Two-dimensional electron-impurity system in a strong magnetic field

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We investigate the electron spectrum and the conductivity of a two-dimensional electron gas (quantum film, inversion channel) with impurities in a strong magnetic field perpendicular to the plane of the system. The cases of a quantizing and a classical magnetic field are considered. For the first case, the spectrum of the system with pointlike impurities is considered. It is shown that at an impurity concentration n less than $1/2\pi a^2$ (a is the magnetic length) the state with energy $\hbar\omega_H(N+1/2)$ is infinitely degenerate [multiplicity $(1/2\pi a^2 - n)S$, where S is the film area]. The static conductivity of such a system vanishes, and the dynamic conductivity at the cyclotron resonance ω_H has an infinitesimally narrow peak. In classical magnetic fields the conductivity is investigated for the case of scattering by impurities whose characteristic dimension is much larger than the electron wavelength. The limits of infrequent short-range centers and of a continuous random potential are considered. It is shown that for both limits there is a percolation level ϵ_{cH} determined by the magnetic field, and the electron trajectory becomes infinite above this level. The conductivity of the system at zero temperature vanishes if the Fermi energy is less than ϵ_{cH}. It is shown that in the limit of infrequent impurities the expansion of the current in powers of the electric field **E** begins with the term $\mathbf{E}|\mathbf{E}|$.

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1. INTRODUCTION

Attention was called in a number of recent papers[1-6] to the properties of a two-dimensional electron gas in a strong magnetic field perpendicular to a film plane.[1)] The most interesting property of such a system is the discrete energy spectrum

$$\varepsilon_N = \hbar\omega_H(N+1/2), \tag{1}$$

where ω_H is the cyclotron frequency. It would be natural to expect the collisions of the electrons with one another, with impurities, and with phonons to lead to a broadening of the discrete levels.[1] The question of the broadening of the electronic states is essential for the determination of the cyclotron-resonance line shape and of the static conductivity of the system. It is known, however, that electron-electron collisions produce no broadening of the cyclotron-resonance line.[7]

We consider in this paper the interaction of "two-dimensional" electrons with impurities in a strong magnetic field. For a quantizing magnetic field, this question was considered earlier in Refs. 3–6. All the cited studies reduce to summation of certain partial series of the perturbation-theory series. It can be shown, however, that in our problem there is no symbolic parameter with respect to which the discarded part of the series is small (see Sec. 3). The reason lies in the infinite multiplicity of the degeneracy of the Landau levels. The construction of a perturbation theory must start with a choice of a correct combination of unperturbed states. In the present paper we draw definite conclusions concerning the spectrum and the conductivity of a two-dimensional system of non-interacting electrons in a strong magnetic field (quantizing and classical), without the aid of a diagram technique. It will be shown that the states of a system of electrons with pointlike impurities break up into two groups. The states of the first group coincide with certain states of free electrons. The states of the second group are localized near the impurities (Sec. 2). This leads absence of static conductivity in a weak electric field (Sec. 3). It is also shown in Sec. 3 that the cyclotron-resonance spectrum contains a discrete line. We discuss its broadening due to simultaneous account of the finite dimensions and finite density of the impurities.

Section 4 deals with the conductivity of a two-dimensional classical electron gas in a continuous random field of impurities and in a field of infrequent short-range impurities. It turns out that some of the electrons move along finite trajectories, and this can lead to zero conductivity in the case of degenerate electrons at zero temperature. It is shown that in this limit, for the case of short-range impurities, the expansion of the current powers of the electric field **E** begins with the term $\mathbf{E}|\mathbf{E}|$.

2. ELECTRON SPECTRUM IN A QUANTIZING MAGNETIC FIELD

Consider the interaction between electrons and a system of randomly distributed impurity centers:

$$U(\rho) = \sum_{i=1}^{\mathcal{N}} u_i \delta(\rho - \rho_i), \tag{2}$$

where $u_i = u_0|\xi(z_i)|^2$ ($\xi(z)$ is the wave function of the ground state for motion across the film), $\mathbf{r}_i = \{\boldsymbol{\rho}_i, z_i\}$ is the coordinate of the i-th impurity, $\mathcal{N} = nS$ is the total number of impurities in a film of area $S = L_x L_y$. The wave functions of the longitudinal motion in a film without impurities, in a gauge $A = 1/2\{-Hy, Hx, 0\}$, is of the form

$$\varphi_{Np}(\rho) = \frac{1}{(L_y a)^{1/2}} \exp\left\{ipy + i\frac{xy}{2a^2}\right\} \chi_N\left(\frac{x+a^2p}{a}\right), \tag{3}$$

where $\chi_N(x)$ are oscillator functions, p is the y-component of the momentum, and the N-th level of the magnetic quantization is degenerate in p with multiplicity

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$S/2\pi a^2$.

We consider linear combinations of the functions (3) in the form

$$\Psi_{Nj}(\rho)=\sum_p C_{Np}^j\varphi_{Np}(\rho), \tag{4}$$

which vanish at the points where the impurities are located. This yields $\mathcal{N}$ homogeneous equations for C_{Np}. Obviously, at $\mathcal{N}< S/2\pi a^2$ we can choose $S/2\pi a^2-\mathcal{N}$ independent systems of C_{Np}^j satisfying these conditions. Consequently the functions (4) are the eigenfunctions of the total Hamiltonian, and the corresponding eigenvalues coincide with (1): $\varepsilon_{Nj}\equiv\varepsilon_N$. The level degeneracy turns out to be $(S/2\pi a^2-\mathfrak{N})$-fold. Thus, at an impurity-center concentration $n<1/2\pi a^2$ the level ε_N is not broadened, but the number of states having this energy decreases with increasing density of the centers[2)] and vanishes at $n=1/2\pi a^2$. It is natural to call these states unsplit, and the remaining $\mathcal{N}$ split. We shall investigate their properties in more detail.

Following Anderson,[9] we regard a state as localized if it decreases rapidly away from a certain point—the localization centers. We shall prove that the unsplit states can be chosen to be the delocalized ones. In fact, assume that a set of unsplit states exists and includes no delocalized states. Then their sum also belongs to the Landau level. On the other hand, in a large system a set of localized state should be statistically homogeneous, i.e., in a large part of the system there should land an identical number of localization centers, regardless of the location of this part. Therefore the sum of the states will not be a localized state, since this wave function will be finite in any place in the system.

Let us now elucidate the meaning of the split states. We shall need hereafter a case of short-range centers more general than (2) with $l\ll a$, where l is the characteristic dimension of the impurity potential), but not necessarily δ-like centers. In the limit $2\pi na^2\ll 1$ we can retain only one center in the first-order approximation in the impurity density. The Hamiltonian of an electron in the field of an axially symmetric impurity, in an eigenfunction representation with a definite projection of the angular momentum m is diagonal in m. Therefore, neglecting the transitions between the Landau levels, the wave functions of the electron in the impurity field coincide with the wave functions in the absence of impurities (Ref. 10)[3)]:

$$\Psi_{Nm}(\rho)=\exp\left\{i\frac{xy_i-yx_i}{2a^2}\right\}\Phi_{Nm}(\rho-\rho_i), \tag{5}$$

where

$$\Phi_{Nm}(\rho)=\left[\frac{n_\rho!}{2\pi a^2(n_\rho+|m|)!}\right]^{1/2}\exp\left\{-im\varphi-\frac{\rho^2}{4a^2}\right\}\left(\frac{\rho^2}{2a^2}\right)^{|m|/2}L_{n_\rho}^{|m|}\left(\frac{\rho^2}{2a^2}\right),$$
$$(n_\rho=N+{}^1/_2(m-|m|),\quad m\geqslant -N),$$

and the corresponding energies are equal to

$$\varepsilon_{Nm}=\hbar\omega_H(N+{}^1/_2)+\Delta_{Nm},$$
$$\Delta_{Nm}=\frac{1}{a^{2|m|+2}}\frac{(|m|+n_\rho)!}{2^{|m|}n_\rho!(|m|!)^2}\int d\rho\,\rho^{2|m|+1}u(\rho).$$

In particular, for a potential in the form $u(\rho)=v_i\times\exp(-\rho^2/2l^2)$

$$\Delta_{Nm}=\frac{v_i}{|m|!}\frac{(|m|+n_\rho)!}{n_\rho!}\left(\frac{l}{a}\right)^{2(|m|+1)}, \tag{6}$$

while for a δ-like center only the level with $m=0$ is split:

$$\Delta_{N0}=u_i/a^2,\quad (u_i=v_il^2;\ v_i\to\infty,\ l^2\to 0). \tag{7}$$

The condition for the applicability of perturbation theory (neglect of transitions between the Landau levels) is

$$\frac{v_il^2}{\hbar\omega_Ha^2}\ln\frac{a}{l}\ll 1,\quad \left(\frac{u_i}{\hbar\omega_Ha^2}\ln\frac{a}{l}\ll 1\right). \tag{8}$$

Owing to the random disposition of the impurities in the film, the split levels form an impurity band. In a film of thickness d, simulated by an infinite rectangular potential well, the density of the split states near the level N is given by

$$\nu_N(\varepsilon)=\frac{n}{2\pi}\frac{\theta((\varepsilon-\varepsilon_N)(\varepsilon_N-\varepsilon+2u_0/a^2d))}{[(\varepsilon-\varepsilon_N)(\varepsilon_N-\varepsilon+2u_0/a^2d)]^{1/2}}. \tag{9}$$

We note that the statement that there is no smearing of the Landau levels is exact in the model of δ-like impurities if $2\pi a^2n<1$. At the same time, expression (9) for the state density is valid only if condition (8) is satisfied and if the impurity density is small, $2\pi a^2n\ll 1$. We discuss now the question of the broadening of the unsplit states at $2\pi a^2n\ll 1$. The levels with $m\neq 0$ for a potential of finite dimensions stem from levels that are not split in the model of a δ-like center. We estimate the maximum value of m at which we can neglect the influence exerted on the state of the electron by all but one impurity. For simplicity we consider the case $N=0$. The characteristic distance from the impurity, for a state with angular-momentum projection m, is the quantity $(2|m|)^{1/2}a$. From the condition that the remaining impurities be located farther from the maximum of the wave function, we find that $(2|m|)^{1/2}a\lesssim 1/2(\pi n)^{1/2}$. The minimum splitting of the state with $N=0$ is therefore

$$\delta=\frac{|u_0|}{a^2d}\left(\frac{l}{a}\right)^{\beta/4\pi a^2n}, \tag{10}$$

where β is a numerical constant. Obviously, it is δ which describes the broadening of the zeroth level. The density of the unsplit states is obtained from (6) by differentiating m with respect to ε. As a result we get

$$\nu(\varepsilon)\approx\frac{n\theta(u_0(\varepsilon-\varepsilon_0))}{2|\varepsilon-\varepsilon_0|\ln(a/l)},\quad |\varepsilon-\varepsilon_0|\gg\delta,$$
$$\nu(\varepsilon)\sim\frac{n\theta(u_0(\varepsilon-\varepsilon_0))}{\delta\ln(a/l)},\quad |\varepsilon-\varepsilon_0|\lesssim\delta,\quad \varepsilon_0=\frac{\hbar\omega_H}{2}. \tag{11}$$

The constant β in (10) can be determined from the condition that the total number of the states is

$$\int d\varepsilon\,\nu(\varepsilon)\approx\frac{1}{2\pi a^2}.$$

Whence $\beta=4$. The state density for the levels $N\sim 1$ has the same character as (11).

Notwithstanding the continuity of the spectrum of the split states, these states are localized in the approximation in which the impurity density is low. In fact, the Anderson localization criterion[12] $40W<u_0/a^2d$ at $2\pi na^2\lesssim 1/5$ to $1/7$ is satisfied automatically, since the overlap integral is

$$W \sim \frac{u_0}{a^2 d} \exp\left(-\frac{1}{4\pi a^2 n}\right).$$

3. CONDUCTIVITY IN A QUANTIZING MAGNETIC FIELD (LINEAR RESPONSE)

Within the framework of the linear theory, the conductivity at frequency ω is described by the Kubo formula[13]

$$\sigma_{\mu\nu}(\omega) = \frac{e^2}{S\hbar}\left\langle \sum_{\lambda\lambda'} \frac{f_\lambda - f_{\lambda'}}{\varepsilon_{\lambda'} - \varepsilon_\lambda} \frac{i}{\varepsilon_\lambda - \varepsilon_{\lambda'} - \hbar\omega + i\delta} v^\mu_{\lambda\lambda'} v^\nu_{\lambda'\lambda} \right\rangle. \quad (12)$$

Here λ are the quantum numbers that determine the state of the electron in the field of the impurities (ε_λ is the energy of this state), v^μ is the operator of the μ-component of the electron velocity ($\mu = x, y$), $f_\lambda \equiv f(\varepsilon_\lambda)$ is the Fermi function; the angle brackets denote averaging over the impurity configurations. We consider the contribution to the conductivity from the unsplit states $\tilde{\sigma}_{\mu\nu}(\omega)$.

Using (4), we get

$$\tilde{\sigma}_{\mu\nu}(\omega) = \frac{ie^2}{\hbar S} \sum_{\substack{N,N' \\ j,j'}} \frac{f_N - f_{N'}}{\varepsilon_{N'} - \varepsilon_N} \sum_{p_1 p_2 p_1' p_2'} \frac{v^\mu_{Np_1; N'p_2} v^\nu_{N'p_1'; Np_2'}}{\varepsilon_N - \varepsilon_{N'} - \hbar\omega + i\delta} \langle C^{j*}_{Np_1} C^{j'}_{N'p_2} C^{j'*}_{N'p_1'} C^{j}_{Np_2'} \rangle. \quad (13)$$

In this formula, $v^\mu_{Np,N'p'}$ are the matrix elements of the velocity over the Landau functions (3):

$$\begin{pmatrix} -iv^x_{Np,N'p'} \\ v^y_{Np,N'p'} \end{pmatrix} = \left(\frac{\hbar\omega_H}{2m}\right)^{1/2} \{N^{1/2}\delta_{N,N'+1} \mp N'^{1/2}\delta_{N',N+1}\}\delta_{pp'}. \quad (14)$$

From (14) and (13) we find that all the components of the conductivity tensor can be expressed in terms of a single constant A:

$$\begin{pmatrix} i\tilde{\sigma}_{xx}(\omega) \\ \tilde{\sigma}_{yx}(\omega) \end{pmatrix} = \frac{e^2}{2mS} A \left\{ \frac{1}{\omega_H + \omega - i\delta} \mp \frac{1}{\omega_H - \omega + i\delta} \right\}, \quad (15)$$

where

$$A = \sum_{N,j,j'} (f_N - f_{N+1})(N+1) \left\langle \left| \sum_p C^{j*}_{N+1,p} C^{j'}_{Np} \right|^2 \right\rangle, \quad A > 0.$$

It is easily seen that the conductivity $\tilde{\sigma}_{xx}(0)$ over the unsplit states at zero frequency vanishes, whereas the Hall component $\tilde{\sigma}_{yx}(0)$ is finite. The contribution made at zero frequency to the conductivity tensor by the split states should vanish because of their localization.[12] We ultimately arrive at the conclusion that $\sigma_{xx}(0) = 0$ and $\sigma_{yx}(0) = \tilde{\sigma}_{yx}(0)$. This result agrees with Ref. 1, in which a nonanalytic dependence of the current on the electric field, which leads to $\sigma_{xx}(0) = 0$, was obtained in the Born approximation in the scattering. Our conclusion, in contrast to Ref. 1, is exact. At the same time the result of Ref. 2, in which a finite value of the conductivity $\sigma_{xx}(0)$ was obtained by taking phenomenologically into account the damping of the electronic states, is incorrect, at least for the case of interaction with pointlike impurities.

It is known that absorption of an electromagnetic wave is determined by $\mathrm{Re}\,\sigma_{xx}(\omega)$ and $\mathrm{Im}\,\sigma_{yx}(\omega)$. From (16) we have

$$\mathrm{Re}\,\tilde{\sigma}_{xx}(\omega) = \mathrm{Im}\,\tilde{\sigma}_{yx}(\omega) = {}^1/_2\pi\omega_H \sigma_{yx}(0)\delta(\omega_H - \omega), \quad (16)$$

i.e., the absorption has an infinitesimally narrow peak at the cyclotron frequency. The physical cause of the vanishing of $\sigma_{xx}(0)$ and of the width of the cyclotron-resonance line is that the nonseparated states are exact eigenstates of the unperturbed Hamiltonian, and therefore the conductivity tensor $\tilde{\sigma}_{\mu\nu}$ has the same properties as the conductivity tensor in the absence of impurities.

Consider electromagnetic-wave absorption connected with the split states. In first order in the parameter $2\pi a^2 n$, the wave functions of the split states are given by expressions (5). The only allowed transitions between the unsplit and split states are those stemming from the neighboring Landau levels. Using (13), (4) and (5), we obtain for the corresponding contribution to the real part of the conductivity at a frequency close to ω_H

$$\mathrm{Re}\,\bar{\sigma}_{xx}(\omega) = \sum_N \{A_N \nu_N(\varepsilon_{N+1} - \hbar\omega)[f(\varepsilon_N + \hbar\omega_H - \hbar\omega) - f_{N+1}] + B_N \nu_{N+1}(\varepsilon_N + \hbar\omega)[f_N - f(\varepsilon_{N+1} - \hbar\omega_H + \hbar\omega)]\},$$

$$\begin{pmatrix} A_N \\ B_N \end{pmatrix} = (\pi a e)^2 \frac{1}{mS}(N+1) \left\{ \left\langle \sum_{\substack{j,i \\ p,p'}} C^{j*}_{N+1,p} C^{j}_{N+1,p'} \varphi^*_{Np}(\rho_i) \varphi_{Np'}(\rho_i) \right\rangle \right\}_{N \leftrightarrow N+1}, \quad (17)$$

$\nu_N(\varepsilon)$ is defined in (9).

Figure 1a shows the transition scheme. Transitions of the type 1–3 are allowed. The transition of type 4 is forbidden by the angular-momentum conservation law; it becomes allowed only if a finite center density is considered, and its intensity is proportional to na^2. Transitions of type 5 and 6 determine the harmonics of the cyclotron resonance. Transitions of type 5 are allowed when account is taken of the finite density of the impurities and of the higher-order corrections in the parameter $u_0/a^2 d$, and type-6 transitions are allowed when account is taken at the same time of the finite dimension of the impurity.

Figure 1b illustrates the cyclotron-resonance line shape. The central maximum is due to transitions between the unsplit states. Its form is determined by (11), and the area under it is connected with the value of $\sigma_{yx}(0)$ (formula (16)). The side maxima are connected with the square-root singularity of the density of the separated states (9) and are broadened by an amount $\sim(u_0/a^2 d)(na^2)$ because of the fluctuation approach of the impurities.

Thus, our results deviate substantially from those obtained in Refs. 3–6 by a diagram technique. In those reference they took into account only several of the simplest partial series of the diagrams (for example,

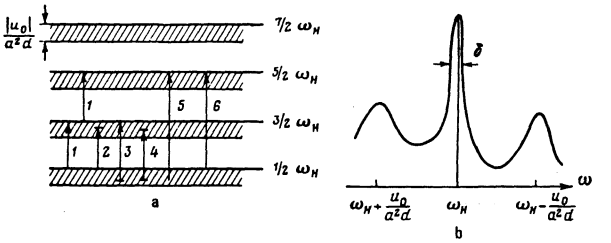

FIG. 1. Transition scheme (a) and cyclotron-resonance line shape (b) in the model of pointlike attracting ($u_0 < 0$) impurities.

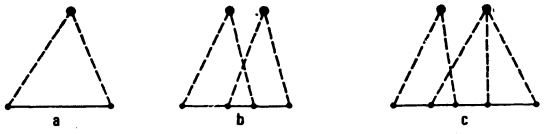

FIG. 2. Diagrams for the self-energy parts. The lines designate total Green's functions.

the diagram of Fig. 2a corresponds to the so-called reduced SCBA approximation, while the diagrams on Figs. 2a–2c and their like correspond to the DSA approximation). Estimates show, however, that it is impossible to confine oneself only to the diagrams that were taken into account. In fact, consider for example the diagrams for the self-energy part $\Sigma(\varepsilon)$ in the ultraquantum limit ($N=0$) for scattering by pointlike impurities. The orders of the diagrams of the type shown in Figs. 2a–2c are, respectively

$$\frac{nu^2}{(\varepsilon-\varepsilon_0-\Sigma)a^2} \quad \frac{n^2u^4}{(\varepsilon-\varepsilon_0-\Sigma)^2a^4} \quad \frac{n^2u^5}{(\varepsilon-\varepsilon_0-\Sigma)^4a^6},$$

where n is the impurity concentration per unit film area, u is the amplitude of the δ-function potential of the impurity, and a is the magnetic length.[4] In the region where $\varepsilon-\varepsilon_0$ is of the order of the lower self-energy part (diagram (a), we have $\varepsilon-\varepsilon_0 \sim n^{1/2}u/a$. Diagrams a and b then turn out to be of the same order, and diagram c differs from them by a factor $(na^2)^{-1/2}$. Therefore to calculate Σ in the region of high impurity density it is necessary to sum diagrams of any order in the density (corresponding to the number of points), and confine oneself to the Born approximation in the interaction with each individual center.[5] At low and intermediate impurity concentrations $na^2 \lesssim 1$, all the more complicated diagrams of type c are essential. It can thus be concluded that the results of Refs. 3–6 are incorrect in the limit $na^2 \lesssim 1$.

4. CLASSICAL MAGNETIC FIELDS

In this section we investigate the case of strong classical magnetic fields, when the Larmor radius $r_H \gg a$. We consider first the motion of an electron in the field of short-range impurities ($\lambda \ll l \ll r_H$, where λ is the characteristic wavelength of the electron). When the electron mean free path in the absence of a magnetic field $L \ll r_H$, the motion of the "two-dimensional" electron does not differ substantially from that of a "three-dimensional" one. It encounters, with overwhelming probability, an impurity on the cyclotron orbit. As a result, the center of its orbit shifts randomly in a plane perpendicular to the magnetic field, by an amount of the order of r_H. This makes possible electron diffusion across the magnetic field.

The situation is different in the limit of strong magnetic fields, when $r_H \ll L$. In the three-dimensional case the diffusion picture remains the same, inasmuch as an electron moving along the magnetic field encounters an impurity sooner or later. A two-dimensional electron cannot move along the magnetic field, but only on two types of trajectory, either on a circle without colliding with impurities at all, or on a rosette of circular trajectories around the impurity (Figs. 3a and 3b).

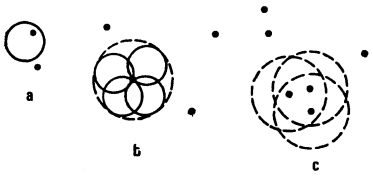

FIG. 3. Electron trajectories in the field of short-range impurities. The dark circles represent the impurities, the solid lines the electron trajectories, and the dashed lines the rosette boundaries.

When it moves over the rosette, the electron "sweeps" a circle of area $4\pi r_H^2$ and remains within the limits of the rosette, unless it collides with another impurity. As a rule this does not happen if the impurity concentration n is much less than $(4\pi r_H^2)^{-1}$. Thus, the motion of the electrons, whose energy satisfies the condition $4\pi r_H^2 n \ll 1$, is finite. From the expression for the diffusion coefficient[13]

$$D_{xx} = \lim_{t\to\infty} \frac{1}{2t}\overline{(x(t)-x(0))^2} \tag{18}$$

($x(t)$ is the x coordinate of the electron trajectory) it follows that the contribution of these electrons to D_{xx} is zero.

There is, of course, a probability of more than one impurity being present inside the rosette. Then finite motion around two or more impurities sets in, over a region constituting a union of rosettes, such that the center of one rosette falls inside another (Fig. 3c). The motion becomes infinite if an infinite cluster of rosettes is possible. This is equivalent to the problem of formation of an infinite cluster of circles of radius r_H with random centers.[14] According to Ref. 14, an infinite cluster is produced under the condition $\pi r_H^2 n \geq 0.68$. This condition determines the energy boundary ε_{cH}, which assumes the role of the percolation level for the magnetized electrons: $\varepsilon_{cH} = 0.68 m\omega_H^2/2\pi n$. We emphasize that even if an infinite cluster were to be produced, the greater part of the electrons, namely $(1-4\pi r_H n l)$, would not collide with the impurities and would take no part in the conduction.

The conductivity of an infinite cluster can be estimated with the aid of a model of overlapping circles with concentration n, inside of which the conductivity has a value σ_0 determined by the diffusion coefficient. This coefficient is of the order of the ratio of the square of the hopping length n^{-1} between the rosettes and the relaxation time $(nlv)^{-1}$. The conductivity of an infinite cluster can be expressed in terms of the conductivity of its element $\sigma_0(\varepsilon)$ and a dimensionless function $F(\varepsilon)$: $\sigma(\varepsilon) = \sigma_0(\varepsilon)F(\varepsilon)$, where $F(\varepsilon) \sim (\varepsilon-\varepsilon_{cH})/\varepsilon_{cH}$ at $\varepsilon-\varepsilon_{cH} \ll \varepsilon_{cH}$ (Ref. 15) and $F(\varepsilon)=1$ at $\varepsilon \gg \varepsilon_{cH}$. As a result we get

$$\sigma_{xx} \sim \left(\frac{\partial \ln n_e}{\partial \zeta}\right)_T \frac{ne^2l^2}{\hbar^2\omega_H} \int_{\varepsilon_{cH}}^{\infty} d\varepsilon\, \varepsilon f(\varepsilon) F(\varepsilon). \tag{19}$$

Here n_e is the electron density and ζ is the chemical potential. If the condition $|\zeta-\varepsilon_{cH}| \gg T$ is satisfied, we get from (19)

$$\sigma_{xx} \sim \left(\frac{\partial \ln n_e}{\partial \zeta}\right)_T \frac{ne^2l^2T^2}{\hbar^2\omega_H} \exp\left(\frac{\zeta-\varepsilon_{cH}}{T}\right) \tag{20}$$

at $\varepsilon_{cH}-\zeta > 0$ and

$$\sigma_{xx}\sim\frac{ne^2l^2}{\hbar^2\omega_H}\frac{(\zeta-\varepsilon_{cH})^2}{\zeta} \tag{21}$$

at $\varepsilon_{cH}-\zeta<0$. As seen from (20), in the degenerate case, at $T=0$ K, there exists a critical magnetic field

$$H_c=\frac{c}{e}\left(\frac{2\pi nm\zeta}{0.68}\right)^{1/2}$$

such that at $H>H_c$ we have $\sigma_{xx}=0$. If $\varepsilon_{cH}-\zeta>0$, then ε_{cH} determines the conductivity activation length.

We estimate now the non-ohmic increments to the current j_x along the electric field in the case $H>H_c$, when the ohmic conductivity is negligibly small. In a weak electric field the electron moves along a helix with a small pitch $2\pi|E|c/H\omega_H$. If the helix pitch is much less than the cross section for the electron scattering by the impurity $\sim l$, then the electron will collide with all the impurities that are contained in a band $2r_H$ wide and perpendicular to the electric field. The encounters with the impurities occur at time intervals $\tau_E\sim H/2\pi r_H|E|$. After encountering an impurity, the electron will describe around it a rosette for a time $lH/c|E|\ll\tau_E$ (since $l\ll r_H$). It will then move away from the impurity in a band shifted a distance $\sim r_H$ away from the old impurity. Thus, τ_E plays the role of the relaxation time in the usual kinetic energy, and to estimate the conductivity we can use the known formula $\sigma_{xx}=n_ee^2/m\omega_H^2\tau$ with $\tau=\tau_E$. We see therefore that in the region of weak electric fields we have $\sigma_{xx}\sim|E|$. This means not only a non-ohmic but also a non-analytic dependence of the current on the field. When E is increased to a value larger than $H\omega_Hl/2\pi c$, the drift manages to carry the electron away from the impurity, so that it experiences only one collision with the given impurity. Therefore the conductivity is determined by the usual relaxation time and ceases to depend on the field (if the field $E^*=H\omega_Hl/2\pi c$ is less than the field at which electron heating sets in).[6)]

We consider now another limiting case, when the impurities produce a large-scale potential relief

$$U(\rho)=\int V(\mathbf{r})|\xi(z)|^2dz$$

with a single characteristic dimension $l(\pi nl^2\gtrsim 1)$. For electrons with energy such that $r_H\ll l$ we can use the drift theory.[16] According to this theory, when the conditions

$$r_H\left|\frac{\partial^2U}{\partial\rho^2}\right|\ll\left|\frac{\partial U}{\partial\rho}\right|, \tag{22}$$

$$\frac{c}{eH}\left|\frac{\partial U}{\partial\rho}\right|\ll v \tag{23}$$

are satisfied the electron motion constitutes revolution along a circle and slow drift along a level line of the potential $U(\rho)$. The trajectory of the orbit center will be finite or infinite, depending on whether or not the level lines are closed. For sufficiently small and large values of the potential, the level lines are obviously closed (in analogy with a section through a mountain or with the shore of a lake). Unclosed level lines are possible for potential values between the upper and lower percolation levels ε_c and ε_c' (Ref. 15). In the two-dimensional case, however, simultaneous percolation over the regions $U>\varepsilon$ and $U<\varepsilon$ is impossible, since these regions must intersect.[15] It follows therefore that $\varepsilon_c=\varepsilon_c'$ and all the electron trajectories, with the exception of one, are finite.[7)] The latter means that such electrons make no contribution to the conductivity. We consider now the motion of an electron at $r_H\gg l$. In a time $2\pi/\omega_H$ the electron will experience $2\pi r_H/l$ collisions and be scattered each time through an angle $\sim\overline{U}/\varepsilon\ll 1$. As a result, after the first revolution the electron trajectory is shifted by an amount

$$\Delta x\sim l\frac{\overline{U}}{m\omega_H^2l^2}\left(\frac{l}{r_H}\right)^{1/2}\ll l.$$

The last inequality means that during the succeeding revolutions the displacements will be the same, until the total displacement is of the order of the characteristic dimension l of the potential. Further displacements will not be correlated with the initial ones and thus, the motion of the center of the orbit acquires the character of diffusion with a diffusion coefficient $\sim l^2(2\pi\omega_H\Delta x/l)$. Since the electrons make no contribution to the conductivity when $r_H\ll l$, the condition $r_H\sim l$ determines the percolation level $\varepsilon_{cH}'\sim m_H^2l^2$. We note that, in contrast to the usual percolation level ε_c, the value of ε_{cH}' is determined only by the dimension of the potential, and ε_{cH}' is much larger than the characteristic amplitude of the potential $\overline{U}\sim\varepsilon_c$.

For the conductivity we obtain a formula similar to (19). In limiting cases we get ($|\zeta-\varepsilon_{cH}'|\gg T$):

$$\sigma_{xx}\sim\left(\frac{\partial\ln n_e}{\partial\zeta}\right)_T\frac{e^2\overline{U}}{\hbar^2}\frac{(ml^2)^{1/2}}{\omega_H^{1/2}}\exp\left(\frac{\zeta-\varepsilon_{cH}'}{T}\right)\cdot\begin{cases}\dfrac{T}{(\varepsilon_{cH}')^{1/2}}, & T\ll\varepsilon_{cH}'\\ T^{1/2}, & T\gg\varepsilon_{cH}'\end{cases} \tag{24}$$

if $\varepsilon_{cH}'-\zeta>0$

$$\sigma_{xx}\sim\frac{e^2\overline{U}}{\hbar^2}\frac{(ml^2)^{1/2}}{\omega_H^{1/2}}\frac{\zeta^{1/2}-(\varepsilon_{cH}')^{1/2}}{\zeta} \tag{25}$$

if $\varepsilon_{cH}'-\zeta<0$. The critical magnetic field is $H_c'\sim(c/el)\times(m\zeta)^{1/2}$.

We emphasize that the results of this section can not be obtained with the aid of the usual kinetic equation. The point is that in the derivation of the kinetic equation it is assumed that the scattering is fully stochastic, i.e., that there is no memory of any preceding scattering act. In the present problem, an electron on a finite trajectory "remembers" the "starting" point for an infinitely long time, and on infinite trajectories it remembers for a long time the preceding collisions.

CONCLUSION

We have demonstrated, using several models, the feasibility of a phase transition into a state with zero conductivity in a two-dimensional electron+impurity system under the influence of a magnetic field. A singularity is observed in the conductivity in the case of a classical system, and additionally in the electron spectrum in a quantum system. The reason for these phenomena lies in the unique property of two-dimensional "magnetized" electrons—in their finite motion (or localization of the states). This distinguishes our problem from either the two-dimensional problem without a magnetic field, or the three-dimensional with a magnetic field, where the electron motion is finite.

We present, using an inversion channel as an example, estimates of the applicability of the different proposed models. The limit of pointlike impurities is realized when the electron gas in the channel is not only quantized but also degenerate ($n_e \sim 10^{12}\,\mathrm{cm}^{-2}$, $T \sim 4$ K). The screening radius of the charged impurities then becomes of the order of the channel thickness $d_{\mathrm{inv}} \sim 30$ Å (case of n-channel, silicon), and $\lambda \sim 300$ Å. In a magnetic field $\sim 10^5$ Oe, one Landau subband is filled. The limit of Sec. 2 is thus realized. The requirement $2\pi n a^2 < 1$ means that $n < 2 \times 10^{11}\,\mathrm{cm}^{-2}$. (By n we must mean here the impurity concentration in the channel and on the surface of the semiconductor.)

The limit of the classical large-scale relief corresponds to an extremely low carrier density in the channel ($n_e \lesssim 10^9\,\mathrm{cm}^{-2}$) and to the absence of degeneracy.[17] The characteristic dimension of the potential is determined in this case by double the thickness of the dielectric, $\sim 1 \times 10^3$ Å, which is twice as large as r_c at $T \sim 70$ K and $H \sim 3 \times 10^4$ G, with $r_H/\lambda \sim 10$.

Vanishing of the conductivity of the inversion channel in a strong magnetic field ~100 kG at $T = 1.4$ K was recently observed[19] at certain values of the voltage on the field electrode. Without going into a detailed discussion of this paper (which calls for allowance for the level splitting due to spin, the presence of many valleys, etc.), we note only that this phenomenon can be explained by our results.

We emphasize in conclusion that the results apply not only to quantum films and inversion channels. They are obviously valid also in cases when the scatterers have a potential that does not depend on one of the coordinate. Such a situation is realized, first, in the case of a crystal that contains oriented edge dislocations, and second in the case of a classical inversion channel, when many subbands of quantization are filled while the potential is produced by impurities located in the depletion layer.

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[1)] For the sake of argument, we refer hereafter to electrons with a quadratic dispersion in a quantizing film with one filled size-effect quantization level. The result apply also to other two-dimensional systems—carriers in an inversion channel, or electrons over the surface of liquid helium.

[2)] We note that our deduction is valid, strictly speaking, only in the case of quadratic dispersion. If account is taken of deviation from a quadratic law, the degeneracy is lifted, but the corresponding level broadening is quite small.[8]

[3)] We take the opportunity to point out that the corresponding formula in Ref. 11 (formula (2), p. 526 of Russian original) is incorrect.

[4)] The estimates of the diagrams were obtained under the condition $\varepsilon - \varepsilon_0 \ll \hbar\omega_H$.

[5)] An attempt to solve the problem for $na^2 \gg 1$, undertaken in Ref. 3, seems unsatisfactory to us. The author replaces the contribution made to Σ by s-point diagrams with a power-law expression with one fit parameter, and bases himself here only on a calculation of diagrams with $s = 2$, 3, and 4.

[6)] The presented qualitative arguments are confirmed by an exact solution of a model problem in which the impurites are replaced by circles of radius l with diffuse surfaces.

[7)] The condition (23) can be satisfied at any point. Condition (22), generally speaking, is equivalent to the condition $r_H \ll l$, but is violated in a region of size r_H near the stationary point $U(\rho)$. The percolation path must of necessity pass through such a point. It can be shown, however, that the exact trajectory of the orbit center differs from the level line $U(\rho)$ by an amount much smaller than r_H, i.e., it coincides with the level line to the same degree of accuracy with which the center of the orbit is determined. The result of the drift theory thus remains valid.

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