tute either a case of a lower marginal dimensionality, or else their dimensionality is even lower than the original one. In these situations the Wilson approach yields practically nothing if it is necessary to resort to exactly solvable models.

In conclusion, it is our pleasant duty to thank Walker and Walsted for kindly supplying the results of their numerical experiment.

1) This term was apparently introduced by Anderson for the description of the situation in spin glasses (see, e.g., Ref. 2).

2) We use in this article vector symbols $\boldsymbol{\varphi}$, $\mathbf{b}_i$, $\boldsymbol{\rho}_i$, and others only for vectors in "isotopic" spin space. For the projections of these vectors we always use Greek superscripts: φ^α, b_i^γ, etc. Latin subscript always denotes Cartesian coordinate, $x_i \equiv x, y, z$. The fact that in our theory the Latin and Greek indices are never mixed means that the interaction is of the exchange type.

3) Dislocations actually generate disclinations in the spin system of not only the simplest two-sublattice antiferromagets, but also of many-sublattice magnets, such as UO_2 (see, e.g., Ref. 12).

4) Actually all the statements that follow hold also for any problem with random distribution of the impurities or with random bonds. For the sake of argument, however, we speak here of spin waves.

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Antiferromagnetic resonance in α-Fe_2O_3 in the absence of an external magnetic field

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Antiferromagnetic resonance was investigated experimentally in the temperature interval 150–320 K and in the wavelength range 4.5–1.5 mm. It is shown that the experimental results are described by two different formulas for $T < T_M = 260.9$ K and $T > T_M$. The experimental results are used to calculate the temperature dependences of the two uniaxial-anisotropy constants within the framework of the generally accepted premises concerning the magnetic properties of α-Fe_2O_3.

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Hematite (α-Fe_2O_3) is an antiferromagnet that can have a weak ferromagnetic moment because of the Dzyaloshinskiĭ interaction.[1] Even though hematite has attracted the attention of many investigators (see the review[2]), many of its important properties remain unclear to this day. In particular, we do not know the mechanism whereby anisotropy constant, as a function of temperature, acquires an anomalous behavior that leads to a phase transition from a easy axis state into an easy plane state. Nor can we explain the extremely small anisotropy constants (≈ 0.2 kOe) that follow from the prevailing theoretical premises concerning the magnetic properties of α-Fe_2O_3.

From the dipole energy calculated by Arman *et al.*[3] it follows that the dipole field is approximately 9 kOe; according to data on the EPR of Fe^{3+} in α-Al_2O_3, the one-ion contribution is about 7 kOe. It must be emphasized that the magnetodipole and one-ion contributions are of the same sign.

According to the accepted premises,[2] the thermodynamic potential of α-Fe_2O_3 can be expressed in the form[1]

$$\Phi = 2M_0[{}^1/_2 E M^2 - {}^1/_2 A_1 L_z^2 - {}^1/_4 A_2 L_z^4 - D(M_x L_y - M_y L_x)], \qquad (1)$$

where $\mathbf{M} = (\mathbf{M}_1 + \mathbf{M}_2)/2M_0$, $\mathbf{L} = (\mathbf{M}_1 - \mathbf{M}_2)/2M_0$, $\mathbf{M}_1$ and $\mathbf{M}_2$ are the sublattice magnetizations, $\mathbf{M}_1^2 = \mathbf{M}_2^2 = M_0^2$. For homogeneous small oscillations of the magnetic system about the equilibrium value we can calculate the frequencies (see, e.g., Ref. 2) of the antiferromagnetic resonance (AFMR) for the low-temperature ($T < T_M = 261$ K, $L_z \neq 0$) and high-temperature ($T > T_M$, $L_z = 0$) states:

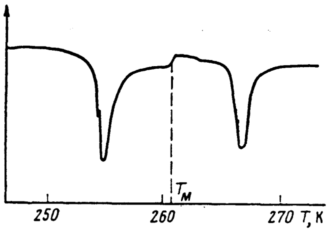

FIG. 1. AFMR absorption lines in α-Fe_2O_3 vs temperature: the electromagnetic-radiation wavelength is $\lambda = 3.02$ mm.

$$\omega_L/\gamma=[E(A_1+A_2)-D^2]^{1/2}, \quad L_z\neq 0, \tag{2}$$

$$\omega_H/\gamma=[-EA_1+D^2]^{1/2}, \quad L_z=0, \tag{3}$$

where E is the effective exchange field, A_1 and A_2 are the anisotropy fields, and D is the Dzyaloshinskiĭ field. For the case $L_2 = 0$ there is one more frequency, equal to zero in the absence of a magnetic field.

By now, AFMR in hematite has been investigated as a function of the magnetic field at room temperature, [2,8,7] and the frequencies corresponding to $H=0$ were obtained by extrapolating the function $\omega(H)$ to zero magnetic field. With such an extrapolation, however, the accuracy with which $\omega(0)$ was determined decreased because the extrapolation was from stronger fields. It was therefore of great interest to measure the temperature dependence of the AFMR frequency without an external magnetic field. Since the AFMR frequency of α-Fe_2O_3 can be varied by varying the temperature, it was possible to record the absorption lines at a fixed frequency by varying the temperature slowly (Fig. 1). The outer peaks are the absorption lines in the low-temperature and high-temperature states, respectively, while the central peak corresponds to the point T_M. The measurements were made at wavelengths from 4.5 to 1.5 mm in accordance with a precedure simlar to that reported before.[8] The presence of the peak at the center made it possible to record for each frequency the transition temperature, which turned out to be 260.9± 0.1 K for the investigated crystals. The temperature dependences of the frequencies ω_L and ω_H are shown in Fig. 2. The vertical line was drawn to agree with the peaks at the point T_M.

Nagai *et al.*, [11] who reported the results of a spin-

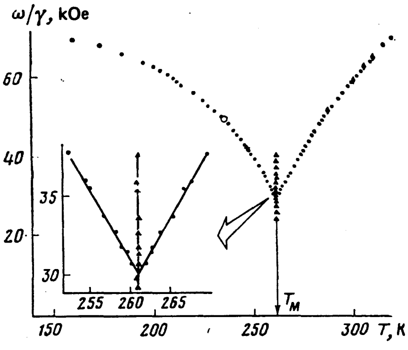

FIG. 2. Temperature dependence of the AFMR spectrum of hematite without an external magnetic field ($H=0$): ●, ▲) present data, ♦) data of Ref. 6, +) data of Ref. 9, ○) data of Ref. 10.

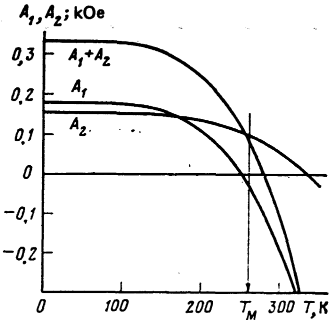

FIG. 3. Temperature dependences of the anisotropy constants A_1 and A_2 calculated from formulas (7) and (8), respectively.

wave calculation for a model Hamiltonian, obtained formulas, according to which the temperature dependences of the frequencies should take the following form:

$$\omega_L^2=\omega_{L0}^2[1-(T/T_L)^4], \tag{4}$$

$$\omega_H^2=\omega_{H0}^2[-1+(T/T_H)^4]. \tag{5}$$

A reduction of the experimental results by (4) and (5) led to the following characteristic frequencies and temperatures (at a g-factor equal to 2):

$$\omega_{L0}/\gamma=74.6\pm0.3 \text{ kOe} \quad T_L=271.7\pm0.2 \text{ K},$$

$$\omega_{H0}/\gamma=52.5\pm0.7 \text{ kOe} \quad T_H=242.6\pm0.7 \text{ K}.$$

The AFMA frequency at $T=4$ K, calculated by formula (4), agrees with the value measured by Roberts and Jacobs.[12] The possibilities of reducing the measurement results with other polynomials, of degree not higher than the fourth, were also verified, but the relative variance of the coefficient in all these cases was much larger than for the reduction of the frequency dependences by formulas (4) and (5). It should be noted that in the temperature interval 260–300 K the frequencies can be obtained by a linear approximation corresponding to expansion of the function (5) in a Taylor series:

$$\omega_H^2=4\omega_{H0}^2(T-T_H)/T_H, \tag{6}$$

but the accuracy of the linear approximation is not very high at temperatures 290-300 K and higher (at 300 K, the deviation of (6) from the experimental points is 30%).

Using the experimental results and formulas (1) and (2), as well as the fact that the Dzyaloshinskiĭ field remains practically unchanged in the investigated temperature interval, [13] we calculated the temperature dependences of the anisotropy constants A_1 and A_2 (Fig. 3), corresponding to the theoretical premises considered in the review of Jacobs *et al.*[2]:

$$A_1=0.182\,[1-(T/253)^4] \text{ kOe} \tag{7}$$

$$A_2=0.156\,[1-(T/334)^4] \text{ kOe} \tag{8}$$

In conclusion, the authors are deeply grateful to A. M. Prokhorov for constant interest and discussions.

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Spectral density of parametrically excited waves

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The spectral density of spin waves excited in ferrites in first-order parametric instability is investigated theoretically and experimentally. It turns out that simultaneous excitation of a large number of degrees of freedom can produce in each individual spin wave appreciable fluctuations (of the order of the amplitude itself) that lead to a substantial nonmonochromaticity of the parametrically excited spin waves. For the case investigated here (single-crystal samples of yttrium iron garnet, room temperature, pump frequency 9.37 GHz), the width of the spectral density of the parametrically excited spin waves is of the order of several kilohertz (at a wave damping decrement of several hundred kilohertz) and depends on the spin-wave damping parameter, on the spin wave vector, and on the supercriticality. The experimental relations are satisfactorily described by the nonlinear theory developed in the paper for parametric excitation of waves in media with a non-decaying dispersion law; these media can be either ferrites or many other physical objects.

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INTRODUCTION

Parametric excitation is the simplest method of generating waves of high amplitudes with wave vector $\mathbf{k}\neq 0$ in a solid. However, even in the first experiments on single-crystal ferrites it was noted that the oscillations of parametrically excited spin waves (PSW) are not monochromatic—their frequencies are distributed in a certain interval $\Delta\omega$ about $\omega_p/2$, where ω_p is the frequency of microwave magnetic pumping field. This has led to an increase of the noise temperature of the nondegenerate magnetostatic ferrite amplifier[1] and to parasitic modulation of the amplitude at the output of ferrite limiters.[2] These examples show that information on the frequency distribution of the PSW is quite essential for the design of ferrite devices in which spin waves are parametrically excited.

The presently existing nonlinear theory of parametric wave excitation[3,4] does not explain the observed phenomena and calls therefore for further development, all the more since effects that are analogous in many respects to parametric processes in ferrites have been observed and are presently studied in plasma, in ferroelectrics, in antiferromagnets, and in other nonlinear media.

We have investigated experimentally and theoretically the PSW frequency distribution $N(\omega)$:

$$N(\omega)=\int n_{\mathbf{k}\omega}\,d\mathbf{k},$$
$$\langle a_{\mathbf{k}\omega}a^{*}_{\mathbf{k}'\omega'}\rangle=n_{\mathbf{k}\omega}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'),$$
$$\langle a_{\mathbf{k}\omega}a^{*}_{\mathbf{k}'\omega'}\rangle=\sigma_{\mathbf{k}\omega}\delta(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega'-\omega_p): \qquad (1)$$

here $a_{\mathbf{k}\omega}$ is the Fourier component of the complex amplitude $a_{\mathbf{k}}(t)$ of a spin wave with wave vector $\mathbf{k}$.

The procedure for the measurement of $N(\omega)$ and the experimental results are presented in Sec. 1. The measurements were made by the parallel-pumping method at a frequency $\omega_p=2\pi\cdot 9.37$ GHz on single-crystal yttrium iron garnet (YIG) spheres having a PSW relaxation frequency $\gamma_k=g\Delta H_k/2\approx 1$ MHz. It was established that even in the absence of self-oscillations of the magnetization the width $\Delta\omega$ of the frequency spectrum $N(\omega)$ is of the order of several kilohertz and depends on the supercriticality, on the values of the PSW wave vectors, and on the parameter ΔH_k. The observed

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