liquid crystals (CLC) of the homologic series of cholesteryl alkanoates. The dependence of the orientational order of CLC on the temperature and structure of the mesogen molecule is investigated and it is shown that this order decreases regularly with increasing length of the alkyl chain of the ester. A section is found, not predicted by the theory, where the optical-anisotropy parameter drops jumpwise in the vicinity of the cholesteric–smectic-*A* phase transition. The temperature dependences of the relative change of the translational order of the CLC are obtained, the growth of the order being accompanied by untwisting of the cholesteric helix.

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INTRODUCTION

One of the least investigated problems of liquid-crystal physics is the structural ordering of the molecules in the cholesteric mesophase. In particular, one cannot regard as finally settled so very important an aspect of the hypomolecular structure of cholesteric liquid crystals (CLC) as the quantitative characteristics of the orientational ordering, the onset of translational order near the cholesteric–smectic-*A* transition, as well as the effect exerted on the structural ordering by macroscopic inhomogeneities of the planar structure.

Orientational ordering in the cholesteric mesophase can be naturally described with the aid of the orientational-order parameter $\eta=\langle P_2(\cos\theta)\rangle$, where θ is the angle between the long axis of the molecule and the axis of the predominant molecular orientation (director), and P_2 is a Legendre polynomial. The presence of the helical twist makes it inconvenient to use for CLC most of the experimental methods employed to determine η in nematics. The theoretical $\eta(T)$ temperature dependence was verified experimentally a number of times on nematic liquid crystals (NLC) (see, e.g., Refs. 1–3), and a connection was established between the subtle features of the structure of mesogenic

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