in the case of relaxation of nuclear spin.

In conclusion, the authors thank V. V. Menaĭlenko for assistance in the numerical calculations on an electronic computer, and also V. L. Ginzburg and the participants in the seminar conducted by him for useful discussion of the present paper.

1) Thus the experimental data[8,9] for Ni in which the coupling constant is not small, agree with the results obtained below.

2) We note that an experimental measurement of the spin-lattice relaxation time is always made in the presence of an external magnetic field.

3) We are considering the case of strong absorption, in which $\delta \ll v_F/T_s$; δ is the depth of penetration of the field.

[1]B. A. Volkov and Yu. V. Kopaev, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 168 (1974) [JETP Lett. **19**, 104 (1974)].

[2]B. A. Volkov, Yu. V. Kopaev, and A. I. Rusinov, Zh. Eksp. Teor. Fiz. **68**, 1899 (1975) [Sov. Phys. JETP **41**, 952 (1975)].

[3]B. A. Volkov, A. I. Rusinov, and R. Kh. Timerov, Zh. Eksp. Teor. Fiz. **70**, 1130 (1976) [Sov. Phys. JETP **43**, 589 (1976)].

[4]L. V. Keldysh and Yu. V. Kopaev, Fiz. Tverd. Tela **6**, 2791 (1964) [Sov. Phys. Solid State **6**, 2219 (1965)].

[5]A. N. Kozlov and L. A. Maksimov, Zh. Eksp. Teor. Fiz. **48**, 1184 (1965) [Sov. Phys. JETP **21**, 790 (1965)].

[6]A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. **43**, 1456 (1962) [Sov. Phys. JETP **16**, 1030 (1963)]; D. C. Mattis and W. D. Langer, Phys. Rev. Lett. **25**, 376 (1970).

[7]E. C. Stoner, Proc. R. Soc. Lond. **A165**, 372 (1938).

[8]B. Golding and M. Barmatz, Phys. Rev. Lett. **23**, 223 (1969).

[9]M. Shaham, J. Barak, U. El-Hanany, and W. W. Warren, Jr., Phys. Rev. Lett. **39**, 570 (1977); (errata) **40**, 1112 (1978).

[10]B. G. Idlis and Yu. V. Kopaev, Fiz. Tverd. Tela **20**, 1383 (1978) [Sov. Phys. Solid State **20** (1978)].

[11]T. Hioki and Y. Masuda, J. Phys. Soc. Jpn. **43**, 1200 (1977).

[12]M. Kontani, T. Hicki, and Y. Masuda, Solid State Commun. **18**, 1251 (1976).

[13]C. Herring, Magnetism (ed. G. T. Rado and H. Suhl), Vol. 4, Academic Press, New York, 1966.

[14]D. J. Kim, Phys. Rev. Lett. **39**, 98 (1977).

[15]Yu. V. Kopaev and V. V. Tugushev, Fiz. Tverd. Tela **19**, 1086 (1977) [Sov. Phys. Solid State **19**, 632 (1977)].

[16]A. Ogawa, Physica BC **91**, 82 (1977).

[17]F. Borsa and R. G. Lecander, Solid State Commun. **20**, 389 (1976).

[18]S. H. Liu, L. Kopp, W. B. England, and H. W. Myron, Phys. Rev. **B11**, 3463 (1975).

[19]P. A. Fedders and P. C. Martin, Phys. Rev. **143**, 245 (1966).

[20]V. A. Volkov, Yu. V. Kopaev, and V. V. Tugushev, Zh. Eksp. Teor. Fiz. **75**, 735 (1978) [Sov. Phys. JETP **48**, 371 (1978)].

[21]N. E. Alekseevskiĭ, N. M. Dobrovolskii, V. I. Tsebro, and V. F. Shamraĭ, Pis'ma Zh. Eksp. Teor. Fiz. **24**, 417 (1976) [JETP Lett. **24**, 382 (1976)].

[22]A. A. Abrikosov, Vvedenie v teoriyu normal'nykh metallov (Introduction to the Theory of Noraml Metals), "Nauka", 1972 (transl., Academic Press, Solid State Physics, Suppl. 12, 1972).

[23]V. G. Kamenskiĭ, Zh. Eksp. Teor. Fiz. **59**, 2244 (1970) [Sov. Phys. JETP **32**, 1214 (1971)].

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Size effects in the magnetoresistance of antimony whisker crystals

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We investigated the magnetoresistance of antimony whisker crystals in the form of thin platelets with thicknesses $d = 0.1–2.6$ μm, at temperatures 4.2–80 K and in magnetic fields 0–70 kOe at various orientations of the magnetic field relative to the measuring current and the surface of the platelets. Similar measurements were made for a bulk ($d = 1$ mm) sample having the same crystallographic orientation as the platelet. For a transverse magnetic field $\mathbf{H} \perp \mathbf{I}$ perpendicular to the surface of the platelet, the character of the dependence of the resistance in the magnetic field and on the temperature in thin platelets and in the bulk sample is the same. A substantial difference in the behavior of the resistance of the bulk sample and the thin platelets was observed only when the field was parallel to the platelet surface (this leads, in particular, to a strong resistance anisotropy, not observed in the bulk sample, of thin platelets in a magnetic field). The function $\rho_{\parallel}(H)$ has two singularities in fields corresponding to $r \approx d$ and $2r \approx d$ (r is the Larmor radius of the electrons). In fields with $r > d$, a logarithmic increase of the resistance with increasing field is observed. In contrast to the bulk sample, an increase of the magnetoresistance with increasing temperature is observed for thin platelets when the magnetic field is oriented parallel to the surface. The experimental results confirmed most clearly and reliably the principal ideas of the theory of static skin effect.

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1. INTRODUCTION

A theoretical analysis of galvanomagnetic size effects, both for the case of diffuse reflection of electrons from the surface of the metal and for the case of specular and almost specular reflection, under the condition $\gamma = r/l \ll 1$, was carried out by Azbel' and Peschanskiĭ[1-4] ($r = cp_F/eH$ is the Larmor radius of the electron trajec-

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tory and l is the electron mean free path in the metal). The main results of this theory, which pertain to the magnetoresistance of thin plates with $d \ll l$ (d is the sample thickness), for a metal with equal numbers of electrons and holes, are shown in Fig. 1.

For thin samples, from the point of view of the size effect, a distinction is made between the region of weak fields, when $r \gtrsim d$ and the region of strong fields, $r \ll d$.

In weak fields, according to the theory, the character of the reflection of the electrons from the surface does not affect the conductivity of the sample. In a magnetic field **H** perpendicular to the electric current **I**, at $r > d$, the resistivity of a thin plate is given by

$$\rho^d(H) \infty \rho^\infty(0) \frac{l}{d} \left(\ln \frac{r\alpha}{d}\right)^{-1}, \quad \alpha \sim 1, \tag{1.1}$$

where $\rho^\infty(0)$ is the resistivity in the volume of the metal in a field $H = 0$.

In the case of a transverse magnetic field parallel to the plane of the plate, in the region $d/2 < r < d$, the theory predicts a decrease of the magnetoresistance (Fig. 1) as $2r \to d$, in accordance with the linear law

$$\rho_\parallel^d(H) \infty \rho^\infty(0) \left\{1 + \alpha \frac{l}{d} \left(1 - \frac{d}{2r}\right)\right\}. \tag{1.2}$$

In strong magnetic fields, $r \ll d$, a static skin effect is produced for the current and is caused by the fact that the electrons colliding with the surface have a higher mobility than the electrons in the interior of the metal, so that the electric current is concentrated near the surface in a layer of depth r. (The static skin effect can be produced in a plate only if the magnetoresistance of the bulk sample of the same material increases quadratically with the field: $\rho^\infty(H) \infty H^2$.) Under the conditions of the static skin effect, the character of the reflection of the electrons from the surface plays the decisive role. The theory makes use of the scattering characteristic averaged over all possible angles φ of encounter between the electrons and the surface—the diffusivity coefficient q_1. The following relation holds true:

$$\frac{1}{\gamma + q_1} = \int_0^\pi \frac{\varphi^2 \, d\varphi}{1 - q(\varphi) + \varphi\gamma} \tag{1.3}$$

where $q(\varphi)$ is the dependence of the probability of specular reflection on the angle φ. The specularity coefficient q is determined from the condition $q = 1 - q_1$.

For the case of almost specular reflection $0 < q_1 \ll 1$, the conductivity of the plate in a transverse magnetic field $\mathbf{H} \perp \mathbf{I}$ parallel to the surface of the plate is given by

$$\sigma_\parallel^d(H) = \sigma^s \frac{r}{d} + \sigma^\infty = \sigma_0^\infty \frac{r}{d} \frac{\gamma}{q_1 + \gamma} + \sigma_0^\infty \gamma^2, \tag{1.4}$$

where σ^s is the conductivity of the surface layer, and $\sigma_0^\infty = 1/\rho^\infty(0)$. From (1.4) we get a formula for the resistivity of a thin plate with $d \ll l$ under the condition of weak diffusivity $q_1 \ll 1$:

$$\rho_\parallel^d(H) = \rho^\infty(H) \Big/ \left(1 + \frac{l}{d} \frac{1}{q_1 + \gamma}\right) = \rho^\infty(0) \frac{ld}{r^2} q_1 + \rho^\infty(0) \frac{d}{r}. \tag{1.5}$$

It is seen from this expression that under the condition $r \gg lq_1$, which can be defined as the condition of specular

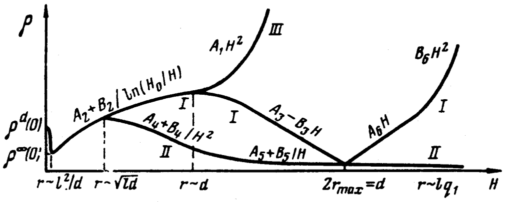

FIG. 1. Theoretical plots of the magnetoresistance against the magnetic field for a thin plate with equal concentration of electrons and holes: I—field **H** perpendicular to the current and parallel to the plane of the plate; II—field **H** parallel to the current; III—field perpendicular to the current and inclined or perpendicular to the plane of the plate.

reflection from the surface, the magnetoresistance has a linear dependence on the field **H** (Fig. 1):

$$\rho_\parallel^d(H) \infty \frac{d}{l} H, \quad r \gg lq_1.$$

If $r \ll lq_1$ (diffuse reflection), the dependence of the magnetoresistance on H is quadratic:

$$\rho_\parallel^d(H) \infty q_1 d H^2, \quad r \ll lq_1.$$

For the case of a strong magnetic field $\mathbf{H} \perp \mathbf{I}$ perpendicular or inclined to the surface of the plate, the theory predicts (just as in the bulky conductor), a quadratic increase of the magnetoresistance regardless of the type of reflection:

$$\rho^d_{\perp,\text{oblique}} \approx \rho^\infty(0) \frac{l}{d} \left(\frac{d}{r}\right)^2 \infty r^{-2} d \infty H^2. \tag{1.6}$$

In a longitudinal magnetic field, $\mathbf{H} \parallel \mathbf{I}$, at $r > d$ the theory predicts a slight increase of the magnetoresistance to a value of the order of $\rho^d(0)$ at $r \sim \sqrt{ld}$, followed by a decrease to $\rho^\infty(0)$ as $2r \to d$ (Fig. 1). In a strong longitudinal magnetic field at $r \ll d$, owing to the twisting of electron trajectories, the surface has no effect on the magnetoresistance of thin samples and $\rho^d(H) \sim \rho^\infty(0)$.

The static skin effect in Zn and Cd whisker crystals was investigated in Refs. 5–8. The investigation of the dependences of the magnetoresistance of thin samples on the magnetic field revealed a disparity between theory and experiment in the case of a longitudinal magnetic field, and also in a weak magnetic field when $r > d$, wherein the magnetoresistance increased like $\rho^d(H) \propto H^{0.65}$ rather than in accordance with the theoretical relation (1.1). In these references the Fuchs parameter (see, e.g., Ref. 9) of the investigated samples was estimated at $\mathcal{P} \approx 0.5$–0.8, i.e., partial specular reflection of the electrons from the surface took place. Attempts to confirm the main predictions of the theory of the static skin effect were made also by Panchenko *et al.*[10]

The present paper is devoted to an investigation of the size effect in the electric conductivity of thin antimony samples in a magnetic field, aimed at the study of the character of the interaction of the conduction electrons with the surface of the metal, and to a comparison of the obtained data with the theory.[1-4]

2. SAMPLES

We chose for the investigations whisker crystals of antimony. It is well known that whiskers have, besides the small dimensions $d \ll l$ that offer a convenient ratio of the surface part of the conductivity to the bulk part, also exceedingly high perfection of the structure of the volume and of the surface, as well as high chemical purity. The most suitable material was recognized to be antimony for the following reasons: a) the small Fermi momenta in semimetals makes for a high probability of specular reflection from the surface; b) the electron and hole densities in antimony are equal, thus ensuring the possibility of observing the static skin effect ($\rho^{\infty}(H) \propto H^2$); c) the Fermi energy of Sb (in contrast to Bi), $\varepsilon_F \sim 2000$ K, is high enough compared with the temperature spread at $T \sim 80$ K, an important factor when $\rho_H^d(T)$ dependences are measured; d) the Fermi surface of antimony is relatively simple and has been thoroughly investigated, an important factor in the numerical interpretation of the results.

The antimony whiskers were obtained by us, for the first time ever, by growing from the gas phase.[11] The whiskers took the form of platelets (ribbons) with the following dimensions: thickness $d = 0.1-2.6\ \mu$m, width $\Delta = 1.1-130\ \mu$m, and length from 0.2 to 3 mm. The platelets were oriented in the basal plane of the crystal (the normal to the surface of the platelet was parallel to the C_3 axis), the growth was along the binary axis, along which the measuring current I was directed. The whisker quality is characterized by a Dingle temperature $T_D \leq 1$ K (electron orbit, $\mathbf{H} \parallel C_1$) and is higher than that of bulk crystals grown from the same initial material: $T_D^{\text{bulk}} = 3.4 \pm 1$ K. The bulk sample used for comparison with the whiskers was cut from a single-crystal block along the binary axis, measured $10 \times 1 \times 1$ mm, and had a resistivity ratio $\rho_{293}/\rho_{4.2} \approx 1700$. The method of determining the thickness and the mounting of the samples are described in Ref. 12.

3. MAGNETORESISTANCE OF THIN PLATELETS OF ANTIMONY IN A MAGNETIC FIELD PERPENDICULAR TO THE ELECTRIC CURRENT

A. Results of experiments

3.1. In the experiment we plotted the dependence of the transverse magnetoresistance of thin platelets on the field H: $\Delta\rho^d(H) = \rho^d(H) - \rho^d(0)$. When the magnetic field is perpendicular to the surface of the platelet, no singularities whatever are observed on the $\Delta\rho^d(H)$ curves, and in this case $\Delta\rho_\perp^d(H) \propto H^{1.8}$ in the entire range of variation of the magnetic field. A similar power-law dependence was observed also for the bulk sample at $\mathbf{H} \parallel C_3$ as well as $\mathbf{H} \parallel C_1$. The $\Delta\rho^d(H)$ curves assume an entirely different form if the magnetic field is parallel to the surface of the platelet. Figure 2 shows a plot of $\Delta\rho_\parallel(H)$ for sample Sb-48 ($d = 0.13\ \mu$m, $\Delta = 5.3\ \mu$m); this plot is typical of all the measured samples.

Curve 1 reveals two inflection or break points, H_{b1} and H_{b2}, between which the plot of $\Delta\rho_\parallel^d(H)$ is approximately linear. In other field intervals the curve can be represented in the form

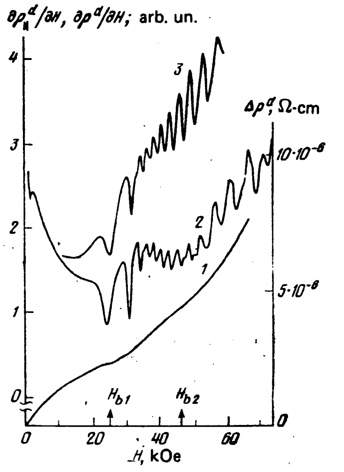

FIG. 2. Results of measurements on whisker sample Sb-48, $d = 0.13\ \mu$m, $\Delta = 5.3\ \mu$m, $T = 4.2$ K; curve 1–plot of the magnetoresistance against the magnetic field; $\mathbf{H} \perp \mathbf{I}$ and is parallel to the plane of the platelet (right-hand scale); 2–plot of $\partial\rho_\parallel^d/\partial H$ against the field H, experimental geometry the same as for curve 1; 3–plot of $\partial\rho^d/\partial H$ against the field H; $\mathbf{H} \perp \mathbf{I}$ and makes an angle $\vartheta = 18°$ to the plane of the platelet.

$$\Delta\rho_\parallel^d(H) \propto \begin{cases} A_2 + \dfrac{B_2}{\ln(H_0/H)}, & 0 < H < H_{b1} \\ H^n,\ n \sim 1.8-1.9, & H > H_{b2} \end{cases}, \qquad (3.1)$$
$$\Delta\rho_\parallel^d(H_{b1}) < \Delta\rho_\parallel^d(H_{b2});$$

with H_{b1} and H_{b2} smaller the thicker the sample.

For a more detailed study of the behavior $\Delta\rho_\parallel^d(H)$, we have examined the behavior of the monotonic part of the derivative $\partial\rho_\parallel^d(H)/\partial H$. A typical plot of this derivative is shown also in Fig. 2 (curve 2).[1)] The monotonic part of the derivative of the magnetoresistance has two minima (the second is less strongly pronounced), corresponding to the two breaks on the $\Delta\rho_\parallel^d(H)$ curve, namely in the fields H_{b1} and H_{b2}.

Curve 3 of Fig. 2 shows a plot of $\partial\rho^d(H)/\partial H$ when the field H is inclined an angle $\vartheta = 18°$ ($\mathbf{H} \to C_3$, the angle $\vartheta = 0$ corresponds to the field H parallel to the plane of the plate). It is seen that the deflection smoothes out the minimum. At $\vartheta > 30°$, the monotonic part of the $\partial\rho^d(H)/\partial H$ does not have the aforementioned singularities, and the plot of $\Delta\rho^d(H)$ is similar to that of $\Delta\rho_\perp^d(H)$.

The appearance of inflections on the $\Delta\rho_\parallel^d(H)$ curves is due to the size effect. To prove this, we have plotted in Fig. 3 the dependence of the inflection points H_{b1} and H_{b2} against the reciprocal of the sample thickness. (The values of H_{b1} and H_{b2} were made more precise by using the $\partial\rho_\parallel^d(H)/\partial H$ curves.) The following relations hold

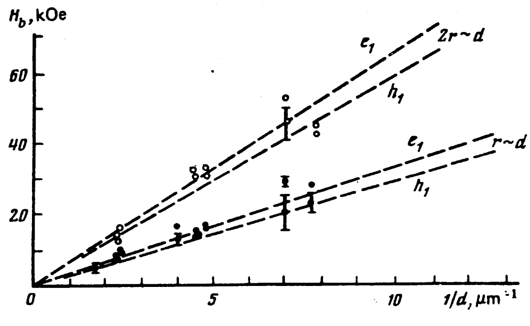

FIG. 3. Values of the inflection (break) fields of the $\rho_\parallel{}^d(H)$ plot for samples of various thicknesses: ● – H_{b1}, ○ – H_{b2}. The accuracy with which H_b was determined is indicated for some samples.

true:

$$H_{b1,2} \propto d^{-1}, \quad H_{b2} \approx 2H_{b1}.$$

Because of these relations, it was possible to reduce the plots of $\Delta\rho_{\parallel}^{d}(H)$ for ten thin samples with $d = 0.13$–0.44 m to a single plot in the coordinates $\varkappa\Delta\rho(H) = F(Hd)$. The values of the coefficient $\varkappa$ are listed in the table and are accurate to ±10%.

3.2. The angular dependence of the magnetoresistance of thin plates is shown in Fig. 4 together with the angular dependence for a bulky sample of antimony having the same orientation. The anisotropy of the magnetoresistance of the whiskers depends both on their thickness and on the magnetic field. It is seen from Fig. 5 that the anisotropy of the bulky sample is practically independent of the magnetic field, whereas the anisotropy of the thin platelets increases strongly with the field, and eventually saturates in strong fields. The dependence of the magnetoresistance anisotropy on the whisker thickness in fields such that the field dependence saturates is shown in Fig. 6. Despite the scatter of the points, a strong dependence of the anisotropy on the sample thickness is discerned.

B. Discussion of results

3.3. Size effect. We shall show that the inflections at the points H_{b1} and H_{b2}, observed on the $\Delta\rho_{\parallel}^{d}(H)$ curves, take place in cases predicted by theory, namely, at $r \approx d$ and $2r \approx d$.

At $\mathbf{H} \parallel C_1$ antimony can have two hole orbits and two electron orbits of unequal size. Figure 3 shows the values of H_{b1} and H_{b2} as functions of d^{-1} as well as straight lines corresponding to the conditions $r \approx d$ and $2r \approx d$, with $r = cp_2/eH$, for the small electron orbit ($p_2 = 5.2 \times 10^{-21}$ g-cm/sec) and for the small hole orbit ($p_2 = 4.2 \times 10^{-21}$ g-cm/sec). It is seen that the experimental points fit these lines with accuracy ±20%.

For the two other electron and hole orbits, the dimensions p_2 are approximately 1.8 times larger, and the fields H_{b1} and H_{b2} for them should be 1.8 times larger; this does not agree with experiment. Thus, the observed singularities of $\Delta\rho_{\parallel}^{d}(H)$ must apparently be attributed to the small sections of the Fermi surface.

We note that in the investigated field region, where $r \lesssim d$, the contribution to the conductivity from the electrons that collide with the narrow lateral faces of the samples can be neglected if $d \ll \Delta$, and the samples can be regarded as platelets independently of the ratio of the mean free path of the electrons $l_{4.2\,\mathrm{K}} \gg r$ to the

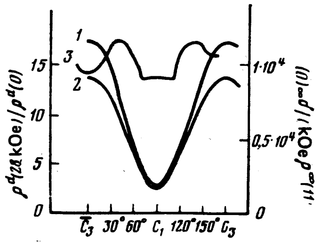

FIG. 4. Angular dependences of the magnetoresistance at $T = 4.2$ K. 1—whisker Sb-43, $d = 0.14$ μm, $\Delta = 11$ μm; 2—whisker Sb-62, $d = 0.43$ μm, $\Delta = 12$ μm; 3—bulk antimony sample.

width Δ. The finite width of the platelets should possibly influence the form of the angular dependences of the magnetoresistance, but should not affect noticably the quantitative results.

3.4. The field H is perpendicular to the plane of the platelet. In fields where $r < d$, the theory is in full agreement with experiment: $\rho_{\perp}^{d}(H) \propto \rho^{\infty}(H) \propto H^{1.8}$. In the weak-field region near $H = 0$, experiment reveals a slower growth of the magnetoresistance with a field than predicted by the theory (inasmuch as the function $\rho \propto H^{1.8}$ increases near $H = 0$ more slowly than $\rho \propto (\ln 1/H)^{-1}$. This circumstance can be attributed to the fact that when the magnetic field is perpendicular to the plane of the platelet a weak magnetic field bends the electron trajectories in the plane of the platelet without influencing the collisions of the electrons with the surface. At the same time, in a magnetic field parallel to the surface of the platelet, the bending of the trajectories of the electrons leads to the onset of additional scattering by the surface of the plate (to an increase of the angles at which the electrons encounter the surface). In the former case, therefore, the magnetoresistance of the plate should increase more slowly than in the latter.

3.5. The field H is parallel to the plane of the platelet. It has been observed that in weak fields $r > d$ the experimental curves are well described by the relation

$$\rho_{\parallel}^{d}(H) \propto A + \frac{B}{d}\left(\ln\frac{r\alpha}{d}\right)^{-1}, \tag{3.2}$$

which agrees with the theoretical relation (1.1). The parameters A, B, and α were obtained from experiments for eight samples at 4.2 and 1.4 K:

$$(A \pm 0.5\,A) = \rho^{d}(0) \approx 1\cdot 10^{-6}\,\Omega\text{-cm},$$
$$(B \pm 0.4\,B) = 0.7\cdot 10^{-10}\,\Omega\text{-cm}^2 \sim \rho^{\infty}(0)\,l \sim p_F/ne^2 = 4\cdot 10^{-10}\,\Omega\text{-cm}^2,$$
$$(\alpha \pm 0.1\alpha) = 6.$$

In fields such that $d/2 < r < d$, experiment reveals a

Sample	d, μm	$q_1+\gamma$	$\varkappa \pm 10\%$	Sample	d, μm	$q_1+\gamma$	$\varkappa \pm 10\%$
Sb-48	0.13	–	1	Sb-63	0.44	0.09	2.9
Sb-31	0.14	0.07	0.8	Sb-62	0.43	0.11	
Sb-34	0.14	0.08	1	Sb-24	0.44	0.19	
Sb-68	0.18	0.1		Sb-45	0.42	0.26	0.7
Sb-69	0.21	0.07	2	Sb-67	0.59	0.12	
Sb-76	0.21	0.19		Sb-55	0.77	0.46	
Sb-72	0.22	0.1	0.7	Sb-52	1.14	0.54	
Sb-35	0.22	0.09	1.2	Sb-37	1.43	0.71	
Sb-25	0.26	0.11		Sb-70	2.2	0.2	

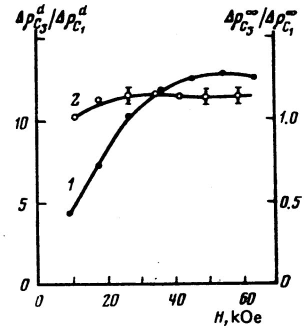

FIG. 5. Dependence of the magnetoresistance anisotropy on the magnetic field: 1—whisker Sb34, $d = 0.14$ μm—left-hand scale, 2—antimony bulk sample, right-hand scale.

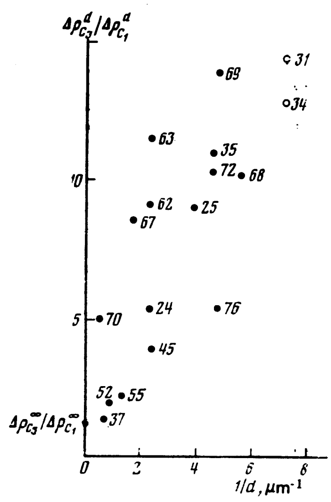

FIG. 6. Anisotropy of the magnetoresistance as a function of the sample thickness: ●–$H=28$ kOe, O–$H=46$ kOe; the numbers at the points are the number of the samples.

linear increase of $\rho_\parallel^d(H)$, whereas theory predicts a decrease of the magnetoresistance like $\rho_\parallel^d(H) \propto (A_3 - B_3 H)$, to a value $\rho^d(H_d) = \rho^\infty(0)$ (Fig. 1). Allowance for the other group of carriers can apparently not justify this disparity, inasmuch as for small cross sections, where $r<d$, the magnetoresitance should theoretically decrease linearly, whereas for large cross sections, where $r>d$, the magnetoresistance should continue its weak logarithmic growth, which cannot greatly influence the total conductivity of the sample, i.e., cannot be the cause of the observed rapid growth of the magnetoresistance in the region $d/2<r<d$.

In strong fields $2r<d$, a comparison of the experimental results ($\rho_\parallel^d(H) \propto H^{1.8}$) with the theory makes it possible to estimate the diffusivity coefficient at $q_1 > d/2l > 10^{-3}$ ($d \sim 0.1$–$0.4\,\mu$m, $l_{max} \sim 200\,\mu$m, Refs. 14 and 15).

3.6. Diffusivity coefficient. The value of q_1 can be calculated from the magnetoresistance anisotropy. The behavior of the anisotropy of thin platelets as a function of the magnetic field (Fig. 5) can be easily explained in accordance with Items 3.4 and 3.5, namely, at $2r>d$ the ratio $\Delta\rho_\perp^d/\Delta\rho_\parallel^d$ increases with increasing H, and at $2r<d$ it tends to a constant value. Thus, in strong fields at $2r<d$, using (1.5) and (1.6), we get

$$\Delta\rho_\perp{}^d/\Delta\rho_\parallel{}^d \approx 1/(q_1+\gamma). \tag{3.3}$$

The values of q_1 calculated by formula (3.3) using the data of Fig. 6 are given in the table. The quantity γ can be neglected compared with q_1: $\gamma|_{H=28\,\mathrm{kOe}} = r/l < 10^{-2}$ ($l>10\,\mu$m, Refs. 14 and 15).

On the average, for thin samples with $d<0.5\,\mu$m, the specular reflection coefficient is $q \sim 0.9$. A substantial difference between the coefficients q for thin samples and the Sb70 sample ($d=2.2$ m) and the samples Sb-37, 52, 55 ($d \sim 1\,\mu$m) is more readily evidence of the poor surface quality of samples Sb37, 52, and 55.

4. MAGNETORESISTANCE OF ANTIMONY WHISKERS IN A MAGNETIC FIELD PARALLEL TO THE ELECTRIC CURRENT

A. Experimental results

We measured the dependences of the magnetoresistance on the magnetic field H at 4.2 K for samples of various thickness, including a bulk sample. Variation of the form of the $\Delta\rho^d(H)$ with thickness was observed.

Figure 7 shows a plot of the longitudinal magnetoresistance against the field H for two whiskers of different thickness. The figure shows also the plot for a bulk sample (curve 1) with the same crystallographic orientation as the whiskers. It is seen from the figure that the decrease of the magnetoresistance and the tendency of $\rho^\infty(H)$ to $\rho^\infty(0)$ in strong fields, which is characteristic of bulk samples, does not appear in thin whiskers and, on the contrary, the magnetoresistance increases more the thinner the sample.

B. Discussion of results

We now compare the curves of Fig. 7 with the theory. In Fig. 1, curve II, which corresponds to the case of longitudinal magnetoresistance, has a maximum at $r \sim \sqrt{ld}$ (for whiskers with $d \lesssim 1\,\mu$m this corresponds approximately to fields $H \sim 1$ kOe), and in strong fields the magnetoresistance should tend to the value $\rho^\infty(0)$. No such behavior of the magnetoresistance is observed in thin samples.

A growth of the longitudinal magnetoresistance in magnetic fields was obtained by Chopra[16] in thin silver films at $2r<d$. This phenomenon was attributed to the presence of specular scattering of the electrons from the film surface. However, the explanation given in Ref. 16 for the growth of the magnetoresistance cannot be used for thin antimony platelets for the following reason.

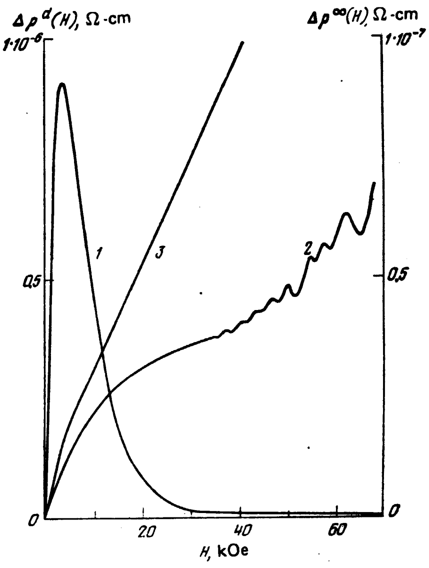

FIG. 7. Dependence of the longitudinal magnetoresistance on the magnetic field at $T-4.2$ K: 1–bulk sample, 2–Sb-33, $d=0.7\,\mu$m, $\Delta=2.6\,\mu$m; 3–Sb-44, $d=0.1\,\mu$m, $\Delta=4.6\,\mu$m.

It is based on the fact that in the case of specular reflection the longitudinal magnetoresistance of thin silver films behaves in the same manner as in bulk samples, namely, it increases gradually by an amount

$$\Delta\rho^d_{spec}(H\to\infty)\sim\Delta\rho^\infty(H\to\infty)\sim\rho^\infty(H=0).$$

For antimony, however, the longitudinal magnetoresistance of bulk samples has an anomalous behavior, which was observed long ago by Steele[17] and is illustrated in Fig. 7.

It is seen from Fig. 7 that the behavior of the magnetoresistance of thin atimony plates is, first, not similar to that of the magnetoresistance of a bulk sample, as should be the case if the reflection from the surface is specular; second, it is not similar to the theoretical field dependence of the magnetoresistance of a thin sample in the case of diffuse reflection (Ref. 18 and curve II of Fig. 1). The assumption that was made in Ref. 17, that the domain structure of bulk antimony crystals is the cause of the anomalous behavior of the magnetoresistance, cannot explain satisfactorily the results of the measurement of the magnetoresistance of whiskers. It is possible to explain the behavior of the longitudinal magnetoresistance of thin platelets of antimony if the behavior of the longitudinal magnetoresistance of the bulk antimony crystals is explained.

5. DEPENDENCE OF THE MAGNETORESISTANCE OF THIN ANTIMONY PLATELETS ON THE TEMPERATURE IN A STRONG MAGNETIC FIELD PERPENDICULAR TO THE CURRENT

A. Experimental results

Some of the experimental results were published in Ref. 19. Here we supplement them somewhat and present a theoretical explanation.

The gist of the observed phenomenon is the following. We measured the $\Delta\rho^d_T(H)$ dependences at different fixed temperatures in the interval 4.2–80 K. The field was oriented either parallel or perpendicular to the plane of the platelets. For the whiskers in the case of H perpendicular to their plane, a decrease of the magnetoresistance with increasing temperature was observed (Fig. 8, curve 2), in analogy with bulk samples. An exception is the case of thin platelets with $d\lesssim 2\ \mu$m, when the vector H lies in the plane of the plate. In this special case, an increase of the magnetoresistance with increasing temperature is observed in the entire range of magnetic fields. For samples of large thickness, $d\gtrsim 2.5\ \mu$m (Fig. 8, curve 1), the growth of $\Delta\rho^d_H(T)$ gives way to a decrease at higher temperatures.

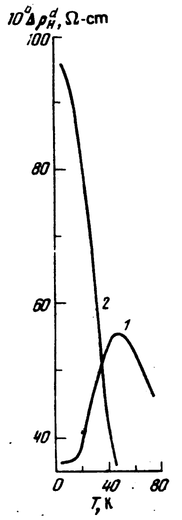

FIG. 8. Temperature dependence of the transverse magnetoresistance $\Delta\rho_H(T)$ at $H=40$ kOe for the Sb28 whisker, $d=2.5\ \mu$m; 1—field in the plane of the sample, 2—field perpendicular to the plane of the sample.

B. Discussion of results

In the case when the field H is parallel to the plane of the plate and the relations $r\ll d$ and $r\ll l$ are satisfied, the conductivity of the plate can be expressed in accordance with (1.14) in the form $\sigma^d=\sigma^s r/d+\sigma^\infty$, where

$$\sigma^s\frac{r}{d}\sim\frac{ne^2}{p}\frac{r}{d}\frac{rl}{lq_1+r},\quad \sigma^\infty\sim\frac{ne^2}{p}\frac{r^2}{l}. \qquad 5.1$$

We assume that the specularity coefficient is constant, and that the temperature dependence of the magnetoresistance is determined by the change of the mean free path $l(T)$. It is then necessary to investigate the behavior of the function $\sigma^d(l)$. Figure 9 shows plots of $(r/d)\sigma^s(l)$ and $\sigma^\infty(l)$ corresponding to formulas (5.1). We see the following: a) at low temperatures, when $l\gg d$, we have $\sigma^d\approx\sigma^s r/d$, and then

$$\partial\rho^d/\partial T\infty-\partial\sigma^d/\partial T>0,$$

i.e., the magnetoresistance increases with increasing T; b) at high temperatures $l\ll d$, $\sigma^d\approx\sigma^\infty$, and then

$$\partial\rho^d/\partial T<0,$$

i.e., the magnetoresistance decreases with increasing T. The transition from the case a) to the case b) corresponds to the condition

$$-\frac{r}{d}\frac{\partial\sigma^s}{\partial T}=\frac{\partial\sigma^\infty}{\partial T}.$$

The solution of this equation under the condition of weak diffusivity of the reflection $q_1<r/d\ll 1$ is

$$l(T_0)\approx d(q_1+\sqrt{r/d})\approx\sqrt{rd}. \qquad 5.2$$

Using expression (5.2), we can estimate from the data of Fig. 8 the electron mean free path at $T_0\approx 50$ K, namely, $l(50\text{ K})\approx 0.4\ \mu$m (we assume here $q_1<r/d\sim 3\cdot 10^{-2}$).

Thus, the increase of the magnetoresistance with increasing temperature in thin platelets is attributed to the fact that the surface part of the conductivity predominates at $T<T_0$ (i.e., $l(T)>l(T_0)$), and the condition under

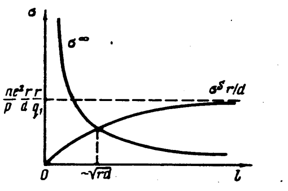

FIG. 9. Theoretical plots of the surface at $(\sigma^s r/d)$ and bulk (σ^∞) conductivities of the sample against the electron mean free path l. The point $l\sim\sqrt{rd}$ corresponds to the condition $(r/d)\partial\sigma^s/\partial l\approx\partial\sigma^\infty/\partial l$ (and simultaneously to the condition $(r/d)\sigma^s\approx\sigma^\infty$) at $q_1<r/d\ll 1$.

which $\rho_H^d(T)$ increases is almost specular reflection of the electrons from the surface: $q_1 < r/d \ll 1$. On the other hand, if the reflection is diffuse, $q_1 = 1$, then the surface conductivity does not depend at all on the temperature,

$$\frac{r}{d}\sigma^s\Big|_{q_1=1} = \frac{ne^2}{p}\frac{r^2}{d}, \quad \frac{\partial\sigma^s}{\partial T} = 0,$$

and in this case the magnetoresistance of a thin platelet will decrease with increasing T regardless of the orientation of **H** relative to its plane.

In conclusion, we wish to note the following. The experimental results obtained in this paper, as is frequently the case when theory is compared with experiment, can be interpreted in two ways. If we consider the theory of the static skin effect,[1-4] which is a correct reflection of reality, then the results described here and in Ref. 19 should indicate, above all, that the conduction electrons are reflected almost specularly from the surface. From another point of view, assuming *a priori* the specularity coefficient in the whisker crystals to be high, the same results can be regarded as an experimental confirmation of the theory of the static skin effect.

However, the existence of high specularity in whisker crystals was observed unequivocally in experiment by an independent method, with the aid of quantization on the electron orbits, truncated by the plate, in a magnetic field.[12,13] We assume therefore that the experiments described above and in Ref. 19 confirm most clearly and reliably the ideas of the theory of the static skin effect.

[1)] The singularities in the bahavior of the Shubnikov–de Haas oscillations in thin conductors are considered in Refs. 12 and 13.

[1] M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **44**, 1262 (1963) [Sov. Phys. JETP **17**, 851 (1963)].

[2] M. Ya. Azbel' and V. G. Peschanskiĭ, Zh. Eksp. Teor. Fiz. **49**, 572 (1965) [Sov. Phys. JETP **22**, 399 (1966)].

[3] V. G. Peschanskiĭ and M. Ya. Azbel', Zh. Eksp. Teor. Fiz. **55**, 1980 (1968) [Sov. Phys. JETP **28**, 1045 (1969)].

[4] V. G. Peschanskiĭ, Dissertation, Low-Temp. Phys. Inst., Khar'kov, 1970.

[5] Yu. P. Gaĭdukov and N. P. Danilova, Zh. Eksp. Teor. Fiz. **64**, 920 (1973) [Sov. Phys. JETP **37**, 467 (1973)].

[6] Yu. P. Gaĭdukov and N. P. Danilova, Zh. Eksp. Teor. Fiz. **65**, 1514 (1973) [Sov. Phys. JETP **38**, 767 (1974)].

[7] Yu. P. Gaĭdukov and N. P. Danilova, Fiz. Tverd. Tela (Leningrad) **15**, 2801 (1973) [Sov. Phys. Solid State **15**, 1867 (1974)].

[8] N. P. Danilova, Dissertation, Moscow University, 1974.

[9] D. C. Larson, transl. of review in: Fizika tonkikh plenok (Physics of Thin Films), Vol. 6, Mir, 1973.

[10] O. A. Panchenko, P. P. Lushitsin, and A. A. Kharlamov, Zh. Eksp. Teor. Fiz. **64**, 2148 (1973) [Sov. Phys. **37**, 1083 (1973)].

[11] Yu. P. Gaĭdukov, E. M. Golyamina, and N. P. Danilova, Prib. Tekh. Eksp. No. 2, 217 (1976).

[12] Yu. P. Gaĭdukov and E. M. Golyamina, Zh. Eksp. Teor. Fiz. **74**, 1936 (1978) [Sov. Phys. JETP **47**, 1008 (1978)].

[13] Yu. P. Gaĭdukov and E. M. Golyamina, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 336 (1976) [JETP Lett. **23**, 301 (1976)].

[14] M. S. Bresler and N. A. Red'ko, Zh. Eksp. Teor. Fiz. **61**, 287 (1971) [Sov. Phys. JETP **34**, 149 (1972)].

[15] Yu. A. Bogod, V. B. Krasovitskiĭ, and L. M. Kondrat'eva, Fiz. Tverd. Tela (Leningrad) **16**, 1347 (1974) [Sov. Phys. Solid State **16**, 869 (1974)].

[16] K. L. Chopra, Phys. Rev. **155**, 660 (1967).

[17] M. C. Steele, Phys. Rev. **97**, 1720 (1955).

[18] Yi-Han Kao, Phys. Rev. **138**, A1412 (1965).

[19] Yu. P. Gaĭdukov, E. M. Golyamina, and N. P. Danilova, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 231 (1975) [JETP Lett. **22**, 107 (1975)].

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