tivity of disordered semiconductors, requires a more rigorous analysis.

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[1)]A similar situation occurs in compensated semiconductors under impurity recombination conditions.[20,21]

[2)]This approach is analogous to a procedure developed elsewhere[20,21,26] in calculations of the probability of occupancy of acceptor states in the theory of interimpurity radiative recombination.

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Fine structure of cyclotron-resonance lines

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It is indicated that a spectrum of bound states of an electron in the field of an attracting center of small but finite radius and in a strong magnetic field in an arbitrary Landau band exists and leads to a serial structure of the cyclotron-resonance lines. The contribution made to the absorption curve by all the electron transitions between the bound states and the continuum state is calculated for the single-center problem. Cyclotron resonance in parallel fields is considered. The possibility of observing cyclotron resonance on the bound states of an electron in a field of neutral impurities in a semiconductor is discussed.

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1. INTRODUCTION

In 1957 Boyle and Brailsford observed cyclotron resonance (CR) in InSb on bound Landau electron states in the field of the Coulomb potential of charged impurities.[1] They offered also a qualitative explanation of the observed effect. The resultant bound-state spectrum was theoretically analyzed by Hasegawa and Howard.[2] Also considered was resonant absorption of the electromagnetic field by such bound states (atomic CR) in

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semiconductors in the presence of broadening by impurities[3] and by acoustic phonons.[4]

It is pointed out in the present paper that atomic CR can be observed on bound states of an electron in the attraction field of a small but finite radius a in a strong magnetic field ($a \ll l = (c\hbar/|e|H)^{1/2}$ is the magnetic length). It is shown that in an attraction field of arbitrarily small but finite radius and in a quantizing magnetic field there is produced an electron bound-state spectrum that lies lower than the bottom of any Landau band. The spectrum is infinite and condenses towards the bottom of the Landau band. States lying below the bottom of the ground band are truly undamped, while in the remaining bands they decay. A system of wave functions is constructed for the bound states and continuous spectrum of the electron in the one-center problem. This system of functions serves as the basis for the calculation of the contribution made to the CR absorption curve for four types of transitions:

1) bound states → bound states ($b \to b$),

2) bound states → continuum ($b \to c$)

3) continuum → bound states ($c \to b$),

4) continuum → continuum ($c \to c$).

It is indicated further that atomic CR can be observed on the bound states of a short-range potential in a strong magnetic field not only in perpendicular ($\mathbf{E} \perp \mathbf{H}$) but also in parallel ($\mathbf{E} \parallel \mathbf{H}$) electric and magnetic fields, for both intraband and interband transitions.

We note that neutral hydrogenlike impurities in semiconductors can be regarded as one example of a short-range attraction potential, so that the effect considered here should be observed in typical semiconductors such as Ge or Si at sufficiently high density of such impurities (see Sec. 6).

2. BOUND-STATE SPECTRUM AND CONTINUOUS SPECTRUM OF THE ELECTRON ENERGY IN THE ONE-CENTER PROBLEM

Consider an electron situated in a homogeneous magnetic field $\mathbf{H} \parallel z$ with a vector potential $A_\varphi = (\frac{1}{2})H\rho$, $A_z = A_\rho = 0$ and in a spherically symmetrical attraction field[1] $U(|\mathbf{r}|)$. The wave function of the stationary state of such an electron satisfies the Schrödinger equation

$$[\hat{H}_0 + U(|\mathbf{r}|)]\psi(\mathbf{r}) = E\psi(\mathbf{r}), \tag{1}$$

$\hat{H}_0$ is the Hamiltonian of the electron in the magnetic field.

The projection m of the orbital angular momentum of the electron on the magnetic-field direction is conserved. The wave functions can therefore by characterized by the values of m (Ref. 2) and we can seek for each given m a solution of (1) in the form

$$\psi^m(\mathbf{r}) = \sum_{n=0}^{\infty} C_{nm}(z) R_{nm}(\rho) e^{im\varphi}/(2\pi)^{1/2}, \tag{2}$$

where n is the radial quantum number.

The form of the radial wave functions $R_{nm}(\rho)$ is well known.[5] From (1) and (2) we obtain a system of equations for the coefficients $C_{nm}(z)$:

$$\frac{d^2C_{nm}}{dz^2} + \frac{2m^*}{\hbar^2}\left[E - \hbar\omega_H\left(N + \frac{1}{2}\right)\right]C_{nm} = -2\sum_{k=0}^{\infty} C_{km} J_{nk}^m(z), \tag{3}$$

m^* is the electron mass $N = n + \frac{1}{2}(|m| + m)$, $\omega_H = |e|H/m^*c$, and

$$J_{nk}^m(z) = \frac{m^*}{\hbar^2}\int_0^\infty \rho\, d\rho\, R_{nm}(\rho)\, |U(\rho, z)|\, R_{km}(\rho). \tag{4}$$

By virtue of the proportionality $R_{nm}(\rho) \propto \rho^{|m|}$ at small $\rho \ll l$,[5] the functions $J_{nk}^m(z)$ are of the order of

$$J_{nk}^m(z) \sim (a/l)^{2|m|} f_m(z). \tag{5}$$

The customarily employed[6,7] replacement of the short-range potential by a δ-function (in ρ), which leads to the appearance of one (!) bound state located below the bottom of the Landau band,[7] is valid only if $m = 0$, for at $m \neq 0$, by virtue of (5), this replacement leads to vanishing of the right-hand side of (3) and to vanishing of the bound-state spectrum. It is important, however, that the functions $J_{nk}^m(z)$ decrease with z over characteristic distances of the order of the effective radius a of the field. If the well is shallow enough,

$$\int_0^\infty \rho\, d\rho\, R_{nm}(\rho)\, U(\rho, z_{\text{eff}})\, R_{km}(\rho) \ll \frac{\hbar^2}{m^* a^2}, \tag{6}$$

then we can make, without loss of generality, the substitution

$$J_{nk}^m(z) \to \alpha_{nk}^m l^{-1}\delta(z), \tag{7}$$

$$\alpha_{nk}^m = \int_{-\infty}^{\infty} dz\, J_{nk}^m(z)\, l. \tag{8}$$

In the general case, the solution of the system (3)–(8) is a difficult task, but if the interaction of the electron with the attraction center is weak enough,

$$\alpha_B = \frac{m^* l^{-1}}{2\pi\hbar^2}\int dV\, U(\mathbf{r}) = \frac{f_B(0)}{l} \ll 1 \tag{9}$$

($f_B(0)$ is the Born amplitude of electron scattering through zero angle in the absence of a magnetic field), then the system (3)–(8) can be easily solved.

Consider the spectrum of the electron in the Landau band N. Assume (this is confirmed by subsequent calculation) that the electron longitudinal-motion energy is small compared with $\hbar\omega_H$

$$-E_\parallel = E - \hbar\omega_H(N + {}^1/_2) \ll \hbar\omega_H. \tag{10}$$

By virtue of (9) and (19), the coefficient C_{nm} $cn + \frac{1}{2}(|m| + m) = N$ is the largest, and the remaining coefficients $C_{km}(k \neq n)$ are of order of smallness $\alpha_B \ll 1$ in comparison. Taking the foregoing into account, we can uncouple the system (3):

$$\frac{d^2C_{nm}}{dz^2} - \frac{2m^*}{\hbar^2}E_\parallel C_{nm} = -2C_{nm}l^{-1}\alpha_n{}^m\delta(z) - 2\sum_{k\neq n} C_{km}\alpha_{kn}{}^m l^{-1}\delta(z), \tag{11}$$

$$\frac{d^2C_{km}}{dz^2} + \frac{2m^*}{\hbar^2}\omega_H(n-k)C_{km} = -2C_{nm}l^{-1}\alpha_{kn}^m\delta(z), \tag{12}$$

where

$$k + {}^1/_2(|m| + m) \neq N,\ \alpha_n{}^m \equiv \alpha_{nn}{}^m,\ \alpha_{00}{}^0 \equiv \alpha.$$

In (12) we have left out the small term $E_\parallel \ll \hbar\omega_H$. The

sum over $k \neq n$ must be retained in (11) in order to take into account the quasistationary character of the bound-state spectrum in a Landau band $N \neq 0$.

A more detailed analysis shows that the condition for the validity of the uncoupling of the system (3) into (10) and (11) is the smallness of

$$\alpha_f = f/l \ll 1,$$

where f is the amplitude for scattering of an electron with zero energy by a potential $U(|\mathbf{r}|)$ in the absence of a magnetic field.

Using the customary method of solving equations with a δ potential,[8] solving (12), expressing the coefficients $C_{km}(z)$ in terms of $C_{nm}(0)$, substituting $C_{km}(z)$ in the right-hand side of (11), and solving the resultant equation, we arrive at the following expression for the energy of the bound state of an electron with a definite angular-momentum projection m on the direction of the magnetic field in the Landau band N:

$$E_N^{m\leq 0} = \hbar\omega_H \left[N + \frac{1}{2} - \frac{1}{2} \left(|\alpha_N{}^m|^2 + 2^{1/2} i \alpha_N{}^m \sum_{k=0}^{N-1} \frac{|\alpha_{Nk}{}^m|^2}{(N-k)^{1/2}} \right) \right], \tag{13}$$

where $m \leq 0$, and for $m > 0$ we have

$$E_N^{m>0} = \hbar\omega_H \left[N + \frac{1}{2} - \frac{1}{2} \left(|\alpha_{N-m}^m|^2 + 2^{1/2} i \alpha_{N-m}^m \times \sum_{k=0}^{N-m-1} \frac{|\alpha_{N-m,k}^m|^2}{(N-m-k)^{1/2}} \theta(N-m-1) \right) \right], \tag{14}$$

where θ is the Heaviside function.

It follows from (13) and (14) that the longitudinal-motion energy spectrum $E_\parallel < 0$ is quantized in the field of the attraction center. This spectrum is infinite and condenses from below to the bottom of the N-th Landau band, and has a ground state $E_\parallel^{\min} = -\frac{1}{2}\alpha^2\hbar\omega_H$ that corresponds to $m = 0$ and coincides with the corresponding result of Ref. 7 taken at $\alpha \ll 1$.

In the general case $m \neq 0$ the longitudinal-motion energy is characterized by the values of m and is of the order of

$$E_\parallel{}^m \approx -{}^1/_2 \alpha^2 \hbar\omega_H \left(\frac{a}{l} \right)^{4|m|} \tag{15}$$

with a width

$$\gamma_N{}^m \sim \alpha^3 \hbar\omega_H \left(\frac{a}{l} \right)^{6|m|} \cdot \begin{cases} (N-m)^{1/2}, & 0 \leq m \leq N, \\ N^{1/2}, & m \leq 0. \end{cases} \tag{16}$$

We note that in any Landau band $n \neq 0$ (!) there is one state with $m = N > 0$ which is strictly stationary. The physical reason is that an electron with $m = N > 0$ can not drop to a lower Landau band $N' < N$, since the latter has no states with orbital momentum projections $m > N'$.[2] On the other hand the widths of the levels (13) and (14) are determined precisely by such electron transitions. Without writing out the general unwieldy expression for the wave function of the quasistationary state, we note also that the characteristic region of its variation along $z \| H$ is within the range $z_{\text{eff}} \approx l/\alpha_N^m$.

Besides the spectrum of the energies lying below the bottom of any Landau band N, the Schrödinger equation (1) has at $E_\parallel = E - \hbar\omega_H(N + \frac{1}{2}) > 0$ solutions that constitute the system of wave functions of the scattered electron (or their superposition), which can also be characterized by the quantum numbers $E_\parallel$, m, and N, the energy spectrum being continuous in $E_\parallel$. It is convenient to choose this system of functions in a form such that on the left (or on the right) of the center there is a scattered wave that decreases along z, and on the right (or left) there is only the scattered wave. Since at the asymptotic distance $z \to \pm\infty$ we have a free particle, the energy of the longitudinal motion can be set equal to $E_\parallel = p^2/2m^*$. The solution of (3) in the approximation used above can then be obtained in elementary fashion.

It is easy to show that the wave function of the continuum has in complex p plane a pole at

$$p = \frac{i\hbar}{l} \left[\alpha_N{}^m + \frac{i}{2^{1/2}} \sum_{n=0}^{N-1} \frac{|\alpha_{Nn}{}^m|^2}{(N-n)^{1/2}} \right], \quad m \leq 0, \tag{17}$$

which corresponds to a bound state of an electron in the field of a center with arbitrary energy $E_\parallel = p^2/2m^*$, given by expression (13). The last term in (17) yields in this case the width of the corresponding quasistationary state. An analysis of the wave function of the continuous spectrum with $m > 0$ shows that they have in complex p plane poles corresponding to the bound states (14). The system of the wave functions of the continuous spectrum for $m = 0$ coincides (after changing to another gauge) with Skobov's[6] system of wave functions at $p^2/2m^* \ll \hbar\omega_H$. We note that since Skobov used in fact a δ-function potential, his paper[9] does not contain states (17) with $m \neq 0$.

3. ABSORPTION IN $b \to b$ AND $b \to c$ TRANSITIONS

We investigate now the absorption of an electromagnetic field $E_0 \sin \omega t (\omega - \omega_H = \Delta\omega \ll \omega)$ by an electron on bound states in transverse fields $\mathbf{E}_0 \perp \mathbf{H}$. We assume a zero electron temperature T, no interaction between the electrons, and a weak electric field.

Under the influence of the weak electric field, the electron undergoes transitions from the ground state $n = m = 0$ into bound states of the first Landau band ($n = 0$, $m = +1$ and $n = 1, m = -1$). The probability of these transitions is proportional to the square of the modulus of the matrix element of the interaction of the electron with the electromagnetic field

$$V_F(\mathbf{r}, t) = -e\rho E_0 \sin \omega t, \tag{18}$$

taken over the corresponding wave functions of the bound state. In this case the transition ($n = m = 0 \to n = 1, m = -1$) is of higher order of smallness in α than the transition ($n = m = 0 \to n = 0, m = +1$). Simple calculations yield, in the first nonvanishing approximation in α, the following formula for the absorption coefficient of the transition ($n = m = 0 \to n = 0, m = +1$)

$$\eta_{0\to+1}^{b-b}(\omega) = \frac{4\pi^2}{\hbar} \cdot 2^{1/2} n_e' e^2 \omega l^2 \frac{\alpha_0^{+1}}{\alpha} \delta(\omega - \omega_1), \tag{19}$$

where

$$\omega_1 = \omega_H + {}^1/_2 \omega_H (\alpha^2 - |\alpha_0{}^1|^2),$$

n_e' is the concentration of the electrons captured by neutral impurities in the semiconductor, and e is the electron charge.

The reason why the absorption peak of the $b \to b$ transition is a δ-function is that the state $n = 0, m = +1$ in the

first Landau band is stationary and cannot decay with a transition into the continuum of the ground Landau band. The absorption peak in (19) becomes smeared out as a result of electron interaction with acoustic phonons and neighboring impurities.[3,4] However, the calculation of these effects is beyond the scope of the present paper.

In analogy with (19), we obtain a term that describes the contribution made to the absorption coefficient by the transitions of the second type, $b \to c$, taking the matrix element over the functions of the bound states and the continuum:

$$\eta_{0\to+1}^{b\to c}(\omega)=\frac{2^{1/2}\pi^2}{m^{*3/2}}\frac{e^2\alpha^3 l^{-1}\hbar^{1/2}\omega}{(\omega-\omega_H)^2}n_e'\theta(\omega-\omega_2)\cdot\frac{(\omega-\omega_2)^{1/2}}{(\omega-\omega_2+{}^1/_2\omega_H|\alpha_0^{+1}|^2)}, \quad (20)$$

where $\omega_2=\omega_H+\alpha^2\omega_H/2$.

The component of the absorption coefficient (20) in the region of small frequency detunings

$$0<\omega-\omega_2\ll{}^1/_2\omega_H|\alpha_0^{+1}|^2\sim{}^1/_2\alpha^2\omega_H(a/l)^4 \quad (21)$$

is proportional to $\sim(\omega-\omega_2)^{1/2}\to 0$, while in the region of large frequency detunings

$$\omega-\omega_2\gg{}^1/_2\omega_H|\alpha_0^{+1}|^2 \quad (22)$$

it has a square-root decrease

$$\eta\sim(\omega-\omega_2)^{-1/2}. \quad (23)$$

The maximum of (20) occurs at the frequency

$$\omega_{max}=\omega_H+{}^1/_2\omega_H(\alpha^2+|\alpha_0^{+1}|^2) \quad (24)$$

and is equal to

$$\eta_{max}^{b\to c}=\frac{4\pi^2}{m^*}\alpha^{-2}\left(\frac{l}{a}\right)^2\frac{n_e'}{\omega_H}e^2. \quad (25)$$

4. ABSORPTION IN $c \to b$ AND $c \to c$ TRANSITIONS

The probability that an electric field $\mathbf{E}_0\perp\mathbf{H}$ can cause an electron transition from the continuum state of a lower Landau band ($n=0, m\leq 0, p$) into bound states of the Landau band ($n=1, m+1$) and ($n=0, m=+1$) is determined by the matrix element (18) taken over the wave functions of the indicated states. Transitions with decrease of $m(m\to m-1)$ are of higher order in α and will not be considered. Simple calculation yields for the probability of the allowed transitions per unit time

$$w_{(m\to m+1,\ n=0\to n=1)}^{c\to b}=\alpha_i^{m+1}l^{-3}(4\pi)^{-1}\left(\frac{eE_0l}{\hbar}\right)^2\frac{p^2}{\hbar^3}$$
$$\times\frac{(\alpha_0^m-\alpha_i^{m+1})^2}{[|\alpha_i^{m+1}|^2l^{-2}+p^2/\hbar^2]^2}\frac{2\pi\hbar L^{-1}}{[|\alpha_0^m|^2l^{-2}+p^2/\hbar^2]}\frac{\Delta_m}{(\omega-\omega_m+p^2/2m^*\hbar)^2+\Delta_m^2}. \quad (26)$$

Here

$$\Delta_m=2^{-1/2}\omega_H\alpha_1^m|\alpha_{10}^m|^2, \quad \omega_m=\omega_H(1-{}^1/_2|\alpha_1^{m+1}|^2), \quad (27)$$

and $L=V^{1/3}$, where V is the volume.

The transition easiest to resolve experimentally ($a/l\ll 1$) corresponds to an initial state $m=-1$:

$$w_{-1\to 0}^{c\to b}=\alpha^3\frac{l}{L}\left(\frac{eE_0l}{\hbar}\right)^2\frac{(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)}{[(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)+{}^1/_2|\alpha_0^{-1}|^2\omega_H]}$$
$$\times\frac{\omega_H^2}{(\omega_H-\omega)^2}\delta\left[\omega-\omega_H+{}^1/_2\alpha^2\omega_H+\frac{p^2}{2m^*\hbar}\right]. \quad (28)$$

Equation (28) describes the electron transition probability per unit time when the electron interacts with a single atom located at the center of a Larmor circle of area πl^2. The number of atoms falling into this area is $n_0L\pi l^2$, (n_0 is the concentration of the atoms). Therefore the total probability of electron transition form the continuum state ($n=0, m=-1, p$) to the bound state ($n=1, m=0$) is obtained by multiplying (28) by $n_0L\pi l^2$.

We can next calculate the absorption-coefficient component due to $c\to b$ transitions. To this end we must know the form of the equilibrium distribution function of the electrons. For the case of free electrons and arbitrary statistics, this function was obtained by Zil'berman[10] and for Boltzmann statistics by Levinson.[11] At sufficiently low impurity concentration, $n_0l^3\ll 1$, it is legitimate to use the functions from Refs. 10 and 11.

In the case of Boltzmann statistics, the electron longitudinal-energy distribution takes the form[11]

$$f\left(\frac{p^2}{2m^*}\right)=\frac{n_e}{(2\pi m^*T)^{1/2}}\exp\left(-\frac{p^2}{2m^*T}\right) \quad (29)$$

and the absorption coefficient is given by

$$\eta^{c\to b}(\omega)=2\pi^{1/2}n_0l^3\alpha^3\frac{e^2l^2}{\hbar^{1/2}}\frac{n_e}{T^{1/2}}\frac{\omega\omega_H^2}{(\omega_H-\omega)^2}\theta\left[\omega_H-\frac{1}{2}\alpha^2\omega_H-\omega\right]\cdot$$
$$\times\frac{(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)^{1/2}}{[(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)+{}^1/_2|\alpha_0^{-1}|^2\omega_H]}\exp\left[-\frac{\hbar}{T}\left(\omega_H-\frac{\alpha^2}{2}\omega_H-\omega\right)\right]. \quad (30)$$

In the region of small frequency detunings

$$0<\omega_H-{}^1/_2\alpha^2\omega_H-\omega\ll{}^1/_2|\alpha_0^{-1}|^2\omega_H\ll T/\hbar \quad (31)$$

Eq. (30) has a square-root dependence on the frequency detuning:

$$\eta^{c\to b}\sim(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)^{1/2}. \quad (32)$$

In the intermediate region

$${}^1/_2|\alpha_0^{-1}|^2\omega_H\ll\omega_H-{}^1/_2\alpha^2\omega_H-\omega\ll T/\hbar \quad (33)$$

the absorption coefficient decreases in square-root fashion

$$\eta^{c\to b}\sim(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)^{-1/2}, \quad (34)$$

and finally, in the frequency region

$$T/\hbar\ll\omega_H-\frac{\alpha^2}{2}\omega_H-\omega\ll\omega_H \quad (35)$$

the square-root dependence (34) gives way to the exponential one

$$\eta^{c\to b}\sim\exp\left[-\frac{\hbar}{T}(\omega_H-{}^1/_2\alpha^2\omega_H-\omega)\right]. \quad (36)$$

The probabilities of transitions of the fourth type $c\to c$, allowed, in the lowest order in α, between the states ($n=0, m=0, p_z\to n=0, m=+1, p$) and ($n=0, m<0, p_z\to n=1, m+1, p$), are given respectively by

$$w_{0\to+1}^{c\to c}=\frac{\pi}{2}\left(\frac{eE_0l}{\hbar}\right)^2\delta(\Delta\omega)\frac{(\alpha\alpha_0^{+1}l^{-2}+p_z^2/\hbar^2)^2}{(\alpha^2l^{-2}+p_z^2/\hbar^2)(|\alpha_0^{+1}|^2l^{-2}+p_z^2/\hbar^2)}, \quad (37)$$

$$w_{m\to m+1}^{c\to c}=\frac{\pi}{2}\left(\frac{eE_0l}{\hbar}\right)^2\delta(\Delta\omega)\frac{(\alpha_0^m\alpha_1^{m+1}l^{-2}+p_z^2/\hbar^2)^2}{[|\alpha_0^m|^2l^{-2}+p_z^2/\hbar^2][|\alpha_0^{m+1}|^2l^{-2}+p_z^2/\hbar^2]}. \quad (38)$$

It follows from (37) and (38) that scattering by an individual short-range potential does not influence the absorption-line contours in electron transitions in the continuum—just as in the case of CR on free electrons, it has a δ-function dependence on the frequency detun-

ing. Therefore the result obtained by Gurvich[12] in the Born approximation, namely that there is no inhomogeneous broadening of the CR absorption curve when an electron is scattered by a short-range potential, remains valid also in the exact solution of the scattering problem.

To obtain the component of the absorption coefficient in the $c \to c$ transitions it is necessary to sum (37) and (38) over all m. For a short-range potential we have in this case $\alpha^m \sim (a/l)^{2|m|} \ll 1$, and we arrive at an expression that differs insignificantly from the usual one (see, e.g., Ref. 12).

5. ABSORPTION IN PARALLEL FIELDS

We consider now the absorption of the electromagnetic field by an electron in parallel ($\mathbf{E}_0 \| \mathbf{H}$) fields. The Hamiltonian of the interaction of the electron with the field takes in this case the form

$$V_F(\mathbf{r}, t) = -eE_0 z \sin \omega t. \tag{39}$$

In the absence of a center, there is no absorption due to transitions of the electron between states of different Landau bands, in view of the orthogonality of the radial wave functions

$$\langle R_{km} | R_{nm} \rangle = \delta_{kn}.$$

The interaction of the electron with the center leads, first, to the appearance of a bound-state spectrum below the bottom of any Landau band, and second, to a mixing of states of different Landau bands having the same value of m. By virtue of these circumstances, two types of absorption are possible in parallel fields: low-frequency intraband absorption at a frequency $\omega \approx \frac{1}{2}\alpha^2\omega_H \ll \omega_H$ and interband absorption at a frequency $\omega \sim \omega_H$.

The intraband absorption at $T=0$ is due to electron transitions between states of the lower Landau band ($n=0, m=0 \to n=0, m=0, p$), and the contribution made by such transitions to the absorption coefficient is

$$\eta_{\parallel}^{\text{intra}}(\omega) = 2^{1/2}\pi n_e' \alpha^3 \frac{e^2 l^2}{\hbar} \frac{\omega_H^{5/2}}{\omega^3} \left(\omega - \frac{1}{2}\alpha^2\omega_H\right)^{1/2} \theta\left(\omega - \frac{1}{2}\alpha^2\omega_H\right). \tag{40}$$

The absorption coefficient $\eta_{\parallel}^{\text{intra}}$ has a maximum at the frequency

$$\omega_{max} = {}^3/_4 \alpha^2 \omega_H, \tag{41}$$

equal to

$$\eta_{\parallel}^{max} = 2^{1/2}\pi \left(\frac{4}{3}\right)^3 \frac{n_e' e^2 l^2}{\alpha^2 \hbar}. \tag{42}$$

At high frequencies

$$\omega \gg \omega_{max} \tag{43}$$

the absorption coefficient decreases in proportion to $\omega^{-5/2}$:

$$\eta^{\text{intra}}(\omega \gg \omega_{max}) \approx 2^{1/2}\pi n_e' \alpha^3 \frac{e^2 l^2}{\hbar} \left(\frac{\omega_H}{\omega}\right)^{5/2}. \tag{44}$$

while at low frequencies

$$\omega < {}^3/_4 \alpha^2 \omega_H \tag{45}$$

the coefficient tends to zero in proportion to $\sim(\omega - \alpha^2\omega_H/2)^{1/2}$.

Finally, for the interband transition ($n=m=0 \to 1$, $m=0, p$) the absorption coefficient in parallel fields is given by

$$\eta_{\parallel}^{\text{inter}}(\omega) = 2^{1/2}\pi \frac{n_e' e^2}{m^*} \frac{\omega_H^{3/2}}{\omega^3} |\alpha_{01}|^2 \alpha \left(\omega - \omega_H - \frac{\alpha^2}{2}\omega_H\right)^{1/2} \theta\left(\omega - \omega_H - \frac{\alpha^2}{2}\omega_H\right). \tag{46}$$

At low frequencies

$$0 < \omega - \omega_H - \alpha^2\omega_H/2 \ll \omega_H$$

we have for the absorption coefficient

$$\eta_{\parallel}^{\text{inter}}(\omega) \sim (\omega - \omega_H - \alpha^2\omega_H/2)^{1/2}. \tag{47}$$

The maximum of the absorption coefficient $\eta_{\parallel}^{\text{inter}}$ lies in the frequency region $\omega \approx (\frac{6}{5})\omega_H$.

6. DISCUSSION OF RESULTS

The effects considered here are realized in semiconductors such as Si and Ge when an electron interacts with hydrogenlike neutral impurities. However, the possibility of observing atomic CR on bound states of an electron in the field of such impurities is limited by a number of conditions.

First, the produced bound-state spectrum becomes smeared by the interaction of the electron with the phonons as well as with the surrounding neutral and ionized impurities. At low temperatures, the principal role is played by the interaction with the acoustic phonons. The characteristic widths of the electron-phonon interaction are of the order of $\hbar\gamma \sim 6.3\times10^{-6}$ eV. The ionization energy of hydrogenlike neutral impurities in Ge is of the order of 10^{-2} eV. Similar neutral impurities can capture electrons to form type-d^- impurities. The binding energy of the captured electron is 1/20 of the ionization energy of the neutral-impurity electron. The characteristic radius of such impurities if of the order of $a \approx 0.5\times10^{-6}$ cm. Therefore at $H = 4\times10^4$ G the dimensionless interaction constant is $\alpha = 0.5$ and the magnetic length is $l = 10^{-6}$ cm. The ground energy level (in the Landau band N) with $m=0$ lies $E_{\parallel}^{m=0} = -2.5\times10^{-4}$ eV below the bottom of the band, and the levels with $m \neq 0$ are lower by $E_{\parallel}^{m} = -2.5\times10^{-4}$ eV. Comparison of the obtained values of $E_{\parallel}^{m}$ and $\hbar\gamma$ yields $E_{\parallel}^{m=0}/\hbar\gamma = 40$, $E_{\parallel}^{m=1}/\hbar\gamma = 2.5$, $E_{\parallel}^{m=-2}/\hbar\gamma = 0.16$, i.e., one should expect the levels with $m=0$ and $m=\pm1$ to be resolvable.

Second, the powerful peak of the CR (on the free electrons) has a half-width of order $\hbar\gamma$, as a result of which the observation of absorption, with a maximum at frequencies of the order of $\omega_H - \alpha^2\omega_H/2$, is made difficult for the $c \to b$ transitions [see (30)] by the presence of an absorption wing for $c \to c$ transitions. They can be resolved if the maximum of the absorption in the $c \to b$ transition is large compared with the height of the absorption curve in the $c \to c$ transition on the wing at the very same frequency; this takes place if the following inequality holds:

$$n l^3 \gg \frac{1}{2^{1/2}\pi^{3/2}} \frac{\hbar^{1/2}\gamma(\omega_H T)^{1/2}}{m^* \alpha^2 l^2 \omega_H^3} \left(\frac{a}{l}\right)^2 \tag{48}$$

At temperatures $T \approx 4$ K and at the assumed values of the parameters, a numerical estimates yields $n_0 \gg 10^{14}$ cm^{-3}. In real situations, the requirement (48) becomes somewhat more stringent because the $c \to b$ absorption

curve is also smeared by the electron phonon interaction. An estimate shows that allowance for this interaction worsens (48) by at most a factor of two.

The broadening of the absorption of the absorption lines on account of the interaction of the electron with neutral and ionized impurities can be estimated by using the appropriate expressions obtained in Refs. 12 and 13 for the relaxation times. In our case this broadening is small compared with the phonon broadening up to an impurity concentration on the order of 10^{16} cm^{-3}.

Third, the intensity of the absorption on bound state is proportional to the concentration of the electrons captured by neutral impurities. To facilitate the observation of atomic CR by such impurities it is expedient to use weakky compensated semiconductors, for which there exists a temperature region in which the donors have not yet been ionized, but the electrons are already uniformly distributed over the impurity band[3,14]; this increases the number of electrons captured by the neutral impurities.

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[1)] It is assumed that there are no bound states in the potential $U(\vec{r})$ in the absence of a magnetic field.

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Dispersion law and structural transitions in crystalline films

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An investigation has made it possible to explain the existence of an experimentally observed minimum of the resistance in size-quantizing films. Methods are proposed that permit investigations of the Fermi-line geometry. The question of quantization of the acoustic spectrum is examined. It is shown that the speed of sound oscillates with changing film thickness.

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We investigate in this paper the question of the dispersion law in size-quantizing crystal films. It is known that size quantization can be observed in thin crystal film. This effect is highly sensitive to the film quality. The best conditions for its observation are realized in semimetal films, where a number of factors (such as the low electron density) cause the de Broglie wavelength to exceed greatly the atomic dimensions, and it is this which makes the surface in fact specular. This is why the size-quantization effect was in fact discovered first in semi-metallic Bi and Sb films. The observation of this effect in thin metallic films (of Sn, Al, Pb, Mg, Au, and Ag) is possible, although more complicated.

We confine ourselves hereafter to semimetallic films in which the size quantization effect is most pronounced. In size-quantizing films, the energy $\varepsilon(\varkappa,n)$ is determined by the longitudinal two-dimensional quasimomentum and by the transverse quantum number n. Instead of the Fermi surface we have a group of two-dimensional subbands. Strictly speaking, the momentum projection k_z perpendicular to the plane of the film is not defined. It can be approximately assumed, however, that the different subbands have different values of $k_z = \pi n/L$ (see, e.g., Ref. 1).

The most interesting situation occurs in the case when the concentration satisfies the condition $n \lesssim L^{-3}$. In this case, only the lowest subband is filled. The film, which remains a three-dimensional system in coordinate space ($L \gg a$, where a is the lattice period), becomes a two-dimensional system in momentum space. This situation is realized, for example, in Bi films at $L \lesssim 5 \times 10^2$ Å. The electrons are characterized in this case not by a Fermi surface, but by the Fermi line

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