Magnetophonon resonance in zero-gap semiconductors under conditions of electron and hole heating

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A theoretical investigation is made of the longitudinal magnetoresistance in a strong electric field and a quantizing magnetic field in the case when the optical phonon energy is equal to the gap between the electron and hole energy levels. The theory is based on an analysis of the energy and carrier density balance equations. It is shown that the bottleneck effect appears in the system and it increases the effective temperature of holes to the electron temperature. Intrinsic and donor-doped semiconductors are consdered in the case when the longitudinal magnetoresistance is independent of the carrier density and the nature of the longitudinal magnetoresistance singularities is governed exclusively by the behavior of the electron temperature in the magnetic field. The singularities are basically different for transitions to states with different projections of the total momentum.

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1. INTRODUCTION

Magnetophonon oscillations of the magnetoresistance of semiconductors[1] at low temperatures, when optical phonons are frozen out, have been observed mainly under conditions of electron heating by an electric field.[2,3] Gel'mont *et al.*[4,5] observed a new type of magnetophonon oscillations of the longitudinal magnetoresistance of HgTe in a strong electric field: this type of oscillation is due to an increase in the probability of recombination of an electron and a hole accompanied by the emission of an optical phonon in the case when the phonon energy is equal to the gap between the electron and hole Landau levels. It is assumed[4,5] that the observed magnetophonon oscillations are exclusively due to oscillations of the electron density. This assumption is used in developing a theory based on an analysis of the balance equation for the electron density.

It should be pointed out that, in the case of strong heating of the electron gas when the resonance condition is satisfied by interband transitions accompanied by optical phonon emission, there are singularities not only in the particle-number balance equation but also in the equations of the energy balance in the electron–hole system. Consequently, magnetoresistance oscillations observed experimentally may be due to oscillations of the carrier density or temperature.

We shall develop a theory of magnetophonon oscillations by analyzing a system of balance equations for the particles and energy in the electron–hole system using the approximation of effective parameters. We shall consider the case of low lattice temperatures. Optical phonons are then frozen out and holes are heated to the electron temperature because of the bottleneck effect.[6]

The interaction of electrons and holes with polar lattice vibrations is described by the Fröhlich Hamiltonian. For this type of interaction the probabilities of transitions with and without spin flip depend in different ways on the electron quasimomentum, which gives rise to basically different types of magnetoresistance oscillations.

2. BASIC EQUATIONS

We shall consider a zero-gap semiconductor of the HgTe type with an inverted energy band structure[7] subjected to a strong electric and a quantizing magnetic field, both in the longitudinal direction. In the investigated range of electric and magnetic fields both electrons and holes obey the Boltzmann statistics. In the approximation of effective parameters, we have

$$f_v^{e(h)} = \exp[(\mu_{e(h)} - \varepsilon)/T_{e(h)}], \tag{1}$$

where $T_{e(h)}$ and $\mu_{e(h)}$ are the temperature and chemical potential of electrons and holes.

We shall determine the parameters T and μ from the energy balance equation, particle balance equations for electrons and holes, and electrical neutrality equation:

$$\sigma_e E^2 = P_e^{\text{opt}} + P_{eh} + P_e^{\text{ac}}, \tag{2a}$$

$$\sigma_h E^2 = P_h^{\text{opt}} - P_{eh} + P_h^{\text{ac}}, \tag{2b}$$

$$\partial\tilde{n}/\partial t = (\partial n/\partial t)^{\text{opt}}, \tag{2c}$$

$$n = p + N_d. \tag{2d}$$

Here, E is the electric field intensity; $\sigma_{e(h)}$ is the electrical conductivity due to electrons (holes); $p_{e(h)}^{\text{opt}}$ is the rate of loss of energy by electrons (holes) due to the emission of optical phonons in recombination processes; P_{eh} is the rate of energy loss by electrons in the inelastic scattering by heavy holes; $P_{e(h)}^{\text{ac}}$ is the power of the acoustic phonon radiation emitted by electrons (holes). The term $(\partial n/\partial t)^{\text{opt}}$ represents the rate of change in the number of electrons in recombination of electron–hole pairs assisted by optical phonons; $\partial\tilde{n}/\partial t$ is the rate of change in the number of particles as a result of impact and Auger recombination processes; n and p are the electron and hole densities; N_d is the concentration of ionized impurities. We shall confine ourselves to the case of electron conductivity ($\sigma_e \gg \sigma_h$) and ignore the true heating of holes, i.e., we shall assume that the left-hand side of Eq. (2b) vanishes.

Adding Eqs. (2a) and (2b), we obtain the energy balance equation for the electron–hole system:

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$$\sigma_e E^2 = \hbar\omega_0 (\partial n/\partial t)^{\text{opt}} + P_h^{\text{ac}}. \quad (3)$$

In the derivation of Eq. (3) it is assumed that

$$P_e^{\text{opt}} + P_h^{\text{opt}} = \hbar\omega_0 (\partial n/\partial t)^{\text{opt}},$$

where ω_0 is the limiting frequency of optical phonons. The term P_e^{ac} can be ignored compared with P_h^{ac} because it is small in respect of the parameter m_e/m_h (m_e and m_h are the effective masses of an electron and a hole). We shall now consider in greater detail the structure of the balance equations. For a given type of the distribution function (1), we have

$$\left(\frac{\partial n}{\partial t}\right)^{\text{opt}} = -\frac{2\pi}{\hbar}\sum_{\nu\mu\mathbf{q}} |C_q^{\text{opt}}|^2 |\langle e\nu|e^{i\mathbf{qr}}|\nu\mu\rangle|^2 f_\nu^e f_\mu^h \delta(E_\nu^e + E_\mu^h - \hbar\omega_0), \quad (4)$$

$$P_h^{\text{ac}} = \sum_{\mathbf{q}} \hbar\omega_q \left[N_q\left(\frac{\hbar\omega_q}{T_h}\right) - N_q\left(\frac{\hbar\omega_q}{T}\right)\right] \Omega_{fh}, \quad (5)$$

where

$$\Omega_{fh} = -\frac{2\pi}{\hbar}|C_q^{\text{ac}}|^2 \sum_{\mu\mu'} (f_\mu^h - f_{\mu'}^h) |\langle\nu\mu|e^{i\mathbf{qr}}|\nu\mu'\rangle|^2 \delta(E_\mu^h - E_{\mu'}^h - \hbar\omega_q) \quad (6)$$

is the frequency of phonon–hole collisions and

$$P_{eh} = -\frac{\pi}{\hbar}\sum_{\substack{\mu\mu'\mathbf{q}\\ \nu\nu'}} |C_q|^2 (E_\mu^e - E_{\nu'}^e) |\langle e\nu|e^{i\mathbf{qr}}|e\nu'\rangle|^2$$

$$\times |\langle\nu\mu|e^{i\mathbf{qr}}|\nu\mu'\rangle|^2 \{f_{\nu'}^e f_\mu^h - f_\nu^e f_{\mu'}^h\} \delta(E_{\nu'}^e - E_\nu^e + E_\mu^h - E_{\mu'}^h). \quad (7)$$

Here,

$$N_q(x) = (e^x - 1)^{-1}, \quad E_\nu^e = E_i(n) + \varepsilon_{ni}(k_z) \quad (8)$$

is the energy of an electron in a state with quantum numbers $\{n, k_y, k_z, i\}$ ($n = 1, 2, 3, \ldots$; $i = a, b$ for $n = 1$ and $i = a^-$, b^- for $n \geq 2$),

$$E_\mu^h = E_j(l) + \varepsilon_{lj}(k_z)$$

is the energy of a hole in a state with quantum numbers $\{l, k_y, k_z, j\}$ ($l = 0, 2, 3, \ldots$; $j = a, b$ for $l = 0$ and $j = a^+, b^+$ for $l \geq 2$) (Ref. 8), $|C_q^{ac}|^2$ and $|C_q|^2$ are the squares of the Fourier transform of the potential of the interaction of holes with phonons and of electrons with holes. The quantity $|C_q^{ac}|^2$ for the interaction of holes with phonons can be expressed in the usual way in terms of the effective deformation potential constant E_{eff} for an energy band with the p-type symmetry.[9] In terms of the Dirac notation, $|e(\nu)\mu\rangle$ is the wave function of an electron (hole). Energy is measured from the point where the bands touch for $H = 0$.

The matrix elements of $e^{i\mathbf{q}\cdot\mathbf{r}}$ occuring in Eqs. (4)–(7) can be calculated sufficiently accurately using a simple model in which the interaction between the nearest three bands is taken into account. The explicit form of the wave functions obtained in this approximation is given by Gel'mont,[10] and by Liu and Tan.[10] Simple calculations yield

$$|\langle e(\nu)\mu|e^{i\mathbf{qr}}|e(\nu)\mu'\rangle|^2 = M^{e(\nu)\mu}_{e(\nu)\mu'}(k_z, k_z', q_\perp^2)\,\delta(k_y', k_y - q_y)\,\delta(k_z', k_z - q_z). \quad (9)$$

The expressions for M are given explicitly in the Appendix.

In the calculation of a spectrum in the three-band approximation the heavy-hole band has an infinite degree of degeneracy in $E = 0$, which is lifted only by allowance for the interaction with higher energy bands. The dispersion law of heavy holes calculated in this approximation is fairly complex.[11] The highest hole band belongs to the series b and its number is $l = 2$. For electrons the lowest band is in the series a and its number is $n = 1$. Retaining the correct systematics of levels at the point Γ, we shall introduce a simplification for $k_z \neq 0$ assuming the quadratic dispersion law for electrons and holes:

$$\varepsilon_{ni} = \hbar^2 k_z^2/2m_e, \quad \varepsilon_{lj} = \hbar^2 k_z^2/2m_h. \quad (10)$$

We shall estimate expressions which occur in the balance equation. Since in the range of magnetic fields under consideration we have $\hbar\Omega_e \gg T_e$, electrons occupy the lowest Landau level with $i = a$ and $n = 1$. The expression for $(\partial n/\partial t)^{\text{opt}}$ is obtained by substituting Eq. (9) into Eq. (4):

$$\left(\frac{\partial n}{\partial t}\right)^{\text{opt}} = \frac{e^2\omega_0 m_e^{1/2} m_h^{1/2}}{8\pi^2\hbar^2\alpha^2}\left(\frac{1}{\varkappa_\infty} - \frac{1}{\varkappa_0}\right)\exp\left(\frac{\mu_e}{T_e} + \frac{\mu_h}{T_h}\right)$$

$$\times \sum_{lj}\int_0^{\Delta_{lj}} d\varepsilon \exp\left[-\frac{\hbar\omega_0}{T_e} + E_{j(l)}\left(\frac{1}{T_e} - \frac{1}{T_h}\right) + \varepsilon\left(\frac{1}{T_e} - \frac{1}{T_h}\right)\right]$$

$$\times \int_0^\infty dq_\perp^2 \frac{M_{ljl}^{ea}[(2m_e(\Delta-\varepsilon)/\hbar^2)^{1/2}; (2m_h\varepsilon/\hbar^2)^{1/2}; q_\perp^2]}{(\Delta_{jl}-\varepsilon)^{1/2}\varepsilon^{1/2}}$$

$$\times \left\{\left[\frac{2m_e}{\hbar}\left[(\Delta-\varepsilon)^{1/2} + \left(\frac{m_h}{m_e}\varepsilon\right)^{1/2}\right]^2 + q_\perp^2\right]^{-1}\right.$$

$$\left. + \left[\frac{2m_e}{\hbar^2}\left[(\Delta-\varepsilon)^{1/2} - \left(\frac{m_h}{m_e}\varepsilon\right)^{1/2}\right]^2 + q_\perp^2\right]^{-1}\right\}, \quad (11)$$

where $\alpha = (c\hbar/eH)^{1/2}$ is the magnetic length.

The expressions for P_{eh} and P_h^{ac} will be considered in the ultraquantum limit for holes and electrons. In this case the scattering of electrons by holes is quasielastic and the initial expressions can be simplified by expanding in terms of the small inelasticity parameter:

$$\frac{\varepsilon_1(k_z') - \varepsilon_1(k_z)}{\varepsilon_1(k_z)} \approx \left(\frac{m_e}{m_h}\right)^{1/2}.$$

Since in the situation under consideration the characteristic quantities k_z and k_z' are small compared with $q_\perp$, it follows that we can calculate the integrals in the matrix elements M retaining only the first nonvanishing terms in k_z and k_z':

$$P_{eh} = -\frac{2^{9/2}\pi^{1/2} m_e^{3/2} e^4\alpha^2}{m_h\hbar^2\varkappa^2 T_e^{1/2}} np(T_e - T_h)\int \frac{dx\, M_{a1}^{ea}(x/2) M_{b+2}^{b^+2}(x/2)}{(x+\alpha^2 q_e^2)^2}, \quad (12)$$

$$\Omega_{fh} = \frac{(2\pi)^{1/2} E_{\text{eff}}^2 m_h^{1/2}}{\hbar\rho s T_h^{1/2}} p\,\text{sh}\left(\frac{\hbar\omega_{q_\perp}}{2T_h}\right) q_\perp M_{b+2}^{b+2}(q_\perp^2)|q_z|^{-1}$$

$$\times \exp\left[-\frac{\hbar^2 q_z^2}{8m_h T_h} - \frac{m_h q_\perp^2 s^2}{2T_h q_z^2}\right]. \quad (13)$$

Substituting Eq. (13) into Eq. (5), we obtain the expression for P_h^{ac} (employing the expansion in terms of $\hbar\omega_q/T \ll 1$ accurate to within terms of the second order of smallness):

$$P_h^{\text{ac}} = -\frac{E_{\text{eff}}^2 m_h^{3/2}}{2^{1/2}\pi^{1/2}\rho\alpha^2 T^{1/2}} p\left(\frac{T}{T_h}\right)^{1/2}\left(1 - \frac{T}{T_h}\right)\int_0^\infty dx\, x M_{b+2}^{b+2}(x)\ln\left(\frac{4\alpha T_h}{\hbar s e^C}\right). \quad (14)$$

Here, $C = 0.577\ldots$ is the Euler constant.

The left-hand side of the particle-number balance equation is a sum of two terms: the rate of generation of particles in impact ionization processes and the rate of change in the number of particles in Auger recombination; in the approximation of effective parameters this

sum can be represented in the form[12]

$$\left(\frac{\partial \bar{n}}{\partial t}\right)=\left(1-\exp\left(\frac{\mu_e}{T_e}+\frac{\mu_h}{T_h}\right)\right)\left(\frac{\partial n}{\partial t}\right)^{\text{imp. ion}}. \quad (15)$$

[For $T_e=T_h$, Eq. (15) becomes exact.]

The expression for $(\partial n/\partial t)^{\text{imp. ion}}$ can be represented in the form[4]

$$\left(\frac{\partial n}{\partial t}\right)^{\text{imp. ion}}=\sum_{\nu}\gamma(k_z)f_\nu.$$

It follows from the laws of conservation that the impact ionization coefficient $\gamma(k_z)$ differs from zero, beginning from a certain threshold value of the momentum

$$k_z^0=\pm[2m_e\Delta E(H)/\hbar^2]^{1/2},$$

where $\Delta E(H)$ is the gap between the conduction and valence bands in a magnetic field. Above the threshold the coefficient $\gamma(k_z)$ depends weakly on the momentum and this dependence can be ignored. We shall assume that the quantity γ is a parameter of the theory. In the adopted approximation, we have

$$\left(\frac{\partial n}{\partial t}\right)^{\text{imp. ion}}=n\gamma\left(\frac{T_e}{\pi\Delta E(H)}\right)^{1/2}\exp\left(-\frac{\Delta E(H)}{T_e}\right). \quad (16)$$

3. INVESTIGATION OF BALANCE EQUATIONS NEAR A RESONANCE UNDER BOTTLENECK CONDITIONS

An effective temperature T_h is established in the system of holes by exchange of energy with two thermostats: the electron system, whose temperature is T_e, and the lattice, whose temperature is T. We shall consider the case when the energy relaxation channel between holes and the lattice is "narrower" than the channel along which energy is transferred from electrons to holes. In this case we may assume that $T_e=T_h=T^*$ and we can find T^*, n, and p simply from Eqs. (2c), (2d), and (3). Under the bottleneck conditions the expression for $(\partial n/\partial t)^{\text{opt}}$ in the vicinity of a resonance $\Delta_{jl}\ll\hbar\Omega_e,\hbar\Omega_h$ becomes

$$\left(\frac{\partial n}{\partial t}\right)^{\text{opt}}=\frac{2\pi^2e^2\alpha^2\omega_0}{T_e}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)np\exp\left[\frac{\Delta E(H)}{T_e}-\frac{\hbar\omega_0}{T_e}\right]\Phi(H), \quad (17)$$

where

$$\Phi(H)=\left\{\vartheta(\Delta_{e0})+\sum_{l=2}^{\infty}\frac{3}{(4l-3)}\vartheta(\Delta_{e^*l})+K_{l^*2}^{e1}+\frac{2}{\hbar\Omega_h}\sum_{l=2}^{\infty}\frac{3l}{(l-1)(l-2)(4l-1)}\Delta_{b^*l}\vartheta(\Delta_{b^*l})\right\}, \quad (17a)$$

$$K_{b^*2}^{e1}=\frac{\Delta_{b^*2}}{\hbar\Omega_h}\vartheta(\Delta_{b^*2})\int_0^{\infty}dx\int_0^{\infty}dt\frac{[(1-x)^{1/2}-(m_hx/m_e)^{1/2}(1-t)]^2}{(1-x)^{1/2}x^{1/2}}\times\left\{\left[t+\frac{2\Delta_{b^*2}}{\hbar\Omega_e}\left[(1-x)^{1/2}+\left(\frac{m_h}{m_e}x\right)^{1/2}\right]^2\right]^{-1}+\left[t+\frac{2\Delta_{b^*2}}{\hbar\Omega_e}\left[(1-x)^{1/2}-\left(\frac{m_h}{m_e}x\right)^{1/2}\right]^2\right]^{-1}\right\}, \quad (17b)$$

$$\vartheta(x)=\begin{cases}1, & x>0\\ 0, & x<0\end{cases}. \quad (17c)$$

In the derivation of Eq. (17) all the terms in the sum over l and j-except for the term $j=b^*$, $l=2$, which diverges-are simplified by dropping the following quantities from the denominators containing q_1^2:

$$\frac{2m_e}{\hbar^2}\left[(\Delta-\varepsilon)^{1/2}+\left(\frac{m_h}{m_e}\varepsilon\right)^{1/2}\right]^2.$$

The expression $(\partial n/\partial t)^{\text{opt}}$ in Eq. (17) has a singularity every time the following condition for the magnetophonon resonance is satisfied:

$$\Delta_{jl}=\hbar\omega_0-E_e(1)-E_j(l)=0. \quad (18)$$

For transitions to the series a the expression $(\partial n/\partial t)^{\text{opt}}$ corresponding to $\Delta_{a1}=0$ changes discontinuously by a finite amount. For transitions to the series b the function $(\partial n/\partial t)^{\text{opt}}$ is continuous at the resonance point ($\Delta_{b1}=0$), but the first derivative has a discontinuity. Expression (17) is obtained for $T_e=T_h$. The nature of the dependence of $(\partial n/\partial t)^{\text{opt}}$ on the magnetic field in the case $T_e\neq T_h$ was studied by Gel'mont *et al.*[4] and is more complex.

Substituting Eqs. (16) and (17) into the balance equation (2c) we obtain the following explicit particle-number balance equation:

$$n\left[1-\frac{np}{N_eN_h}\exp\left(\frac{\Delta E(H)}{T^*}\right)\right]\gamma\left(\frac{T^*}{\pi\Delta E(H)}\right)^{1/2}\exp\left(-\frac{\Delta E(H)}{T^*}\right)=np\frac{2\pi^2e^2\alpha^2\omega_0^2}{T^*}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)\exp\left[\frac{\Delta E(H)}{T^*}-\frac{\hbar\omega_0}{T^*}\right]\Phi(H). \quad (19)$$

Here,

$$N_{e(h)}=\left(\frac{m_{e(h)}T^*}{2\pi\hbar^2}\right)^{1/2}\frac{\hbar\Omega_{e(h)}}{T^*}.$$

Depending on the impact ionization coefficient γ, a steady-state number of electrons may be established either because of recombination involving optical phonons or because of Auger recombination.

a) Intrinsic semiconductor ($n=p$). The Auger recombination process predominates when

$$\frac{\pi^2e^4\omega_0m_e^{1/2}m_h^{1/2}}{8\hbar^2\gamma^2}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)\Delta E(H)\Phi^2(H)\exp\left[3\frac{\Delta E(H)}{T^*}-2\frac{\hbar\omega_0}{T^*}\right]\ll1. \quad (20)$$

The electron density is given by

$$n\approx(N_eN_h)^{1/2}\exp[-\Delta E(H)/2T^*]. \quad (21)$$

When the inequality is opposite to that given by Eq. (20), the recombination process assisted by optical phonons predominates. We then have

$$n\approx\frac{\gamma T^{*3/2}\exp[\hbar\omega_0/T^*-2\Delta E(H)/T^*]}{2\pi^{1/2}e^2\alpha^2\omega_0(1/\varkappa_\infty-1/\varkappa_0)\Delta E^{1/2}(H)\Phi(H)}. \quad (21')$$

b) Extrinsic donor-doped semiconductor ($n=p+N_d$, $p\ll N_d$). The Auger recombination process predominates if

$$\frac{e^2\omega_0m_e^{1/2}m_h^{1/2}}{4\gamma\hbar^2\alpha^2T^{*2}}\left(\frac{1}{\varkappa_\infty}-\frac{1}{\varkappa_0}\right)\Delta E(H)\Phi(H)\ll1. \quad (22)$$

The hole density is given by

$$p=\frac{N_eN_h}{N_d}\exp\left[-\frac{\Delta E(H)}{T^*}\right]. \quad (23)$$

In the case of recombination accompanied by optical phonon emission the expression for the hole density is identical with the expression for $p=n$ in the case of an intrinsic semiconductor [Eq. (21)].

The behavior of the electron temperature near a resonance is found from the energy balance equation (3) using the expressions for n and p. At low temperatures

the electron scattering occurs mainly on charged centers. In this case the electrical conductivity is independent of the electron density and, therefore, $\sigma=\sigma_0(T_e/T)^{3/2}$, where σ_0 is the electrical conductivity in the limit $E\to 0$. We shall now write Eq. (3) in the dimensionless form for the case when the loss of energy occurs in one of the real channels (losses due to the interaction with acoustic or optical phonons):

$$\left(\frac{E}{E_0}\right)^2[\Phi(H)]^r=t\left(\frac{T}{T^*}\right)\exp s\frac{T}{T^*}. \tag{24}$$

The quantities E_0, t, s, and r are described by the following expressions.

In the case of the interaction with acoustic lattice vibrations and for recombination accompanied by the emission of optical phonons, we have

$$E_0^2=\frac{E_{\text{eff}}^2 m_h^{1/2}\gamma_1 T\ln(4\alpha T/\hbar s e^C)}{2^{1/2}\pi^2\sigma_0\alpha^6 e^2\omega_0(1/\varkappa_\infty-1/\varkappa_0)\Delta E(H)},$$

$$t(T/T^*)=(T/T^*)^{1/2}(1-T/T^*),\ s=[\hbar\omega_0-2\Delta E(H)]/T,\ r=+1;$$

for $s<0$, $|s|\gg 1$,

$$(T^*/T)_{cr}=2|s|,\quad E_{cr}=E_0\Phi^{-1/2}(H)(2|s|)^{-1/2}e^{-1/2},$$

for $s>0$, $|s|\gg 1$,

$$(T^*/T)_{cr}=1+s^{-1},\quad E_{cr}=E_0\Phi^{-1/2}(H)e^1/s.$$

For the interaction with polar optical vibrations and for the recombination in the Auger channel

$$E_0^2=\frac{e^2\omega_0^2(m_e m_h)^{1/2}(1/\varkappa_\infty-1/\varkappa_0)}{8\pi\sigma_0\hbar\alpha^2},$$

$$t(T/T^*)=(T/T^*)^{1/2},\quad s=-\hbar\omega_0/T,\quad r=-1,$$

$$(T^*/T)_{cr}=2/3|s|,\quad E_{cr}=E_0\Phi^{1/2}(H)(3/2|s|)^{3/2}e^{-3/4}.$$

For the interaction with polar optical vibrations and recombination accompanied by optical phonon emission,

$$E_0^2=\frac{\gamma^2T^2}{2\pi^3\sigma_0 e^2\alpha^2(1/\varkappa_\infty-1/\varkappa_0)^2[\Delta E(H)]^2},$$

$$t(T/T^*)=(T^*/T)^{1/2},\quad s=[\hbar\omega_0-3\Delta E(H)]/T,\quad r=+1.$$

We note that the function $t(T/T^*)$ should tend to zero in the limit $T^*\to T$. However, in those cases when the energy loss channel involves optical phonon emission, $t(T/T^*)$ does not have this property because we are ignoring the processes accompanied by phonon absoprtion. In such cases the results are valid if the heating is sufficiently strong.

The values of E_0, t, s, and r are identical for intrinsic and extrinsic semiconductors in all cases, with one exception when the major relaxation and the recombination of carriers are due to the interaction with optical phonons. In this case, an extrinsic semiconductor is characterized by $r=0$ and, therefore, T^* has no singularities under resonance conditions.

A graphical analysis of Eq. (24) gives the results shown in Fig. 1: it demonstrates that there is a maximum (critical) value of the electric field E_{cr} in which a steady state of the system is still possible and that in the range $E>E_{cr}$ the energy balance equation breaks down and the system of electrons and holes is heated without limit. In this range of electric fields Eq. (24) has no solutions. The field E_{cr} corresponds to T_{cr}^*, the critical value of the electron temperature which divides the range of electron temperatures into the ranges

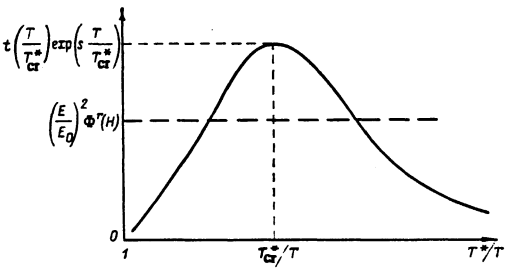

FIG. 1. The continuous curve represents the right-hand side of Eq. (24) considered as a function of T^*/T. The left-hand side of the equation, which has a discontinuity at the resonance point, is shown dashed.

$T^*<T_{cr}^*$ and $T^*>T_{cr}^*$. In the temperature range $T^*<T_{cr}$ there is a root T_1^* of the equation and this root rises with the electric field. In the temperature range $T^*>T_{cr}^*$ the root T_2^* lies in the falling branch (Fig. 1) and its value decreases on increase of the electric field. At the critical point $T^*=T_{cr}^*$ the two roots coincide.

The equation suitable for finding T_{cr}^* is obtained by equating to zero the derivative of the right-hand side of Eq. (24):

$$t'+ts=0. \tag{25}$$

The critical values of the electric field and temperature deduced from Eqs. (24) and (25) are given above. The nature of the singularity of T^* at resonance is governed by the sign of r and by the branch on which T^* is located. The amplitude of the temperature oscillations depends on the steepness of the branch and has its maximum value in the vicinity of the singularity $T^*=T_{cr}^*$.

Under resonance conditions for transitions to the series a the function $\Phi(H)$ has a discontinuity. It is clear from Fig. 1 that in this case in the range $T^*<T_{cr}^*$ the value of T^* suddenly decreases for $r>0$ and increases for $r<0$. If $T^*>T_{cr}^*$, the value of T^* suddenly increases for $r<0$ and decreases for $r>0$. For transitions to the series b the function $\Phi(H)$ remains continuous with a kink at the resonance point, but its first derivative exhibits a jump. In this case the value of T^* also has a kink at the resonance point. Thus, when the resonance condition (18) is satisfied, the singularities for transitions to the series a are stronger than for transitions to the series b. Such a considerable difference between the nature of the oscillations is due to the different dependences of the matrix elements on k_z for transitions between the levels belonging to the same series and transitions between the levels belonging to different series. This feature is characteristic of the matrix elements describing the interaction with polar lattice vibrations. The elements describing the scattering of carriers by deformation optical vibrations are of the same order in k_z for transitions in either series.[13] In the case when the scattering by deformation optical vibrations predominates, the oscillation singularities at resonance should be the same for transitions to different series.

Since at low lattice temperatures, when the momentum is dissipated mainly by interaction with charged centers, the magnetoresistance is $\rho_{\parallel}=\sigma_{\parallel}^{-1}\propto T_e^{3/2}$, for

intrinsic and donor-doped semiconductors, it follows that the singularity of the magnetoresistance under resonance conditions is determined entirely by the behavior of the electron temperature.

4. RANGE OF VALIDITY OF THE THEORY

We shall now obtain the criterion for a bottleneck in the investigated situation. It follows from the energy balance equation (2) that

$$T_h=T_e\frac{\Omega_{eh}}{\Omega_{eh}+\Omega_h^{ac}+\Omega_h^{opt}}+T\frac{\Omega_h^{ac}}{\Omega_{eh}+\Omega_h^{ac}+\Omega_h^{opt}}, \tag{26}$$

where the energy relaxation frequencies are given by

$$P_{eh}=(T_e-T_h)\Omega_{eh},\quad P_h^{ac}=(T_h-T)\Omega_h^{ac},\quad P_h^{opt}=T_h\Omega_h^{opt}.$$

We shall now obtain expressions for the relaxation frequencies Ω_{eh} and Ω_h^{ac} in the ultraquantum limit. We shall also estimate Ω_h^{opt} assuming that $\Phi(H)=1$ and $T_c=T_h$:

$$\Omega_h^{opt}\approx\frac{\hbar\omega_0}{T_h}\left(\frac{\partial n}{\partial t}\right)^{opt}.$$

We shall consider the relative width of the energy exchange channels. We shall do this employing the relationships

$$\frac{\Omega_h^{ac}}{\Omega_{eh}}\approx\frac{E_{eff}^2\hbar^2\varkappa_0^2}{32\pi^2\rho\alpha^6e^4T_hn}\left(\frac{m_h}{m_e}\right)^{3/2}\left(\frac{T_e}{T_h}\right)^{1/2}\ln\left(\frac{4\alpha T_h}{\hbar se^c}\right)$$
$$\times\int_0^\infty dx\,M_{b^+2}^{b^+2}(x)\left[\int_0^\infty dx(x+\alpha^2q_s^2)^{-2}M_{a1}^{a1}(x)M_{b^+2}^{b^+2}(x)\right]^{-1}, \tag{27}$$

$$\frac{\Omega_h^{opt}}{\Omega_{eh}}\approx\frac{\pi^{1/2}}{16}\left(\frac{\hbar\omega_0}{T_h}\right)^2\left(\frac{\varkappa_0}{\varkappa_\infty}-1\right)\left(\frac{m_h}{m_e}\right)\left(\frac{T_e}{\varepsilon_0}\right)^{1/2}\exp\left[\frac{\Delta E(H)}{T_h}-\frac{\hbar\omega_0}{T_h}\right]$$
$$\times\left[\int_0^\infty dx(x+\alpha^2q_s^2)^{-2}M_{a1}^{a1}(x)M_{b^+2}^{b^+2}(x)\right]^{-1}, \tag{28}$$

where $\epsilon_0=m_ee^4/2\varkappa^2\hbar^2$ is the Bohr energy for electrons in a crystal and $q_s=(4\pi e^2n/T_e)^{1/2}$ is the reciprocal of the screening radius. If $\alpha^2q_s^2\ll1$, then

$$\int_0^\infty dx(x+\alpha^2q_s^2)^{-2}M_{a1}^{a1}(x)M_{b^+2}^{b^+2}(x)\approx\frac{1}{\alpha^2q_s^2}.$$

We shall obtain estimates using the parameters of HgTe ($E_{eff}\sim4-5$ eV, which is typical for semiconductors with the zinc-blende structure,[14] $\varkappa_0\approx21$, $\varkappa_\infty\approx15$, $\rho\approx8$ g/cm^3, $m_h/m_e\approx20$, $\hbar\omega_0\approx17$ meV, $n\approx10^{14}-10^{15}$ cm^{-3}). For $T/k\approx4$°K, $T_e/k\approx10$°K, and $H\approx10^5$ Oe the ratio of the relaxation frequencies is of the order of

$$\frac{\Omega_h^{ac}}{\Omega_{eh}}\approx10^{-2},\quad\frac{\Omega_h^{opt}}{\Omega_{eh}}\approx50\exp\left[\frac{\Delta E(H)}{T_h}-\frac{\hbar\omega_0}{T_h}\right].$$

Hence, it follows that the bottleneck effect ($T_e=T_h$) occurs when the following inequality is satisfied:

$$[\hbar\omega_0-\Delta E(H)]/T\gg1.$$

This criterion may possibly break down only near the last resonance, when the hole temperature differs considerably from the electron temperature.

We recall that the results obtained above apply to nondegenerate electrons and holes and that the range of magnetic fields is limited from above by the quantization condition for heavy holes ($\hbar\Omega_h\gtrsim T_h$) and from above by the last resonance condition [$\hbar\omega_0\gtrsim\Delta E(H)$].

APPENDIX. CALCULATION OF MATRIX ELEMENTS $\langle\mu|e^{i\mathbf{qr}}|\mu'\rangle$

According to Luttinger,[8] the wave functions transforming at the point $k=0$ in accordance with the Γ_8 representation are

$$|\mu\rangle=N^{-1/2}\exp\{i(k_{\mu y}y+k_{\mu z}z)\}\sum_j A_{\mu j}\Phi_\mu U_j, \tag{A.1}$$

where U_j is a Bloch function, Φ_μ are the eigenfunctions of the harmonic oscillator, and the coefficients $A_{\mu j}$ are given by Gel'mont,[10] and by Liu and Tan.[10]

Using the orthogonality relationships for the Bloch functions, we can express the square of the matrix element in terms of the coefficients $A_{\mu j}$ and matrix elements $I_{\mu\mu'}=\langle\Phi_\mu|e^{i\mathbf{qr}}|\Phi_{\mu'}\rangle$:

$$|\langle\mu|e^{i\mathbf{qr}}|\mu'\rangle|^2=M_{\mu\mu'}\delta(k_y',k_y-q_y)\delta(k_z',k_z-q_z),$$
$$M_{\mu\mu'}=\left|\sum_j A_{\mu j}A_{\mu' j}I_{\mu\mu'}\right|^2. \tag{A.2}$$

The explicit form of the matrix elements $I_{\mu\mu'}$ is given in the monograph of Zyryanov and Klinger:[15]

$$I_{\mu\mu'}=\left[\frac{\alpha}{\sqrt2}(q_x+iq_y)\right]^{|n_\mu-n_{\mu'}|}\frac{\bar n!}{(n_\mu!\,n_{\mu'}!)^{1/2}}\exp\left(-\frac{\alpha^2q_\perp^2}{4}\right)L_{\bar n}^{|n_\mu-n_{\mu'}|}\left(\frac{\alpha^2q_\perp^2}{2}\right).$$

Here, $\bar n=\min\{n_\mu,n_\mu'\}$ and $L_n^s(x)$ is a Laguerre polynomial. The explicit forms of the matrix elements for transitions allowed for in the present paper are as follows:

$$M_{ea1}^{ea1}=\frac{[(1-y^2)+2|x||x'|]^2}{(2x^2+1)(2x'^2+1)}e^{-y^2},\quad M_{ea0}^{ea1}=(2x^2+1)^{-1}y^2e^{-y^2},$$
$$M_{ea^+l}^{ea1}=\frac{3[(l-y^2)+2|x||x'|]^2y^{2l-2}}{(l-2)!(2x^2+1)(2x'^2+l)(2x'^2+4l-3)}e^{-y^2},\quad l\geqslant2,$$
$$M_{eb^+l}^{ea1}=\frac{6l[(l-1)|x|-(l-1-y^2)|x'|]^2y^{2l-4}}{(l-1)!(2x^2+1)(2x'^2+l-1)(2x'^2+4l-1)}e^{-y^2},\quad l\geqslant2, \tag{A.3}$$
$$M_{eb^+2}^{eb^+2}=\frac{(2x^2+1)(2x'^2+1)}{(2x^2+7)(2x'^2+7)}\left[\left(1-2y^2+\frac12y^4\right)\right.$$
$$\left.+\frac{12|x||x'|}{(2x^2+1)(2x'^2+1)}(1-y^2)+\frac{6}{(2x^2+1)(2x'^2+1)}\right]^2e^{-y^2},$$

where

$$x=\alpha k_z,\quad x'=\alpha k_z',\quad y=\alpha q_\perp/\sqrt2.$$

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Dislocation damping by electrons in the ultraquantum limit

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The damping of dislocations by conduction electrons in a strong magnetic field corresponding to the ultraquantum limit is investigated. A substantial increase of the "electron friction" with increasing magnetic field ($\propto H^{7/2}$) is observed. The strongest slowing down is experienced by dislocations oriented at small angles Φ to the **H** direction. For these dislocations, the dependence of the damping force on the velocity and on the angle Φ can have a nonmonotonic character.

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1. INTRODUCTION

1. The plastic properties of metals at low temperatures are determined to a considerable degree by the damping of the dislocations by the conduction electrons. The energy of a moving dislocation is consumed in excitation of the electrons, i.e., in the raising of the electrons from states with lower energy to states with higher energy, and is then converted into heat when the electron system relaxes. This is how a fraction of the energy of the external loads that cause the dislocation motion is dissipated.

We are interested in the energy Q absorbed by the electrons per unit time. It is determined by two factors. First, Q depends on the parameters of the field of the dislocation strains; second, the dissipated energy is expressed in terms of characteristics of the electron system. An external magnetic field **H** alters the wave functions and the chemical potential of the electrons, i.e., the problem arises of determining the dependence of Q on **H**. The influence of a magnetic field on the dislocation damping force was investigated in Refs. 1–4 with a simple single-band model of electrons with a quadratic isotropic dispersion law as an example. In Refs. 1 and 3 they considered a special situation, where a linear dislocation was oriented parallel to **H**. This problem was solved for an arbitrary geometry in Ref. 4. One of the principal results of the latter reference is the conclusion in most cases that the damping force depends little on the magnetic field. The point is that the magnetic field alters the character of the electron motion only in a plane perpendicular to the vector **H**. Along the magnetic field, the electrons move as free particles. It is precisely this drift motion which plays the decisive role in the absorption of the dislocation energy. Owing to the drift along **H**, the center of the electron orbit manages, during the cyclotron period, to cross many times the fronts of the elastic wave generated by the dislocation. Therefore the absorption of such a dislocation phonon has a collisionless character and does not depend on **H**. This result is valid if the magnetic field is not too strong, when the distance $\hbar\Omega$ between the Landau levels ($\Omega \equiv eH/mc$ is the cyclotron frequency and e is the absolute value of the electron charge) is much less than the Fermi energy ε_F.

In very strong magnetic fields, in semimetals and semiconductors with low carrier density, the inverse limiting case

$$\hbar\Omega/\varepsilon_F \gtrsim 1 \qquad (1.1)$$

may be realized, and is called "ultraquantum." Here the cyclotron period $2\pi/\Omega$ becomes so small that during this time the electron moves in a field of practically homogeneous strains. Therefore in the case (1.1) one should expect a strong effect of the magnetic field on the damping force.[1)] An analysis of this question is the subject of the present paper.

2. Just as in Ref. 4, we confine ourselves to an electron gas with quadratic isotropic dispersion. The state of the electrons in the external constant and homogeneous magnetic field is classified with the aid of four quantum numbers. Three of them, namely, the momentum projection p_H on the direction of the magnetic field, the principal quantum number n, and the

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