Recombination radiation spectrum of a disordered semiconductor

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(Submitted 30 March 1978)
Zh. Eksp. Teor. Fiz. **75**, 1044–1055 (September 1978)

A theory is given of radiative recombination of nonequilibrium carriers whose energies correspond to the density-of-states tails formed in a disordered semiconductor near the forbidden band edges. A direct-gap heavily doped compensated semiconductor with very different effective carrier masses is considered. It is shown that a small effective mass makes it possible to describe the distribution of nonequilibrium electrons between states in the conduction band tail by introducing a quasi-Fermi level, whereas the distribution of holes between localized states in the valence band tail differs considerably from the quasi-equilibrium form because of radiative transitions resulting from the tunneling of electrons. The spectral intensity of the interband recombination radiation is calculated and the energy of its maximum is found for various temperatures and excitation rates. It is shown that in the Gaussian region the optimal fluctuations, which govern the recombination radiation intensity $\Phi(\omega)$, differ from fluctuations optimal from the point of view of the absorption coefficient $\alpha(\omega)$ and the density of electron states $\rho(\epsilon)$. Consequently, the exponential falls of $\Phi(\omega)$, $\alpha(\omega)$, and $\rho(\epsilon)$ at long wavelengths are of different nature and the spectral intensity of the recombination radiation decreases more rapidly on reduction of the frequency than does the absorption coefficient. These results make it possible to explain the observed features of the recombination radiation spectra of disordered semiconductors.

PACS numbers: 78.60. – b, 71.25.Cx

The main attention in the theory of disordered semiconductors is concentrated on the density of electron states $\rho(\varepsilon)$ (Refs. 1 and 2) and on the conductivity.[3,4] However, the function $\rho(\varepsilon)$ is an average characteristic whose knowledge is insufficient for the description of the majority of the observed dependences and, in particular, of the absorption $\alpha(\omega)$ and recombination radiation $\Phi(\omega)$ spectra. A strong (exponential) dependence of the matrix element of an interband radiative transition on the parameters of a fluctuation well has the effect that fluctuations dominating the density of states are generally nonoptimal from the point of view of the absorption coefficient. Consequently, there are differences between the dependences $\alpha(\omega)$ and $\rho(\varepsilon)$ (Refs. 1, 5, and 6).

In contrast to the absorption of light, the radiative recombination is a strongly nonequilibrium process associated with the presence of excess carriers which appear in a semiconductor on illumination or injection. Therefore, one frequently encounters the problem of finding the function describing the distribution of carriers between fluctuation levels. This problem has been solved in an investigation of the luminescence of heavily doped semiconductors,[7,8] where it is shown that the distribution of nonequilibrium carriers between localized tail states may differ considerably from the quasiequilibrium form. This shows that carrier recombination may have an important influence on the energy distribution. However, in Refs 7 and 8 the probability of radiative recombination is assumed to depend weakly on the carrier energy and parameters of a fluctuation well, which is justified for degenerate (weakly compensated) heavily doped semiconductors. The characteristic electron energy in such materials is equal to the Fermi energy and the characteristic size of a fluctuation well is equal to the screening radius in a degenerate electron gas.

The main features of radiative recombination in disordered semiconductors appear clearly in heavily doped and strongly compensated semiconductors, which have been subjected to very thorough experimental studies.[9-16] The interest in heavily doped and strongly compensated semiconductors is stimulated also by the fact that they represent one of the models of amorphous semiconductors.[17] An increase in the degree of compensation reduces the characteristic electron energy and increases the screening radius. Therefore, the matrix element of a radiative tunnel transition and, consequently, the probability of occupancy of the appropriate fluctuation well begin to depend strongly on the well parameters. Consequently, fluctuations governing the luminescence spectra of such disordered semiconductors differ from fluctuations optimal from the point of view of $\alpha(\omega)$ and $\rho(\varepsilon)$. This means that the long-wavelength luminescence spectrum does not repeat the frequency dependence of the absorption coefficient, as found experimentally.[11,12]

1. DISTRIBUTION OF NONEQUILIBRIUM CARRIERS BETWEEN FLUCTUATION LEVELS

We shall consider a heavily doped semiconductor with approximately equal donor N_D and acceptor N_A concentrations. Such a heavily doped and strongly compensated semiconductor is characterized by extended density-of-states tails which form near the forbidden band edges because of fluctuations in the impurity concentrations.[1,2] Shallower tail levels are nonlocalized states, whereas deeper ones are localized. The boundary separating these two types of state is the percolation level[3,4] denoted by E_c and E_v in Fig. 1. For example, electrons of energies $E_e > E_c$ are free but those of energies $E_e < E_c$ are localized in the parts of a crystal where fluctuations of the impurity concentration produce fairly deep potential wells (Fig. 1). The capture of carriers by these wells is a cascade process, similar to the capture by an attractive impurity center.[18]

 0038-5646/78/090527-06$02.40

In investigations of the recombination of carriers localized in the density-of-states tails we can, as in the theory of impurity luminescence,[18] allow only for the ground state in each potential well. This approximation is usually employed also in the calculation of the density of states in heavily doped semiconductors.[1,2]

In the case of the wells whose occupancy probability is $q \ll 1$, it is sufficient to consider the capture of the first carrier. Then, if the size of the optimal well R_m is such that $nR_m^3, pR_m^3 < 1$, we can assume that the densities of free electrons n and holes p are independent of the coordinate.[7,8]

It follows from the above discussion that the probability of electron occupancy q_e of a well whose ground-state energy is E_e satisfies the equation

$$\frac{\partial q_e}{\partial t} = nW_n(1-q_e) - N_c W_n \exp\left(\frac{E_e - E_c}{T}\right) q_e - p\tilde{W}_p q_e - \sum_j \nu(E_e, E_{hj}, r_j)\, q_{hj} q_e. \quad (1)$$

Here, nW_n is the probability of capture of a free electron by a "donor" well, where W_n depends weakly on the ionization energy of the well and on temperature;[18] $p\tilde{W}_p$ is the probability of radiative recombination of a trapped electron with a free (band) hole (TB transition in Fig. 1). The second term in Eq. (1) describes the thermal release of an electron from the well to nonlocalized states (N_c is the effective number of states of the continuous spectrum near the electron percolation level E_c –Ref. 19); the last term represents the probability of radiative recombination of an electron trap in the "donor" well under consideration with one of the holes of energy E_{hj} trapped in a neighboring "acceptor" well localized at a distance r_j from the "donor" well (TT transition in Fig. 1).

Similarly, the equation for the probability of the hole occupancy q_h of an "acceptor" well of ground-state energy E_h is

$$\frac{\partial q_h}{\partial t} = pW_p(1-q_h) - N_v W_p \exp\left(\frac{E_v - E_h}{T}\right) q_h - n\tilde{W}_n q_h - \sum_i \nu(E_{ei}, E_h, r_i)\, q_h q_{ei}, \quad (2)$$

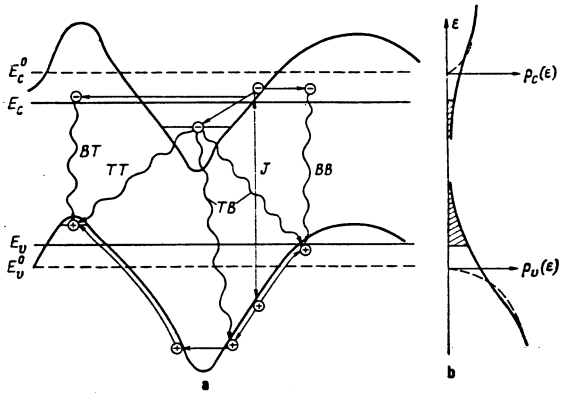

FIG. 1. Energy band diagram (a) and density of states distribution (b) in a disordered semiconductor. The shaded regions represent localized carrier states (E_c and E_v are the electron and hole percolation levels, E_c^0 and E_v^0 are the edges of the forbidden band of the undoped semiconductor). The wavy lines represent possible types of radiative transition.

where N_v is the effective number of states in the continuous spectrum near the hole percolation level E_v (Ref. 19); pW_p is the probability of nonradiative capture of a free hole by the "acceptor" well. The last two terms on the right-hand side of Eq. (2) describe the loss of a hole from a localized state because of its recombination with band or trapped electrons (BT and TT transitions in Fig. 1).

Equation (2) is simplified by dropping the terms representing the jumps of holes between the "acceptor" wells because–by definition–the effective hole mass m_h is large and, therefore, at moderately low temperatures the probability of a jump of a heavy hole to a neighboring well is low compared with the probability of its thermal release to a band, i.e., to states with $E_h < E_v$ (Fig. 1). However, at low temperatures the probability of a change in the occupancy q_h because of jumps is low compared with the probability of the recombination loss of a hole. In fact, the loss of a hole from a given well to a deeper one is unlikely because there are few deep wells and they are very likely to be filled. Moreover, shallow wells, to which jumps at low temperatures are not likely because of their thermally activated nature, located reasonably close to a given well are also unfilled.[3,4] In view of the small value of m_e, electron jumps between the neighboring "donor" wells may be significant, but they are not included in Eq. (1). Naturally, the distribution of electrons tends to become thermalized due to such jumps between wells. However, we shall show that even without allowance for the jumps the electron distribution between localized states in the conduction band tail is in quasiequilibrium.

In fact, under steady-state conditions, Eqs. (1) and (2) yield

$$q_e = nW_n\left[nW_n + N_c W_n \exp\left(\frac{E_e - E_c}{T}\right) + p\tilde{W}_p + \sum_j \nu(E_e, E_{hj}, r_j)\, q_{hj}\right]^{-1}, \quad (3)$$

$$q_h = pW_p\left[pW_p + N_c W_p \exp\left(\frac{E_c - E_h}{T}\right) + n\tilde{W}_n + \sum_i \nu(E_{ei}, E_h, r_i)\, q_{ei}\right]^{-1}. \quad (4)$$

The distribution of electrons is of quasiequilibrium type

$$q_e = f_e = \left[1 + \exp\left(\frac{E_e - F_n}{T}\right)\right]^{-1} \quad (5)$$

[$F_n = E_c - T\ln(N_c/n)$ is the electron quasi-Fermi level] when the first two terms in the denominator of Eq. (3) are larger than the other terms. It follows from Eqs. (3) and (4) that this is true if

$$W_n\left[n + N_c \exp\left(\frac{E_e - E_c}{T}\right)\right] \gg p\left[\tilde{W}_p + W_p \sum \nu(E_e, E_{hj}, r_j) \times \left(pW_p + N_c W_p \exp\left(\frac{E_c - E_h}{T}\right) + n\tilde{W}_n + \sum_i \nu(E_{ei}, E_h, r_i) f_e\right)^{-1}\right]. \quad (6)$$

We recall that the term $p\tilde{W}_p$ describes radiative recombination associated with the tunneling of a heavy hole (TB transition in Fig. 1) and, therefore, $W_p \ll W_n$ (Ref. 18). We can then easily show that the condition (6) applies when $nW_n > pW_p$. Since $W_n \approx W_p$, the latter inequality follows in fact from the condition $m_e \ll m_h$. In fact, if $m_e \ll m_h$, localized states in the valence band tail are deeper and their number is greater than the number of localized states in the conduction band tail. Therefore, at all temperatures we have $n \gg p$.

The physical meaning of the quasiequilibrium distribution of electrons between localized states is as follows. A consequence of the neutrality condition is that the density of localized electrons is approximately equal to the density of localized holes since the densities of free carriers are low compared with the densities of those which are localized. This means that at moderate excitation rates, when only some "donor" wells are filled, the majority of the "acceptor" wells near each filled "donor" well are empty. Consequently, a localized hole is, on the average, at a considerable distance from a filled "donor" well and, therefore, even at low temperatures the probability of thermal release of a localized electron to a "band" is higher than the probability of its radiative recombination with a hole. In other words, carrier recombination has little influence on the thermal redistribution of electrons between localized tail states. On the other hand, holes are localized in deeper wells from which thermal release is difficult. Moreover, electrons easily tunnel because of their small mass. Consequently, in the case of localized holes the probability of recombination may exceed the probability of thermal release and, therefore, the distribution function of holes between localized states in the valence band tail $q_h(E_h)$, given by Eq. (4), may differ considerably from the quasiequilibrium function.[1)]

2. RECOMBINATION RADIATION SPECTRUM

We may expect[7,8] that at low temperatures and relatively high excitation rates the luminescence is mainly due to the recombination of carriers whose energies lie near the electron E_c and hole E_v percolation levels (Fig. 1) in regions of small (Gaussian) fluctuations of the impurity potential. The density of states in the band tails corresponding to these fluctuations is[1]

$$\rho(\varepsilon)=\rho_0\exp(-\varepsilon^2/2\gamma^2), \tag{7}$$

where

$$\gamma=(2\pi N r_0^3)^{1/2}e^2/\varkappa r_0 \tag{8}$$

is the characteristic depth and r_0 is the characteristic size of a potential well; $N=N_A+N_D$; $\varkappa$ is the permittivity of the material.

The value of r_0 for a heavily doped and strongly compensated semiconductor depends greatly on the degree of correlation in the distribution of donors and acceptors. For a random distribution of the impurities the order-of-magnitude relationship is[22]

$$r_0\approx N_D^{1/6}\tilde{n}^{-1/2} \quad (N_D>N_A), \tag{9}$$

i.e., the characteristic size of a potential well is governed by the total carrier density

$$\tilde{n}=\int_{E_v}^{\infty}\rho_e f_e\,dE_e.$$

The opposite situation appears in the case of a correlated distribution when r_0 and γ are independent of the degree of compensation, temperature, and rate of excitation, i.e., they are independent of $\tilde{n}$. A correlation in the distribution of impurities may be due to, in particular, their Coulomb interaction.[23,24,8] If the free-carrier density at the impurity "freezeout" temperature T_0 during the process of preparation of a compensated semiconductor with $N_A\approx N_D$ does not exceed the total impurity concentration, then

$$r_0=(\varkappa T_0/4\pi N e^2)^{1/2}. \tag{10}$$

It should be noted that characteristics of the radiative recombination in heavily doped and strongly compensated semiconductors have been investigated most thoroughly in the case of GaAs (Refs. 9–15) and, as indicated by theoretical estimates[8] and experimental studies,[15] the distribution of impurities in strongly compensated GaAs with $N\gtrsim 10^{18}$ cm^{-3} is correlated. On the other hand, strongly compensated Ge is characterized by a random distribution of impurities.[16]

The intensity of recombination radiation $\Phi(\omega)$ emitted from a heavily doped semiconductor depends exponentially on the photon energy ω (Refs. 7 and 8). The argument of the exponential function in $\Phi(\omega)$ can be found most simply (to within a constant factor of the order of unity), by a method analogous to the approximation of a uniformly charged sphere.[1,25] The value of the relevant factor can then be obtained using the results of a rigorous theory of the absorption of light by a heavily doped and strongly compensated semiconductor.[5,6]

The energy of a photon emitted as a result of recombination of an electron with a localized hole is

$$\omega=E_e-E_h=E_g^0-\Delta, \quad (E_g^0=E_c^0-E_v^0), \tag{11}$$

where E_e and E_h are the energies of an electron and a hole, respectively. We shall assume that in the region of Gaussian fluctuations the energy of a localized hole is close to the bottom of an "acceptor" well, i.e., $\hbar^2/m_h r_0^2 \ll E_h - E_v^0$ (this is the "classical" case).[25] Then, the recombination probability

$$\nu(E_e,E_h,R)=\nu_0\exp\left[-\eta\frac{R(m_e\Delta)^{1/2}}{\hbar}\right] \tag{12}$$

is determined by the tunneling of an electron under a potential barrier of height Δ and radius R (Fig. 2). The excess number of impurities needed to create such a barrier is approximately $Z=\varkappa R\Delta/e^2$ and, therefore, the probability of its appearance in the $R\leqslant r_0$ case is[1]

$$P(E_e,E_h,R)=P_0\exp\left(-\zeta\frac{Z^2}{NR^3}\right)=P_0\exp\left(-\zeta\frac{\varkappa^2\Delta^2}{e^4NR}\right). \tag{13}$$

In Eqs. (12) and (13) the coefficients ζ and η are of the order of unity and their values will be determined later.

The average probability of occupancy of an "accep-

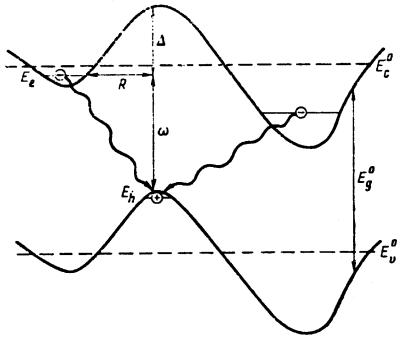

FIG. 2. Schematic representation of the emission of a photon in the case of an energy deficit Δ.

tor" well $\langle q_h \rangle_R$ by a hole whose recombination results in the emission of a photon of energy ω can be found by considering a configuration in which a "donor" well with an electron energy $E_h+\omega$ (Fig. 2) is closest to a given "acceptor" well and then averaging q_h of Eq. (4) over all possible configurations of other "donor" wells located at distances exceeding R; this has to be done allowing for their occupancy.[2)] In view of the quasi-equilibrium nature of the electron distribution (5), there is no need to separate electrons into free (band) and localized, i.e., there is no need to separate the term $n\tilde{W}_n$ in the expression for q_h given by Eq. (4). Using Eq. (5), we shall follow earlier work[20] in averaging approximately Eq. (4):

$$\langle q_h \rangle_R = pW_p \left[pW_p + N_v W_p \exp\left(\frac{E_v - E_h}{T}\right) + \langle v(E_h) f_e \rangle_R + v(E_e, E_h, R) f_e(E_e) \right]^{-1}. \quad (14)$$

Equation (14) is valid if

$$\langle v(E_h) f_e \rangle_R = 4\pi \int_R^\infty dR' R'^2 \int_{E_c}^\infty dE_e v(E_e, E_h, R') P(E_e, E_h, R') \quad (15)$$

is small compared with the sum of the other terms in the denominator of the expression for $\langle q_h \rangle_R$.

When we know $\langle q_h \rangle_R$, the spectral intensity of the recombination radiation in the case of Gaussian fluctuations can be written in the form

$$\Phi(\omega) \propto \int dE_e \int dE_h \delta(E_e - E_h - \omega) f_e(E_e) \times \langle \langle q_h(E_e, E_h) \rangle_R v(E_e, E_h, R) P(E_e, E_h, R) \rangle, \quad (16)$$

where the symbol $\langle \ldots \rangle$ denotes averaging over the configurations of the potential barriers making a contribution to the recombination radiation of photon energy ω. In this approximation the barriers all have the same height $\Delta = E_g^0 - \omega$ given by Eq. (11) but they differ in respect of the radius R. Since the product of the functions (12)–(14) being averaged has a sharp maximum in its dependence on R, the average value of the products of such functions should be replaced with its most probable value.

A similar procedure is used to calculate the spectral dependence of the interband absorption coefficient of a heavily doped and strongly compensated semiconductor,[1,5] which for

$$\omega > \omega_1 = E_g^0 - I_D (r_0/a_e)^{4/3} (Na_e^3)^{2/3} \quad (17)$$

has the form

$$\alpha(\omega) \propto \langle v(\Delta, R) P(\Delta, R) \rangle \propto \exp\left[-\beta \left(\frac{\Delta}{I_D (Na_e^3)^{2/3}} \right)^{1/4} \right], \quad (18)$$

and in this case the size of optimal fluctuations is

$$\tilde{R} = (\zeta/\eta)^{1/4} a_e (\Delta/I_D)^{1/4} (Na_e^3)^{-1/4}.$$

A more rigorous approach[5] based on the optimal fluctuation method[2,27] makes it possible to determine the exact value of the coefficient β:

$$\beta = (\zeta\eta)^{1/4} (2^{-1/2} + 4^{-1}) = 2 \cdot 5^{-1} \pi^{-1/2}$$

and it shows[6] that the ionization energy of a shallow donor $I_D = \hbar^2/(2m_e a_e^2)$ and its Bohr radius a_e in Eq. (18) have to be replaced with the corresponding parameters of an exciton. In contrast to Eq. (18) for $\alpha(\omega)$, the expression (16) for $\Phi(\omega)$ has in $\langle \ldots \rangle$ an additional factor $\langle q_h \rangle_R$, which—as is clear from Eq. (14)—may depend strongly on R.

In the calculation of the integral (16) the Fermi function f_e can be replaced by a step, which gives

$$\Phi(\omega) \propto \left\langle \frac{pW_p v(R, \Delta) P(R, \Delta)}{pW_p + N_v W_p \exp[(E_g^0 - F_n + E_v - \Delta)/T] + v(R, \Delta)} \right\rangle. \quad (19)$$

Since the maximum of the expression in Eq. (19) in $\langle \ldots \rangle$ occurs in the range $\tilde{R} \leq R_m \leq r_0$, it then follows from Eq. (15) that the quantity $\langle v f_e \rangle_R$ can be ignored compared with $v(R, \Delta) f_e$ if $\Delta \gtrsim I_D$. Thus, in the investigated range of frequencies the recombination loss of a hole is controlled mainly by the nearest "donor" well. This approximation is equivalent to allowance for the nearest donor–acceptor pairs in the theory of inter-impurity radiative recombination.[20,21,26]

It follows from Eq. (19) that $\Phi(\omega)$ reaches its maximum value at

$$\omega_{m1} = E_g - T \ln\left(\frac{N_c}{n}\right) - T \ln \frac{N_v}{p + v(R_m, E_g^0 - \omega_{m1})/W_p} \quad (E_g = E_c - E_v), \quad (20)$$

whereas in the short-wavelength range ($\omega > \omega_{m1}$) it falls as

$$\Phi(\omega) \propto \exp\left[\frac{E_g^0 - \omega}{T} - \beta \left(\frac{E_g^0 - \omega}{I_D (Na_e^3)^{2/3}} \right)^{1/4} \right], \quad (21)$$

i.e., the characteristic scale of the fall is given by T.

Since in the derivation of Eq. (4) and, consequently, of Eqs. (14) and (19), use is made of the condition $pR_m^3 < 1$, it follows that for $\omega < \omega_{m1}$, and low rates of excitation when

$$p \ll \frac{v_0}{W_p} \exp\left(-\eta \frac{r_0 [m_e (E_g^0 - \omega)]^{1/2}}{\hbar} \right) r_0^{-3},$$

we have

$$q_h = \frac{pW_p}{v(R, \Delta)} \ll 1$$

and $\langle \ldots \rangle$ in Eq. (19) reduces simply to the function $P(R, \Delta)$. This function has a maximum at $R_m = r_0$ and the value of $\langle P \rangle$ governs the density of states (7) (Refs. 1, 25, and 28). This makes it possible to reconstruct the numerical values of the coefficients ζ and η:

$$\zeta = 2^{-3/2} \pi^{-1/2}, \quad \eta = 2^{9/2} \cdot 5^{-2} \pi^{-1/2} (3^{1/3} + 2^{-1/2})^{-1}$$

and to write down the spectral dependence of the luminescence for $\omega < \omega_{m1}$ in the form (Fig. 3)

$$\Phi(\omega) \propto \exp[-(E_g^0 - \omega)^2/2\gamma^2]. \quad (22)$$

At high excitation rates all the localized states are filled with carriers, i.e., $q_h = 1$, and, therefore, within the averaging symbols in Eq. (19) there are all the quantities which occur in Eq. (18) for $\alpha(\omega)$. Hence, it follows that $\Phi(\omega) \sim \alpha(\omega)$, i.e., for $\omega_1 < \omega < \omega_{m1}$ we have $R_m = \tilde{R}$ and the luminescence spectrum is

$$\Phi(\omega) \propto \exp\left[-\beta \left(\frac{E_g^0 - \omega}{I_D (Na_e^3)^{2/3}} \right)^{1/4} \right], \quad (23)$$

whereas for $\omega < \omega_1$ it is described by the Gaussian function (22) since in this case we have $R_m = r_0$ (Ref. 5). It should be noted that similar considerations are used above in deriving Eq. (21).

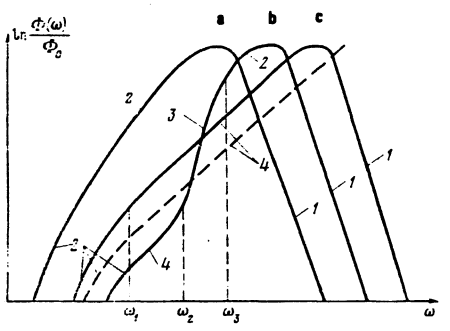

FIG. 3. Qualitative appearance of the luminescence spectra at some fairly low temperature and various rates of excitation: a) low; b) moderate; c) high. The numbers identify the parts of the curves described by various formulas: 1) Eq. (21); 2) Eq. (22); 3) Eq. (26); 4) Eq. (23). The dashed curve represents the frequency dependence of the absorption coefficient.[5] The energy ω_1 is found from Eq. (17) and the energies ω_2 and ω_3 correspond to the limits of the interval (25).

At moderate excitation rates

$$\frac{\nu_0}{W_p}\exp\left[-\left(\frac{r_0}{a_e}\right)^{5/3}(Na_e^3)^{1/3}\right]<p<\frac{\nu_0}{W_p}\exp\left[-\left(\frac{E_g^0-\omega_{m1}}{I_D(Na_e^3)^{2/3}}\right)^{4/3}\right],\quad R_m^{-3}, \tag{24}$$

we have

$$R_m=a_e\left(\frac{I_D}{\Delta}\right)^{1/3}\ln\left(\frac{\nu_0}{pW_p}\right)$$

and there is a fairly wide range of energy deficit

$$E_g^0-\omega_{m1}, I_D\left(\frac{a_e}{r_0}\ln\frac{\nu_0}{pW_p}\right)^2<\Delta<I_D(Na_e^3)^{2/3}\left(\ln\frac{\nu_0}{pW_p}\right)^{4/3}, \tag{25}$$

where the spectral intensity obeys approximately

$$\Phi(\omega)\propto\exp\left\{-\xi\left[\frac{E_g^0-\omega}{I_D(Na_e^3\ln(\nu_0/pW_p))^{2/3}}\right]^{3/2}\right\}; \tag{26}$$

here, ξ is a coefficient of the order of unity. The range of excitation rates (24) and energy deficit (25) in which Eq. (26) is valid increases with the heavy doping parameter Na_e^3 and with increasing screening radius r_0.

Thus, only at high excitation rates when all the states in the valence band tail are occupied does the fall of the luminescence spectrum in the long wavelength range (23) repeat the spectral dependence of the absorption coefficient (18). In the opposite case, as is clear from a comparison of Eq. (18) with Eqs. (22) and (26), $\Phi(\omega)$ in the range $\omega<\omega_{m1}$ falls more rapidly on reduction of the frequency than does $\alpha(\omega)$. This feature demonstrates the absence of quasiequilibrium in the excess carrier distribution and it has been discovered in an analysis of the experimental data on the photoluminescence and electroluminescence of compensated GaAs (Refs. 11 and 12).

It should be noted that at moderate excitation rates given by Eq. (24), when Eq. (26) applies, the characteristic energy of the fall of $\Phi(\omega)$ depends on the nonequilibrium density of mobile holes, i.e., in the final analysis it depends on the temperature of a sample and the rate of excitation. In the case of narrow-gap heavily doped strongly compensated semiconductors, such as Ge, InSb, InAs, and GaSb, such a dependence should be observed also at lower excitation rates when (22) is valid because in the case of these materials we may expect the distribution of the impurities to be random[24] so that the quantities r_0 of Eq. (9) and γ of Eq. (8) depend on the excitation rate.

At low temperatures the energy of a spectral maximum decreases practically linearly, in accordance with Eq. (20), on increase of temperature and this is due to thermal liberation of carriers from "donor" and "acceptor" wells of increasing depth. When the rate of excitation is increased, the value of ω_{m1} given by Eq. (20) rises because of the filling of the shallow wells, whose number is greater than that of deep wells and which are characterized by a higher probability of radiative recombination. Such dependences of ω_m on the temperature and rate of excitation at low temperatures are in agreement with the experimental data.[9-16]

It follows formally from Eq. (21) that $\Phi(\omega)$ has a maximum at

$$\omega_{m2}=E_g^0-(Na_e^3)^2I_D^3(2\sqrt{\pi}/T)^4. \tag{27}$$

Since Eq. (20) is obtained for energies $\omega>\omega_{m1}$, it follows that the value of ω_{m2} governs the maximum only at high temperatures when $\omega_{m2}>\omega_{m1}$. Since we are considering optical transitions associated solely with Gaussian fluctuations, it follows that Eqs. (20) and (27) are valid at the temperatures corresponding to ω_{m1} and ω_{m2}, not very different from E_g^0 (Refs. 1 and 2). In other words, Eq. (20) describes correctly the dependence of ω_m at sufficiently low temperatures, whereas Eq. (27) applies at high temperatures. It follows from these equations that the temperature dependence of ω_m is nonmonotonic and that at high temperatures T the value of ω_m ceases to depend on the rate of excitation, as observed experimentally.[10-12,14,16] The shift of ω_m toward shorter wavelengths on increase of temperature in accordance with Eq. (27) is due to the fact that the difference between the probabilities of occupancy of deep and shallow wells decreases on increase of T and, therefore, shallow wells begin to dominate the recombination process.

Free carriers of energies $E_c<E_e<E_c^0$ and $E_v^0<E_h<E_v$ are located in various parts of a heavily doped strongly compensated semiconductor so that the probability of their radiative recombination (BB transition in Fig. 1), which corresponds to $\omega_m\approx E_g=E_c-E_v$ can be much less than for all the materials. In the case of strongly compensated samples characterized by a random distribution of impurities in which the characteristic size of fluctuations r_0 of Eq. (9) is particularly large and also in the case of inhomogeneous semiconductors,[29-31] this results, in particular, in the "frozen-in" photoconductivity.[31] As shown above, radiative recombination in heavily doped and strongly compensated semiconductors is governed by fluctuation wells whose size is usually much smaller than r_0. The distribution of carriers between such wells cannot be regarded as of quasiequilibrium type and the corresponding probability of radiative recombination does not indicate a thermally activated process. Thus, the assumptions underlying the work of Tkach[29] and Shik[30] are invalid, at least for direct-gap semiconductors at moderate temperatures. Therefore, the determination of the dependences n and p on the rate of excitation and temperature, i.e., of the photoconduc-

tivity of disordered semiconductors, requires a more rigorous analysis.

The authors are deeply grateful to V. L. Bonch-Bruevich, A. P. Levanyuk, B. I. Shklovskiĭ, and A. L. Éfros for discussing the results.

[1)]A similar situation occurs in compensated semiconductors under impurity recombination conditions.[20,21]

[2)]This approach is analogous to a procedure developed elsewhere[20,21,26] in calculations of the probability of occupancy of acceptor states in the theory of interimpurity radiative recombination.

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Translated by A. Tybulewicz

Fine structure of cyclotron-resonance lines

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(Submitted 31 March 1978)
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It is indicated that a spectrum of bound states of an electron in the field of an attracting center of small but finite radius and in a strong magnetic field in an arbitrary Landau band exists and leads to a serial structure of the cyclotron-resonance lines. The contribution made to the absorption curve by all the electron transitions between the bound states and the continuum state is calculated for the single-center problem. Cyclotron resonance in parallel fields is considered. The possibility of observing cyclotron resonance on the bound states of an electron in a field of neutral impurities in a semiconductor is discussed.

PACS numbers: 76.40. + b

1. INTRODUCTION

In 1957 Boyle and Brailsford observed cyclotron resonance (CR) in InSb on bound Landau electron states in the field of the Coulomb potential of charged impurities.[1] They offered also a qualitative explanation of the observed effect. The resultant bound-state spectrum was theoretically analyzed by Hasegawa and Howard.[2] Also considered was resonant absorption of the electromagnetic field by such bound states (atomic CR) in

 0038-5646/78/090532-06$02.40