

$$\alpha = 0.96\lambda/\xi_0 + 2.38\rho_0\gamma^2 = 0.96\lambda/\xi_0 + 0.7\lambda/l, \quad (5)$$

where  $\xi_0$  is the coherence length of a pure superconductor. The presence of two terms in Eq. (5) makes it possible to distinguish intrinsic (pure) type II superconductors ( $l \gg \xi_0$ ) and "dirty" type II superconductors ( $l \ll \xi_0$ ).

Although Eqs. (4) and (5) are simplified expressions (they are obtained in the one-band approximation), they still give a qualitative idea of the dependences of  $H_{c1}$  and  $H_{c2}$  on the degree of purity of a sample, i.e., on the mean free path of electrons, and they demonstrate the existence of the "upper" limit for  $H_{c1}$  and the "lower" limit for  $H_{c2}$ .

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## Temperature dependence of impurity conductivity of metals at low temperatures

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The kinetic properties of conduction electrons are considered at temperatures so low that the wavelength of the thermal phonon exceeds the electron mean free path for elastic collisions with the impurities. An important role is played in this case by the interference between the phonon and impurity mechanisms of impurities, so that the Matthiessen rule is violated. Since the impurities influence the electron-phonon interaction, the kinetic equation in its usual form is not valid. An equation analogous to the kinetic equation is derived and makes it possible to take the interference effects into account. It is used to study the temperature dependence of resistivity and the energy relaxation of the electrons.

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### 1. INTRODUCTION

The resistivity of normal metals at sufficiently low temperature is determined by the scattering of the electrons by the impurities. However, the temperature dependence of the resistivity and the energy relaxation of the electrons, and some other important kinetic characteristics of the electron system, cannot be obtained without taking into account the scattering of the elec-

trons by phonons. At sufficiently high temperature, the Matthiessen rule, wherein the impurity and phonon scattering are independent, is valid (see, e.g., Ref. 1).

At low temperatures, this rule is violated.<sup>2</sup> One of the mechanisms of its violation is considered in Ref. 3. It was noted there that if the characteristic momentum  $\bar{q}$  transferred by the phonon to the electron becomes comparable with the quantity  $1/l$  ( $l$  is the impurity mean free

path of the electron; Planck's constant is assumed equal to unity), then the character of the electron-phonon interaction changes: the phonon interacts with an electron that diffuses in the impurity field, and not with an electron that travels with Fermi velocity  $v_F$ . This changes the characteristic time  $\tau_{\text{int}}^{\text{ep}}$  of the electron-phonon interaction.<sup>4</sup> At  $\bar{q}l \gg 1$  the characteristic time is  $\tau_{\text{int}}^{\text{ep}} \sim 1/v_F \bar{q}$ , and at  $\bar{q}l \ll 1$  we have  $\tau_{\text{int}}^{\text{ep}} \sim 1/d\bar{q}^2$ , where  $d = 1/3v_F l$  is the electron diffusion coefficient. Since  $\bar{q} \sim T/w$ , where  $w$  is the speed of sound in the metal and  $T$  is the temperature, it follows that the temperature region in which this effect is significant is determined by the relation

$$T \leq T_1 = \frac{w}{l} \sim \frac{a}{l} \theta \sim \frac{w}{v_F \tau} \sim \frac{\theta}{\mu \tau}. \quad (1)$$

Here  $a$  is the lattice constant,  $\theta$  is the Debye temperature,  $\mu$  is the chemical potential, and  $\tau = l/v_F$  is the impurity lifetime of the electron.

With further decrease of the temperature  $\tau_{\text{int}}^{\text{ep}}$  becomes of the order of  $1/\omega \sim 1/T$ , where  $\omega$  is the phonon frequency. This takes place at

$$\frac{1}{\omega} \leq \frac{1}{d\bar{q}^2}, \quad T \leq T_2 = \frac{w^2}{d} = 3 \frac{w}{v_F} T_1 \sim 3 \frac{\theta^2}{\mu^2 \tau}. \quad (2)$$

An attempt was made in Ref. 3 to take this effect into account within the framework of an equation similar to the kinetic equation. However, some errors in that reference affected the result. Furthermore, no attempts were made to derive an equation with which to calculate any other quantity besides the resistivity.

In the present paper, an equation is derived, similar to the kinetic equation both in the form and in its capabilities, with account taken of the interference between the phonon and impurity scattering mechanisms. Just as in Ref. 3, besides satisfying the usual kinetic-equation criterion

$$\mu \tau \gg 1, \quad (3)$$

it is assumed also that the phonon contribution to the electron damping is small compared with the impurity contribution, and the metal is assumed for simplicity to be isotropic. No other assumptions are made with respect to the phonons, so that all the results can be easily generalized to include the case of any other long-wave scattering process. We derive the temperature dependence of the conductivity and the energy dependence of the departure time of the energy relaxation.

## 2. KINETIC EQUATION

To derive the kinetic equation we use the Keldysh diagram technique,<sup>5</sup> wherein the electron Green's function constitutes a matrix

$$\hat{G}(x_2, x_1) = \begin{pmatrix} 0 & G^a(x_2, x_1) \\ G^r(x_2, x_1) & F(x_2, x_1) \end{pmatrix}. \quad (4)$$

The matrix structure of the self-energy operator takes the form

$$\hat{\Sigma}(x_2, x_1) = \begin{pmatrix} \Omega(x_2, x_1) & \Sigma^r(x_2, x_1) \\ \Sigma^a(x_2, x_1) & 0 \end{pmatrix}. \quad (5)$$

The Dyson equation in the differential form can be written in two ways:

$$G_0^{-1}(x_2) \hat{G}(x_2, x_1) = \partial_x \left[ \delta^4(x_2 - x_1) + \int d^4 y \hat{\Sigma}(x_2 y) \hat{G}(y, x_1) \right], \quad (6a)$$

$$G_0^{-1}(x_1) \hat{G}(x_2, x_1) = \left[ \delta^4(x_2 - x_1) + \int d^4 y \hat{G}(x_2 y) \hat{\Sigma}(y, x_1) \right] \partial_x, \quad (6b)$$

where

$$x_2 = (t_2, r_2), \quad G_0^{-1}(x_2) = i \frac{\partial}{\partial t_2} - \varepsilon_F \left( -i \nabla_2 - \frac{e}{c} \mathbf{A}(x_2) \right) - e \Phi(x_2). \quad (7)$$

Here  $e$  is the electron charge,  $c$  is the speed of light, and  $\mathbf{A}$  and  $\Phi$  are the vector and scalar potentials of the external magnetic field, assumed to be classical.

The derivation presented in Ref. 5 for the kinetic equation consists in setting up the difference between Eqs. (6a) and (6b), making the change of variables

$$X = \frac{x_2 + x_1}{2} = (t, r), \quad x = x_2 - x_1 = (\theta, \rho) \quad (8)$$

and taking the Fourier transform with respect to the difference:

$$G(\varepsilon, \mathbf{p}, X) = \int d\theta d\rho G(x, X) \exp \left\{ i\theta [\varepsilon + e\Phi(X)] - i\rho \left[ \mathbf{p} + \frac{e}{c} \mathbf{A}(X) \right] \right\}. \quad (9)$$

Here  $\mathbf{p}$  and  $\varepsilon$  are the kinematic momentum and kinematic energy, in contrast to the canonical variables defined by the expressions in the square brackets.

As a result we obtain in the left-hand side of the equation the field operator that acts on the Green's function:

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v} \nabla_r + e \mathbf{v} \mathbf{E} \frac{\partial}{\partial \varepsilon} + \left( e \mathbf{E} + \frac{e}{c} [\mathbf{v} \times \mathbf{H}] \right) \nabla_p \right\} G(\varepsilon, \mathbf{p}, r, t) = \Pi G, \quad (10)$$

where  $\mathbf{v} = d\varepsilon_p/d\mathbf{p}$ , and  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic fields.

We note that the same procedure, applied to the sum of Eqs. (6), yields in the left-hand side

$$2(\varepsilon - \varepsilon_p) G. \quad (11)$$

As for the right-hand side of the kinetic equation, the quantities  $G$  and  $\Sigma$  are taken in Ref. 5 in the zeroth and first order of perturbation theory, respectively. This is not enough for our purposes. We confine ourselves to the lowest orders of perturbation theory in the electron-phonon interaction, but the scattering of the electrons by the impurities must be taken into account exactly, using only the condition (3).

To obtain the right-hand side of the equation, we consider an integral in the form

$$h(x_2, x_1) = \int d^4 y f(x_2, y) g(y, x_1). \quad (12)$$

It was easy to show that

$$h(\varepsilon, \mathbf{p}, X) = f(\varepsilon, \mathbf{p}, X) g(\varepsilon, \mathbf{p}, X) + i \{f, g\}. \quad (13)$$

Here  $\{f, g\}$  are the Poisson brackets of the quantities  $f$  and  $g$ :

$$\{f, g\} = \left( \frac{\partial f}{\partial \varepsilon} \frac{\partial g}{\partial t} - \frac{\partial g}{\partial \varepsilon} \frac{\partial f}{\partial t} \right) + \left( \frac{\partial f}{\partial \mathbf{p}} \frac{\partial g}{\partial \mathbf{r}} - \frac{\partial g}{\partial \mathbf{p}} \frac{\partial f}{\partial \mathbf{r}} \right), \quad (14)$$

where

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} - \frac{e}{c} \frac{\partial \mathbf{A}}{\partial t} \nabla_p - e \frac{\partial \Phi}{\partial t} \frac{\partial}{\partial \varepsilon}, \quad (15)$$

$$\frac{\partial}{\partial r_i} = \frac{\partial}{\partial r_i} - \frac{e}{c} \frac{\partial A_i}{\partial r_i} \frac{\partial}{\partial p_i} - e \frac{\partial \Phi}{\partial r_i} \frac{\partial}{\partial \varepsilon}.$$

It was easy to verify that  $\{f, g\}$  can be written in the

form

$$\{f, g\} = \frac{\partial f}{\partial \epsilon} \left\{ \frac{\partial}{\partial t} + v_r \nabla_r + \left( e \mathbf{E} v_r - \frac{\partial f / \partial t}{\partial f / \partial \epsilon} \right) \frac{\partial}{\partial \epsilon} \right. \\ \left. + \left( e \mathbf{E} + \frac{e}{c} [\mathbf{v} \times \mathbf{H}] + \frac{\partial f / \partial \mathbf{r}}{\partial f / \partial \epsilon} \right) \nabla_r \right\} g, \quad (16)$$

where

$$v_r = - \frac{\partial f / \partial p_r}{\partial f / \partial \epsilon}. \quad (17)$$

Thus,

$$\Pi G = \{ \epsilon - \epsilon_p, G \}. \quad (18)$$

Generally speaking, we have obtained six equations for three Green's functions. It is easy to show, however, the relations obtained from the sum and difference of the equations in (6) are equivalent. It is more convenient to obtain the equation for  $G^a$  or  $G^r$  from the sum:

$$(e - \epsilon_p) G^{a,r} = 1 + G^{a,r} \Sigma^{a,r}, \\ G^{a,r} = \frac{1}{\epsilon - \epsilon_p - \Sigma^{a,r}} = (G^{a,r})', \quad (19)$$

and the equation for  $F$  from the difference between (6b) and (6d):

$$i \{ \epsilon - \epsilon_p, F \} = \Omega (G^a - G^r) - F (\Sigma^a - \Sigma^r) + 1/2 i \{ \Omega, G^a + G^r \} + 1/2 i \{ \Sigma^a + \Sigma^r, F \}. \quad (20)$$

It is known that at equilibrium we have

$$F(\epsilon, \mathbf{p}) = s_0(\epsilon) (G^a - G^r), \quad s_0(\epsilon) = \text{th} \frac{\mu - \epsilon}{2T}. \quad (21)$$

Under disequilibrium conditions it is convenient to seek  $F$  in the form<sup>6</sup>

$$F(\epsilon, \mathbf{p}, X) = s(\epsilon, \mathbf{p}, X) (G^a - G^r) + 1/2 i \{ s, G^a + G^r \}, \quad (22)$$

which takes in the coordinate representation the natural form

$$F(x_0, x_1) = \int d^3 y [s(x_0, y) G^a(y, x_1) - G^r(x_0, y) s(y, x_1)]. \quad (23)$$

We now substitute (22) in (20). Using (19) and the properties of the Poisson brackets, we obtain

$$i \{ \epsilon - \epsilon_p, s \} (G^a - G^r) = [\Omega - s (\Sigma^a - \Sigma^r) + 1/2 i \{ \Sigma^a + \Sigma^r, s \}] (G^a - G^r) \\ + 1/2 i \{ \Omega - s (\Sigma^a - \Sigma^r), G^a + G^r \}. \quad (24)$$

The last term in the right-hand side of (24) should be neglected, since it can be written in the form of double Poisson brackets and constitutes therefore the allowance for the influence of the external field on the collisions.

We note also that in the calculations of the quantities  $\hat{\Sigma}$  it is impossible to use the usual diagram technique in the momentum representation: the diagrams for  $\hat{\Sigma}$ , which in the coordinate representation are combinations of integrals of the type (12), yield following the Fourier transformation, besides the usual terms, also increments in the form of Poisson brackets:  $\hat{\Sigma} = \hat{\Sigma}_0 + \delta \Sigma$ . Bearing this circumstance in mind, we write down the kinetic equation in the following final form

$$\left\{ \frac{\partial}{\partial t} + v_r \nabla_r + e \mathbf{E} v_r + \left( e \mathbf{E} + \frac{e}{c} [\mathbf{v} \times \mathbf{H}] \right) \nabla_r \right\} s = I(s) + \Delta(s), \quad (25)$$

where

$$I(s) = -i [\Omega_0 - s (\Sigma_0^a - \Sigma_0^r)], \quad (26)$$

$$\Delta(s) = -i [\delta \Omega - s (\delta \Sigma^a - \delta \Sigma^r)] + 1/2 i \{ \Sigma^a + \Sigma^r, s \}. \quad (27)$$

### 3. PHONON-IMPURITY INTERFERENCE

In the calculation of  $\hat{\Sigma}$  it must be borne in mind that the diagrams containing crossings of impurity lines are small relative to the parameter  $(\mu \tau)^{-1}$  (Ref. 7), and therefore

$$\hat{\Sigma}(p, \epsilon) = \overset{\curvearrowright}{\text{---}} + \overset{\curvearrowleft}{\text{---}} = \hat{\Sigma}_{im} + \hat{\Sigma}_{ph}. \quad (28)$$

The dashed line denotes here the impurity while the wavy lines stand for the phonon propagator

$$\text{---} = \hat{D} = \begin{pmatrix} 0 & D^a \\ D^r & D^k \end{pmatrix}. \quad (29)$$

A triangle denotes an exact electron-phonon vertex. We shall study it later on, and examine for the time being the impurity contribution to the collision integral  $I_{im}$ .

For simplicity we assume that the impurity center, which is located at the point  $\mathbf{r}_0$  produces a potential  $g_{im} \delta^3(\mathbf{r} - \mathbf{r}_0)$ . An impurity vertex corresponds to the matrix  $\sigma_x$ ,

$$\hat{\Sigma}_{im}(\epsilon) = N_i g_{im}^2 \int \frac{d^3 k}{(2\pi)^3} \sigma_x G(\mathbf{k}, \epsilon) \sigma_x, \quad (30)$$

where  $N_i$  is the impurity concentration. Substituting (22) and (30) in (26), we obtain

$$I_{im} = 2N_i g_{im}^2 \int \frac{d^3 k}{(2\pi)^3} [s(\mathbf{k}, \epsilon) - s(\mathbf{p}, \epsilon)] \text{Im} G^a(\mathbf{k}, \epsilon) \\ = \pi N_i g_{im}^2 \nu(\epsilon) [\bar{s}_0 - s(\mathbf{p}, \epsilon)]. \quad (31)$$

Here  $\nu(\epsilon)$  is the electron state density:

$$\nu(\epsilon) = \frac{2}{\pi} \int \frac{d^3 k}{(2\pi)^3} \text{Im} G^a(\mathbf{k}, \epsilon), \\ \bar{s}_0 = \frac{\int d^3 k \text{Im} G^a(\mathbf{k}, \epsilon) s(\mathbf{k}, \epsilon)}{4\pi^3 \nu(\epsilon)}. \quad (32)$$

If we disregard the phonon renormalizations of the electron propagators, then we obtain directly from (19) and (30)

$$G^a(\mathbf{p}, \epsilon) = G^r(\mathbf{p}, \epsilon) = \frac{1}{\epsilon - \epsilon_p - i/2\tau}, \quad (33)$$

$$\tau^{-1} = N_i g_{im}^2 \pi \nu(\epsilon), \quad \nu(\epsilon) = \frac{p^3}{\pi^2 v} \Big|_{\epsilon, \mu}. \quad (34)$$

The collision integral can be written in the form

$$I_{im}^0 = \frac{\bar{s}_0 - s(\mathbf{p}, \epsilon)}{\tau}. \quad (35)$$

We shall henceforth take  $\hat{G}$  to mean electron propagators not renormalized by phonons. Denoting by  $\delta_{ph} G$  the phonon increment to  $G$  and using (19), we rewrite (31) in the form

$$I_{im} = I_{im}^0 + 2N_i g_{im}^2 \int \frac{d^3 k}{(2\pi)^3} [s(\mathbf{k}, \epsilon) - s(\mathbf{p}, \epsilon)] \text{Im} \delta_{ph} G^a(\mathbf{k}, \epsilon), \quad (36) \\ \delta_{ph} G^a = G^a \Sigma_{ph}^a, \quad \Delta_{im} = 0.$$

Neither the phonon renormalization of the electron density of states, which is accounted for by the second term in the right-hand side of (36), nor the first term of the right-hand side of (27), was taken into account in Ref. 3.

We now examine the phonon part of the collision integral. As already noted above, the diagrams containing crossings of impurity lines are negligibly small. On the other hand, a diagram in which a phonon line crosses impurity lines is not small. The reason is the small-

ness of the characteristic momentum  $\bar{q}$  transferred by the phonon. In the expression for  $\hat{\Sigma}_{ph}$

$$\Sigma_{ph}^{ii} = i \int \frac{d^3q d\omega}{(2\pi)^4} D^i(\mathbf{q}\omega) [\Gamma_{i'j'}^i(-\mathbf{q}, -\omega, \mathbf{e} + \omega) G_{i'j'}(\mathbf{p} - \mathbf{q}, \mathbf{e} - \omega) \cdot \Gamma_{ij}^i(\mathbf{q}, \omega, \mathbf{e}) + 1/2 i \Gamma_{i'j'}^i \{G_{i'j'}(\mathbf{e}, \Gamma_{ij}^i) + 1/2 i \{ \Gamma_{i'j'}^i, G_{i'j'} \} \Gamma_{ij}^i \}] \quad (37)$$

we must therefore take  $\hat{\Gamma} = \hat{\Gamma}_0 + \delta\hat{\Gamma}$  to mean the electron-phonon vertex that is exact with respect to the impurities; this vertex can be determined from the equation

$$\Gamma = \Delta = \circ + \triangle$$

$$\hat{\Gamma}_0(\mathbf{q}, \omega, \mathbf{e}) = \hat{\gamma}(\mathbf{q}) + N_e g_{im}^2 \int \frac{d^3k}{(2\pi)^3} \delta_\epsilon \hat{G}(\mathbf{k} - \mathbf{q}, \mathbf{e} - \omega) \hat{\Gamma}_0(\mathbf{q}, \omega, \mathbf{e}) \hat{G}(\mathbf{k}, \mathbf{e}) \delta_\epsilon$$

$$\delta\hat{\Gamma}(\mathbf{q}, \omega, \mathbf{e}) = N_e g_{im}^2 \int \frac{d^3k}{(2\pi)^3} \delta_\epsilon \{ \hat{G}(\mathbf{k} - \mathbf{q}, \mathbf{e} - \omega) \delta\hat{\Gamma}(\mathbf{k}, \mathbf{e}) \} \quad (38)$$

$$+ 1/2 i \{ \hat{G}(\mathbf{k} - \mathbf{q}, \mathbf{e} - \omega), \hat{\Gamma}_0 \} \hat{G}(\mathbf{k}, \mathbf{e}) + 1/2 i \{ \hat{G}(\mathbf{k} - \mathbf{q}, \mathbf{e} - \omega) \hat{\Gamma}_0, \hat{G}(\mathbf{k}, \mathbf{e}) \} ]$$

Here  $\gamma_{ij}^i(\mathbf{q})$  is the unrenormalized electron-phonon vertex:

$$\gamma_{ij}^i = \frac{g(\mathbf{q})}{2^i} \delta_{ij}, \quad \gamma_{ij}^i = \frac{g(\mathbf{q})}{2^i} (\sigma_z)_{ij} \quad (39)$$

As usual, we assume that the phonon system is at equilibrium. In this case relations similar to (21) are valid and relate different phonon propagators:

$$D^r(\mathbf{q}, \omega) = [D^a(\mathbf{q}, \omega)]^* = D^a(-\mathbf{q}, -\omega), \quad (40)$$

$$D^a(\mathbf{q}, \omega) = (2N_e + 1) [D^r - D^a], \quad N_e = (e^{i\omega\tau} - 1)^{-1}$$

Leaving out the straight forward albeit long calculations, which consist of solving (38), substituting this solution in (37), and subsequently substituting  $\hat{\Sigma}_{ph}$  in (26), we write down the total phonon increment to the collision integral in the form

$$\delta_{ph} I = I^{el} + I^{inel} \quad (41)$$

Here  $I^{el}$  is the elastic part of the collision integral, which vanishes if  $s$  is an arbitrary function of the energy, and  $I^{inel}$  is the inelastic part, which vanishes only at  $s = s_0(\mathbf{e})$ :

$$I^{inel} = 8 \text{Im} \int \frac{d^3q d\omega}{(2\pi)^4} |g(\mathbf{q})|^2 \text{Im} D^a(\mathbf{q}\omega) G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) \times \left\{ R_{i,p}^{a,q} + \frac{N_e g_{im}^2}{1 - \zeta(\mathbf{q}, \omega, \mathbf{e})} \int \frac{d^3k}{(2\pi)^3} R_{i,k}^{a,q} G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}, \mathbf{e}) \right\} \quad (42)$$

Here

$$R_{i,p}^{a,q} = N_e n(\mathbf{e}, \mathbf{p}) [1 - n(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})] - (1 + N_e) [1 - n(\mathbf{e}, \mathbf{p})] \times n(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q}), \quad n(\mathbf{e}, \mathbf{p}) = (s(\mathbf{e}, \mathbf{p}) + 1)/2, \quad (43)$$

$$\zeta(\mathbf{q}, \omega, \mathbf{e}) = N_e g_{im}^2 \int \frac{d^3p}{(2\pi)^3} G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) \times G^a(\mathbf{p}, \mathbf{e}) = \frac{i}{2ql} \ln \frac{ql + \omega\tau + i}{-ql + \omega\tau + i} \quad (44)$$

We note that in the case  $ql \ll 1$  and  $\omega\tau \ll 1$  we have

$$[1 - \zeta(\mathbf{q}, \omega, \mathbf{e})]^{-1} = [\tau(-i\omega + dq^2)]^{-1} \quad (45)$$

We obtained, as expected, the usual diffusion pole. In the opposite limiting case  $ql \gg 1$  we have  $\zeta \sim (ql)^{-1}$ .

The first term in  $I^{inel}$  differs from the usual phonon collision integral only in the presence of damping in the electron Green's functions. The second term, on the other hand, can be regarded as a manifestation of the

scattering of a certain impurity-caused electronic state by the phonon.

In addition to  $\delta_{ph} I_{im}$ , the quantity  $I^{el}$  includes also a certain part of  $I_{ph}$ . Thus, the influence of the phonons on the elastic impurity scattering of the electrons manifests itself both via renormalization of the electron state density, and via renormalization of the impurity vertex by the phonon. Calculations show that

$$I^{el} = N_e g_{im} \text{Re} \int \frac{d^3q d\omega d^3k}{(2\pi)^7} |g(\mathbf{q})|^2 D^r(\mathbf{q}, \omega) \times G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}, \mathbf{e}) \left\{ [s(\mathbf{e}, \mathbf{p}) - s(\mathbf{e}, \mathbf{k})] \times \left[ \frac{G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) + G^a(\mathbf{k}, \mathbf{e})}{[1 - \zeta(\mathbf{q}, \omega, \mathbf{e})]^2} N_e g_{im}^2 \int \frac{d^3k_1}{(2\pi)^3} G^r(\mathbf{k}_1 + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}_1, \mathbf{e}) \right. \right. \\ \times s(\mathbf{e} + \omega, \mathbf{k}_1 + \mathbf{q}) + \frac{2G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) + G^a(\mathbf{k}, \mathbf{e})}{1 - \zeta} s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) \\ \left. \left. + 2i \text{Im} G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) \frac{s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) - s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})}{1 - \zeta} \right] \right. \\ \left. + N_e g_{im}^2 \int \frac{d^3k_1}{(2\pi)^3} G^a(\mathbf{k}_1 + \mathbf{q}, \mathbf{e} + \omega) G^r(\mathbf{k}_1, \mathbf{e}) [s(\mathbf{e}, \mathbf{k}) - s(\mathbf{e}, \mathbf{k}_1)] \right. \\ \left. \times [s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) - s(\mathbf{e} + \omega, \mathbf{k}_1 + \mathbf{q})] \frac{G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega)}{|1 - \zeta|^2} \right\} \quad (46)$$

For  $\Delta$  we can obtain

$$\Delta = \text{Im} \int \frac{d^3q d\omega}{(2\pi)^4} |g(\mathbf{q})|^2 D^r(\mathbf{q}, \omega) \left[ s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q}) \times \{ G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega), s(\mathbf{e}, \mathbf{p}) \} + \frac{N_e g_{im}^2}{1 - \zeta} \int \frac{d^3k}{(2\pi)^3} s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) \times \{ G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega), s(\mathbf{e}, \mathbf{p}) \} G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}, \mathbf{e}) - \left( \frac{N_e g_{im}^2}{1 - \zeta} \right)^2 \right. \\ \times \int \frac{d^3k d^3k_1}{(2\pi)^6} s(\mathbf{e}, \mathbf{p}) G^r(\mathbf{k}_1 + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}_1, \mathbf{e}) \\ \left. \times \{ (G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega), s(\mathbf{e} + \omega, \mathbf{k}_1 + \mathbf{q})) G^a(\mathbf{k}, \mathbf{e}) \} \right. \\ \left. + \{ G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) s(\mathbf{e} + \omega, \mathbf{k}_1 + \mathbf{q}), G^a(\mathbf{k}, \mathbf{e}) \} \right] \quad (47)$$

We write down also an expression for  $\Sigma_{ph}^a = (\Sigma_{ph}^a)_0 + \delta\Sigma_{ph}^a$ :

$$(\Sigma_{ph}^a)_0 = 1/2 i \int \frac{d^3q d\omega}{(2\pi)^4} |g(\mathbf{q})|^2 \left\{ G^a(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) D^a(\mathbf{q}, \omega) - \frac{D^r(\mathbf{q}, \omega)}{1 - \zeta} N_e g_{im}^2 \int \frac{d^3k}{(2\pi)^3} G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}, \mathbf{e}) \right. \\ \times \left[ G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) \left( \frac{s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q})}{1 - \zeta} + \frac{s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})}{\zeta} \right) \right. \\ \left. \left. + G^a(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega) \left( s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) - \frac{s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})}{\zeta} \right) \right] \right\} \quad (48)$$

$$\delta\Sigma_{ph}^a = \frac{N_e g_{im}^2}{4} \int \frac{d^3q d\omega d^3k}{(2\pi)^7} |g(\mathbf{q})|^2 \frac{D^r(\mathbf{q}, \omega)}{1 - \zeta} \times G^r(\mathbf{k} + \mathbf{q}, \mathbf{e} + \omega) G^a(\mathbf{k}, \mathbf{e}) \left[ \left\{ G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega), \frac{s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q})}{1 - \zeta} \right. \right. \\ \left. \left. + \frac{s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})}{\zeta} \right\} - \left\{ G^a(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega), s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}) - \frac{s(\mathbf{e} + \omega, \mathbf{p} + \mathbf{q})}{\zeta} \right\} \right] \\ + \frac{G^r(\mathbf{p} + \mathbf{q}, \mathbf{e} + \omega)}{(1 - \zeta)^2} N_e g_{im}^2 \int \frac{d^3k_1}{(2\pi)^3} \{ (G^r(\mathbf{k}_1 + \mathbf{q}, \mathbf{e} + \omega), s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q})) \\ \times G^a(\mathbf{k}_1, \mathbf{e}) + (G^r(\mathbf{k}_1 + \mathbf{q}, \mathbf{e} + \omega) s(\mathbf{e} + \omega, \mathbf{k} + \mathbf{q}), G^a(\mathbf{k}_1, \mathbf{e})) \} \quad (48')$$

Eq. (25) together with (35), (41), (42), (46), (47) describes completely the kinetic properties of the system. It can be verified that the conservation laws for the number of electrons and for the sum of the electron and phonon energies follow from this equation (an analogous equation holds true for phonons).

#### 4. ENERGY RELAXATION OF ELECTRONS

As the first example of the use of the derived kinetic equation, we consider the energy relaxation of the electrons. Assume no external field or spatial inhomogeneity. During a time on the order of  $\tau$ , momentum relaxation takes place, and the energy relaxation connected with the phonons will be much slower. Therefore, in the study of the latter, we can assume that

$$s(\epsilon, \mathbf{p}) = \bar{s}_\epsilon. \quad (49)$$

In this case  $J_{im}^0 = I^{el} = 0$ . The quantity  $\Delta$  need likewise not be taken into account, since we are considering first-order perturbation theory with respect to the phonons. For  $n_\epsilon = (\xi_\epsilon + 1)/2$  we obtain the equation

$$\frac{\partial n_\epsilon}{\partial t} = 2\tau \int \frac{d^3q d\omega}{(2\pi)^4} |g(q)|^2 \text{Re} \frac{\xi}{1-\xi} \text{Im} D^r(q, \omega) \times [N_{n_\epsilon}(1-n_{\epsilon+\omega}) - (1+N_\omega)(1-n_\epsilon)n_{\epsilon+\omega}]. \quad (50)$$

It is easily seen that the relaxation time of a state with energy  $\epsilon$  is determined by the integral

$$\frac{1}{\tau_\epsilon} = \tau \int \frac{q^2 dq}{\pi^2} |g(q)|^2 \int_{-\infty}^{\infty} d\omega \text{Re} \frac{\xi}{1-\xi} \text{Im} D^r(q, \omega). \quad (51)$$

If the Debye approximation is valid and there is no electron damping, then

$$D^r(q, \omega) = \frac{1}{\omega - \omega_q + i0} - \frac{1}{\omega + \omega_q + i0}, \quad \omega_q = \omega q. \quad (52)$$

We assume furthermore, as usual, that

$$|g(q)|^2 = \frac{\pi}{2} \alpha \omega_q \nu(\mu), \quad (53)$$

where  $\alpha$  is the dimensionless electron-phonon coupling constant. We can then show that

$$\frac{1}{\tau_\epsilon} = \begin{cases} \frac{16\alpha}{\pi} \frac{\mu}{\Theta} \frac{(e-\mu)^4}{T_2 \Theta^2}, & (e-\mu) \ll T_2, \\ \frac{3\alpha}{2\pi} \frac{T_1 (e-\mu)^2}{\Theta^2}, & T_2 \ll e-\mu \ll T_1, \\ \frac{\alpha}{3} \frac{(e-\mu)^2}{\Theta^2}, & (e-\mu) \gg T_1. \end{cases} \quad (54)$$

#### 5. CONDUCTIVITY

We now calculate the temperature-dependence corrections to the electric conductivity of the metal. The electric current is equal to

$$\mathbf{j} = 2e \int \mathbf{v} \tilde{G}^+(\mathbf{p}, \epsilon) \frac{d^3p d\epsilon}{(2\pi)^4} = 2e \int \mathbf{v} s(\epsilon, \mathbf{p}) \text{Im} \tilde{G}^+(\mathbf{p}, \epsilon) \frac{d^3p d\epsilon}{(2\pi)^4}. \quad (55)$$

The tilde over the Green's functions means here that they take into account the phonon renormalization. In the case of a constant electric field, in the approximation linear in  $\varphi(\epsilon, \mathbf{p}) = s(\epsilon, \mathbf{p}) - s_0(\epsilon)$ , the kinetic equation is of the form

$$eE\mathbf{v} \frac{\partial s_0(\epsilon)}{\partial \epsilon} = I\{\varphi\} + \Delta\{s_0\}. \quad (56)$$

In the absence of phonons

$$\varphi_0(\epsilon, \mathbf{p}) = -e\tau\mathbf{v}E \frac{\partial s_0}{\partial \epsilon}, \quad \mathbf{j}_0 = 2e \int \mathbf{v} \varphi_0(\epsilon, \mathbf{p}) \text{Im} G^+(\mathbf{p}, \epsilon) \frac{d^3p d\epsilon}{(2\pi)^4}. \quad (57)$$

In the lowest order of perturbation theory with respect to the phonons we have

$$\delta_{ph}\varphi(\epsilon, \mathbf{p}) = \tau[\delta_{ph}I\{\varphi_0\} + \Delta\{s_0\}], \quad (58)$$

$$\delta_{ph}\mathbf{j} = 2e \int \mathbf{v} [\delta_{ph}\varphi \text{Im} G^+(\mathbf{p}, \epsilon) + \varphi_0 \text{Im} \delta_{ph}G^+(\mathbf{p}, \epsilon) + s_0 \text{Im} \delta G^+(\mathbf{p}, \epsilon) + s_0 \text{Im} \delta_{ph}G^+(\mathbf{p}, \epsilon)] \frac{d^3p d\epsilon}{(2\pi)^4}. \quad (59)$$

The symbols  $\delta$  and  $\delta_{ph}$  denote, as before, the increments due to the Poisson brackets and the renormalizations by the phonons, while  $\delta_{ph}G^+$  is the nonequilibrium increment to  $G^+$  on account of the direct dependence of this Green's function on  $s$  (due to the dependence of  $\Sigma_{ph}^+$  on  $s$ ).

Calculations show that the ratio of the phonon increment to the conductivity and the impurity value of the conductivity can be written in the form

$$\frac{\delta_{ph}\sigma}{\sigma_0} = -\frac{1}{v_F^2} \text{Im} \int \frac{d^3q d\omega}{(2\pi)^4} j\left(\frac{\omega}{T}\right) D^r(q, \omega) J(q, \omega, \epsilon) + \frac{\delta_{ph}^{inc}\sigma}{\sigma_0}, \quad (60)$$

$$j\left(\frac{\omega}{T}\right) = \frac{1}{2} \int s_0(\epsilon+\omega) \frac{ds_0(\epsilon)}{d\epsilon} d\epsilon = N_+ - N_- + \frac{2\omega}{T} N_+ N_- \quad (61)$$

The second term in (60) is the contribution of the inelastic part of the collision integral, which we considered previously.

We shall not write down the long expression for the  $J(q, \omega, \epsilon)$ , which is valid for all  $q$  and  $\omega$ , and consider two limiting cases  $ql \ll 1$  and  $ql \gg 1$ . In the first case it is necessary to take into account only terms with the very lowest power of  $q$ , i.e., those which contain the diffusion pole  $(1-\xi)^{-1}$  to the highest degree—squared. It turns out that in the first term of (59) these terms have "extra"  $q^2$ , since they contain the factor

$$\int d\epsilon_s G^+(\mathbf{p}, \epsilon) G^r(\mathbf{p}, \epsilon) [G^s(\mathbf{p}, \epsilon) + G^r(\mathbf{p}, \epsilon)] = -i\tau \int d\epsilon_s [G^s(\mathbf{p}, \epsilon) - G^r(\mathbf{p}, \epsilon)],$$

which vanishes as a result of the analytic properties of the functions  $G^s$  and  $G^r$ . The second term of (59) is similarly cancelled by that part of the third term which is connected with the first term of (48). The main contribution to the conductivity is made by the remaining part of the third term of (69). At  $ql \ll 1$  and  $\omega\tau \ll 1$  we have

$$J(q, \omega, \epsilon) = \frac{|g(q)|^2}{(1-\xi)^2} \left(\frac{\partial \xi}{\partial q}\right)^2, \quad (62)$$

and at  $ql \gg 1$  we have

$$J(q, \omega, \epsilon) \sim l^2 \xi^2 |g(q)|^2. \quad (63)$$

We note immediately that since  $J(q, \omega, \epsilon)$  does not depend on temperature, and  $f(\omega/T)$  tends to unity as  $\omega \rightarrow \infty$ , the quantity  $\delta_{ph}\sigma/\sigma_0$  contains a temperature-independent part  $c_0$ . Formally, the integral with respect to  $q$  in (60) diverges at the upper limit. It is clear, however, that the integration with respect to  $q$  is carried out up to  $q_{max} \approx \pi/a$ . It turns out that  $c_0 \sim \Theta/\mu$ .

Substituting (52), (53), and (62) in (60) we obtain at  $T \ll T_1$

$$\frac{\delta_{ph}\sigma}{\sigma_0} = F\left(\frac{T}{T_1}\right), \quad (64)$$

$$F(x) = \frac{3\alpha}{16} \frac{T_1}{\mu^2 \tau} \text{Re} \int \frac{dy}{y} f(y) \frac{8-15(i/xy)^2 + 10(i/xy)^4 - 3(i/xy)^6}{(1-iyx)^3}.$$

At  $T \ll T_2$  we have

$$\frac{\delta_1 \sigma}{\sigma_0} = c_0 - \frac{3 \cdot 6^{1/2}}{8} \alpha c_1 \frac{T_1}{\mu^2 \tau} \left( \frac{T}{T_1} \right)^{1/2}, \quad (65)$$

$$c_1 = \int_0^{\infty} dx \frac{1-f(x)}{x^{3/2}} \approx 2.5,$$

and at  $T_2 \ll T \ll T_1$

$$\frac{\delta_1 \sigma}{\sigma_0} = c_0 - \frac{3 \alpha T_1}{2 \mu^2 \tau} \left( \text{const} + \ln \frac{T}{T_1} \right). \quad (66)$$

In the temperature region  $T \gg T_1$  the substitution of (63) in (60) yields

$$\frac{\delta_1 \sigma}{\sigma_0} - c_0 \sim \frac{T^2}{\mu \Theta}. \quad (67)$$

Thus, when the temperature changes from zero to  $T_2$ , the resistivity of the metal increases like  $T^{1/2}$ , and increases logarithmically up to  $T_1$ ; with further increase of temperature, the resistance increases like  $T^2$ .

The effects considered by us are best observed in beryllium, where  $\Theta \approx 1500^\circ\text{K}$  and  $\mu = 0.7 \text{ eV}$ .<sup>8</sup> Estimates based on formulas (64) and (65) yield  $\delta\sigma/\sigma_0 \sim 10^{-4} - 10^{-5}$  at  $\tau^{-1} \sim 100^\circ\text{K}$ .

We note that the considered increment to the resistivity, which represents the influence of the interaction with the virtual phonon on the impurity scattering can be estimated from the formula

$$\delta_1 \sigma / \sigma_0 \sim \tau_{\text{int}}^{\text{ep}} / \tau_{\text{ph}}. \quad (68)$$

Here  $\tau_{\text{ph}}$  is the characteristic phonon emission time

$$\tau_{\text{ph}}^{-1} \sim |g(\bar{q})|^2 \tau_{\text{int}}^{\text{ep}} \bar{\omega} \bar{D} \bar{q}^2.$$

At  $T \ll T_2$  we have  $\tau_{\text{int}}^{\text{ep}} \sim 1/\omega \sim 1/T$ ; inasmuch as the main contribution to the integration with respect to  $q$  in (60) is made by the region near the diffusion pole  $\bar{q} \sim (T/d)^{1/2}$ , it follows that  $\bar{D} \sim (\omega \bar{q})^{-1}$  and  $\tau_{\text{ph}}^{-1} \sim \alpha T^{3/2} / \mu^2 \tau^{3/2}$ . In the two other cases the main contribution is made by the region near the pole of the phonon Green's function  $D$ , namely  $\bar{q} \sim T/w$ , in which case  $\bar{\omega} \bar{D} = 1$  and  $\tau_{\text{ph}} \sim \tau_e [(\varepsilon - \mu) \sim T]$ .

We consider now the contribution of the inelastic part of the collision integral to  $\delta_{\text{ph}} \sigma$ . It is easy to show that only the first term in (42) contributes to the conductivity. Calculations show that

$$\frac{\delta_{\text{ph}}^{\text{inel}} \sigma}{\sigma_0} \sim \begin{cases} \alpha \tau \frac{T^2}{\Theta^2}, & T \gg T_1, \\ \alpha \tau^2 \mu \frac{T^2}{\Theta^2}, & T \ll T_1, \end{cases} \quad (69)$$

This is natural, inasmuch as the entire change amounts to replacing  $\tau_{\text{int}}^{\text{ep}} \sim 1/\bar{q} v_F \sim w/v_F T$  by a quantity of the order of  $\tau$  at  $T \ll T_1$ .

Thus, the contribution made to the conductivity by the effects connected with the change of the impurity scattering on account of the phonons is larger than the usual Bloch contribution, up to temperatures on the order of  $\Theta(\mu\tau)^{-1/2}$ .

## 6. CRITERIA OF THE APPLICABILITY OF THE KINETIC EQUATION

In the derivation of the kinetic equation we have assumed all the phonon corrections to be small. As seen

from (68), this means that  $\tau_{\text{int}}^{\text{ep}} \ll \tau_{\text{ph}}$ . The condition  $\mu\tau \gg 1$  has the same meaning as  $\tau_{\text{int}}^{\text{ep}} \sim \mu^{-1}$ . Thus, the equation is valid if the characteristic interaction time for each scattering process is much shorter than the relaxation time due to this process alone:

$$\tau_{\text{int}}^i \ll \tau_i. \quad (70)$$

In addition, discarding terms such as the last term in the right side of (24), we have resorted to the fact that the external field is classical and therefore does not influence the interaction process. This means, for example, that the energy given up by the electric field to the electron during the interaction time should be much smaller than  $\mu$ :

$$e E v \tau / \mu \ll 1, \quad (71)$$

where

$$\tau^i = \begin{cases} \tau_{\text{int}}^i, & \tau_{\text{int}}^i \ll \tau_p, \\ (\tau_{\text{int}}^i \tau_p)^{1/2}, & \tau_{\text{int}}^i \gg \tau_p, \end{cases} \quad (72)$$

$\tau_p$  is the characteristic momentum relaxation time, and in our case it is equal to  $\tau$ . We note that

$$\tau^i = (\bar{q} v_p)^{-1}, \quad \bar{q} \sim \begin{cases} (T/d)^{1/2}, & T \ll T_2, \\ T/w, & T \gg T_1. \end{cases} \quad (73)$$

Since  $\tau \ll (\tau \tau_e)^{1/2}$ , the kinetic equation makes it possible to solve all the problems connected with the heating of the electrons.

The condition imposed on the magnetic field takes the form

$$\Omega_e \tau \ll 1, \quad \Omega_e = \frac{e v_p H}{c p}. \quad (74)$$

Unfortunately, in investigations of say, the Hall effect, the phonons cannot be taken into account within the framework of our kinetic equation: when  $\tau_{\text{int}}^{\text{ep}} \leq \tau$  the quantum phonon corrections become of the same order as the classical corrections.

In order that the external field be classical, it is necessary also that its wave numbers  $k_f$  and frequencies  $\omega_f$  be small enough. These limitations are quite natural:

$$k_f \ll \bar{q}, \quad \omega_f \tau \ll 1. \quad (75)$$

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