CONCLUSION

Thus, the structures of shock waves propagating in a plasma, along the magnetic field depend on the values of the Mach numbers M_a and M. In the region outside the "wedge" in Fig. 1, the shock wave is gas-dynamic and the magnetic field plays no role, generally dropping out of the equations. Inside the wedge, the gas-dynamical shock wave is unstable, and the shock wave is magnetohydrodynamic with front width equal either to the diffusion length of the magnetic field, $c^2/4\pi\sigma v_1$ in the case of an unmagnetized plasma, or to a length that is characteristic for the electronic thermal conductivity l/ε for a magnetized plasma. The characteristic size of the oscillations of the magnetic field is connected with the Hall terms and the thermal emf, leading to a dispersion of the magnetosonic waves propagating at an angle to the magnetic field; the corresponding scale is $\Delta \approx M\delta l/M_a^2 = c/\omega_{pi}$.

In the present work, we have not considered the problem of the stability of the actuating sound wave, which is discussed in a series of theoretical works (Refs. 12–14). In particular, it has been shown by Roikhvarger and Syrovatskii[14] that while the actuating wave is evolutionary, i.e., there exists for it a unique solution of the problem of small perturbations, the actuating shock wave is non-evolutionary in the linear approximations, i.e., i is unstable to the spontaneous emission of Alfven waves. The instability of the actuating shock wave is evidently connected with the fact that azimuthal symmetry of the original unperturbed flow is disrupted in it. An arbitrarily small azimuthal asymmetry ahead of the shock front removes such a degeneracy in the intermediate region. The solution of the problem of the stability of the actuating shock wave in such an arrangement is the object of a separate paper.

In conclusion, I express my gratitude to A. L. Velikovich for numerous discussions.

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Scattering and interaction of sound with sound in a turbulent medium

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The effect of developed hydrodynamic turbulence on sound is studied. The correlation time and length of the acoustic field, the isotropization length and the frequency diffusion coefficient for the acoustic wave packet are calculated. The region of applicability of the kinetic equation for sound with a linear dispersion law are found. The parameter $k_s LM$ (k_s is the sound wave vector, L is the energy-containing scale, M is the Mach number) is of interest in principle for solution of the aforementioned problems. Precisely this parameter determines whether the second-order perturbation theory is sufficient or an infinite set of diagrams must be summed (*i.e.*, transport must be taken into account) in studies of the interaction between sound and hydrodynamic turbulence.

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INTRODUCTION

Various aspects of the problem of the interaction of sound with hydrodynamic turbulence have been studied in a number of researches.[1-5] Thus, the propagation of sound in a turbulent atmosphere was considered in the work of Tatarskiĭ[2] under conditions in which the principal role is played by processes of elastic scattering of a monochromatic sound wave; in our previous work,[3] processes of absorption and emission of sound by homo-

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geneous turbulence were considered. In the present paper we consider the evolution of sound packets (acoustic turbulence) in a medium with homogeneous isotropic hydrodynamic turbulence. For definiteness, we assume that its spectrum is a Kolmogorov one: $J_k \sim k^{-11/3}$ in the inertial interval $L > k^{-1} > l_0$, where L and $l_0 = L\ \mathrm{Re}^{3/4}$ are the external and internal turbulence scales. At low sound intensities, we cannot consider processes of the interaction of sound with sound, and are limited to the study of the role of processes of interaction of sound with turbulence.

In Sec. 2, we consider the scattering of sound in a turbulent medium. As is well known,[2,4] these processes are almost elastic and lead to the isotropization of the acoustic packet with respect to directions. For very narrow packets ($L\Delta k_s < 1$), the process of scattering from vortices of energy-containing scale L with characteristic time

$$\tau_r^{-1} \approx k_s v_T (k_s LM), \tag{1}$$

are important. Here k_s is the acoustic wave vector, v_T is the characteristic value of the turbulence velocity, $M = v_T/c_s$ is the Mach number. Scattering takes place here at the small angle $\Delta\theta \approx (k_s L)^{-1}$. But this does not mean that the evolution of broad packets $L\Delta k_s \gg 1$ can be considered in the differential approximation. The fact is that the small-angle scattering does not materially change the shape of the broad packet and the basic role is played by scattering through angles of the order of its width. As a result, the characteristic time of change of the width of the packet increases and, in place of (1) we obtain

$$\tau^{-1}(\Delta k_s) \approx k_s v_T (k_s LM)(\Delta k_s L)^{-5/3}. \tag{2}$$

The total time of isotropization τ_{is} is determined by the evolution of the packet at the last stage, when $\Delta k_s \approx k_s$, whence

$$\tau_{is}^{-1} \approx \frac{v_T}{L} M (k_s L)^{1/3}. \tag{3}$$

This expression is valid if k_s lies in the inertial interval of scales $l_0 < k_s^{-1} < L$. At $k_s L < 1$, the scattering processes are strongly suppressed because of the small intensity of vortices with $k_T L < 1$; in the case $k_s l_0 \gg 1$, the scattering takes place at a small angle and the differential approximation is valid; the time of isotropization is determined by the scattering from vortices of scale l_0:

$$\tau_{is}^{-1} \approx \frac{v_T}{L} M\, \mathrm{Re}^{-1/4}. \tag{4}$$

Within the isotropization time, the sound is not able to give up the turbulence energy because the time of sound absorption by the turbulence τ_{dis}, which is calculated in Ref. 3, turns out to be very large:

$$\tau_{dis}^{-1} \approx v_T M^2/L. \tag{5}$$

It is interesting that τ_{dis} does not depend on the sound wave vector; therefore the initial shape of the sound energy distribution function over the frequencies does not change in the absorption process. The frequency evolution of the acoustic packet of low intensity is therefore determined by the inelastic part of the scattering of sound by the vortices. As is shown in Sec. 2, under the conditions $l_0 < k_s^{-1} < L$, the characteristic time of frequency evolution τ_{diff} is

$$\tau_{diff}^{-1} \approx \frac{v_T}{L} M^3 (k_s L)^{1/3} \quad \text{at} \quad k_s LM < 1, \tag{6}$$

$$\tau_{diff}^{-1} \approx \frac{v_T}{L} M^{7/6} (k_s L)^{-1/6} \quad \text{at} \quad k_s LM > 1. \tag{7}$$

The time τ_{diff} was calculated earlier in the work of Krasil'nikov and Pavlov[4] under the conditions $k_s LM < 1$, $k_s l_0 > 1$. With increase in the sound intensity, the necessity arises of taking into account the interaction of sound with sound (ss interaction) and the problem is how this is to be done. It is known that the approximation of almost random phases (the kinetic equation) is inapplicable for the description of acoustic turbulence in the case of a linear dispersion law $\omega_k = c_s k_s$. The fact is that all the waves propagating in one direction have the same velocity, and the interaction between them leads to a strong phase correlation. In the presence of dispersion, a spreading of the acoustic packet takes place and the kinetic equation is applicable if the time of ss-interaction

$$\tau_{ss}^{-1} \approx k_s c_s E_s / \rho_0 c_s^2 \tag{8}$$

is longer than the time of randomization of the phases in the packet τ_d: $\tau_d^{-1} = \omega''(\Delta k^2)$. Another reason exists for the randomization of the phases for sound in a turbulent medium—its scattering from the random vortex field. It is therefore natural to estimate the time of randomization from the time of scattering of the sound by the vortices, i.e., to assume $\tau_d^{-1} = \tau_r^{-1} = (k_s L)(k_s LM)$ [(see (1)]. Thus the kinetic equation is applicable if $\tau_{ss} > \tau_r$, i.e.,

$$E_s < \rho_0 v_T^2 (k_s L). \tag{9}$$

In the region $k_s LM < 1$ this criterion is obtained in Sec. 3 by the analysis of the diagram series for renormalization of the vertex that describes the interaction of sound with sound. At $k_s LM > 1$, this analysis leads to another criterion for the applicability of the kinetic equation

$$E_s < \rho_0 v_T^2 M^{-1} (k_s LM)^{-1/4}. \tag{10}$$

Let us clarify the reason for this difference. The parameter $k_s LM$ has the meaning of a phase lag $\Delta\varphi$ over the distance L, which arises because of its interaction with the vortex velocity field of scale L. At $k_s LM < 1$, $\Delta\varphi < 1$ and the time of destruction of the phase correlation τ_{cor} is determined by the random distribution of the phase over a large number of vortices; at $k_s LM > 1$, the phase shift over the path L is large and the destruction of the correlations takes place over a distance Λ_{cor} that is smaller than L. We can therefore assume that the acoustic packet is transported as a whole in the almost homogeneous velocity field of the large-scale vortices; the time Λ_{cor} is determined by the Doppler effect from these vortices and $\tau_{cor} \approx (k_s v_T)^{-1}$.

The transport of the packet as a whole does not destroy the correlation of phases between the waves inside the packet and therefore the criterion of applicability of the kinetic equation is not determined by the time τ_{cor}. To obtain this criterion, as we have shown in Sec. 3, it is necessary to compare the interaction length

TABLE I. Characteristic frequencies describing the interaction of sound with hydrodynamic turbulence.

Frequency of the process	Region of applicability		
	$L^{-1} < k_s < (LM)^{-1}$	$(LM)^{-1} < k_s < l_0$	$k_s > l_0$
τ_d^{-1}	$k_s v_T (k_s LM)$	$k_s v_T$	
c_s/Λ_{cor}	$k_s v_T (k_s LM)$	$k_s v_T (k_s LM)^{-1/4}$	
τ_{is}^{-1}	$v_T L^{-1} (k_s L)^{1/3}$		$v_T L^{-1} M\ \mathrm{Re}^{-1/4}$
τ_{diff}^{-1}	$v_T L^{-1} M^3 (k_s L)^{1/3}$	$v_T L^{-1} M^{8/3} (k_s LM)^{-1/6}$	$v_T L^{-1} M^2 (\mathrm{Re}^{-1} k_s LM)^{1/3}$

Note. For comparison, we give the sound damping decrement: $\tau_{dis} \approx v_T L^{-1} M^2$ at $ML^{-1} < k_s < ML^{-1}\mathrm{Re}^{1/2}$, $\tau_{dis} \approx \nu k_s^2$ at $k_s > ML^{-1}\mathrm{Re}^{1/2}$.

$\Lambda_{int} = c_s \tau_{ss}$ with the distance Λ_{cor} in which the phase correlation in the wave is destroyed. The correlation length Λ_{cor}, as is shown in Sec. 2, is determined by the sound scattering from vortices of scale $\Lambda_{cor} < L$; determining it self-consistently, we obtain

$$\Lambda_{cor} \approx L(k_s LM)^{-3/4}. \tag{11}$$

The relation $\Lambda_{int} > \Lambda_{cor}$ is equivalent to the criterion (10). It is seen from all that has been said above that the interaction of the sound with the hydrodynamic turbulence is characterized on the whole by a set of times which describe the sound attenuation, the scattering and correlation properties of the acoustic packets and so on. For convenience in comparing these, we have collected the corresponding expressions for the frequencies τ^{-1} in a table.

In the region of weak turbulence [upon satisfaction of the criteria (9) and (10)] the interaction of sound with sound appears first in the frequency evolution of the packet. It becomes decisive when

$$E_s > \rho_0 v_T^2 M^2 (k_s L)^{-2/3}. \tag{12}$$

If now $\tau_{ss} < \tau_{dis}$, i.e.,

$$\begin{aligned} &E_s > \rho_0 v_T^2 M (k_s L)^{-1}, \quad M < k_s L < M\,\mathrm{Re}^{1/2}, \\ &E_s > \rho_0 v_T^2 (k_s L) M^{-1}\,\mathrm{Re}^{-1}, \quad k_s L > M\,\mathrm{Re}^{1/2}, \end{aligned} \tag{13}$$

then the role of turbulence is reduced merely to the isotropization of the packet and is determined in other respects by the kinetic equation of the interaction of sound with sound.

If we consider the problem of the spectrum of acoustic turbulence, then the Zakharov-Sagdeev spectrum $E_s \sim k^{-3/2}$ will be produced in the region (9), (10) and (13) as the exact solution of the kinetic equation. If the sound intensity is large and the criterion (9), (10) is violated, a region of strong acoustic turbulence begins, the effect of the vortices can be neglected[1)] and the Kadomtsev-Petviashvili spectrum $E_s \sim k^{-2}$ will be realized.

In the study of the questions enumerated above, we started from the Hamiltonian equations of motion

$$i\dot{a}_k = \delta H/\delta a_k^*, \quad i\dot{b}_k = \delta H/\delta b_k^* \tag{14}$$

for the vortical, a_k, and potential, b_k, barotropic flows of a compressible fluid. The Hamiltonian of the problem is given in Sec. 1—formulas (1.1)–(1.7). Here the diagram technique of Wyld is described briefly. We use it for the statistical description of nonlinear interacting fields a_k, b_k.

1. FUNDAMENTAL EQUATIONS

1. Dynamic description. The Hamiltonian

The Euler equations for barotropic flows of a compressible fluid can be represented in Hamiltonian form with the help of the Clebsch variables.[6] At small Mach numbers, we[3] have constructed a canonical transformation to new variables, in which the vortex motion of the liquid (a_k^*, a_k) and the potential motion (b_k^*, b_k) are separated to a maximum extent. In these variables, the equations of hydrodynamics have the form (14) with the Hamiltonian

$$H = H_s + H_t + H_{st}. \tag{1.1}$$

Here H_s is the Hamiltonian for sound in the quiescent liquid[7]:

$$H_s = \int \omega_k b_k^* b_k dk + \frac{1}{2}\int V_{1,23}(b_1^* b_2 b_3 + \text{c.c.})\,\delta(1-2-3)\,d1\,d2\,d3, \tag{1.2}$$

where

$$\omega_k = kc_s,\ V_{1,23} = \frac{1}{2(2\pi)^{3/2}}\left(\frac{c_s}{2\rho_0}\right)^{1/2}(k_1 k_2 k_3)^{1/2}\,[(\mathbf{n}_1\mathbf{n}_2 + \mathbf{n}_2\mathbf{n}_3 + \mathbf{n}_1\mathbf{n}_3) + (\gamma - 2)], \tag{1.3}$$

$\mathbf{n} = \mathbf{k}/k$, γ is the adiabatic exponent, $\gamma = c_p/c_v$; H_t is the Hamiltonian for turbulent pulsations of an incompressible liquid:[8]

$$H_t = \frac{1}{4}\int T_{12,34} a_1^* a_2^* a_3 a_4 \delta(1+2-3-4)\,d1\,d2d3d4, \tag{1.4}$$

in which

$$\begin{aligned} &T_{12,34} = \rho_0(\varphi_{13}\varphi_{24} + \varphi_{14}\varphi_{23}), \\ &\varphi_{kk'} = \frac{1}{2\rho_0}\frac{1}{(2\pi)^{3/2}}\left[\mathbf{k}+\mathbf{k}' - (\mathbf{k}-\mathbf{k}')\frac{k^2 - k'^2}{|\mathbf{k}-\mathbf{k}'|^2}\right]; \end{aligned} \tag{1.5}$$

H_{st} is the Hamiltonian of interaction of the sound with turbulence;

$$H_{st} = \int S_{12,34} a_1^* a_2 b_3^* b_4 \delta(1-2+3-4)\,d1\,d2\,d3\,d4$$
$$+ \frac{1}{4}\int W_{k,12|34}(b_k + b_{-k}^*)\,a_1 a_2 a_3^* a_4^* \delta(\mathbf{k}-1-2+3+4)\,d\mathbf{k}\,d1\,d2d3d4. \tag{1.6}$$

The first component describes the scattering of sound by the turbulence, the second, the processes of emission and absorption of sound by the turbulence. The matrix elements have the form

$$\begin{aligned} &S_{12,34} = \frac{1}{2}\frac{(k_3 k_4)^{1/2}}{(2\pi)^{3/2}}\varphi_{12}(\mathbf{n}_3 + \mathbf{n}_4), \\ &W_{q,12|34} = \frac{1}{(2\pi)^{3/2}}\left(\frac{2\rho_0 q}{c_s}\right)^{1/2}[(\varphi_{13}\mathbf{n}_q)(\varphi_{24}\mathbf{n}_q) + (\varphi_{23}\mathbf{n}_q)(\varphi_{14}\mathbf{n}_q)]. \end{aligned} \tag{1.7}$$

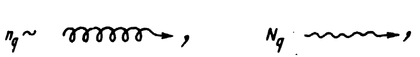
Components of the type $S^{(n)} a_1 a_2^* b^{n+2}$ and $W^{(n)} a^2 a^{*2} b^{n+1}$, which are not significant for what follows, are not written out in Eq. (1.7). They differ from Saa^*bb^* and $Wa^2a^{2*}b$ by the small parameter $b^n (k_s/\rho_0 c_s)^{n/2}$.

2. Statistical description

Just as before,[3] we use the canonical diagram technique of Wyld for the statistical description of the nonlinear fields a_k, b_k. This was analyzed in detail in Ref. 8. The averaged equations contain the pair correlations

n_q and N_q:

$$\delta(q-q')n_q=\langle a_q a_{q'}^*\rangle,\quad \delta(q-q')N_q=\langle b_q b_{q'}^*\rangle,$$



(1.8)

$$q=\mathbf{k},\ \omega,\quad \delta(q)=\delta(\omega)\delta(\mathbf{k})$$

as also the Green's functions G_q, g_q, which have the meaning of the linear response of the system to the forces F_q and f_q:

$$\delta(q-q')G_q=\langle\delta b_q/\delta F_{q'}\rangle,\quad \delta(q-q')g_q=\langle\delta a_q/\delta f_{q'}\rangle,$$

$$G_q \sim \text{- - - - - - -}\rightarrow,\qquad g_q \sim \longrightarrow. \tag{1.9}$$

These quantities satisfy the Dyson equations

$$G_q^{-1}=\omega-\omega_q-\Sigma_q,\quad g_q^{-1}=\omega-\sigma_q,$$
$$N_q=|G_q|^2\Phi_q,\quad n_q=|g_q|^2\varphi_q, \tag{1.10}$$

where $\Sigma_q, \sigma_q, \Phi_q, \varphi_q$ are the sums of the corresponding irreducible diagrams. In particular, $\mathrm{Im}\Sigma_q$ has the meaning of the damping decrement of the plane wave.

If the interaction is in some sense weak, then we can limit ourselves in the series for Σ and Φ to diagrams of second order in the vertices, which correspond to the kinetic-equation approximation for sound. In this case, Φ_q describes the arrival terms of this equation. For the study of sound propagation in a turbulent medium, this approximation frequently turns out to be insufficient because of the fact that the hydrodynamic turbulence is strong.

Difficulties develop here that are specific for the theory of strong hydrodynamic turbulence of an incompressible liquid, in particular, the long-wave divergences of the integrals, which are connected with the fact that the fundamental interaction reduces to the kinematic effect of transport of the small-scale vortices by large-scale motion. Since these difficulties have not been entirely overcome at the present time, we shall use an approximate description of hydrodynamic turbulence: we shall sum the most strongly diverging "transport" diagrams exactly as was done in the work of one of the authors,[9] after which the diagram interaction of the vortices will be taken into account in the "direct interaction approximation," limiting ourselves to some simple sequence of diagrams. In this approximation, the diagram equations have solutions in the form of a Kolmogorov spectrum:

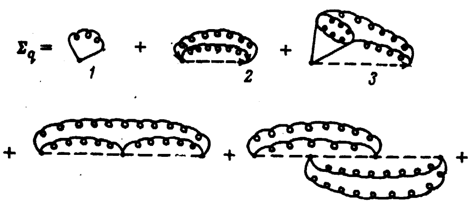
$$n_{\mathbf{k}\omega}=\langle\tilde n_{\mathbf{k},\omega-\mathbf{k}\mathbf{v}}\rangle_{\mathbf{v}},\quad g_{\mathbf{k}\omega}=\langle\tilde g_{\mathbf{k},\omega-\mathbf{k}\mathbf{v}}\rangle_{\mathbf{v}}, \tag{1.11}$$

where $\langle\ldots\rangle$ denotes averaging over the ensemble of turbulent velocity $\mathbf{v}(\mathbf{r}, t)$ at an arbitrary point $\mathbf{r}, t$, and

$$\tilde n_{\mathbf{k}\omega}=\rho_0 k^{-5}f\left(\frac{\omega L}{v_T(kL)^{2/3}}\right),\quad \tilde g_{\mathbf{k}\omega}=\frac{1}{\omega}g\left(\frac{\omega L}{v_T(kL)^{2/3}}\right). \tag{1.12}$$

2. SCATTERING OF SOUND IN A TURBULENT MEDIUM

1. Scattering of a plane wave. The damping decrement $\Gamma_\mathbf{k}=\Sigma''_{\mathbf{k},\omega_\mathbf{k}}$ of a plane wave in a turbulent medium appears first as a result of the direct absorption of sound by the turbulent pulsations and, second, because of scattering of the sound. The contribution to $\Sigma''_{\mathbf{k},\omega_\mathbf{k}}$ due to direct absorption of sound that arises from the diagrams containing the vertex W, was analyzed in detail previously.[3] The diagrams $\Sigma_{\mathbf{k},\omega}$, which describe the scattering of sound by turbulent pulsations, have the following structure:



(2.1)

Diagrams with a single vertex S (for example, 1) describe the Doppler shift of the frequency $k\langle v_T\rangle$ and are equal to zero in a system of coordinates where the liquid is at rest on the average.

In the region $k_s L<1$, diagrams of the series (2.1) make no contribution to $\Sigma_{\mathbf{k}\omega}$ because of the laws of energy-momentum conservation, under the assumption that there are no turbulent pulsations with scale $kL<1$.

At $1<k_s L<M^{-1}$, it is necessary to take into account only diagrams of the order of S^2: numbers 2, 3, etc., from series (2.1). It is not difficult to see that the sum of this series has the form

$$\Sigma^{(2)}=\frac{1}{4(2\pi)^3}\int I^{\alpha\beta}_{\mathbf{k}''\omega''}G_{\omega'\mathbf{k}'}\left(\frac{k^\alpha}{|\mathbf{k}|}+\frac{k'^\alpha}{|\mathbf{k}'|}\right)\left(\frac{k^\beta}{|\mathbf{k}|}+\frac{k'^\beta}{|\mathbf{k}'|}\right)$$
$$\times\delta(\mathbf{k}-\mathbf{k}'-\mathbf{k}'')\,\delta(\omega-\omega'-\omega'')\,d^3\mathbf{k}'\,d\omega'\,d^3\mathbf{k}''\,d\omega''. \tag{2.2}$$

Here $I^{\alpha\beta}$ is the pair correlator of the velocity:

$$\langle v_{\mathbf{k}\omega}^\alpha v_{\mathbf{k}'\omega'}^\beta\rangle=I_{\mathbf{k}\omega}^{\alpha\beta}\delta(\omega+\omega')\delta(\mathbf{k}+\mathbf{k}'). \tag{2.3}$$

In the calculation of the correction to the sound dispersion law $\Delta\omega_k=\mathrm{Re}\Sigma_{\mathbf{k},\omega_\mathbf{k}}$, it is necessary to substitute the complete Green's function in (2.2). As a result,

$$\Delta\omega_\mathbf{k}=kv_T M^2\ln[(kL)^2 M]. \tag{2.4}$$

We now estimate $\mathrm{Im}\Sigma_{\mathbf{k},\omega_\mathbf{k}}$ on the mass shell, substituting the bare Green's function in (2.2). The principal contribution to the integration over k'' is made by the energy-containing region $k''\approx L^{-1}$. As a result,

$$\Gamma_\mathbf{k}=-\mathrm{Im}\,\Sigma_{\mathbf{k},\omega_\mathbf{k}}\approx kv_T(kLM). \tag{2.5}$$

In the scattering processes, the frequency and, consequently, the energy of the sound is conserved (the inelasticity is small in the parameter M), broadening of the Green's function and of the pair correlator in ω is due to loss of phase coherence. Strictly speaking, the time $\tau=\Gamma^{-1}$ is the time of phase correlation and is not at all connected with the sound energy dissipation in the turbulent medium.

We now obtain the estimate of (2.5) qualitatively, considering the sound scattering from vortices of scale k_T^{-1}. In a single scattering act, the phase of the wave is altered, because of the Doppler frequency shift, by an amount

$$\Delta\varphi\approx\frac{k_s}{k_T}M(k_T L)^{-1/3},$$

where $v_T(k_T L)^{-1/2}$ is the circumferential velocity of the vortex. In a time τ, $N=k_T c_s\tau$ acts of scattering take place and the phase change increases by $N^{1/2}$. The time of loss of the correlations in scattering from vortices of scale k_T^{-1} is determined from the condition $\Delta\varphi N^{1/2}\approx 1$:

$$\Gamma_{k_s}(k_T)=\tau_{k_s}^{-1}(k_T)\approx k_s^2 vLM/(k_T L)^{1/3}. \quad (2.6)$$

The principal contribution to $\Gamma_k(k_T)$ is made by vortical motion with characteristic scale $k_T \sim L^{-1}$, which corresponds to the estimate (2.5).

In the shorter-wave region, when $(k_s LM)>1$, the physical picture of the scattering of the wave is changed, since the Doppler phase shift by a single vortex of energy-containing scale exceeds π. Formally, this is expressed by the fact that the diagrams (2.1) with two, three and more vertices S turn out to be of the same order of magnitude. The principal contribution to $\Sigma_{k\omega}$ is made by those diagrams for which the external momentum is carried along the "backbone" of the sound Green's function, oriented from left to right, while integration is carried out over the remaining turbulent lines n_q, g_q in the energy-containing region $Lk\approx 1$. This sequence is summed by the method described in Ref. 9. As a result,

$$G_{k\omega}=\left\langle\frac{1}{\omega-\omega_k-\mathbf{k}\mathbf{v}+i\delta}\right\rangle_{\mathbf{v}}. \quad (2.7)$$

This Green's function describes the transport of the field of the sound wave as a whole with random velocity.

The width in ω determines the lifetime of the phase correlations $\tau_{cor}^{-1}\approx k_s v_T$. However, $\Lambda_{cor}\neq c_s\tau_{cor}$, because the sound wave vector is conserved in the approximation (2.7) corresponding to transport of the spatially homogeneous velocity field. Therefore, the correlation function $K(R,0)$ is proportional to $\exp(ik_s R)$ and the spatial correlation of the phases is not destroyed.

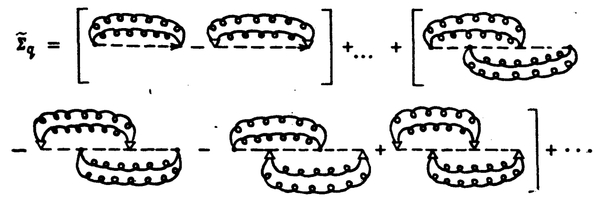
2. *Spatial correlation length.* At $k_s LM \ll 1$, the distance at which the phase of the wave falls off by π,

$$\Lambda\approx c_s\tau_{cor} \quad (2.8)$$

is greater than the energy-conserving scale L and is the correlation length Λ_{cor}. At $kLM>1$, we cannot assume the quantity (2.8) to be the correlation length, as has already been noted above, since the almost homogeneous transport of the scale (2.8) by vortices of the scale L, making the principal contribution to τ_{cor}, does not lead to a destruction of the spatial correlations. We refine the approximation (2.7) seeking $G_{k\omega}$ in the form

$$(G_{k\omega}=\langle[\omega-\omega_k-\mathbf{k}\mathbf{v}-\tilde{\Sigma}_{k,\omega-\mathbf{k}\mathbf{v}}]^{-1}\rangle_{\mathbf{v}}. \quad (2.9)$$

The diagram series for $\tilde{\Sigma}$ has the form



where the vertex-triangle is $S_{12,34}\delta(\mathbf{k}_2-\mathbf{k}_4)\delta(\omega_2-\omega_4)$ and differs from the usual vertex-point, which has the form $S_{12,34}\delta(q_1-q_2-q_3-q_4)$. Subtraction leads to cancellation of terms in $\tilde{\Sigma}$ from the region of integration over $k_T<\varkappa$ which is determined from the condition

$$\varkappa c_s\approx\Sigma''_{k\omega_k}. \quad (2.10)$$

With account of this, estimate of Σ from the first bracket gives

$$\Sigma_{k\omega_k}\approx(k_s v_T)^2/(\varkappa L)^{2/3}\,\Sigma_{k,\omega_k}. \quad (2.11)$$

All the remaining diagrams here are of the same order of magnitude. Thus,

$$\Sigma_{k,\omega_k}\approx(kv_T)(kLM)^{-1/4}. \quad (2.12)$$

This contribution to $\tilde{\Sigma}$ arises because of the spatial inhomogeneity of the velocity field and therefore leads to a destruction of the spatial correlations:

$$\Lambda_{cor}(k)\approx c_s\Sigma''^{-1}_{k,\omega_k}\approx L(kLM)^{-3/4}. \quad (2.13)$$

We note that $\Lambda_{cor}\approx\varkappa^{-1}$, where $\varkappa$ is determined from (2.10). Thus, Λ_{cor} is of the order of the size of the vortices which make the principal contribution in the destruction of the spatial correlations.

We now obtain an estimate for Λ_{cor} qualitatively. It is obvious that vortices with size $k_T\Lambda_{cor}<1$ do not destroy the phase correlations at a distance Λ_{cor}. Their effect reduces to the uniform transport of the sound field in the volume Λ_{cor}^3, while the shortwave vortices with k_T $k_T\Lambda_{cor}>1$ weakly disrupt the correlations [see (2.60)]. Therefore, we can understand why the basic contribution to Λ_{cor} is made by vortices with size $k_T\Lambda_{cor}\approx 1$.

From the qualitative estimate of (2.6), we obtain the following for the correlation length:

$$\Lambda_{cor}(k_T)=c_s\tau_{k_s}(k_T)=\frac{L}{(k_s L)^2}\frac{(k_T L)^{1/3}}{M^2}.$$

Substituting Λ_{cor}^{-1} as k_T and solving the resultant equation relative to Λ_{cor}, we obtain the desired estimate (2.13).

3. *Evolution of acoustic packet in direction.* With the help of the nonstationary kinetic equation

$$\tfrac{1}{2}dN_k/dt=-\Gamma_k N_k+\pi\Phi_{k,\omega_k}, \quad (2.14)$$

we obtain the evolution of the acoustic packet in the linear approximation from the intensity of the sound. At $k_s L<1$, there is no sound scattering at all; in the range $M<k_s LM<1$, as has already been noted, we can limit ourselves to diagrams that are quadratic in the vertices S. Using the expression (2.2) for Γ_k and summing the diagram series for $\Phi_{k\omega}$ in analogous fashion,

we obtain

$$\frac{dN_k}{dt}=\frac{1}{16\pi^2}\int I^{\alpha\beta}_{k'',\omega''}\left(\frac{k^\alpha}{|\mathbf{k}|}+\frac{k'^\alpha}{|\mathbf{k}'|}\right)\left(\frac{k^\beta}{|\mathbf{k}|}+\frac{k'^\beta}{|\mathbf{k}'|}\right)(N_k-N_{k'})$$
$$\times\delta(\mathbf{k}-\mathbf{k}'+\mathbf{k}'')\delta(\omega_k-\omega_{k'}+\omega'')d\mathbf{k}'\,d\mathbf{k}''\,d\omega''. \quad (2.15)$$

This equation was discussed previously in the work of Krasilnikov and Pavlov.[4] For isotropic turbulence in the approximation $M\ll 1$, this can be simplified:

$$\frac{dN_k}{dt}=\frac{1}{16\pi^2}\int I_{k'',\omega''}\left(4-\frac{k''^2}{kk'}\right)(N_k-N_{k'})\delta(\mathbf{k}-\mathbf{k}'+\mathbf{k}'')$$
$$\times\delta(\omega_k-\omega_{k'}+\omega'')d\mathbf{k}'\,d\mathbf{k}''\,d\omega''. \quad (2.16)$$

At the beginning of this section, in the calculation of the damping decrement of a plane wave in scattering processes, it was shown that the principal contribution to Γ_k is made by large-scale vortices, which lead to scattering at small angles of the order of $(kL)^{-1}$. Therefore, we can show that the diffusion approximation is valid over the angles,[4] i.e., the process of scattering at large angles is the result of small angle scattering in stages. However, analysis of Eq. (2.16) shows that the diffusion approximation over the angles is valid if the sound wavelength is greater than the internal scale of the turbulence. The fact is that the contribution of the scattering at small angles $\Delta\theta$ and the evolution of the acoustic packet of width Δk_s is strongly suppressed (as a result of the scattering, the wave does not emerge from the packet) if $\Delta\theta < (\Delta k_s/k_s)$. Therefore, scattering from vortices with $k_T L > 1$ begins to play a role in the inertial interval. As a result, the evolution is essentially determined by the scattering at angles $\Delta\theta$ of the order of the width of the acoustic packet: $\Delta\theta \approx \Delta k_s/k_s$. Formally, all this means that a strong cancellation takes place in the integral (2.16) in the region $k_T'' < \Delta k_s$ and the fundamental contribution is made by $k_T'' \approx \Delta k_s$. With account of this, the damping decrement $\Gamma_{ks}(k_s)$ of a packet of width Δk_s turns out to be of the order of

$$\Gamma_{ks}(\Delta k_s) \approx k_s^2 vLM(\Delta k_s L)^{-5/3}. \tag{2.17}$$

This result is easily obtained from qualitative considerations by considering scattering from vortices of scale $k_T \approx \Delta k_s$. Taking it into account that the peripheral speed in these vortices is $v_{k_T} = v_T (k_T L)^{-1/2}$, it is not difficult to obtain an estimate for the scattering angle from a single vortex: $\Delta\theta_{ks} \approx M(k_T L)^{-1/2}$. By virtue of the random character of the scattering some $k_T^2/(k_s \Delta\theta_{ks})$ acts are necessary for scattering at an angle of the order of k_T/k_s. Then the length of the path of the sound relative to the scattering processes from the vortices k_T has the form

$$\Lambda_{ts}(k_T) \approx \frac{k_T}{k_s^2 \Delta\theta_{k_s}^2} \approx \frac{k_T}{k_s^2} \frac{(k_T L)^{2/3}}{M^2}. \tag{2.18}$$

Its corresponding damping decrement, $\Gamma_{k_s}(\Delta k_s) = c_s \Lambda^{-1}(k_T)$, is identical with the estimate (2.17). It is clear from this consideration that we must take L^{-1} as k_T if $\Delta k_s < L^{-1}$. Then (2.17) is identical with the damping decrement of a plane wave (2.5). At $\Delta k_s > L^{-1}$, $k_T \approx \Delta k_s$. For broad angle packets $\Delta k_s \sim k_s$ and

$$\Gamma_{k_s} \approx k_s vM(k_s L)^{-2/3}. \tag{2.19}$$

This estimate determines the isotropization time of the packet. Its corresponding path length determines the distance Λ_{is} over which the direction of propagation changes by an angle of order π:

$$\Lambda_{is} \approx LM^{-2}(k_s L)^{-1/3}; \tag{2.20}$$

if $l_0 \Delta k > 1$ (l_0 is the internal scale), then, in place of (2.20) we have

$$\Lambda_{ts} \approx LM^2 (L/l_0)^{1/3} \approx LM^{-2}\,\mathrm{Re}^{-1/4}.$$

Here and in (2.20), we have assumed that the isotropization length is less than the viscous damping length.

4. *Criterion for the transparency of the turbulent layer.* In propagating through a turbulent layer, sound is chiefly absorbed by two mechanisms; in the region $M < k_s L < M\,\mathrm{Re}^{1/2}$, as a consequence of direct absorption of the sound by the turbulence, with decrement $\Gamma_k = v_T L^{-1} M^2$; in the region $k_s L > M\,\mathrm{Re}^{1/2}$, because of viscosity and thermal conductivity of the medium. If $k_s L < 1$, then the sound is propagated in a straight line and a layer of thickness

$$\Lambda > \Lambda_{ts} = \frac{c_s}{\Gamma_{ts}} = \begin{cases} LM^3 & k_s L < M\,\mathrm{Re}^{1/2} \\ L(k_s L)^{-2} M^{-1} & k_s L > M\,\mathrm{Re}^{1/2} \end{cases}$$

turns out to be opaque because of the sound absorption.

At $L^{-1} < k < l_0^{-1}$ it is necessary to take into account processes of elastic scattering of the sound, which lead to a random walk of the phonons in the turbulent medium. After traveling a path $\Lambda_{tr} \gg \Lambda_{is}$, the phonon moves away from the initial point to a distance of the order of

$$\Lambda_{ts}(\Lambda_{dis}/\Lambda_{ts})^{1/2} \approx (\Lambda_{ts}\Lambda_{dis})^{1/2}.$$

Thus the turbulence will be opaque if

$$\Lambda > \Lambda_{tr} = (\Lambda_{ts}\Lambda_{dis})^{1/2} = \begin{cases} LM^{-5/2}(kL)^{-1/6}, & 1 < kL < M\,\mathrm{Re}^{1/2} \\ LM^{-3/2}(kL)^{-7/6}, & kL > M\,\mathrm{Re}^{1/2} \end{cases}. \tag{2.21}$$

5. *Frequency evolution of the acoustic packets.* Since isotropization takes place rapidly, there is sense in limiting ourselves to isotropic distributions in the study of the frequency evolution of the wave packets. The process of sound scattering from turbulence is almost elastic, $\Delta\omega \approx M\omega$; therefore the differential frequency approximation is valid. Averaging Eq. (2.15) over the directions, we obtain

$$\frac{\partial N_k}{\partial t} = \frac{\partial}{\partial \omega_k} D_k \frac{\partial}{\partial \omega_k} N_k.$$

Here

$$D_k = \frac{k^2}{16\pi c^3} \int_0^{2k} dk' \int_{-\infty}^{\infty} d\omega'\, k'\omega'^2 \left(4 - \frac{k'^2}{k^2}\right) I_{k'\omega'}.$$

At $k_s l_0 < 1$, this expression differs from the expression for D_k obtained in Ref. 4 with another upper limit of integration in k' ($2k$, and not ∞): it is obvious that the vortices with $k' > 2k_s$ cannot participate in almost elastic scattering because of the energy-momentum conservation laws. Taking it into account that the basic contribution to the integral is made by the region $k' \sim k_s$, we obtain the estimate

$$D_k \approx \omega_{k_s}^3 M^4 (kL)^{-2/3}.$$

This diffusion coefficient corresponds to a characteristic time of change of the distribution function $\tau_{diff} = \Gamma_{diff}^{-1}$, where

$$\Gamma_{diff} \approx D_k / c_s^2 k^2 \approx \omega_s M^4/(kL)^{2/3}. \tag{2.22}$$

We obtain this result qualitatively by considering the interaction with vortices of scale k_s^{-1}. The change in the frequency in the scattering is connected with motion of the vortices relative to the observer. In a single act,

$$\frac{\Delta\omega_s}{\omega_s} \approx \frac{\tilde{v}_T}{c_s} \frac{M}{(k_s L)^{1/2}}, \tag{2.23}$$

where $\tilde{v}_T$ is the fluctuation of the velocity of vortex motion over the distance of phase correlation of the sound. At $kLM < 1$, it is obvious that $\tilde{v}_T = v_T$ and $\Delta\omega_s/\omega_s \approx M^2/(k_s L)^{1/2}$. Thus, $\Delta\omega/\omega$ is smaller by a factor of M than $\Delta k/k$ in a single act (2.23). Therefore, Γ_{diff} is smaller by a factor M^2 than the isotropization decrement (2.19):

$$\Gamma_{diff} \approx \frac{v_T}{L} M^3 \mathrm{Re}^{-1/4} \quad \text{at} \quad kl_0 > 1.$$

At $k_s LM > 1$ we can not restrict ourselves to the diagrams of the kinetic equation (2.15), i.e., to diagrams that are proportional to S^2. We sum the entire diagram series with the help of the method described in Sec. 2. As a result, we find that the estimates (2.19) and (2.20) for the elastic part of the scattering do not change. Actually, it follows from the qualitative considerations given above that the physical picture of elastic scattering does not depend on the parameter kLM.

For the diffusion coefficient in the range $k_s LM > 1$, we get

$$D_k \approx \omega_s^{\,3} M^{7/4} (kL)^{1/4}. \qquad (2.24)$$

The difference of (2.24) from (2.22) arises because at $kLM > 1$ we have

$$\tilde{v}_T \approx v_T (kLM)^{-1/4}; \quad D_k \approx \frac{\omega_k^2 v_T}{L} M^2 \left(\frac{kLM}{\mathrm{Re}}\right)^{1/2} \quad \text{at} \quad kl_0 > 1.$$

3. INTERACTION OF SOUND WITH SOUND IN A TURBULENT MEDIUM

In the study of the evolution of the sound field, we have up to now neglected the interaction of sound with sound (ss interaction) in comparison with the interaction with hydrodynamic turbulence. The characteristic inverse time of the ss interaction Γ_{ss} is easily estimated from the kinetic equation for the sound:

$$\Gamma_{ss} \approx N_{k_s} k_s^{\,5} \rho_0^{-1} \approx (k_s c_s) \frac{E_s}{(\rho_0 c_s^{\,2})}; \qquad (3.1)$$

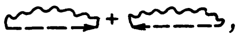
at low sound intensities, Γ_{ss} is small.

For isotropic acoustic turbulence, the approximation used above (which is linear in the sound amplitude) is valid if Γ_{ss} is less than the sound damping decrement due to the turbulence Γ_{diss} (2.20), i.e.,

$$\frac{N_k k^5}{\rho_0} < \frac{v}{L} M^2 \quad \text{or} \quad E_s < \rho_0 v_T^{\,2} \frac{M}{kL}, \qquad (3.2)$$

where E_s is the sound energy density.

In the opposite case, the ss scattering is decisive. Limiting ourselves to the first diagrams for Σ_k and Φ_k:

$\Sigma_k =$ , $\Phi_k =$ ,

we obtain the kinetic-equation approximation for the sound.[7] This approximation is valid if the subsequent diagrams, which renormalize the vertex of the ss interaction, are small. For sound with dispersion, propagating in a nonturbulent medium,[8] we have

$\Gamma^{ss}_{123} =$ 

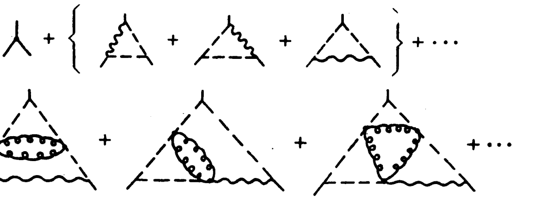
For an acoustic packet of width Δk, calculation of the diagrams yields

$$\Gamma_{123} = V_{123}(1 + \Gamma_{ss}/\omega''(\Delta k)^2 + \ldots).$$

The well-known criterion for the applicability of the kinetic equation for sound then follows: $\Gamma_{ss} < \omega''(\Delta k)^2$.

It can be shown that the length of the ss interaction, $\Lambda_{ss} = c_s \Gamma_{ss}^{-1}$ should be large in comparison with the correlation length for the phase: $\Lambda_{cor} \approx c_s/\omega''(\Delta k)^2$. The phase mismatch of the waves in the packet arises from the fact that waves with different k propagate with different group velocities $\Delta v_{gr} = \omega'' \Delta k$.

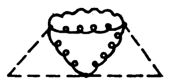
In a turbulent medium, the series for Γ^{ss}_{123} has the following form:

$\Gamma^{ss}_{123} =$ 

At $kLM < 1$, the diagrams which contain the turbulent lines have an additional smallness parameter: $(k_s LM)^2$. Calculation of the diagram in the curly brackets with substitution of the solid lines $N_{k\omega}$ and $G_{k\omega}$, which take into account the effect of the turbulence, yields the following criterion for the applicability of the kinetic equation:

$$\Gamma_{ss} \ll \Gamma \approx k_s v k_s LM. \qquad (3.3)$$

In the range $k_s LM > 1$, the situation becomes more complicated and, as usual, it is necessary to take into account the whole series of diagrams in powers of the hydrodynamic velocity. It is also impossible to establish the fact that the role of hydrodynamic turbulence reduces to the renormalization of the functions $N_{k\omega}, G_{k\omega}$ and the vertex of the ss interaction. For example, diagrams of the type

$\Sigma_{k\omega} =$  $+ \cdots$

are important. Nevertheless, the whole series of the "kinetic" equation can be summed by the method described in Ref. 9, through transition to the "randomly moving reference system," which eliminates from the Green's function the Doppler shift $\mathbf{k}\cdot\mathbf{v}$ from vortices with scale greater than Λ_{cor}. As a result, we obtain the following criterion of applicability of the kinetic equation for sound in a turbulent medium in the range $k_s LM > 1$:

$$\Gamma_{ss} < \Sigma \approx \frac{c_s}{L}(kLM)^{3/4}. \qquad (3.4)$$

The inequalities (3.3) and (3.4) have a simple meaning—the length of the ss interaction should be large in comparison with the length of the phase correlation. In this case, the phases have time to become stochastic and the kinetic equation is valid. We emphasize that the width of the acoustic packet does not enter into the criteria (3.3) and (3.4). The randomization of the phase in a turbulent medium at a distance Λ_{cor} takes place even for a single wave.

It is of interest to express the criteria (3.3) and (3.4) in terms of the energy density of the acoustic field:

$$E_s < (\rho_0 v_T^{\,2}) kL, \quad M < kLM < 1,$$
$$E_s < (\rho_0 v_T^{\,2}) (kLM)^{-1/4} M^{-1}, \quad kLM > 1. \qquad (3.5)$$

In conclusion, we discuss the problem of the spectrum of the acoustic turbulence excited by an external source. If the sound intensity satisfies the criterion (3.2), then no energy redistribution over the spectrum takes place, since the sound attentuation due to turbulence exceeds the inelastic scattering of sound (3.1). In this case, the acoustic spectrum outside the pumping region will be "equilibrium"[3]: $E(\omega_s)\sim\omega_s^{-7/2}$. If the sound intensity exceeds the threshold (3.2), but all the criteria (3.5) are satisfied, then the interaction of the sound with sound can be studied in the approximation of the kinetic equation, and the sound interaction with the turbulence leads to isotropization of the acoustic spectrum with a time $\tau^{-1}\approx k_s vM(k_s L)^{-2/3}$ [see (2.19)]. In this case, the isotropic spectrum of Zakharov-Sagdeev is established[7]

$$E_k\sim k^{-3/2}$$

as the exact solution of the kinetic equation.

At higher intensities, the criterion (3.5) is violated and we fall into the region of strong acoustic turbulence where the Kadomtsev-Petviashvili spectrum exists.[10]

[1)] Another viewpoint is expressed in Ref. 5, the authors of which assume that the Zakharov-Sagdeev spectrum has a much wider region of existence than the region of applicability of the kinetic equation.

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Translated by R. T. Beyer

Equation of state of molecular hydrogen. Phase transition into the metallic state

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The parameters of semi-empirical model equations of state of the solid and liquid phases of molecular hydrogen are obtained on the basis of experimental data on the isentropic compression and thermodynamic properties at atmospheric pressure. It follows from the equation of state of the molecular phases, which is derived in the present paper, and from the equation of state of the metallic phase as given by Kagan, Pushkarev, and Kholas [Sov. Phys. JETP 46, 511, (1977)] that the phase transition into the metallic state at $T = 0$ K can take place at a pressure from 2 to 4 Mbar. Two variants of the phase diagram of the solid and liquid molecular and metallic hydrogen are calculated.

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1. INTRODUCTION

One of the essential problems in the study of metallic hydrogen, is the determination of the state parameters at which the molecular hydrogen becomes metallic. The rather wide range of theoretical estimates obtained until recently for the possible transition pressures was due mainly to the uncertainty of the thermodynamic potential of molecular hydrogen, which could not be calculated reliably enough theoretically. Therefore in this paper, just as before,[1,2] the equation of state of the molecular phase at high pressures, and its zeroth isotherm, are determined from the aggregate of a variable and experimental data, while the theoretical concepts are used to construct a physically substantiated model-deduced semi-empirical equation of state.

In contrast to the preceding papers,[1,2] we take the rotation into account in the equation of state of molecular phase. The zeroth isotherm and the zero-point oscillation energy in the equation of state of the metallic hydrogen are taken in accord with the data of Kagan,

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