Resistive states in superconductor junctions

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Stationary nonequilibrium states in tunnel junctions, due to injection of quasiparticles, are considered. It is shown that under certain conditions a stationary structure of the electronic system, which is spatially inhomogeneous over the film area, can be produced under certain conditions in the flat films that form the junctions. The coordinate-dependent quantities in this case are the order parameter and chemical potential, so that this inhomogeneous state is resistive.

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The investigation of nonequilibrium states brought about by injection in tunnel junctions of superconductors continues to attract the attention of experimenters.[1,2] Kommers and Clarke[1] used a tunnel procedure to observe directly the change of the energy gap under the influence of excitation injection, and found qualitative agreement with the predictions of the theory.[3] Dynes, Nasaganamurti, and Garno[2] described a curious phenomenon which they observed in aluminum junctions. Spatial inhomogeneity was produced in a planar junction only at injection currents above a certain value. It appears that the transition of a superconducting state with a relatively large disequilibrium level into an inhomogeneous state is a rather universal fact. Such a transition was observed also in the experiments of Gray and Willemsen[4] with tunnel junctions.

The present paper deals with stationary nonequilibrium states produced in superconducting films by tunnel injection. The temperature dependence of the order parameter exhibits in this case hysteresis, which can explain, for example, the break observed in Ref. 2. In addition, at certain temperatures the film goes over jumpwise into a resistive state, with formation of a spatial stationary structure of the order parameter and of the correction to the chemical potential; this signifies in fact the onset of a dc coordinate-dependent voltage.

1. ASYMMETRICAL *SIS'* JUNCTION

Consider a tunnel junction of two *SIS'* superconductors. We observe the film *S* and assume that its voltage is zero, while the voltage of film *S'* is *V*. The excitations arriving at *S* relax under the influence of the phonons, and since the electron-phonon coupling is quite weak, the stationary deviation of the electron system from equilibrium can be appreciable. An alternating Josephson current of frequency $2V$ flows in the system and exerts little influence on the nonequilibrium stationary state.

If the junction is planer and its area large enough to be able to speak of inhomogeneity, then the electron distribution function is obtained from the kinetic equation[5]

$$-D\nabla^2 n_\varepsilon + I_\varepsilon = \left(\frac{\partial n}{\partial t}\right)^T, \qquad (1)$$

where the integral for the collisions with the phonons is

$$I_\varepsilon = \frac{g}{\omega_D{}^2} \left\{ \int_\varepsilon^\infty (\varepsilon'-\varepsilon)^2 d\varepsilon' (u_\varepsilon u_{\varepsilon'} - v_\varepsilon v_{\varepsilon'} + 1) [n_\varepsilon(1-n_{\varepsilon'})N_{\varepsilon'-\varepsilon} \right.$$

$$-n_{\varepsilon'}(1-n_\varepsilon)(1+N_{\varepsilon'-\varepsilon})] + \int_{-\infty}^\varepsilon (\varepsilon-\varepsilon')^2 d\varepsilon' (u_\varepsilon u_{\varepsilon'} - v_\varepsilon v_{\varepsilon'} + 1)$$

$$\times [n_\varepsilon(1-n_{\varepsilon'})(1+N_{\varepsilon-\varepsilon'}) - n_{\varepsilon'}(1-n_\varepsilon)N_{\varepsilon-\varepsilon'}] + \int_{-\infty}^{-\varepsilon} (\varepsilon+\varepsilon')^2 d\varepsilon' (u_\varepsilon u_{\varepsilon'}$$

$$+v_\varepsilon v_{\varepsilon'} - 1) [n_\varepsilon n_{\varepsilon'} N_{\varepsilon+\varepsilon'} - (1-n_\varepsilon)(1-n_{\varepsilon'})(1+N_{\varepsilon+\varepsilon'})]$$

$$\left. + \int_{-\varepsilon}^\infty (\varepsilon+\varepsilon')^2 d\varepsilon' (u_\varepsilon u_{\varepsilon'} + v_\varepsilon v_{\varepsilon'} - 1) [n_\varepsilon n_{\varepsilon'} (1+N_{\varepsilon+\varepsilon'}) - (1-n_\varepsilon)(1-n_{\varepsilon'})N_{\varepsilon+\varepsilon'}] \right\}, \qquad (2)$$

$$u_\varepsilon = \varepsilon v_\varepsilon/\Delta = |\varepsilon|(\varepsilon^2-\Delta^2)^{-1/2}\theta(\varepsilon^2-\Delta^2).$$

This integral takes into account the relaxation of the difference between the populations of the electron and hole branches,[6] and if the two are symmetrical, when $n_\varepsilon = 1 - n_{-\varepsilon}$, it goes over into the integral obtained by Éliashberg.[7]

We introduce the functions $\beta_\varepsilon = n_\varepsilon - n_{-\varepsilon}$ and $\alpha_\varepsilon = n_\varepsilon + n_{-\varepsilon} - 1$; α_ε expresses the unbalance of the branches, and $\beta_\varepsilon = -\tanh(\varepsilon/2T)$ in the equilibrium case. The tunnel source in (1) can then be expressed in the form

$$\left(\frac{\partial n}{\partial t}\right)^T = R_\varepsilon + L_\varepsilon; \qquad (3)$$

$$R_\varepsilon = w[u_{\varepsilon-v}(\beta_{\varepsilon-v}-\beta_\varepsilon) + u_{\varepsilon+v}(\beta_\varepsilon - \beta_{\varepsilon+v}) + (\alpha_{\varepsilon-v} - \alpha_{\varepsilon+v}) - u_\varepsilon(u_{\varepsilon-v} + u_{\varepsilon+v})\alpha_\varepsilon],$$

$$L_\varepsilon = w[u_\varepsilon u_{\varepsilon-v}(\beta_{\varepsilon-v}-\beta_\varepsilon) + u_\varepsilon u_{\varepsilon+v}(\beta_{\varepsilon+v}-\beta_\varepsilon) + u_\varepsilon(\alpha_{\varepsilon-v}-\alpha_{\varepsilon+v})$$
$$+ (u_{\varepsilon+v} - u_{\varepsilon-v})\alpha_\varepsilon].$$

Here *w* is proportional to the tunneling probability, and all the quantities with the shifted arguments $\varepsilon \pm V$ pertain to the film *S'*. The tunnel current through the junction is

$$I = \frac{1}{2R} \int_0^\infty d\varepsilon [u_\varepsilon u_{\varepsilon-v}(\beta_{\varepsilon-v}-\beta_\varepsilon) + u_\varepsilon u_{\varepsilon+v}(\beta_\varepsilon - \beta_{\varepsilon+v})$$

$$-(u_{\varepsilon-v} + u_{\varepsilon+v})\alpha_\varepsilon + u_\varepsilon(\alpha_{\varepsilon-v} + \alpha_{\varepsilon+v})]. \qquad (4)$$

It is expedient next to divide the kinetic equation (1) into parts even and odd in ε:

$$\tfrac{1}{2}(I_\varepsilon + I_{-\varepsilon}) + \tfrac{1}{2}Dk^2\alpha_\varepsilon = R_\varepsilon, \qquad (5a)$$

$$\tfrac{1}{2}(I_\varepsilon - I_{-\varepsilon}) + \tfrac{1}{2}Dk^2\beta_\varepsilon = L_\varepsilon. \qquad (5b)$$

We make now additional assumption concerning the junction. Let the film *S'* be so thick or let the inelastic interaction in it be so strong that the deviation from equilibrium in *S'* is small. In addition, as will be made clear later, the next higher terms of the expansion in the barrier transparency *w* need be taken into account

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only in α, and in the right-hand side of (5) we can put $\beta_\varepsilon = -\tanh(\varepsilon/2T)$. The spatial dispersion turns out to be such that the diffusion term need be retained only in (5a), and the odd part of the distribution function β_ε adjusts itself to α_ε in this case "adiabatically."

As shown by Artemenko and Volkov,[8] the function α_ε can be sought in the form

$$\alpha = \frac{1}{u_\varepsilon} \frac{\Phi}{2T \, \mathrm{ch}^2(\varepsilon/2T)}. \qquad (6)$$

We then obtain for the quantity Φ, which serves as a correction to the chemical potential,

$$\left(\frac{Dk^2}{\gamma} + a + \frac{\pi\Delta}{8T}\right)\Phi = aV, \quad a = \frac{w}{\gamma}, \qquad (7)$$

where $\gamma = 7\zeta(3)gT^3/\omega_D^2$ and is proportional to the probability of the electron-phonon interaction.

In the collision-integral part that is odd in ε we need to retain the part linear in $\beta_\varepsilon' = \beta_\varepsilon + \tanh(\varepsilon/2T)$ and quadratic in α_ε:

$$\tfrac{1}{2}(I_\varepsilon - I_{-\varepsilon}) = \tfrac{1}{2}(I_\varepsilon - I_{-\varepsilon})_{lin} + A_\varepsilon,$$

$$\frac{1}{2}(I_\varepsilon - I_{-\varepsilon})_{lin} = \frac{g}{\omega_D^2}\left\{\int_\varepsilon^\infty d\varepsilon' (\varepsilon' - \varepsilon)^2 (u_\varepsilon u_{\varepsilon'} - v_\varepsilon v_{\varepsilon'})[\beta_{\varepsilon'}'(n_{\varepsilon'}{}^F\right.$$

$$+ N_{\varepsilon'-\varepsilon}) - \beta_{\varepsilon}'(1 - n_{\varepsilon'}{}^F + N_{\varepsilon'-\varepsilon})]$$

$$+ \int_\Delta^\varepsilon d\varepsilon'(\varepsilon - \varepsilon')^2(u_\varepsilon u_{\varepsilon'} - v_\varepsilon v_{\varepsilon'})[\beta_{\varepsilon'}'(1 - n_\varepsilon{}^F + N_{\varepsilon-\varepsilon'}) - \beta_{\varepsilon}'(n_\varepsilon{}^F + N_{\varepsilon-\varepsilon'})] \qquad (8)$$

$$+ \int_\Delta^\infty d\varepsilon'(\varepsilon + \varepsilon')^2(u_\varepsilon u_{\varepsilon'} + v_\varepsilon v_{\varepsilon'})[\beta_{\varepsilon'}'(n_\varepsilon{}^F + N_{\varepsilon+\varepsilon'}) + \beta_{\varepsilon'}'(n_\varepsilon{}^F + N_{\varepsilon+\varepsilon'})]\right\},$$

$$A_\varepsilon = -\frac{g}{\omega_D^2}\alpha_\varepsilon\left[\int_\Delta^\varepsilon (\varepsilon^2 + \varepsilon'^2)\alpha_{\varepsilon'} d\varepsilon' + 2\varepsilon\int_\varepsilon^\infty \varepsilon' \alpha_{\varepsilon'} d\varepsilon'\right].$$

Substituting (8) in (5b) we get

$$\tfrac{1}{2}(I_\varepsilon - I_{-\varepsilon})_{lin} = L_\varepsilon + A_\varepsilon, \qquad (9)$$

i.e., a linear inhomogeneous equation. Its solution enters in the equation for the order parameter

$$\frac{T_\varepsilon - T}{T} - \frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2} + F = 0,$$

$$F = -\int_0^\infty \frac{d\varepsilon}{\varepsilon} u_\varepsilon \beta_\varepsilon'. \qquad (10)$$

The solution corresponding to L_ε is concentrated at energies $\Delta \ll T$ (the temperature is assumed close to critical). To find this solution, we can therefore represent the collision integral in the τ approximation. At energies of the order of the temperature, the solution corresponding to A_ε is

$$\beta_\varepsilon' = -\frac{\Phi^2}{4T^2}\,\mathrm{th}\,\frac{\varepsilon}{2T}\,\mathrm{ch}^{-1}\frac{\varepsilon}{2T}$$

and makes a contribution of the order of Φ^2/T^2. The existence of such a term in the equation for Δ was pointed out by Galaĭko.[9] Finally we get

$$F = -C\frac{\Phi^2}{T^2} - \frac{a}{2T}\int_0^\infty \frac{d\varepsilon}{\varepsilon}u_\varepsilon\left(V - \frac{\Phi^2}{u_\varepsilon}\right)(u_{\varepsilon-V} - u_{\varepsilon+V}),$$

$$C = \frac{7\zeta(3)}{4\pi^2}. \qquad (11)$$

Equation (10) together with (11) and (17) makes it pos-

sible to solve problems with spatial variations of the gap and of the chemical potential, but the wave vector must satisfy in this case the condition $Dk^2 \ll \gamma$. It is seen from (7) that the main source of the nonlinearity is the quantity α_ε, since the nonlinearity sets in here already at a $\sim \Delta/T$.

Because of the specific form of the tunnel source, the order parameter and the change of the chemical potential are interrelated. Let the critical temperatures of the superconductors making up the junction differ by a small amount: $0 < (T_c - T_{c'}) \ll T_c$. In the homogeneous case (Δ' is the gap of the superconductor S') we then find from (11) at $V \ll \Delta$ and $V \ll \Delta'$ that at $|\Delta - \Delta'| \ll \Delta$ we have

$$F = a\frac{V}{4T}\ln\frac{\Delta - \Delta' - V}{\Delta - \Delta' + V},$$

$$\Phi = \Phi_0 = V\frac{8aT}{8aT + \pi\Delta}. \qquad (12)$$

Together with the temperature dependence of the gap in the superconductor S' we have

$$\frac{7\zeta(3)}{8\pi^2}\frac{\Delta'^2}{T^2} = \frac{T_{c'} - T}{T}. \qquad (13)$$

Relations (10) and (12) determine the temperature dependence of the gap in S. This dependence is shown in Fig. 1. Its character is connected with the nonmonotonic dependence of F on Δ. The ambiguity in Fig. 1 occurs at those temperatures at which $|\Delta - \Delta'| \sim V$. The temperature of the break at the point A is therefore given by

$$\frac{T_c - T_0}{T_c} = \frac{2\pi^2}{7\zeta(3)}\left(\frac{T_c - T_{c'}}{V}\right)^2. \qquad (14)$$

The same temperature scale determines the entire region of ambiguity. The width of this region in terms of Δ is of the order of V. The dashed curve in the figure is the equilibrium dependence of the gap.

We now test this regime, which is homogeneous in the coordinate, for stability to small fluctuations:

$$\Delta \to \Delta + \Delta_1(z), \quad \Phi \to \Phi_0 + \Phi_1(z), \quad \Phi_1 \sim \Delta_1 \sim e^{\Omega t}.$$

From (7) we get a connection between the fluctuations of the chemical potential and the gap:

$$\Phi_1(k) = -\Phi_0\frac{\pi}{8T}\Delta_1(k)\left[a + \frac{\pi\Delta}{8T} + \frac{Dk^2}{4\gamma}\right]^{-1}. \qquad (15)$$

Varying (10) with respect to Δ and Φ with allowance for (15), we obtain a dispersion equation that determines the possible k:

$$\Delta\frac{\partial}{\partial\Delta}\left[-\frac{7\zeta(3)}{8\pi^2}\frac{\Delta^2}{T^2} + F\right] = \frac{\Phi_0^2}{4T^2}\frac{\pi Dk^2}{8\gamma}\left[a + \frac{\pi\Delta}{8T} + \frac{Dk^2}{4\gamma}\right]^{-1}\left(\frac{16C\Delta}{8aT + \pi\Delta} + B\right). \qquad (16)$$

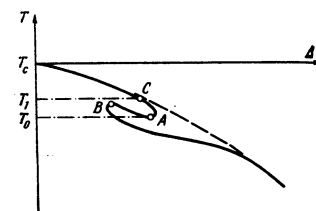FIG. 1.

On the left is the variation of the equation for the gap, and B is given by the integral

$$B = \frac{\Delta}{2V}\int_0^{\infty}\frac{d\varepsilon}{\varepsilon^2}(\varepsilon^2-\Delta^2)^{1/2}(u_{\varepsilon+V}-u_{\varepsilon-V}). \tag{17}$$

In the limiting cases we have

$$B = -\frac{1}{2}\ln\left|\frac{\Delta}{\Delta-\Delta'}\right|, \quad V\ll|\Delta-\Delta'|\ll\Delta,$$
$$B = -\frac{1}{2}\ln\frac{\Delta}{V}, \quad |\Delta-\Delta'|\ll V. \tag{18}$$

Equation (16) has as solutions real k, when we are on the physical sections of the curve Fig. 1. (The unphysical section is AB and is unstable and cannot be realized in the experiment.) It is seen that near the extremum (point A) the corresponding k are small, but with increasing distance to the extremum k increases at some point C the real solutions drop out because of saturation in the right-hand side of (16).

Thus, if we decrease the temperature starting with T_c, then a transition to an inhomogeneous stationary state occurs jumpwise at the point C, with spatial fluctuations of the gap and of the chemical potential. The characteristic wave vector of these fluctuations is

$$Dk^2 \sim \gamma. \tag{19}$$

It can be shown that below the point C the homogeneous state is absolutely unstable in time.

Calculating the temperature T_1 corresponding to the point C under the assumption that $V \ll [T_c(T_c-T_{c'})]^{1/2}$ and $aV \ll (T_c-T_{c'})$, we get

$$\frac{T_c-T_1}{T_c} = \frac{T_c-T_0}{T_c}\left[1-\left(\frac{V^2}{T_c(T_c-T_{c'})}\right)^{\prime}\left(\frac{aV}{T_c-T_{c'}}\right)^{\prime}\right]. \tag{20}$$

In order for the break of the regime at the point A and the transition to the inhomogeneous space to occur at sufficiently different temperatures we must have, as seen from (20),

$$V \lesssim [T_c(T_c-T_{c'})]^{1/2}, \quad a \lesssim \left(\frac{T_c-T_{c'}}{T_c}\right)^{1/2}. \tag{21}$$

These values are optimal for this effect.

2. SYMMETRICAL S/S JUNCTION

In this case $\Delta = \Delta'$ and the thickness of the superconductors and the inelastic interactions are assumed equal, so that the nonequilibrium state develops on both sides of the junction. The functions β_ε and β'_ε which are odd in energy are equal in this case, and the even ones are connected by the relation

$$\alpha_\varepsilon = -\alpha'_\varepsilon = \frac{1}{u_\varepsilon}\frac{\Phi}{2T\,\mathrm{ch}^2(\varepsilon/2T)}. \tag{22}$$

As follows from (5a), the correction to the chemical potential is

$$\Phi = V\frac{8aT}{16aT+\pi\Delta}. \tag{23}$$

Φ is small in the approximation linear in the transparence. On the contrary, for a sufficiently transparent barrier we have $\Phi = V/2$ for the film S and $\Phi' = -V/2$ for the film S', thus cancelling out the voltage V. We note that in the preceding case we had for large transparency
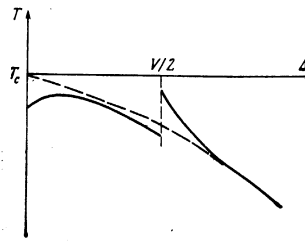
$\Phi = V$ and $\Phi' = 0$, leading likewise to cancellation. Near T_c the current through the junction, as follows from (4), is

$$I = \frac{1}{R}(V-\Phi+\Phi') = \frac{V}{R}\frac{\pi\Delta}{\pi\Delta+16aT}. \tag{24}$$

The effective voltage, in the case of direct current is $V - \Phi + \Phi'$, whereas in the case of alternating Josephson current the frequency is determined by double the difference $(2V)$ between the chemical potentials of the pair.

The distribution-function component odd in the energy is determined from (5b):

$$\beta_\varepsilon' = \frac{a}{2T}\left[V(u_{\varepsilon-V}-u_{\varepsilon+V})+\Phi\left(\frac{u_{\varepsilon+V}-u_{\varepsilon-V}}{u_\varepsilon^2}+\frac{1}{u_{\varepsilon+V}}-\frac{1}{u_{\varepsilon-V}}\right)\right]. \tag{25}$$

The anomalous term in the Ginzburg–Landau equation (10), when (23) is taken into account is therefore

$$F = \frac{V\Phi}{2T}\int_{V/2}^{\infty}\frac{d\varepsilon\,\mathrm{sign}(\varepsilon-V)}{(\varepsilon^2-\Delta^2)^{1/2}[(\varepsilon-V)^2-\Delta^2]^{1/2}}$$
$$\times\left[\frac{\pi\Delta}{8T}+a\frac{\Delta^2}{\varepsilon^2}+a\frac{\Delta^2}{(\varepsilon-V)^2}\right]\theta(\varepsilon^2-\Delta^2)\theta[(\varepsilon-V)^2-\Delta^2]C\frac{\Phi^2}{T^2}. \tag{26}$$

In the limiting cases we have

$$F = \frac{aV^2}{4\Delta T}\ln\frac{\Delta}{V}, \quad V\ll\Delta; \tag{27a}$$
$$F = -\frac{2\pi}{3^{3/2}}a\frac{\Phi}{T}+\frac{\pi}{2}a\frac{\Delta}{T}\theta(2\Delta-V)-C\frac{\Phi^2}{T^2}, \quad |V-2\Delta|\ll\Delta; \tag{27b}$$
$$F = -\frac{\pi}{2}a\frac{\Delta}{T}\frac{\Phi}{V}-C\frac{\Phi^2}{T^2}, \quad \Delta\ll V. \tag{27c}$$

The inhomogeneity indicated above does not occur in a symmetrical junction.

The temperature dependence of the order parameter is shown in Fig. 2. Nonequilibrium states in junctions were considered by Peskovatskii̇ and Seminozhenko,[10] but they did not take into account the change of the chemical potential, and their results are therefore in error.

Here, too, there is a discontinuity, but at a voltage $V = 2\Delta$. This is quite probably the cause of the transition observed in Ref. 2.

FIG. 2.

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Superconducting transition of an inhomogeneous bridge in a microwave field

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It is shown that, in agreement with the experimental results, the transition of a long inhomogeneous bridge to the superconducting state may decrease in width in a microwave field until it becomes a step. This happens because stimulation of the superconductivity varies with the value of T_c characterizing a given region. The influence of the edges increases the width of the transition at high microwave radiation powers. The dependence of the transition width on the microwave power is derived.

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Recent experimental investigations[1] of the superconducting transition in long bridges subjected to a microwave field revealed that an increase in the microwave power causes the superconducting transition to first decrease in width to a step and then to widen again to a finite width which increases with the microwave power. The length of such bridges L is much greater than the size of a pair ξ_0 and, therefore, they consist of regions whose critical temperatures T_c differ somewhat.[2] For this reason the superconducting transition of a long bridge has a finite width along the temperature axis. In a microwave field the electron energy distribution function $n(\varepsilon)$ differs from the Fermi form and this nonuniformity may result in stimulation of the superconductivity in a bridge.[3,4] However, stimulation varies from one region to another because of variation of T_c and, as shown below, this accounts for the observed behavior of the superconducting transition width on the microwave field power.

1. NARROWING OF A SUPERCONDUCTING TRANSITION AT HIGH RADIATION POWERS

The order parameter Δ of a bridge is found from the Ginzburg–Landau equation which, supplemented by the nonequilibrium term $\Phi(\Delta)$, is[4]

$$\frac{\pi D}{8T}\Delta'' - \frac{I_s^2}{4\pi e^2 \rho^2 D S^2 \Delta^2 f(\Delta)} + \frac{T_c - T}{T_c}\Delta$$
$$- \frac{7\zeta(3)}{8\pi^2}\frac{\Delta^3}{T^2} + \Phi(\Delta) = 0, \qquad (1)$$

$$\Phi(\Delta) = \Delta \int_\Delta^\infty \left[f(\varepsilon) - \text{th}\frac{\varepsilon}{2T} \right] \frac{d\varepsilon}{(\varepsilon^2 - \Delta^2)^{1/2}},$$

where $\overline{I_s^2}$ is the average value of the square of the superconducting current through a bridge proportional to the radiation power; the function $f(\varepsilon)$ gives the electron energy distribution $n(\varepsilon) = [1 - f(\varepsilon)]/2$; $D = v_F l_{tr}/3$ is the coefficient of spatial diffusion of electrons; ρ is the density of states; S is the cross-sectional area of the bridge.

The electron energy distribution in a microwave field is found from the transport equation. The electron energy relaxation time τ_ε is long compared with the characteristic time of spatial diffusion of electrons in a bridge D/L^2 even if this bridge is long: $\xi_0 \ll L \ll \xi_0\sqrt{T\tau_\varepsilon}$. If we also assume that this relaxation time is long compared with the field period, we obtain the following equation for the function $f(\varepsilon)$ averaged over the coordinates and time

$$\frac{1}{\tau_\varepsilon}\left[f(\varepsilon) - \text{th}\frac{\varepsilon}{2T} \right] \overline{\langle \varepsilon (\varepsilon^2 - \Delta^2)^{-1/2} \rangle} = \frac{\partial}{\partial \varepsilon}\left(D_\varepsilon \frac{\partial f}{\partial \varepsilon} \right), \qquad (2)$$

where the coefficient D_ε represents the electron energy diffusion due to the direct acceleration of electrons in the electric field[3] and oscillations of the gap characterizing the bridge in a microwave field[4]; the brackets $\langle ... \rangle$ denote averaging over the part of the bridge where $\Delta < \varepsilon$ and the bar represents the time averaging.

The transport equation (2) has to be supplemented by two boundary conditions. The distribution of electrons at the edges of a bridge retains its equilibrium form in a microwave field since the current density at the edges is low. Therefore, electrons with energies greater than the maximum value of the order parameter Δ_{max} may diffuse freely out of the bridge and they acquire an equilibrium energy distribution in contrast to the electrons whose energies are $\varepsilon < \Delta_{max}$ and which are "confined" to the bridge. This condition means