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Electron-electron collisions in a weakly ionized plasma

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Electron-electron collisions in a weakly ionized plasma are studied in the case when relaxation in momentum occurs on neutral atoms. It is shown that if the electron mean free path becomes less than the Debye length, the form of the diffusion coefficient in energy space changes: the Coulomb logarithm is cut off at the mean free path and, in addition, a nonlogarithmic contribution from large impact parameters appears. The results of the work are applicable also to semiconductors when momentum diffusion occurs, for example, in acoustic phonons and the energy relaxation is the result of electron-electron collisions.

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1. INTRODUCTION

It is well known that in a rather highly ionized plasma the relaxation of the electrons in energy is determined by electron-electron collisions, while the relaxation of nonequilibrium electrons in momentum occurs both on electrons and on ions. Since these collisions occur at large impact parameters, i.e., in each event there is a small transfer of energy and momentum, the operator of these collisions can be represented in differential form as was done by Landau.[1]

In a weakly ionized plasma the relaxation of the electrons in momentum occurs in collisions with neutral atoms, while electron-electron collisions continue to remain important for energy relaxation even in this case. The question arises of how the finite electron mean free path affects the form of the Landau operator. It is clear that as long as the electron mean free path l is much greater than the Debye screening radius $\varkappa^{-1}$, collisions with neutral atoms do not affect the nature of electron-electron collisions. In the present work we investigate the opposite limiting case: $\varkappa l \ll 1$, and derive the Landau operator for energy relaxation.

The principal result is that the electron-electron collision operator has the usual form

$$\left(\frac{\partial n_\varepsilon}{\partial t}\right)_{\text{coll}} = \frac{1}{\nu(\varepsilon)}\frac{\partial}{\partial \varepsilon}\int_0^\infty d\varepsilon' \nu(\varepsilon')\nu(\varepsilon) D(\varepsilon,\varepsilon')\left[n_{\varepsilon'}\frac{\partial n_\varepsilon}{\partial \varepsilon} - n_\varepsilon \frac{\partial n_{\varepsilon'}}{\partial \varepsilon'}\right], \quad (1)$$

where $\nu(\varepsilon) = (2m^3\varepsilon)^{1/2}/\pi^2\hbar^3$ is the density of electron states and n_ε is the electron distribution function. However, $D(\varepsilon,\varepsilon')$ differs from the standard expression.[1] It turns out that

$$D(\varepsilon,\varepsilon') = {}^2/_3 e^4 (vv')^{-1/2} f(v'/v), \quad (2)$$

where $v = (2\varepsilon/m)^{1/2}$ is the electron velocity. The function $f(v'/v)$ has the obvious property $f(v'/v) = f(v/v')$ and for $v' \leqslant v$ we have the form

$$f\left(\frac{v'}{v}\right) = \left(\frac{v'}{v}\right)^{3/2} \ln\left(0.751\frac{l}{\rho_0}\right) + \varphi\left(\frac{v'}{v}\right). \quad (3)$$

Here ρ_0 is of the order of the electron wavelength for $e^2/\hbar v \ll 1$, and in the reverse case it is equal to the value of the impact parameter of the Coulomb scattering problem, at which the scattering angle becomes of the order of π.[2]

A plot of the function $\varphi(v'/v)$ is shown in Fig. 1. It is important to note that in the case considered φ does not depend on the parameter l/ρ_0 (with accuracy to terms of order ρ_0/l). For small v'/v we have

$$\varphi(v'/v) = \sqrt{3}v'/v. \quad (4)$$

It is evident from Eqs. (2) and (3) that, in contrast to the usual case, the Coulomb integral is cut off at the mean free path l and not at the Debye length. In addition, $D(\varepsilon,\varepsilon')$ contains a nonlogarithmic contribution which for a sufficiently high velocity ratio becomes greater than the logarithmic contribution. Physically the presence of the two terms in Eq. (3) is due to the fact that as long as the impact distances are small in comparison with the mean free path l, Coulomb colli-

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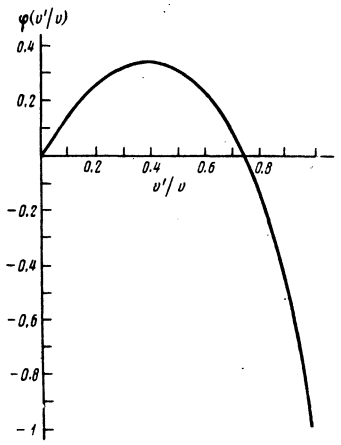

FIG. 1.

sions occur in the same way as in the absence of collisions with neutral atoms. For impact distances greater than the mean free path, Coulomb collisions occur not between free electrons, but between electrons diffusing in the field of neutral atoms.

The question arises of when the electron-electron relaxation plays the principal role, compared to slightly inelastic collisions with neutral atoms. These collisions also are described by the Fokker-Planck equation with an energy diffusion coefficient $B_n \sim \varepsilon^2 mNv\sigma/M$, where σ is the cross section for scattering of an electron by a neutral atom, N is the concentration of these atoms, and M is the mass of the atom. Comparison of Eqs. (1) and (2) with B_N shows that electron-electron collisions play the principal role if

$$n/N \gg (\varepsilon/\varepsilon_0)^2 m/M \tag{5}$$

where $\varepsilon_0 = e^2/\sqrt{\sigma}$, and n is the electron concentration. On the other hand, the inequality $\varkappa l \ll 1$ must be satisfied. These inequalities are compatible if $l \ll \rho_0 M/m$.

It is clear that the situation considered in the present work can occur also in nondegenerate semiconductors where diffusion in momentum occurs, for example, by means of acoustic phonons and the relaxation in energy occurs as the result of electron-electron collisions. Here $B_{ph} \sim ms^2/\tau_p$, where s is the velocity of sound and τ_p is the momentum relaxation time. In this case the condition (5) takes the form

$$n \gg \frac{1}{\rho_0^2 l}\frac{s^2}{v^2},$$

and the condition of compatibility is $l \ll \rho_0 v^2/s^2$.

2. DERIVATION OF THE KINETIC EQUATION FOR A WEAKLY IONIZED PLASMA

In deriving the kinetic equation for the case in which the electron frequently encounters neutral atoms, it is convenient to use the Keldysh diagram technique[3] for highly nonequilibrium processes. In this technique the Green's function is the matrix

$$\hat{G} = \begin{pmatrix} 0 & G^a \\ G^r & F \end{pmatrix}, \tag{6}$$

where G^a and G^r are respectively the advanced and retarded Green's functions and F is related to G^a and G^r by the expression

$$F = (2n_\varepsilon - 1)(G^a - G^r). \tag{7}$$

We shall investigate only electron energy relaxation processes, and therefore the distribution function of the electrons n_ε depends only on the energy variable ε.

The self-energy operator $\hat{\Sigma}$ is also a matrix:

$$\hat{\Sigma} = \begin{pmatrix} \Omega & \Sigma^r \\ \Sigma^a & a \end{pmatrix}.$$

The kinetic equation in this representation has the form

$$\frac{\partial n_\varepsilon}{\partial t} = \frac{1}{2\pi\nu(\varepsilon)}\int \frac{d^3p}{(2\pi)^3}[F(\Sigma^r - \Sigma^a) - \Omega(G^a - G^r)], \tag{8}$$

where

$$\nu(\varepsilon) = \frac{1}{\pi i}\int \frac{d^3p}{(2\pi)^3}(G^a - G^r)$$

is the density of states of electrons.

The self-energy part associated with the electron-electron interaction is shown in Fig. 2. The broken line is the renormalized Coulomb interaction of the electrons and the wavy line is the free Coulomb propagator $4\pi e^2 \hat{\sigma}_x/q^2$ ($\hat{\sigma}_i$ are the Pauli matrices). Since scattering of electrons by each other occurs with small transfers of energy and momentum, it is necessary, as in the normal skin-effect problem,[4] to renormalize the vertex part and the Green's function of the electron with allowance for interaction with neutral atoms. As a result of the fact that the atoms are heavy, the collisions of electrons with them are quasielastic and the neutral-atom loop shown in Fig. 3 by the dashed line does not carry energy. Therefore taking into account the interaction with atoms is equivalent to the problem of scattering by random short-range impurities.[4] The Green's function of an electron scattered by atoms, which is shown in Figs. 2 and 3 by the solid line, has the form

$$G^r(p, \varepsilon) = G^{a*}(p, \varepsilon) = \frac{1}{\varepsilon - p^2/2m + i/2\tau_\varepsilon}, \tag{9}$$

where $\tau_\varepsilon^{-1} = N\sigma\bar{v}_\varepsilon$ and p is the electron momentum.

The equation for the vertex part, illustrated in Fig. 3, has the form[5]

$$\hat{\Gamma}^h(q, \varepsilon) = \hat{\gamma}^h + \frac{1}{\pi\tau_\varepsilon\nu(\varepsilon)}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\hat{\sigma}_x \hat{G}(p-q)\hat{\Gamma}^h(q, \varepsilon)\hat{G}(p)\hat{\sigma}_x. \tag{10}$$

Here $q = (\mathbf{q}, \omega), p = (\mathbf{p}, \varepsilon)$, and

$$\gamma_{ij}^1 = \delta_{ij}/\sqrt{2}, \quad \gamma_{ij}^2 = (\sigma_x)_{ij}/\sqrt{2} \tag{11}$$

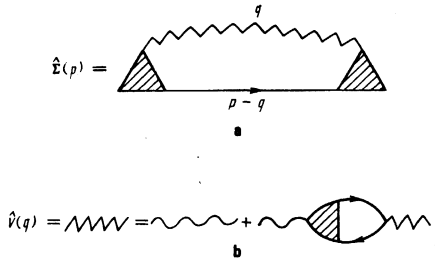

FIG. 2. a) Self-energy part, b) equation for renormalized Coulomb interaction.

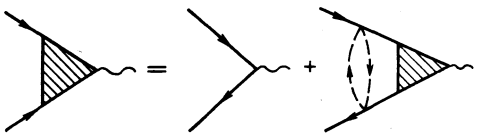

FIG. 3. Equation for the vertex function. The dashed line shows the Green's function for neutral atoms.

are the bare vertices.

In renormalization of the vertex parts it is sufficient to take into account only the diagrams which have ladders of atomic loops, since the diagrams with rescatterings are small in the parameter $1/p_\varepsilon l \ll 1$. This is due to the fact that scattering by neutral atoms occurs at arbitrary angles and the momentum transfer is of the order p_ε. For the same reason Γ^k_{ij} does not depend on $\mathbf{p}$ and the matrix integral equation (10) reduces to a system of eight linear algebraic equations which is easily solved. As a result we have for $\Gamma^k_{ij}(q,\varepsilon)$

$$\Gamma_{22}{}^{2}=0,\quad \Gamma_{11}{}^{2}=\frac{\sqrt{2}\,\zeta_\varepsilon^*}{1-\zeta_\varepsilon^*}(n_\varepsilon-n_{\varepsilon-\omega}),\quad \Gamma_{21}{}^{2}=(\Gamma_{12}{}^{2})^*=\frac{1}{\sqrt{2}(1-\zeta_\varepsilon{}^r)},$$
$$\Gamma_{22}{}^{1}=\frac{1}{\sqrt{2}(1-\zeta_\varepsilon)},\quad \Gamma_{21}{}^{1}=\frac{2n_\varepsilon-1}{\sqrt{2}}\frac{\zeta_\varepsilon}{1-\zeta},$$
$$\Gamma_{12}{}^{1}=-\frac{2n_{\varepsilon-\omega}-1}{\sqrt{2}}\frac{\zeta_\varepsilon}{1-\zeta_\varepsilon},$$
$$\Gamma_{11}{}^{1}=\frac{1}{\sqrt{2}(1-\zeta_\varepsilon^*)}-\sqrt{2}(2n_\varepsilon-1)(2n_{\varepsilon-\omega}-1)\,\mathrm{Re}\frac{\zeta_\varepsilon}{1-\zeta_\varepsilon}, \tag{12}$$

where

$$\zeta_\varepsilon{}^r=\frac{1}{\pi\tau_\varepsilon\nu(\varepsilon)}\int\frac{d^3\mathbf{p}}{(2\pi)^3}G^r(p-q)G^r(p)=\frac{i}{2p_\varepsilon l},$$
$$\zeta_\varepsilon=\frac{1}{\pi\tau_\varepsilon\nu(\varepsilon)}\int\frac{d^3\mathbf{p}}{(2\pi)^3}G^r(p-q)G^r(p) \tag{13}$$
$$=\frac{i}{2ql}\ln\frac{\omega+qv+i/\tau_\varepsilon}{\omega-qv+i/\tau_\varepsilon}.$$

In Eq. (12) we have neglected terms of order $\zeta_\varepsilon^r \sim 1/p_\varepsilon l \ll 1$ in those places where they are not important in what follows.

Substituting the expressions for $\hat{\Gamma}^k$ and $\hat{G}$ into the equation shown in Fig. 2b for

$$V(q)=\begin{pmatrix}0 & V^a\\ V^r & V\end{pmatrix},$$

we obtain the following expressions:

$$V^r(q)=V^{a*}(q)=-\frac{4\pi e^2}{q^2\varepsilon(\omega,\mathbf{q})}, \tag{14}$$

where

$$\varepsilon(\omega,\mathbf{q})=1-\frac{4\pi e^2}{q^2}\int\frac{d\varepsilon}{2\pi i}\pi\tau_\varepsilon\nu(\varepsilon)(2n_\varepsilon-1)\Big[\frac{\zeta_{\varepsilon+\omega}}{1-\zeta_{\varepsilon+\omega}}$$
$$-\frac{\zeta_\varepsilon}{1-\zeta_\varepsilon}+\zeta_\varepsilon{}^{r*}-\zeta_{\varepsilon+\omega}^{\,r}\Big], \tag{15}$$

and

$$V(q)=-\frac{4\pi e^2}{q^2}\frac{\Pi(\omega,\mathbf{q})}{|\varepsilon(\omega,\mathbf{q})|^2}, \tag{16}$$

$$\Pi(\omega,\mathbf{q})=\frac{16\pi e^2}{q^2}\int\frac{d\varepsilon}{2\pi i}[n_\varepsilon+n_{\varepsilon-\omega}-2n_\varepsilon n_{\varepsilon-\omega}]\,\mathrm{Re}\frac{\pi\tau_\varepsilon\nu(\varepsilon)\zeta_\varepsilon}{1-\zeta_\varepsilon}. \tag{17}$$

The expression for $\varepsilon(\omega,\mathbf{q})$ in the limit $\tau_\varepsilon^{-1}\to 0$ goes over into the usual expression for the permittivity of a free electron gas. If we took into account ions, then as usual they would contribute to the permittivity, and since for ions the collisions are elastic, this would lead to the static number $\varkappa_i^2$—the square of the reciprocal Debye length associated with the ions.

In the limit of low frequencies of the wave vectors ($\omega\tau_\varepsilon \ll 1$ and $ql \ll 1$) the expression for $\varepsilon(\omega,q)$ has the form

$$\varepsilon(\omega,\mathbf{q})=1-4\pi e^2\int_0^\infty d\varepsilon\,\nu(\varepsilon)\frac{\partial n_\varepsilon}{\partial\varepsilon}\frac{D_\varepsilon}{-i\omega+D_\varepsilon q^2}, \tag{18}$$

where $D_\varepsilon=v_\varepsilon^2\tau_\varepsilon/3$ is the coefficient of diffusion of electrons with energy ε.

We note here one feature of Eq. (18): it involves the coefficient of diffusion of electrons with a given energy. This is a consequence of the fact that in derivation of $\varepsilon(\omega,\mathbf{q})$ we neglected energy relaxation processes, i.e., we assumed that frequencies $\omega \gg 1/\tau_{\text{energy}}$ are important and therefore $\varepsilon(\omega,\mathbf{q})$ involves partial quantities and differs from the "hydrodynamic" $\varepsilon(\omega,\mathbf{q})$ obtained for $\omega \ll 1/\tau_{\text{energy}}$.

The self-energy part has the form (Fig. 2a)

$$\hat{\Sigma}(p)=i\int\frac{d^4q}{(2\pi)^4}\hat{\Gamma}^i(-q,\varepsilon-\omega)\hat{G}(p-q)\hat{\Gamma}^k(q,\varepsilon)V^{ik}(q). \tag{19}$$

Substituting Eqs. (16), (17), and (12) into (19) and then substituting the expressions obtained for the matrix elements $\hat{\Sigma}$ into the kinetic equation (8), after extensive algebraic transformations we obtain the electron-electron collision operator in the form

$$\Big(\frac{\partial n_\varepsilon}{\partial t}\Big)_{\text{coll}}=\frac{-2}{\nu(\varepsilon)}\int_0^\infty\frac{d\varepsilon'}{2\pi}\int_{-\infty}^\infty\frac{d\omega}{2\pi}\int\frac{d^3q}{(2\pi)^3}\Big|\frac{4\pi e^2}{q^2\varepsilon(\omega,\mathbf{q})}\Big|^2\,\mathrm{Re}\frac{\zeta_{\varepsilon'+\omega}\tau_{\varepsilon'+\omega}\nu(\varepsilon'+\omega)}{1-\zeta_{\varepsilon'+\omega}}$$
$$\times\,\mathrm{Re}\frac{\zeta_\varepsilon\tau_\varepsilon\nu(\varepsilon)}{1-\zeta_\varepsilon}[n_\varepsilon n_{\varepsilon'}(1-n_{\varepsilon-\omega})(1-n_{\varepsilon'+\omega})-n_{\varepsilon-\omega}n_{\varepsilon'+\omega}(1-n_\varepsilon)(1-n_{\varepsilon'})]. \tag{20}$$

The operator (20) for a nondegenerate plasma can be simplified by utilizing the fact that the characteristic energy transfers ω are small in comparison with ε. Expanding in ω and neglecting n_ε in comparison with unity, we obtain

$$\Big(\frac{\partial n_\varepsilon}{\partial t}\Big)_{\text{coll}}=\frac{2}{\nu(\varepsilon)}\frac{\partial}{\partial\varepsilon}\int_0\frac{d\varepsilon'}{2\pi}\nu(\varepsilon')\nu(\varepsilon)\int_{-\infty}\frac{\omega^2d\omega}{2\pi}\int\frac{d^3q}{(2\pi)^3}\Big|\frac{4\pi e^2}{q^2\varepsilon(\omega,\mathbf{q})}\Big|^2$$
$$\times\,\mathrm{Re}\frac{\zeta_\varepsilon\tau_\varepsilon}{1-\zeta_\varepsilon}\mathrm{Re}\frac{\zeta_{\varepsilon'}\tau_{\varepsilon'}}{1-\zeta_{\varepsilon'}}\Big[n_{\varepsilon'}\frac{\partial n_\varepsilon}{\partial\varepsilon}-n_\varepsilon\frac{\partial n_{\varepsilon'}}{\partial\varepsilon'}\Big]. \tag{21}$$

Let us turn now to calculation of $D(\varepsilon,\varepsilon')$, which is equal to

$$D(\varepsilon,\varepsilon')=\frac{1}{\pi}\int\frac{\omega^2d\omega}{2\pi}\int\frac{d^3q}{(2\pi)^3}\Big|\frac{4\pi e^2}{q^2\varepsilon(\omega,\mathbf{q})}\Big|^2\mathrm{Re}\frac{\zeta_\varepsilon\tau_\varepsilon}{1-\zeta_\varepsilon}\mathrm{Re}\frac{\zeta_{\varepsilon'}\tau_{\varepsilon'}}{1-\zeta_{\varepsilon'}}. \tag{22}$$

As will be seen from what follows, values $q \gtrsim l^{-1}$ are important and $\omega \gtrsim 1/\max(\tau_\varepsilon,\tau_{\varepsilon'})$, and therefore in Eq. (22) we can assume that $\varepsilon(\omega,\mathbf{q})=1$. Taking into account the departure of $\varepsilon(\omega,\mathbf{q})$ from unity gives corrections of order $\varkappa l \ll 1$.

Using the explicit expression (13) for ζ_ε and introducing the dimensionless variables

$$\omega\tau_{\varepsilon'}=x,\quad ql=y,\quad \alpha=v'/v,$$

we write Eq. (22) in the form

$$D(\varepsilon,\varepsilon')=\tfrac{2}{3}e^4(vv')^{1/2}f(\alpha), \tag{23}$$

where

$$f(\alpha)=\frac{6}{\pi^2}\alpha^{3/2}\int_0^\infty\frac{dy}{y^2}\int_{-\infty}^{\infty}x^2dx\,\mathrm{Re}\frac{\zeta(x,y)}{1-\zeta(x,y)}\,\mathrm{Re}\frac{\zeta(\alpha x,y)}{1-\zeta(\alpha x,y)}, \tag{24}$$

$$\zeta(x,y)=\frac{i}{2y}\ln\frac{x+y+i}{x-y+i}. \tag{25}$$

It follows from Eqs. (24) and (25) that for $y\gg 1$

$$\mathrm{Re}\frac{\zeta(x,y)}{1-\zeta(x,y)}=\frac{\pi}{2y}\theta(y-|x|), \tag{26}$$

and therefore the integral (24) as usual diverges logarithmically at the upper limit, which in our designation is l/ρ_0.

If $\varkappa l\gg 1$, then Eq. (26) is valid over the entire region of integration and the integral (24) diverges logarithmically at both limits. Here the lower limit is $\varkappa l$, and as a result we obtain the usual Landau expression.

In the region $x\ll 1$, $y\ll 1$ we have

$$\mathrm{Re}\frac{\zeta(x,y)}{1-\zeta(x,y)}=\frac{3y^2}{9x^2+y^4}, \tag{26a}$$

which corresponds to the diffusion approximation. However, direct substitution of (26a) into (24) leads to importance of the region $x\sim 1$ and $y\sim 1$ and therefore it is not possible to calculate (24) for arbitrary α. However, it is possible to find the asymptote of (24) in the region $\alpha\ll 1$.

In this connection we note that the quantities $\zeta(x,y)/[1-\zeta(x,y)]$ have singularities in x all lying in the lower half plane: a pole at the point $x_0=i(y\cot y-1)$ for $y\leqslant\pi$ and branch points at $x_s=-i\pm y$. We represent $\mathrm{Re}\{\zeta(x,y)/[1-\zeta(x,y)]\}$ in the following form:

$$\mathrm{Re}\frac{\zeta(x,y)}{1-\zeta(x,y)}=\frac{1}{2}\left\{\left[\frac{\zeta(x,y)}{1-\zeta(x,y)}-\frac{i}{x}-\frac{iy^2}{3x^2(x-i\delta)}\right]+\left[\frac{\zeta^*(x,y)}{1-\zeta^*(x,y)}+\frac{i}{x}+\frac{iy^2}{3x^2(x-i\delta)}\right]\right\}. \tag{27}$$

Substituting (27) into (24), we obtain $f(\alpha)$ in the form of a sum of two integrals. The contour of integration over x for the integral containing the first square bracket we shall close in the upper half plane (Fig. 4), and that for the other integral we close in the lower half plane, but in such a way that these contours do not cross the cuts due to the function

$$\mathrm{Re}\frac{\zeta(\alpha x,y)}{1-\zeta(\alpha x,y)}.$$

Since these cuts are far from the real axis as the result of the smallness of α, their contribution to the integral can be neglected, and the entire contribution occurs from the pole $x=i\delta$ and the poles $x_0'=\pm x_0/\alpha$ in the function $\mathrm{Re}\{\zeta(\alpha x,y)/[1-\zeta(\alpha x,y)]\}$. Since values $x\sim 1$ are important, the characteristic values are $y\sim\sqrt{\alpha}\ll 1$ and $x_0'=iy^2/3\alpha$. When the velocities of the two particles are greatly different, the fast particle traverses the interaction region without collisions with atoms, but the slow particle diffuses through this region. The diffusion nature of the pole x_0' also corresponds to this. As a result we obtain

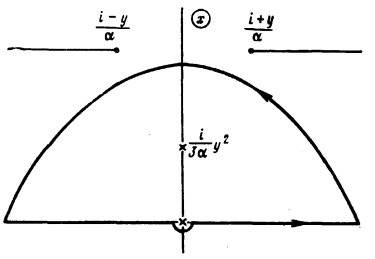

FIG. 4. Contour for integration over x in (24) for $\alpha\ll 1$.

$$f(\alpha)=\sqrt{3}\alpha+\alpha^{3/2}\ln(al/\rho_0), \tag{28}$$

where

$$\ln a=-\frac{6}{\pi}\int_0^\infty dz\left[\frac{\pi/6}{z+1}-\frac{1}{3}\frac{\mathrm{arctg}\,z}{z-\mathrm{arctg}\,z}+\frac{1}{z^2}\right],\quad a\approx 0.751. \tag{29}$$

For arbitrary α the function $f(\alpha)$ can be obtained numerically. Here it is convenient to separate the logarithmic contribution from (24) immediately. The remainder $\varphi(\alpha)$ converges satisfactorily at the upper limit and therefore does not depend on $l/\rho_0\gg 1$.

Thus, as was stated in the Introduction, for $\varkappa l\ll 1$ the Coulomb logarithm is cut off at the electron mean free path and, in addition, a nonlogarithmic contribution appears to the electron diffusion coefficient in energy from large impact distances.

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