spectrum occupies the range $0 \leq \omega \leq 10^{-8}N\ \mathrm{sec}^{-1}$. As the electron component cools, the intensity of the emitted radiation gradually rises and then falls; however, the width of the spectrum decreases monotonically, approaching $\omega = 0$. The application of a homogeneous magnetic field may shift the spectrum completely so that $\omega = 0$ coincides with the cyclotron frequency of the electrons and then all the other results (with the exception of some numerical factors) remain constant on condition that $\tau\omega \gg 1$.

We have ignored the exchange of energy between the electrons, which prevents inversion of the distribution function. In the case of argon at electron energies not too low compared with 1 eV, the criterion of validity of this approximation is $n/N < 10^{-6}$. In designing experiments, we have to bear in mind that an increase in the electron density n first increases the negative conductivity, in accordance with Eq. (1), but, when n is high, Eq. (2) is no longer valid and the effect disappears. The initial establishment of a distribution function $f(v)$ and the application of a probe field for the measurement of the conductivity should ensure the retention of a sufficient degree of spherical symmetry of the distribution function and, consequently, of the stability against plasma oscillations.

The author is grateful to É. I. Rashba for an extremely useful critique and encouraging interest in this investigation and is also very grateful to G. I. Gudzenko and V. I. Shaldyrvan for their help in the calculations.

[1]V. L. Ginzburg and A. V. Gurevich, Usp. Fiz. Nauk **70**, 201 (1960) [Sov. Phys. Usp. **3**, 115 (1960)].
[2]A. V. Rokhlenko, Zh. Eksp. Teor. Fiz. **69**, 169 (1975) [Sov. Phys. JETP **42**, 87 (1975)].
[3]V. L. Ginzburg, Usp. Fiz. Nauk **69**, 537 (1959) [Sov. Phys. Usp. **2**, 874 (1960)]; L. M. Kovrizhnykh and A. A. Rukhadze, Zh. Eksp. Teor. Fiz. **38**, 850 (1960) [Sov. Phys. JETP **11**, 615 (1960)].
[4]N. F. Mott and H. S. W. Massey, The Theory of Atomic Collisions, 3rd ed., Clarendon Press, Oxford, 1965 (Russ. Transl., Mir, M., 1969).

Translated by A. Tybulewicz

Thermal conductivity of pure vanadium in normal, superconducting, and mixed states

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(Submitted 13 January 1978)
Zh. Eksp. Teor. Fiz. **75**, 1320–1329 (October 1978)

The thermal conductivity of very pure vanadium ($\rho_{273}/\rho_{4.2} = 1570$) was investigated in the normal, superconducting, and mixed states. A satisfactory agreement was obtained betwen the experimental and theoretical values of the thermal conductivity in the superconducting state and the half-width of the energy gap $\Delta_0 = 9.5°\mathrm{K}$ was determined. The results obtained demonstrated that vanadium is a superconductor with a weak electron–phonon coupling. An investigation of the thermal conductivity in the mixed state yielded the critical magnetic fields H_{c1} and H_{c2}. A comparison was made of the theory and experiment and the upper limit of the effective electron-scattering width of Abrikosov filaments ($\sigma \lesssim 0.6\times 10^{-6}$ cm was determined. A study of the electrical resistivity in a magnetic field at various temperatures T made it possible to deduce the temperature dependences of the critical magnetic fields H_{c2} and H_{c3}.

PACS numbers: 74.30.Ek, 74.60.Ec, 74.70.Lp, 72.15.Eb

The physical essence of the superconducting state of a metal can be demonstrated by comparing the electrical and thermal conductivities above and below the superconducting transition temperature T_c. The characteristic relationship between these conductivities embodied in the Wiedemann–Franz relationship for the normal state of a metal changes greatly on transition to the superconducting state. The electrical conductivity rapidly tends to infinity on lowering of the temperature T below T_c but the thermal conductivity of a pure metal does not change abruptly but gradually decreases compared with that in the normal state, and in the limit $T \to 0$ it approaches a value typical of an insulator (which forms from the original metal by the gradual exclusion of electrons from the thermal balance of the crystal). This simple experimental picture of the comparative behavior of the electrical and thermal conductivities yields important conclusions on the physical nature of the superconducting state. Therefore, much experimental information has now been accumulated not only about the electrical conductivity but also about the thermal conductivity of superconducting pure metals and alloys.

The thermal conductivity of type II superconductors is of special interest because in this case it is possible to study not only the influence of the superconducting transition on the thermal conductivity but also on the

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little-studied interaction of heat carriers (electrons, phonons) with elementary magnetic flux quanta (Abrikosov filaments) in the mixed state of such a superconductor. A study of the electron scattering (in its pure form without phonon participation) by Abrikosov filaments is best carried out on the purest possible elemental type II superconductors of which there are two well-known examples: niobium and vanadium.

Investigations of niobium have been carried out sufficiently thoroughly and carefully[1] and they have shown that a deep understanding of the process of heat conduction in the mixed state requires further experimental and theoretical studies. The available experimental data on the thermal conductivity of vanadium at low temperatures are scarce and apply mainly to samples with low degrees of purity.[2] Preliminary results of an investigation of the temperature dependences of the thermal conductivity and electrical resistivity of very pure vanadium ($\rho_{273}/\rho_{4.2}=1570$) carried out in the temperature range 2–125°K were reported by us earlier.[3] Recently, Jung, Schmidt, and Danielson[4] published the results of their study of the thermal conductivity of high-purity ($\rho_{273}/\rho_{4.2}=1524$) vanadium in the temperature range 6–300°K and these agreed with ours at temperatures below 125°K. However, none of these investigations was carried out on the mixed state.

We decided to investigate a polycrystalline coarse-grained sample of vanadium with a variable diameter ranging from 1.1 to 1.5 mm (over a distance of 100 mm) characterized by a record purity for this metal: the resistivity ratio was $\rho_{273}/\rho_{4.2}=1570$ and the thermal conductivity K near $T_c=5.4$°K was two orders of magnitude higher than the conductivity of samples investigated earlier.[2] Vanadium with a higher resistivity ratio could not be obtained at present because of considerable difficulties encountered in the purification technology. Our sample was purified using the method of zone melting by an electron beam in a high vacuum followed by the electrotransport method in an argon atmosphere. More detailed information on the technology of deep purification, composition of the residual impurities, and further experimental data on the electrical properties of the vanadium sample investigated in the present study were reported earlier.[5]

The value of K was found by the method of steady-state heat flow through the investigated sample placed in a vacuum chamber. Use was made of carbon resistance thermometers (50 Ω, 1/8 W) in the range 2–30°K and copper–constantan thermocouples (0.1 mm in diameter) in the range 20–125°K; these instruments were first calibrated against the saturated vapor pressures of thermostatting liquids (helium, hydrogen, nitrogen, and oxygen) and against T_c of pure lead. In the liquid helium temperature range the carbon resistance thermometers were calibrated before each measurement made after heating to room temperature. The lower end of a sample was either soldered to the bottom of the vacuum chamber, which was immersed in a thermostatting liquid, or this end passed through an aperture in the bottom of the chamber and was in direct contact with the liquid. In the latter case it was possible to

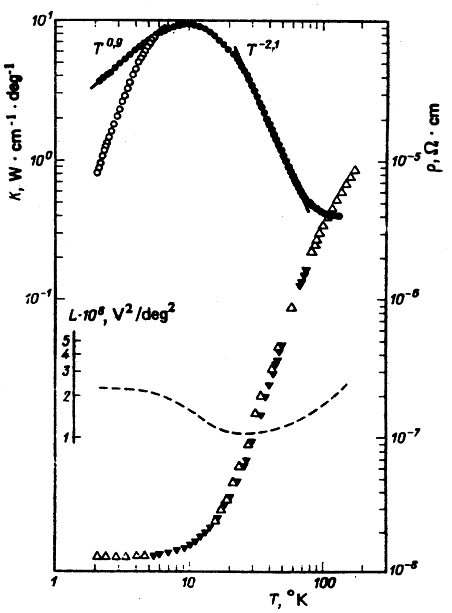

FIG. 1. Temperature dependences of the thermal conductivity K: ●) K_n; ○) K_s, and of the electrical resistivity ρ: △) our results; ▲) data of Aleksandrov *et al.*[5] The dashed curve is the Wiedemann-Franz relationship $L=K\rho/T$.

reduce the average temperature of the sample for a given temperature gradient. Heaters were attached to the lower and upper ends of the sample and thermometers was placed between the heaters. The upper heater created a temperature gradient and the lower one made it possible to increase the average temperature of the sample for a given gradient and to carry out control measurements of the thermal conductivity by the method of a single thermometer.[6] The temperature difference ΔT was 0.1–0.2°K at helium temperatures and up to 10°K at nitrogen or higher temperatures. The value of K was found from

$$Q=K\frac{\Delta T}{\Delta X}S,$$

where Q was the heat flux through the sample; the ratio of the cross-section area S to the distance between the thermometers ΔX was found from the electrical resistance R measured at 273°K and the known resistivity ρ ($\rho_{273}=19.9\times10^{-5}$ Ω·cm). The magnetic field intensity was calibrated and monitored by a Hall probe made of n-type InSb. The inhomogeneity of the magnetic field over the length of the sample did not exceed 2.5%.

Figure 1 shows the results of measurements of $\rho(T)$ and $K(T)$ and of calculations of the ratio $L=K\rho/T$ from the Wiedemann–Franz relationship in the range $T=2$–125°K. At $T<5.4$°K the vanadium was in the superconducting (s) state. Therefore, the values of K_n and ρ_n in the normal (n) state (at $T<5.4$°K) were determined by subjecting a sample to a longitudinal magnetic field H exceeding the second critical field H_{c2} in the determination of K_n and the third critical field H_{c3} in the determination of ρ_n. A further increase of H right up to twice the values of H_{c2} and H_{c3} had practically no effect on K_n and ρ_n. It is clear from Fig. 1 that K_s decreased more rapidly as a result of cooling than did K_n. In the case of K_n the temperature dependence was

$K \propto T^{0.9}$, which was practically identical with the dependence ($K_n \propto T$) expected for this range of temperatures because the heat carriers (normal electrons) were scattered by the impurities. On the other hand, the value of K_s obeyed the exponential dependence $K_s \propto \exp(-8.1/T)$ typical of the superconducting state and indicating the formation of a gap in the energy spectrum of the superconductor.

In the subsequent analysis of the experimental results, with the aim of comparing them with the theory, it was necessary to determine in advance whether not only electrons but also phonons participated in the transport of heat at $T < 5.4$°K, what was the heat carrier scattering mechanism, and which type of the electron–phonon coupling (weak or strong) was typical of the superconducting V. The answer to the first question was given by the graph of the Wiedemann–Franz relationship in Fig. 1. Since the Lorenz number L did not exceed the Sommerfeld value $L_0 = 2.4 \times 10^{-8}\,\mathrm{V^2/deg^2}$, we concluded that the phonon thermal conductivity was insignificant in the investigated range of temperatures. A further analysis of the results confirmed this hypothesis. The experimental temperature dependences $\rho_n(T) = \mathrm{const}$ and $K \propto T$ indicated that the electrons were scattered by impurities. It was interesting to note that at higher temperatures of 25–60°K, when the electrons were scattered by phonons, the temperature dependence was $K_n \propto T^{-2}$ in accordance with the Bloch–Wilson theory.

The type of coupling was determined by estimating the electron–phonon interaction constant

$$g = \left[\ln \frac{1.14\Theta}{T_c}\right]^{-1}$$

for V from the known values of $T_c = 5.4$°K and the Debye temperature $\Theta = 380$°K (Ref. 7). The value $g = 0.23$ obtained in this way allowed us to draw the preliminary conclusion that V was a weakly coupled superconductor.[8]

On the basis of the above factors we selected the Geĭlikman formula[9] for the dependence $K(T)$. This Geĭlikman formula (or the completely identical but written in a different mathematical form Bardeen–Rickayzen–Tewordt formula[10]) made it possible to use the experimental dependences $K_s(T)$ and $K_n(T)$ to determine the energy gap $2\Delta_0$ at $T = 0$:

$$\frac{K_s}{K_n} = \frac{6}{\pi^2}\left\{\frac{\Delta^2(T)}{T^2}\left[\exp\left(\frac{\Delta(T)}{T}\right)+1\right]^{-1} + 2\sum_{s=1}^{\infty}\frac{(-1)^{s+1}}{s^2}\exp\left(-\frac{s\Delta(T)}{T}\right) + 2\frac{\Delta(T)}{T}\ln\left[1+\exp\left(-\frac{\Delta(T)}{T}\right)\right]\right\}. \quad (1)$$

The dependence $\Delta(T)$ was based on the Mühlschlegel data[11] found by calculations carried out using the BCS theory (weak coupling). Figure 2 shows, in relative units, the theoretical curve corresponding to Eq. (1) for $\Delta_0 = 9.5 \pm 0.05$°K, which ensured a satisfactory agreement with the experimental results in the temperature range $t = T/T_c = 0.4$–0.9. This value of Δ_0 was in satisfactory agreement with the values for vanadium found from other independent experimental data: from the tunnel effect, attenuation of ultrasound, and absorp-

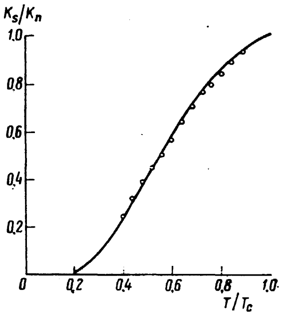

FIG. 2. Comparison of the theoretical curves for the K_s/K_n ratio (continuous line) with the experimental results (circles).

tion in the infrared range.[8] The preliminary conclusion made above on the weak coupling in vanadium was confirmed by an analysis of the thermal conductivities $K_n(T)$ and $K_s(T)$ because the experimental data satisfied Eq. (1) and also because the gap Δ_0 deduced from the experimental data obeyed the BCS relationship $\Delta_0 = 1.76\,T_c$ (Ref. 8).

The dependence of the thermal conductivity of vanadium on the magnetic field H is governed by the fact that a mixed (m) state appears in this metal between the fields H_{c1} and H_{c2} and it is characterized by the presence of a system of magnetic Abrikosov filaments penetrating a sample (approximately in the direction of an external magnetic field). Through each of these filaments passes one magnetic flux quantum $\Phi_0 = hc/2e = 2 \times 10^{-7}\,\mathrm{G \cdot cm^2}$. An increase in H increases the density of these magnetic filaments. The third critical field H_{c3} sets the limit of existence of the surface superconductivity in the specific case of a longitudinal (relative to the current) magnetic field. The surface superconductivity of a bulk sample has practically no effect on heat conduction.

Figure 3a shows the experimental results of an investigation of the influence of longitudinal ($\mathbf{H} \parallel \nabla T$) and transverse ($\mathbf{H} \perp \nabla T$) magnetic fields on the thermal conductivity of vanadium at four temperatures below T_c. The values of H_{c1} and H_{c2} are manifested clearly by these curves. We can see that in the Meissner range $H < H_{c1}$, when the magnetic field does not penetrate the metal, K_s is independent of H. In the $H > H_{c1}$ range, which corresponds to the mixed state, an increase in the field causes $K_m(H)$ to decrease first steeply because of the enhancement of the scattering of electrons by the magnetic filaments which permeate the superconductor in a density increasing with the field H; then, $K_m(H)$ passes through a minimum and begins to rise because of an increase in the mean free path l of the electrons since at high densities of magnetic filaments a further increase in the density makes the medium more homogeneous and also since the approach of H to H_{c2} increases the number of the normal electrons participating in the thermal balance. In the range $H > H_{c2}$, we find that $K = \mathrm{const}$. This behavior of the thermal conductivity can be explained by the fact that the effective field Hl is weak.

The filament pinning effect gives rise to a hysteresis of the field dependence of K_m, as is clear from Fig. 3b, but the hysteresis appears only to the left of the minimum of the dependence $K_m(H)$. This circumstance may

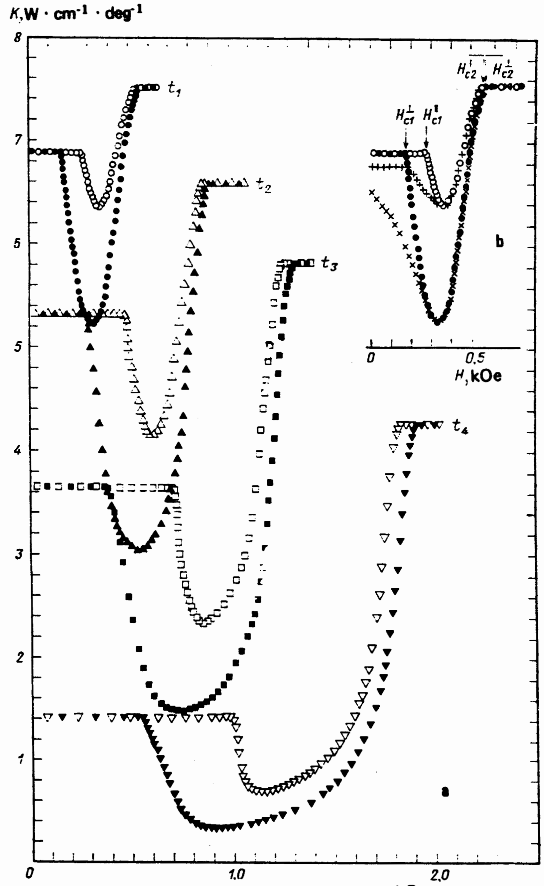

FIG. 3. a) Dependences of the thermal conductivity K on the magnetic field at four relative temperatures: $t_1=0.86$, $t_2=0.76$, $t_3=0.63$, and $t_4=0.45$. The open symbols for K correspond to $\mathbf{H}\parallel\nabla T$ and the black ones to $\mathbf{H}\perp\nabla T$. b) Dependence of the thermal conductivity on the magnetic field obtained by increasing H (○, ●) and reducing it (+, ×); ○), +) $\mathbf{H}\parallel\nabla T$; ●), ×) $\mathbf{H}\perp\nabla T$.

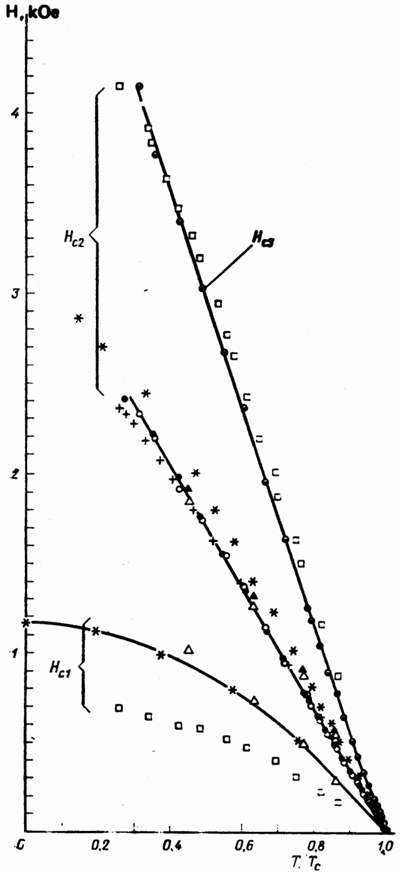

FIG. 4. Temperature dependences of the critical magnetic fields H_{c1}, H_{c2}, and H_{c3}: △), ▲) H_{ci} deduced from the dependences $K(H)$; ◒), ○), ●) H_{ci} deduced from the dependence $R(H)$; △), ○) $\mathbf{H}\parallel\nabla T$, $\mathbf{j}$; ▲), ●) $\mathbf{H}\perp\nabla T$, $\mathbf{j}$; *) results of Radebaugh and Keesom[12]; +) results of Sekula and Kernohan;[13] □) results of Usui *et al.*[20]

affect the value of H_{c1} deduced from the effective dependence $K_m(H)$ obtained after heating above T_c. The value of H_{c1} may be slightly higher than the equilibrium value in the absence of the filament pinning effect. It is clear from Fig. 4 that our experimental data on H_{c1} and H_{c2} agree, to within 10%, with the data of other investigators deduced from measurements of the specific heat[12] and magnetic properties.[13] The experimental observation that the difference between $H_{c1}^{\parallel}$ and $H_{c1}^{\perp}$ (at the same temperature) is not by a factor of 2, as expected after allowance for the demagnetization factor of a cylinder, but a factor of 1.6–1.8 (Fig. 3) requires further analysis. This behavior of the critical fields may be due to the nonparallel orientations of the magnetic field and current (in the $\mathbf{H}\parallel\nabla T$) case because of the variable diameter of the sample and also because of the filament pinning effect which can affect differently the values of $H_{c1}^{\parallel}$ and $H_{c1}^{\perp}$.

There is at present no general quantitative theory describing the dependence $K_m(H)$ throughout the range of fields corresponding to the mixed state of a type II superconductor, i.e., between H_{c1} and H_{c2}. However, there are various theories describing the behavior of $K_m(H)$ near H_{c1} and H_{c2}. The validity of each of these theories in any specific case is governed primarily by the ratio l/ξ, where ξ is the coherence length. In our case the value of l can be determined as follows. The diffusion coefficient is $D=1/3v_F l=(2e^2N\rho_0)^{-1}$, where v_F is the Fermi velocity and N is the density of states. Substituting in $l=3(2e^2N\rho_0 v_F)^{-1}$ the values known to apply to V [$N=5.7\times10^{34}$ erg$^{-1}\cdot$cm^{-3} (Ref. 14), $v_F=1.77\times10^7$ cm/sec (Ref. 12), and $\rho_0=\rho(T=0)=1.3\times10^{-8}\,\Omega\cdot$cm (Fig. 1)], we obtain $l=4.5\times10^{-4}$ cm. Since $\xi\approx4.5\times10^{-6}$ cm (Ref. 12), it follows that $l/\xi\approx100$ and, consequently, we are dealing with the "pure limit" case to which we can apply the Cleary theory[15] near H_{c1} and the Houghton–Maki theory[16] near H_{c2}.

According to the Cleary theory, the electrons in the mixed state near H_{c1} experience not only the impurity but also additional scattering by the relatively few Abrikosov magnetic filaments, and the ratio of K_s and K_m is governed by the characteristic parameter σ, which is the scattering width that can be estimated from the experimental data using the simple approximate Cleary formulas:

$$\left.\frac{K_s}{K_m}\right|^{\perp}=1+2\frac{l\sigma^{\perp}}{\Phi_0}(H-H_{c1}^{\perp}),$$
$$\left.\frac{K_s}{K_m}\right|^{\parallel}=1+\frac{l\sigma^{\parallel}}{\Phi_0}(H-H_{c1}^{\parallel}), \qquad (2)$$

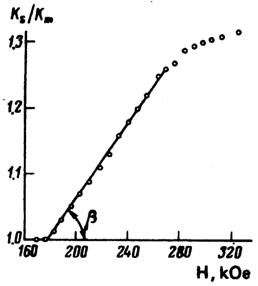

FIG. 5. Dependence of the thermal conductivity ratio K_s/K_m on the magnetic field (for $\mathbf{H}\perp\nabla T$) near H_{c1} at the relative temperature $t=T/T_c=0.86$.

where $\sigma=0$ for $H<H_{c1}$.

Figure 5 shows, by way of example, the dependence K_s/K_m on $\mathrm{H}\perp\nabla T$ near H_{c1}. The angle β can be used to determine σ. At the test temperature $t=T/T_c=0.86$, which is closest to the condition of validity of the Cleary theory $(t\to 1)$, it is found that $\sigma^{\perp}$ and $\sigma^{\parallel}$ have the same (within the limits of 10%) value of 0.6×10^{-6} cm. This value may be somewhat overestimated because of the filament pinning effect and the associated hysteresis but for reasons discussed in connection with Fig. 4 this overestimate should not exceed 10%.

In the Houghton–Maki theory which allows not only for the impurity scattering but also for the scattering of electrons by the spatial inhomogeneity of the order parameter Ψ, the dependence $K_m(H)$ near H_{c2} is governed by the dimensionless parameter

$$\mu=2\pi^{1/2}k_c l(\Psi/\hbar k_c v_F)^2,$$

where $k_c=(2eH/c\hbar)^{1/2}$ is the reciprocal lattice vector of the Abrikosov magnetic filaments and

$$\Psi^2=\frac{1}{2\pi N}\frac{H_{c2}-H}{1,16(2\varkappa_2^2-1)+D}\left(H_{c2}-\frac{t}{2}\frac{dH_{c2}}{dt}\right);$$

$\varkappa_2$ is the second generalized parameter of the Ginzburg–Landau theory introduced by Maki[17] and D is the demagnetization factor. When the conditions $\mu<1$ and $t\to 0$ are satisfied, the change in the thermal conductivity near $H_{c2}(H\leqslant H_{c2})$ for the transverse and longitudinal magnetic fields is described by the formulas

$$\frac{\Delta K^{\perp}}{K_n}=\frac{K_n-K_m}{K_n}\Big|^{\perp}=3\mu\left[\mu^2 I_1+\left(\frac{\pi}{4}-\mu\right)\right],$$

$$\frac{\Delta K^{\parallel}}{K_n}=\frac{K_n-K_m}{K_n}\Big|^{\parallel}=6\mu\left[(1-\mu^2)I_1-\left(\frac{\pi}{4}-\mu\right)\right], \qquad (3)$$

$$I_1=\int_0^{\pi/2}\frac{\cos\theta}{\cos\theta+\mu}d\theta,$$

where I_1 is a tabulated integral.

Taking the dependence $\varkappa_2(t)$ for vanadium from Ref. 12 and allowing for the influence of l on $\varkappa_2(t)$ by means of the Goodman relationship[18] (this correction is very small), we obtain theoretical estimates of $\Delta K^{\perp}/K_n$ and $\Delta K^{\parallel}/K_n$ at various temperatures. Figure 6 gives the ratios of the theoretical estimates to the experimental values of these quantities at different temperatures T. We can see that in the limit $t\to 0$, i.e., on approach to the condition of validity of the Houghton–Maki theory, these ratios tend to unity so that the values approach those predicted by this theory.

An investigation of the electrical resistivity of vanadium in a transverse magnetic field makes it possible

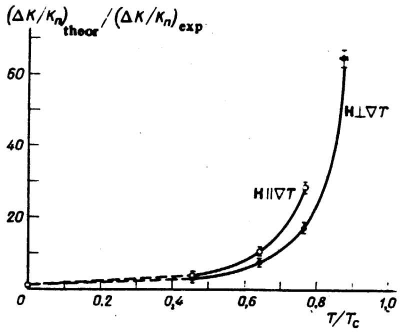

FIG. 6. Comparison of the experimental values of $\Delta K/K_n$ with calculations for different values of $t=T/T_c$. The calculations were carried out on the basis of the Houghton–Maki theory[16] assuming that $\mu=3$.

to find the upper critical fields H_{c2} and a study of the resistivity in a longitudinal magnetic field can yield the critical values H_{c2} and H_{c3}. The critical fields H_{c2} found by us from an investigation of the thermal conductivity and electrical resistivity are in good agreement (Fig. 4). The difference between the values of H_{c2} for $\mathbf{H}\parallel\nabla T$, $\mathbf{j}$ and $\mathbf{H}\perp\nabla T,\mathbf{j}$ can be explained by the fact that the crystal blocks in the investigated sample were relatively large (comparable with the diameter of the sample) and the value of H_{c2} for an anisotropic Fermi surface should be anisotropic even for a cubic crystal.[19] The third critical field H_{c3}, in which the surface superconductivity was destroyed, was found by us from the dependence $R(H)$ and it corresponded to complete recovery of the normal electrical resistance of the sample in a longitudinal magnetic field. Figure 4 includes also the temperature dependences of the critical magnetic fields obtained by other investigators from measurements of the specific heat and magnetic properties of samples of vanadium with a ratio of the room-temperature and helium-temperature resistivities amounting to 140 (Ref. 12), 436 (Ref. 13), and 18 (Ref. 20).

It is clear from the data in Fig. 4 that the upper critical field H_{c2} decreases on increase of the purity of the samples tending to a "lower" limit, whereas the lower critical field H_{c1} rises to an "upper" limit, in qualitative agreement with the Ginzburg–Landau,[21] Abrikosov,[22] and Gor'kov[23] theories, which have been used by de Gennes[8] and Maki[24] to obtain the expressions for $H_{c1}(T)$ and $H_{c2}(T)$:

$$H_{c1}(T)=\frac{H_c(T)}{\sqrt{2}\varkappa_3(T)}\ln[\varkappa_3(T)],$$

$$H_{c2}(T)=\sqrt{2}\varkappa_1(T)H_c(T), \qquad (4)$$

where $H_c(T)$ is the thermodynamic value of the critical magnetic field. The parameters $\varkappa_{1,3}(T)$ approach in the limit $T\to T_c$ the Ginzburg–Landau parameter $\varkappa=\lambda/\sqrt{2}\xi$, where λ is the London penetration depth and ξ is the coherence length. As shown earlier,[23,25,26] this parameter can be expressed in terms of the physical constants characterizing a sample: l (cm), which is the mean free path of electrons, ρ_0 ($\Omega\cdot$cm), which is the residual electrical resistivity, and γ (J$\cdot$cm$^{-3}\cdot$deg^{-1}) which is the coefficient of the linear term in the expression for the specific heat. We find that

$$\varkappa=0.96\lambda/\xi_0+2.38\rho_0\gamma^{1/2}=0.96\lambda/\xi_0+0.7\lambda/l, \quad (5)$$

where ξ_0 is the coherence length of a pure superconductor. The presence of two terms in Eq. (5) makes it possible to distinguish intrinsic (pure) type II superconductors ($l \gg \xi_0$) and "dirty" type II superconductors ($l \ll \xi_0$).

Although Eqs. (4) and (5) are simplified expressions (they are obtained in the one-band approximation), they still give a qualitative idea of the dependences of H_{c1} and H_{c2} on the degree of purity of a sample, i.e., on the mean free path of electrons, and they demonstrate the existence of the "upper" limit for H_{c1} and the "lower" limit for H_{c2}.

The authors are grateful to Yu. M. Gal'perin and É. B. Sonin for discussing the results obtained.

[1]T. Mamyia, A. Oota, and Y. Masuda, Solid State Commun. **15**, 1689 (1974); A. Oota, T. Mamyia, and Y. Masuda, J. Phys. Soc. Jpn. **38**, 1362 (1975).
[2]K. D. Chaudhuri, K. Mendelssohn, and M. W. Thompson, Cryogenics **1**, 47 (1960); A. Calverley, K. Mendelssohn, and P. M. Rowell, Cryogenics **2**, 26 (1961).
[3]B. K. Chakal'skiĭ, V. M. Azhazha, N. A. Red'ko, and S. S. Shalyt, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 513 (1976) [JETP Lett. **23**, 468 (1976)].
[4]W. D. Jung, F. S. Schmidt, and G. C. Danielson, Phys. Rev. B **15**, 659 (1977).
[5]B. N. Aleksandrov, E. D. Semenova, O. I. Petrova, B. P. Chernyĭ, and V. M. Azhazha, Fiz. Nizk. Temp. **1**, 388 (1975) [Sov. J. Low Temp. Phys. **1**, 194 (1975)].
[6]D. K. Wagner, J. C. Garland, and R. Bowers, Phys. Rev. B **3**, 3141 (1971).
[7]E. S. R. Gopal, Specific Heats at Low Temperatures, Heywood, London, 1966, p. 63.
[8]P. G. de Gennes, Superconductivity of Metals and Alloys, Benjamin, New York, 1966 (Russ. Transl., Mir, M., 1968, pp. 115, 126, 70).
[9]B. T. Geĭlikman, Zh. Eksp. Teor. Fiz. **34**, 1042 (1958) [Sov. Phys. JETP **7**, 721 (1958)].
[10]J. Bardeen, G. Rickayzen, and L. Tewordt, Phys. Rev. **113**, 982 (1959).
[11]B. Mühlschlegel, Z. Phys. **155**, 313 (1959).
[12]R. Radebaugh and P. H. Keesom, Phys. Rev. **149**, 217 (1966).
[13]S. T. Sekula and R. H. Kernohan, Phys. Rev. B **5**, 904 (1972).
[14]M. Ishikawa and L. E. Toth, Phys. Rev. B **3**, 1856 (1971).
[15]R. M. Cleary, Phys. Rev. B **1**, 169 (1970).
[16]A. Houghton and K. Maki, Phys. Rev. B **4**, 843 (1971).
[17]K. Maki, Physics (Long Island City, N. Y.) **1**, 21 (1964).
[18]B. B. Goodman, IBM J. Res. Dev. **6**, 63 (1962).
[19]K. Takanaka and T. Nagashima, Prog. Theor. Phys. **43**, 18 (1970).
[20]N. Usui, T. Ogasawara, K. Yasukochi, and S. Tomoda, J. Phys. Soc. Jpn. **27**, 574 (1969).
[21]V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. **20**, 1064 (1950).
[22]A. A. Abrikosov, J. Phys. Chem. Solids **2**, 199 (1957).
[23]L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **34**, 735 (1958); **36**, 1918 (1959); **37**, 833 (1959) [Sov. Phys. JETP **7**, 505 (1958); **9**, 1364 (1959); **10**, 593 (1960)].
[24]K. Maki, Phys. Rev. **148**, 362 (1966).
[25]B. B. Goodman, Rev. Mod. Phys. **36**, 12 (1964).
[26]T. G. Berlincourt, Rev. Mod. Phys. **36**, 19 (1964).

Translated by A. Tybulewicz

Temperature dependence of impurity conductivity of metals at low temperatures

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(Submitted 22 February 1978)
Zh. Eksp. Teor. Fiz. **75**, 1330–1341 (October 1978)

The kinetic properties of conduction electrons are considered at temperatures so low that the wavelength of the thermal phonon exceeds the electron mean free path for elastic collisions with the impurities. An important role is played in this case by the interference between the phonon and impurity mechanisms of impurities, so that the Mathiessen rule is violated. Since the impurities influence the electron-phonon interaction, the kinetic equation in its usual form is not valid. An equation analogous to the kinetic equation is derived and makes it possible to take the interference effects into account. It is used to study the temperature dependence of resistivity and the energy relaxation of the electrons.

PACS numbers: 72.10.Fk

1. INTRODUCTION

The resistivity of normal metals at sufficiently low temperature is determined by the scattering of the electrons by the impurities. However, the temperature dependence of the resistivity and the energy relaxation of the electrons, and some other important kinetic characteristics of the electron system, cannot be obtained without taking into account the scattering of the electrons by phonons. At sufficiently high temperature, the Matthiessen rule, wherein the impurity and phonon scattering are independent, is valid (see, e.g., Ref. 1).

At low temperatures, this rule is violated.[2] One of the mechanisms of its violation is considered in Ref. 3. It was noted there that if the characteristic momentum $\overline{q}$ transferred by the phonon to the electron becomes comparable with the quantity $1/l$ (l is the impurity mean free

 0038-5646/78/100670-06$02.40