InSb and Te (it is convenient to measure the energy in degrees in these estimates).

The amplitude of the potential is $U_0=2\text{ K}\cdot(W/W_0)^{1/2}$ in Ge, and $U_0=5\text{ K}(20\text{ K})\cdot\ \nu_0 W^{1/2}/\nu W_0^{1/2}$ in InSb and Te, respectively; here W is the intensity and the frequency of the sound wave, $W_0=1\text{ W/cm}^2$, and $\nu_0=5-10^9$ Hz. Thus, Eq. (77) is satisfied at a sound intensity on the order of 1 W/cm² in Ge and on the order of several dozen watts per square centimenter in InSb and Te. At these intensities, the condition $\alpha\lesssim 1$ is satisfied at a sound frequency not exceeding 10^{10} Hz.

In addition, we have stipulated that all the electrons be under tight-binding conditions (9), i.e.,

$$T_e,\ \varepsilon_F<U_0. \tag{78}$$

Finally, the condition (10) meant that the screening of the soundwave field by the conduction electrons must not violate the tight-binding condition. At an electron concentration $N\sim 10^{12}\text{ cm}^{-3}$ and a sound frequency $\nu\sim 5\cdot 10^9$ Hz the quantity $e^2N/\varkappa Q^2$, which should be less than U_0, is of the order of 1 K. At these sound intensities the condition (10) should be satisfied in Ge at $N\sim 10^{12}$ and in InSb and Te at $N\sim 10^{13}$. At such concentrations we have $\varepsilon_F\sim 0.01$ K in Ge and ~1 K in InSb and Te, i.e., Eq. (78) sets the temperature limit.

In conclusion, we thank Yu. M. Gal'perin, V. L. Gurevich, and V. D. Kagan for important remarks.

[1)] It is seen from (8) that at $n>2^5/\alpha$ the width Δ_n of the allowed band begins to decrease. But expression (8) is likewise no longer valid at these values.

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Plasma-acoustic waves on the surface of a piezoelectric crystal

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A theoretical analysis is made of the interaction between a two-dimensional electron plasma and shear surface piezoacoustic waves. The cases of a plasma layer on the surface of a piezoelectric crystal in vacuum and of an inversion channel in a metal–insulator–semiconductor structure are considered. Renormalization of the velocity of sound and damping of acoustic waves due to their interaction with a plasma are found. A specific damping mechanism of two-dimensional plasmons associated with the emission of acoustic waves is investigated. In all cases considered the characteristic parameters of the wave processes depend strongly on the surface charge density, which should make it possible to control them in experimental studies.

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INTRODUCTION

Electron processes in quasi-two-dimensional systems are attracting considerable attention. Two types of system are being investigated more than others: electrons above the surface of liquid helium (or helium film) and carriers in inversion channels in metal–insulator–semiconductor (MIS) structures. These two types of system are being investigated because they provide experimental means for varying the characteristic parameters in a wide range so that the main parameter, which is the surface charge density, can be varied over four orders of magnitude.

Recent experiments on inversion layers in silicon[1,2] have revealed the presence of two-dimensional plasmons. This is a very important result because the interaction of two-dimensional plasmons with other oscil-

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lations in crystals opens up new and sometimes unique possibilities for controlling wave processes in solids.

We shall consider the interaction of a two-dimensional electron plasma with shear surface piezoacoustic waves of the Bluestein–Gulyaev type.[3,4] We shall consider two situations in which this interaction takes place: a two-dimensional plasma on the surface of a piezoelectric in vacuum[1)] and an inversion channel (layer) in an MIS structure with a piezoelectric semiconductor (in the latter case one has to allow for the elastic waves in the insulator).

In the case of a collisionless plasma ($\omega\tau \gg 1$, where ω is the frequency and τ is the electron relaxation time) there are two branches of weakly damped oscillations. Acoustic oscillations are damped out as a result of their interaction with the plasma-layer electrons in accordance with a mechanism analogous to the Landau damping. Long-wavelength plasma oscillations experience additional damping due to the emission of acoustic waves. This mechanism is activated when the plasmon group velocity exceeds the bulk sound velocity.

If the electron scattering is sufficiently strong, so that $\omega\tau \ll 1$ in the frequency range under discussion, there is no plasma branch but the interaction with the plasma affects the dispersion law and the damping of surface acoustic waves. Under certain conditions a surface wave with a linear dispersion law at low frequencies can exist only because of the presence of a plasma. We shall develop a quantitative theory of these effects.

1. SURFACE PIEZOACOUSTIC SHEAR WAVE

We shall begin with the simplest model: we shall assume that a piezoelectric crystal of symmetry class C_{6v} occupies the half-space $y>0$; the c_6 axis coincides with the z axis of the selected coordinate system and lies in the plane of the $y=0$ boundary; a layer of electrons of surface density N_s is also located in the $y=0$ plane. A wave travels along the x axis and is polarized parallel to the c_6 axis. Lattice displacements U and an electric potential φ are proportional to $\exp[i(kx-\omega t)-\varkappa y]$, where k is the wave number and $\varkappa>0$ is the spatial damping decrement. The potential φ should also decrease exponentially in the vacuum half-space ($y\to-\infty$).

The equations of motion for a piezoelectric crystal of this symmetry are[5]

$$\left.\begin{aligned}\rho\ddot U_z&=\lambda\Delta_{x,y}U_z-\beta\Delta_{x,y}\varphi,\\ \varepsilon\Delta_{x,y}\varphi&+4\pi\beta\Delta_{x,y}U_z=0,\end{aligned}\right\}\tag{1}$$

where ρ, ε, λ, and β are, respectively, the density, permittivity, shear modulus, and piezoelectric modulus of this crystal; $\Delta_{x,y}=\partial^2/\partial x^2+\partial^2/\partial y^2$. We can easily see that in the selected geometry of the problem, the system (1) contains only one component of the tensors ε_{ik}, λ_{iklm}, and β_{ikl}. We shall omit the index z of U_z. We find from the system (1) that

$$\varkappa_1^2=k^2,\quad \varkappa_2^2=k^2-\omega^2/c_v^2,\tag{2}$$

where c_v is the volume velocity of a shear wave of the same polarization and travelling in the same direction: $c_v^2=\lambda/\rho+4\pi\beta^2/\varepsilon\rho$. We shall assume that $4\pi\beta^2/\varepsilon\rho c_v^2\equiv\gamma$. The general solution of the system (1) is

$$\left.\begin{aligned}U&=Ae^{-\varkappa_2 y},\quad \varphi=-\frac{4\pi\beta}{\varepsilon}Ae^{-\varkappa_2 y}+Be^{-ky},\quad y>0;\\ \varphi&=Ce^{ky},\quad y<0.\end{aligned}\right\}\tag{3}$$

Here, A, B, and C are arbitrary constants; the factor $\exp(ikx-i\omega t)$ is omitted. The boundary conditions lead to three equations for these constants:

$$\left.\begin{aligned}&\lambda\,\partial U/\partial y-\beta\partial\varphi/\partial y\,|_{y=+0}=0,\ \varphi(+0)=\varphi(-0),\\ &\varepsilon\partial\varphi/\partial y+4\pi\beta\partial U/\partial y\,|_{y=+0}-\partial\varphi/\partial y\,|_{y=-0}=-4\pi e\tilde N_s,\end{aligned}\right\}\tag{4}$$

where $\tilde N_s$ is the nonequilibrium correction to the surface charge density N_s.

The system (4) can be closed by determining $\tilde N_s$ from the transport equation. We shall assume that there is only one relaxation time τ and that the collision term describes the relaxation of the electron distribution in a local coordinate system linked to the lattice. The nonequilibrium correction to the electron distribution function f_1 can be found by analogy to Ref. 6:

$$f_1=\frac{i\mathbf{kv}e\tau\varphi-m\dot{\mathbf U}\mathbf v-\tilde N_s\,\partial\mu/\partial N_s}{1-i\omega\tau+i\mathbf{kv}\tau}\frac{\partial f_0}{\partial E}.\tag{5}$$

Here, e, m, E, and $\mathbf v$ are the charge, effective mass, energy, and velocity of electrons; μ is the chemical potential; f_0 is the equilibrium distribution function. The value of $\tilde N_s$ is given by

$$\tilde N_s=\int f_1\,d\mathbf p/2\pi^2;\quad \mathbf p=m\mathbf v,\quad \hbar=1.$$

Bearing in mind that only the x component of the vector $\mathbf k$ differs from zero and that $\dot{\mathbf U}$ contains only the z component, we find that

$$\left.\begin{aligned}\tilde N_s&=-\frac{ik^2\varphi}{e\omega}\frac{\sigma_{xx}}{1-R_x},\\ \sigma_{\alpha\beta}&=-\frac{2e^2\tau}{(2\pi)^2}\int\frac{v_\alpha v_\beta}{1-i\omega\tau+i\mathbf{kv}\tau}\frac{\partial f_0}{\partial E}d\mathbf p,\\ \mathbf R&=-\frac{2}{(2\pi)^2}\frac{\partial\mu}{\partial N_s}\int\frac{\mathbf v}{1-i\omega\tau+i\mathbf{kv}\tau}\frac{\partial f_0}{\partial E}d\mathbf p.\end{aligned}\right\}\tag{6}$$

Substituting Eq. (3) into Eq. (4) and using Eq. (6), we obtain the dispersion equation of coupled surface waves:

$$\left[1+\frac{4i\pi k\sigma_{xx}}{\omega(\varepsilon+1)(1-R_x)}\right](\gamma k-\varkappa_2)=\frac{\varepsilon\gamma k}{\varepsilon+1}.\tag{7}$$

The required roots of Eq. (7) denoted by $\omega(k)$ should have a negative imaginary part and a positive real part $\varkappa_2$ [see Eq. (2)] because we are dealing with a surface wave.

1a. We shall first consider the limiting case of strong scattering: $\omega\tau\ll1$ and $kl\ll1$, where l is the mean free path of electrons. In this case the surface conductivity σ_{xx} is

$$\left.\begin{aligned}\sigma_{xx}&=\sigma=\frac{N_s e^2}{m}\tau(\mu_0),\\ \sigma&=\frac{N_s e^2}{m}\langle\tau\rangle,\quad \langle\tau\rangle=\int\frac{E\tau(E)}{T^2}e^{-E/T}dE\end{aligned}\right\}\tag{8}$$

for the Fermi and Boltzmann statistics, respectively (μ_0 is the Fermi energy and T is the absolute temperature in energy units). In this limiting case, we obtain the following expression for R:

$$\left.\begin{aligned} &R=-4i\pi\sigma k^2 r_0/(\varepsilon+1)\omega,\\ &r_0=\frac{\varepsilon+1}{4\pi e^2}\frac{\partial\mu}{\partial N_s}=\frac{\varepsilon+1}{4me^2}\left[1-\exp\left(-\frac{\pi N_s}{mT}\right)\right]^{-1}.\end{aligned}\right\} \quad (9)$$

Here, r_0 is the screening radius of the Coulomb interaction in a two-dimensional system. The spatial damping decrement is obtained from Eq. (7):

$$\varkappa_2=\frac{\gamma k}{\varepsilon+1}\frac{\varepsilon+1+kr_0-i\xi}{1+kr_0-i\xi}, \quad (10)$$

where $\xi\equiv(\varepsilon+1)\omega/4\pi\sigma k$. If $kr_0\gg\varepsilon$, then—irrespective of the value of ξ—the depth of penetration and the phase velocity s are given by

$$\varkappa_2^{-1}=(\varepsilon+1)/\gamma k, \quad s=s_1\equiv c_v[1-\gamma^2/(\varepsilon+1)^2]^{1/2},$$

which corresponds to a surface wave on a free boundary. In the $kr_0\ll 1$ case, we shall seek a solution in the form $\omega=sk$ and use Eqs. (10) and (2) to ensure that s is independent of k. The equation for s is very cumbersome but we can easily see that $s<c_v$ and that $s\to c_v$ in the limit $\gamma\to 0$, whereas $s\to 0$ in the limit $\gamma\to 1$ (by definition, the condition $0<\gamma<1$ is always satisfied). Since γ is usually much less than unity, then for an arbitrary value of c_v/σ the order of magnitude s is the same as that of c_v. In the $c_v\gg\sigma$ case we again obtain $s=s_1$. It should be noted that in spite of the high density in a surface plasma ($kr_0\ll 1$) the low conductivity makes the result the same as for a free boundary. If $c_v\ll\sigma$, then $s=s_2\equiv c_v(1-\gamma^2)^{1/2}$ (surface wave on a grounded boundary).

We shall now calculate the surface wave damping. We shall use the fact that in all the limiting cases considered here the value of $\omega''=\operatorname{Im}\omega$ is much smaller than $\omega'\equiv\operatorname{Re}\omega$. In the first stage we can assume that ω is real and solve the dispersion equation by successive approximations. Then, assuming that $\varkappa_2=\varkappa_2'+i\varkappa_2''$, we obtain

$$\omega\approx c_v(k^2-\varkappa_2'^2)^{1/2}\left(1-\frac{i\varkappa_2'\varkappa_2''}{k^2-\varkappa_2'^2}\right). \quad (11)$$

We shall now give the results in various limiting cases

$$\sigma\gg c_v,\quad kr_0\ll 1;\quad \varkappa_2'=\gamma k,\quad \omega''=\frac{\gamma^2\varepsilon\omega'}{4\pi\sigma(1-\gamma^2)^{1/2}}\propto\frac{\omega'}{N_s}; \quad (12a)$$

$$\sigma\gg c_v,\quad kr_0\gg\varepsilon;\quad \varkappa_2'=\frac{\gamma k}{\varepsilon+1},\quad \omega''=\frac{\gamma^2\varepsilon[(\varepsilon+1)^2-\gamma^2]^{1/2}c_v^2}{4\pi\sigma r_0^2(\varepsilon+1)^2\omega'}\propto\frac{1}{\omega' N_s}; \quad (12b)$$

$$\sigma\ll c_v,\quad kr_0\ll 1;\quad \varkappa_2'=\frac{\gamma k}{\varepsilon+1},\quad \omega''=\frac{4\pi\varepsilon\gamma^2\sigma\omega'}{[(\varepsilon+1)^2-\gamma^2]^{1/2}c_v}\propto\omega' N_s; \quad (12c)$$

$$\left.\begin{aligned}&\sigma\ll c_v,\quad kr_0\gg\varepsilon;\\ &\varkappa_2'=\frac{\gamma k}{\varepsilon+1},\quad \omega''=\frac{\gamma^2\varepsilon c_v^2\omega'}{s_1}\frac{4\pi\sigma}{(4\pi\sigma kr_0)^2+s_1^2(\varepsilon+1)^2},\quad k=\frac{\omega'}{s_1}.\end{aligned}\right\} \quad (12d)$$

It is clear from Eq. (12) that in the high-conductivity case the maximum of the electron absorption of sound corresponds to $kr_0\sim 1$, whereas in the case of low conductivities it shifts toward $kr_0\sim c_v/\sigma\gg 1$.

1b. Since the characteristic electron velocity is usually much higher than the velocity of sound, it is desirable to consider the intermediate case when the spatial dispersion is important, i.e., $kl=kv\tau\gtrsim 1$, whereas the temporal dispersion can still be ignored ($\omega\tau\ll 1$). Then, instead of Eq. (10), we obtain the following expressions in the Fermi case:

$$\left.\begin{aligned}&\varkappa_2=\frac{\gamma k}{\varepsilon+1}\frac{\varepsilon+1+kr_0-iQ\xi}{1+kr_0-iQ\xi},\\ &Q=\frac{(kv_0\tau)^2[1+(kv_0\tau)^2]^{1/2}}{2[[1+(kv_0\tau)^2]^{1/2}-1]},\quad v_0=\left(\frac{2\mu_0}{m}\right)^{1/2}.\end{aligned}\right\} \quad (13)$$

The appearance of a new parameter $kv_0\tau$ increases considerably the number of possible limiting cases. We shall consider two of them which are most interesting and at the same time most realistic. We shall first assume that the plasma is degenerate. We shall adopt the following values of the characteristic parameters: $N_s\sim 10^{12}$ cm^{-2}, $\tau\sim 10^{-12}$ sec, $c_v\sim 3\times 10^5$ cm/sec, $m\sim 10^{-28}$ g, and $\varepsilon\sim 10$. We then find $v_0\sim 2\times 10^7$ cm/sec, $r_0\sim 10^{-7}$ cm, and $\sigma\sim 2\times 10^9$ cm/sec. We shall assume that $kv_0\tau\gg 1$, $kr_0\ll 1$. We then have $Q\approx(kv_0\tau)^2/2$ and there are two possibilities:

$$\left.\begin{aligned}&(kv_0\tau)^2\ll\frac{\sigma}{c_v},\quad \varkappa_2'=\gamma k,\quad s=s_2,\\ &\omega''=\frac{\gamma^2\varepsilon m v_0^2\omega'^3\tau}{8\pi(1-\gamma^2)^{1/2}e^2c_vN_s}\propto\frac{\omega'^3\tau}{N_s};\end{aligned}\right\} \quad (14a)$$

$$\left.\begin{aligned}&(kv_0\tau)^2\gg\sigma/c_v,\quad \varkappa_2'=\gamma k/(\varepsilon+1),\quad s=s_1,\\ &\omega''=\frac{8\pi\gamma^2\varepsilon e^2c_vN_s}{(\varepsilon+1)^2[(\varepsilon+1)^2-\gamma^2]^{1/2}mv_0^2\omega'\tau}\propto\frac{N_s}{\omega'\tau}.\end{aligned}\right\} \quad (14b)$$

It follows from the system (14) that as a result of the spatial dispersion we find that there is an absorption maximum in the $kr_0\ll 1$ case and it occurs at a frequency $\omega_{max}\propto(\sigma c_v)^{1/2}/v_0\tau\propto(N_s/\tau)^{1/2}$. Comparing Eqs. (12a) and (14a), we can see that at frequencies $\omega<\omega_{max}$ the damping rises first linearly with ω and then as ω^3. Another result of the spatial dispersion is that in the range $\sigma\gg c_v$, $kr_0\ll 1$ the phase velocity of the waves is equal to the velocity in the presence of a free boundary [compare with Eq. (12a)]. When the above parameters are used, Eqs. (14a) and (14b) are valid in the range 10^{10} sec$^{-1}\ll\omega\ll 10^{12}$ sec^{-1} and an absorption maximum appears at $\omega_{max}\sim 10^{11}$ sec^{-1}.

A characteristic frequency dependence of the damping appears in the case of a nondegenerate plasma if the conditions $r_0\gg l$, $\sigma\gg c_v$ are satisfied, which is equivalent to the requirement $\varepsilon v_T/4\pi\gg\sigma\gg c_v$, where v_T is the average thermal velocity of an electron. We shall now assume that $N_s\sim 10^{10}$ cm^{-2}, $\langle\tau\rangle\sim 10^{-13}$ sec, and $T\sim 10^2$ °K, and that all the other parameters have the same values as before; then, $\sigma\sim 2\times 10^6$ cm/sec, $r_0\sim 10^{-5}$ cm, $l\sim 10^{-6}$ cm, and $v_T\sim 2\times 10^7$ cm/sec. The frequency dependence of the damping then has two maxima: the first one is described by Eqs. (12a) and (12b) and it appears at $\omega\sim c_v/r_0$; the second is due to the spatial dispersion and is located at $\omega\sim(\sigma c_v)^{1/2}/l$. To the left of this maximum in the $(kl)^2c_v/\sigma\ll 1$ case the damping rises linearly with ω, whereas for $(kl)^2c_v/\sigma\gg 1$ it decreases as ω^{-1}.

1c. We shall now consider the case of strong spatial and temporal dispersion: $\omega\tau\gg 1$, $kl\gg 1$. We shall use the collisionless approximation in which the Landau damping of two-dimensional plasmons is the only mechanism. The nonequilibrium correction to the surface charge density $\tilde N_s$ can be expressed in terms of the quantum-mechanical response function $\tilde N_s=-e\varphi\Pi(\omega,\mathbf{k})$, where Π is the familiar polarization operator[7]:

$$\Pi(\omega,\mathbf{k})=\frac{1}{2\pi^2}\int\frac{f_0(\mathbf{p}+\mathbf{k})-f_0(\mathbf{p})}{E(\mathbf{p})-E(\mathbf{p}+\mathbf{k})+\omega+i\delta}d\mathbf{p},\quad \delta\to+0. \quad (15)$$

Substituting the value of $\tilde{N}_s$ found in this way in the system (4), we obtain the dispersion equation

$$\left[1+\frac{4\pi e^2\Pi}{k(\varepsilon+1)}\right](\gamma k-\varkappa_2)=\frac{\gamma\varepsilon k}{\varepsilon+1}. \tag{16}$$

We shall consider the range of frequencies corresponding to a surface acoustic wave. In the most realistic situation we may assume that the conditions $k \ll p_e$, $\omega \ll kv_e$ are satisfied, where p_e and v_e are the characteristic electron momentum and velocity. We then find that if $\Pi(\omega, k)$, then

$$\Pi(\omega,k)=\frac{m}{\pi}\left[1-\exp\left(-\frac{\pi N_s}{mT}\right)\right]-\frac{im}{\pi}\int\frac{\omega}{kv}\frac{\partial f}{\partial E}dE. \tag{17}$$

Ignoring the imaginary part of $\Pi(\omega, k)$, we obtain the following expressions for $\varkappa_2$ and $\omega(k)$:

$$\varkappa_2=\gamma k\frac{\varepsilon+kr_0+1}{(\varepsilon+1)(kr_0+1)},\quad \omega(k)=c_v(k^2-\varkappa_2^2)^{1/2}. \tag{18}$$

The velocity s of short-wavelength surface waves characterized by $kr_0 \gg \varepsilon$ is s_1 (surface wave on a free boundary), whereas in the case of long wavelengths corresponding to $kr_0 \ll 1$, it is equal to s_2 (surface wave on a grounded boundary). However, in general, the presence of a plasma gives rise to dispersion of a surface wave because ω is related to k by the nonlinear equation (18).

We can now find the surface wave damping by substituting $\omega(k)$ in the imaginary part of $\Pi(\omega, k)$ and solving Eq. (16) by the iteration method:

$$\left.\begin{aligned}\omega''&=-\frac{k^2(k^2-\omega^2/c_v^2)^{1/2}\varepsilon r_0}{\omega(\varepsilon+1)(kr_0+1)}\varphi(k),\\ \varphi(k)&=\frac{4me^2r_0\omega(k)}{k(\varepsilon+1)(kr_0+1)}\int\frac{1}{v}\frac{\partial f}{\partial E}dE,\quad v=\left(\frac{2E}{m}\right)^{1/2}.\end{aligned}\right\} \tag{19}$$

Elementary transformations give the damping in various asymptotic cases. We shall give only the dependences of ω'' on k, N_s, and T. In the Fermi statistics case, we have

$$\left.\begin{aligned}&kr_0\ll1,\quad \omega''\sim k^2N_s^{-1/2},\\ &kr_0\gg1,\quad \omega''\sim N_s^{-1/2};\end{aligned}\right\} \tag{20a}$$

in the Boltzmann statistics case, we find that

$$\left.\begin{aligned}&kr_0\ll1,\quad \omega''\sim k^2T^{1/2}N_s^{-1},\\ &kr_0\gg1,\quad \omega''\sim T^{-1/2}N_s.\end{aligned}\right\} \tag{20b}$$

It is interesting to note that in the short-wavelength limit the damping is independent of the wave number.

2. EMISSION OF ACOUSTIC WAVES BY TWO-DIMENSIONAL PLASMONS

In addition to the solution of Eq. (16), investigated in Sec. 1c, there is one other branch of surface oscillations. In fact, substitution of $\gamma = 0$ in Eq. (16) gives the dispersion law of free two-dimensional plasmons (i.e., plasmons not coupled to sound). The following expression in Ref. 8 is obtained in the Fermi case[2)]:

$$\omega_p^2(k)=\frac{4\pi e^2N_sk}{m(\varepsilon+1)}\frac{(1+ka_0/2)^2}{1+ka_0/4},\quad \omega_p\gg kv_0,\quad k\ll mv_0, \tag{21}$$

where a_0 is the effective Bohr radius: $a_0 = (\varepsilon + 1)/2me^2$. There is no damping because the condition $\omega_p > kv_0$ is satisfied. The Landau damping applies to free plasmons in the Boltzmann statistics and $\omega_p(k)$ is then given by

$$\left.\begin{aligned}&1+kr_0+i\pi^{1/2}\frac{\omega}{kv_T}\exp\left(-\frac{\omega^2}{k^2v_T^2}\right)\left(1+\frac{2i}{\pi^{1/2}}\int_0^{\omega/kv_T}\exp z^2\,dz\right)=0,\\ &v_T=\left(\frac{2T}{m}\right)^{1/2}.\end{aligned}\right\} \tag{22}$$

The damping is weak only if $kr_0 \ll 1$, $\omega \gg kv_T$. It is then found that

$$\omega_p^2(k)=\frac{4\pi e^2N_sk}{m(\varepsilon+1)}\left[1-i\frac{\pi^{1/2}}{2}(kr_0)^{-3/2}\exp\left(-\frac{1}{2kr_0}-\frac{3}{2}\right)\right]. \tag{23}$$

In the Fermi and Boltzmann cases the group velocity is easily shown to be much less than the velocity of sound for reasonably realistic values of N_s and T. Consequently, the coupling of phonons to elastic waves via the piezoelectric effect gives rise to an additional damping of two-dimensional plasma waves. This damping can be found from Eq. (16) by solving this equation in the range $\omega \gg c_v k$. We can then ignore γk compared with $\varkappa_2 \sim \omega/c_v$ because $\gamma < 1$. Next, we have to assume that $\varkappa_2 = (k^2 - \omega^2/c_v^2)^{1/2} = -i|\varkappa_2|$, so that $\exp(-\varkappa_2 y)$, i.e., plasmons emit acoustic waves which travel into the bulk of the crystal $y > 0$.

The results now are

$$\omega''=\left(1-\frac{k^2v_0^2}{\omega_p^2}\right)^{1/2}\frac{\varepsilon\gamma k}{2c_v(\varepsilon+1)} \tag{24}$$

for a degenerate gas and

$$\omega''=\omega_{p0}\left(\frac{\pi}{8}\right)^{1/2}(kr_0)^{-3/2}\exp\left(-\frac{1}{2kr_0}-\frac{3}{2}\right)-\frac{\varepsilon\gamma k}{2c_v(\varepsilon+1)} \tag{25}$$

for a Boltzmann gas; $\omega_{p0}^2 = 4\pi e^2N_sk/m(\varepsilon + 1)$ is the plasma frequency in the long-wavelength limit. The two terms in Eq. (25) correspond to the above two damping mechanisms. The damping of two-dimensional plasmons is relatively weak in both cases described by Eqs. (24) and (25):

$$\omega''/\omega'\sim\gamma c_vk/\omega\ll1.$$

When the surface charge density is very low and the temperature is sufficiently low, we may find that the conditions $c_v > v_0$ and $c_v > v_T$ are satisfied. Irrespective of whether the Wigner crystallization occurs at such low values of N_s, the longitudinal branch of plasma oscillations of the $\omega = \omega_{p0}(k)$ type always exists[9] and may intersect an acoustic branch. Then, there is no Landau damping in the Fermi case and emission into the bulk of a crystal is possible only if the group velocity of the surface wave exceeds the velocity of bulk sound c_v. Therefore, in the situation under discussion there are two branches of undamped surface oscillations described by the dispersion equation

$$(1-\omega_{p0}^2/\omega^2)(\gamma k-\varkappa_2)=\varepsilon\gamma k/(\varepsilon+1). \tag{26}$$

Equation (26) can be obtained from Eq. (16) if we use the condition $\omega \gg kv_0$. An analysis of Eq. (26) leads to the following conclusions. In the range of very low values of k ($c_vk \ll \omega_{p0}$) there is only one solution $\omega_1(k)$ which has a real and positive value of $\varkappa_2$. Consequently, there is only one surface wave with the dispersion law $\omega_1(k) \approx s_2k$ (the same as for a grounded surface). When k is

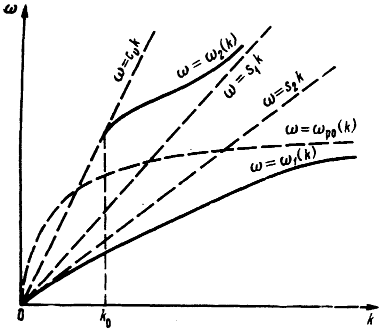

FIG. 1.

increased, the value of ω_{p0} becomes less than $c_v k$ and the branch in question transforms into a two-dimensional plasma wave (subject to a slight renormalization):

$$\omega_1(k)\approx\omega_{p0}(1-\gamma)^{1/2}(1-\gamma/(\varepsilon+1))^{-1/2},\quad \varkappa_2\approx k. \tag{27}$$

As soon as k exceeds a critical value $k_0=4\pi e^2 N_s/mc_v^2$ at which the vanishing of $\varkappa_2$ satisfies Eq. (26), another branch $\omega_2(k)$ appears. In a narrow region near k_0 we have $\omega_2\approx c_v k$ and in the limit $k\to\infty$ we have $\varkappa_2\approx\gamma k/(\varepsilon+1)$, $\omega_2\approx s_1 k$, i.e., $\omega_2(k)$ transforms into a surface wave on a free boundary. The $\omega_1(k)$ and $\omega_2(k)$ curves are shown qualitatively in Fig. 1. In the range $k<k_0$ the second branch is damped out in accordance with the mechanism described above since at $k=k_0$ the group velocity $\partial\omega_2/\partial k$ becomes comparable with c_v, so that the criterion of Cherenkov emission into the bulk of a crystal is satisfied. The penetration depth $\varkappa_2^{-1}$ tends to infinity in the limit $k\to k_0$, i.e., the wave $\omega_2(k)$ is no longer of the surface type. The group velocity of the first branch $\omega_1(k)$ remains less than c_v for any value of k and the depth of penetration $\varkappa_2^{-1}$ is always finite.

3. COUPLED WAVES IN MIS STRUCTURES

We shall now consider a system comprising a piezoelectric crystal of the C_{6v} symmetry $(0<y<\infty)$, an insulator layer of thickness Δ, permittivity ε_f, and velocity of sound c_f $(-\Delta<y<0)$, and a metal plate (field electrode) in the $y=-\Delta$ plane (Fig. 2). We shall assume that the insulator layer is isotropic so there is no piezoelectric effect in this layer. The boundary conditions on the metal are $\varphi=0$ and $T_{ik}n_k=0$ at $y=-\Delta$, where T_{ik} is the stress tensor and $\mathbf{n}$ is the normal to the surface. We shall ignore the thickness and weight of the metal plate. As before, an inversion layer plasma is located at $y=0$. The following conditions are satisfied on this boundary:

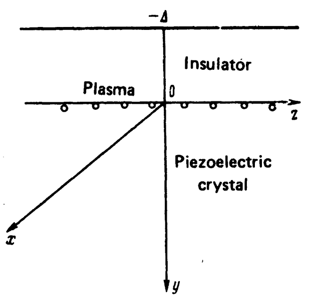

FIG. 2.

$$U_z(-0)=U_z(+0),\quad \lambda\partial U_z/\partial y-\beta\partial\varphi/\partial y|_{+0}=\lambda_f\partial U_z/\partial y|_{-0},$$
$$\varphi(-0)=\varphi(+0),$$

where λ_f is the shear modulus of the insulator layer. These conditions should be supplemented by one describing a discontinuity of the normal component of the electric induction vector, similar to that used above.

The dispersion equation is [compare with Eq. (7)]

$$\left[1+\frac{4i\pi k\sigma_{xx}}{\omega(\varepsilon+\varepsilon_f\operatorname{cth}k\Delta)(1-R_x)}\right]\left(\gamma k+\frac{\lambda_f\varkappa_f}{\rho c_v^2}\operatorname{tg}\varkappa_f\Delta-\varkappa_2\right)=\frac{\varepsilon\gamma k}{\varepsilon+\varepsilon_f\operatorname{cth}k\Delta}, \tag{28}$$
$$\varkappa_f^2=\omega^2/c_f^2-k^2,\quad \varkappa_2^2=k^2-\omega^2/c_v^2.$$

The above analysis corresponds to the limiting case $k\Delta\to\infty$, $\lambda_f\to 0$, $\varepsilon_f=1$. If we assume that $\gamma\to 0$, the second expression in parentheses of the first equation in Eq. (28) gives the well-known dispersion law of an acoustic waveguide: $\lambda_f\varkappa_f\tan\varkappa_f\Delta=\lambda\varkappa_2$. This type of wave exists only in the $c_v>c_f$ case.

3a. We shall now consider the strong scattering case corresponding to $\omega\tau\ll 1$, $kl\ll 1$. We shall bear in mind that electrons on the surface containing the c_6 axis have two components of the effective mass m_x and m_z. For the quantities σ and R defined in Sec. 1a, we now obtain

$$\sigma=\frac{N_s e^2\tau}{m_x},\quad R=-\frac{i\pi\sigma k^2}{e^2(m_x m_z)^{1/2}\omega}\left[1-\exp\left(-\frac{\pi N_s}{(m_x m_z)^{1/2}T}\right)\right]^{-1}, \tag{29}$$

where τ is defined by Eq. (8).

In view of the cumbersome nature of the general solution of Eq. (28), we shall confine our attention to the limit $k\Delta\ll 1$. Substituting Eq. (29) into Eq. (28), we find that the spatial damping decrement is described by

$$\left.\begin{aligned}&\varkappa_2=\gamma k+bk^2\Delta+\gamma\frac{\varepsilon}{\varepsilon_f}\frac{k^2\Delta}{1+a/\Delta-i\tilde{\xi}/k\Delta},\\&\tilde{\xi}=\frac{\varepsilon_f\omega}{4\pi\sigma k},\quad b=\frac{\lambda_f}{\rho c_v^2}\left(\frac{c_v^2}{c_f^2}-1\right),\\&a=\frac{\varepsilon_f}{4(m_x m_z)^{1/2}e^2}\left[1-\exp\left(-\frac{\pi N_s}{(m_x m_z)^{1/2}T}\right)\right]^{-1}.\end{aligned}\right\} \tag{30}$$

In real cases we can always assume that $a\lesssim\Delta$. It is clear from Eqs. (30) and (11) that the surface wave velocity in the $\sigma\gg c_v$ and $\sigma\ll c_v$ cases is equal to s_2 (grounded surface). We shall consider the frequency dependence of the damping in the most realistic case of $\sigma\gg c_v$. A characteristic value of k separating the two damping asymptotes is $k\sim c_v/\sigma\Delta$. It follows from Eqs. (30) and (11) that

$$k\ll\frac{c_v}{\sigma\Delta},\quad \omega''=\frac{4\pi\gamma^2\varepsilon\Delta^2\sigma\omega'^3}{(1-\gamma^2)^{5/2}\varepsilon_f^2 c_v^3}\propto N_s\omega'^3, \tag{31a}$$
$$k\gg\frac{c_v}{\sigma\Delta},\quad \omega''=\frac{\gamma^2\varepsilon c_v\omega'}{4\pi(1-\gamma^2)^{1/2}(1+a/\Delta)^2\sigma}\propto\frac{\omega'}{N_s}. \tag{31b}$$

It is clear from Eqs. (31a) and (31b) that the damping has no maximum within the range defined by $kl\ll 1$, $k\Delta\ll 1$. Clearly, the decrease in the damping on increase of the frequency occurs in the $k\Delta\gg 1$ case, which follows from the results in Sec. 1a.

3b. We shall now consider the collisionless case $(\tau=\infty)$ and, moreover, we shall again assume that $k\ll p_e$, $\omega\ll kv_e$. Then, using the method in Sec. 1c, we find that in the case corresponding to $\varkappa_f\Delta\ll 1$, $k\Delta\ll 1$

Eq. (18) becomes

$$\varkappa_2 \approx \gamma k + bk^2\Delta, \quad \omega = s_2 k(1 + o(k\Delta)). \tag{32}$$

The most important effect of the finite thickness of the insulator layer is a weakening of the coupling between plasma and acoustic waves in the range $k\Delta \ll 1$ [see the right-hand side of Eq. (28)].

The plasmon dispersion in our system has been obtained earlier in the long-wavelength limit[9]: $\tilde{\omega}_{p0}^2 = 4\pi e^2 N_s k/m_x(\varepsilon + \varepsilon_f \coth k\Delta)$. The plasmon damping due to the emission of sound is described by [compare with Eq. (24)]:

$$\omega'' = \frac{\varepsilon\gamma k^2\Delta}{2\varepsilon_f} \quad \text{for} \quad \tilde{\omega}_{p0} \gg c_v k, \quad \frac{\rho_f k\Delta}{\rho} \ll \frac{c_v k}{\tilde{\omega}_{p0}}, \tag{33a}$$

$$\omega'' = \frac{\varepsilon\gamma}{2\varepsilon_f c_v \Delta}\left(\frac{\rho c_v k}{\rho_f \tilde{\omega}_{p0}}\right)^2 \quad \text{for} \quad \tilde{\omega}_{p0} \gg c_v k, \quad \frac{\rho_f k\Delta}{\rho} \gg \frac{c_v k}{\tilde{\omega}_{p0}}, \tag{33b}$$

where ρ_f is the density of the insulator layer. Since $\tilde{\omega}_{p0}$ is proportional to k for $k\Delta \ll 1$, the case (33b) gives rise to damping independent of the plasmon wave number.

If the surface charge density N_s is sufficiently low, a branch of free plasma oscillations of frequency $\tilde{\omega}_{p0}(k)$ may intersect an acoustic branch. Then, strongly coupled plasma-acoustic waves appear in the system and the dispersion equation of these waves is obtained from Eq. (28) by assuming that $kv_s \ll \omega$, $\tau = \infty$:

$$(\tilde{\omega}_{p0}^2 - \omega^2)\left(\varkappa_2 - \frac{\varkappa_f \lambda_f}{\rho c_v^2}\operatorname{tg}\varkappa_f\Delta - \frac{\gamma\varepsilon_f k \operatorname{cth} k\Delta}{\varepsilon_f \operatorname{cth} k\Delta + \varepsilon}\right) = \frac{\gamma\varepsilon k \tilde{\omega}_{p0}^2}{\varepsilon_f \operatorname{cth} k\Delta + \varepsilon} \tag{34}$$

Let us assume that $k\Delta \ll 1$; then, Eq. (34) transforms to

$$\left[\frac{\varkappa_2^2}{k^2} - 1 + \eta(1 - \alpha k\Delta)\right]\left(\frac{\varkappa_2}{k} - \gamma - bk\Delta\right) = \gamma\eta\alpha k\Delta, \tag{35}$$

where $\alpha = \varepsilon/\varepsilon_f$, $\eta = 4\pi N_s e^2\Delta/m_x\varepsilon_f c_v^2$. In the derivation of Eq. (35) we have assumed that $\omega^2 = c_v^2(k^2 - \varkappa_2^2)$. The greatest interest lies in the case when the plasmon and sound velocities are close so that $\eta > 1$ and $\eta - 1 \ll 1$. An analysis of Eq. (35) can be divided into two cases: $c_v > c_f$, when a waveguide solution does exist, and $c_v < c_f$. In both cases there is an oscillation branch $\omega = \omega_1(k)$, which begins from zero with the slope s_2 (wave velocity on a grounded surface) and transforms into a plasma branch on increase in k.

In the waveguide case the solutions of Eq. (35) are in many ways analogous to those of Eq. (26). The second oscillation branch $\omega_2(k)$ begins from

$$k\Delta = k_1\Delta \approx \frac{\eta - 1}{2\eta\alpha} + \left[\left(\frac{\eta - 1}{2\eta\alpha}\right)^2 + \frac{(\eta - 1)\gamma}{2b\eta\alpha}\right]^{1/2} \tag{36}$$

and transforms into an acoustic branch $\omega \approx s_2 k$.

If $c_v < c_f$, then for any k we have no more than one solution of Eq. (35) giving rise to a surface wave. This is due to the fact that in the absence of plasma a surface wave can now exist only in the range $0 < k < \gamma/|b|\Delta$. Therefore, when the condition

$$(\eta - 1)/\alpha\eta > 4\gamma/|b| \tag{37}$$

is satisfied, we have an interval $k_2 < k < k_1$, in which there are no surface waves. Here,

$$k_{1,2}\Delta \approx \frac{\eta - 1}{2\alpha\eta} \pm \left[\left(\frac{\eta - 1}{2\alpha\eta}\right)^2 - \frac{(\eta - 1)\gamma}{2\alpha\eta|b|}\right]^{1/2}. \tag{38}$$

Thus, if $c_v < c_f$, the only branch of surface oscillations begins from $\omega = s_2 k$, it may have a discontinuity if the condition (37) is satisfied, and it then transforms into a plasma branch. At the limits of the interval $[k_1, k_2]$ the depth of penetration becomes infinite ($\varkappa_2 = 0$) and the group velocity becomes c_v, i.e., the wave becomes detached from the surface. We can see from Eqs. (36)–(38) that the range of existence of surface waves and the depth of penetration can be varied by altering N_s because $\eta \propto N_s$.

4. SURFACE WAVES IN A CUBIC CRYSTAL

We shall now consider the (100) face of a cubic piezoelectric crystal. In the absence of a plasma layer when the wave vector is oriented along [010] or [001], there are no pure shear surface waves of the Bluestein–Gulyaev type. However, we may assume that a surface plasma-acoustic wave with a linear dispersion law at low values of k can appear under certain conditions. As in the cases discussed above, this wave is characterized by transverse displacements of the lattice of the piezoelectric crystal and by a longitudinal electric field. All the calculations are fully analogous to those given in Secs. 1 and 2 but the formulas for a cubic crystal are much more cumbersome. Therefore, we shall give only the final results and these can be understood solely in the qualitative sense.

In the collisionless approximation and for sufficiently high values of N_s (a criterion is given below) there is a plasma branch whose damping is due to the two mechanisms described in Sec. 2. Clearly, for $k \gtrsim N_s^{1/2}$ there cannot be any plasma waves. Therefore, the interaction of a plasma branch with an acoustic one is most effective in the case when the intersection condition $\omega_p = c_v k$ is satisfied in the range $k < N_s^{1/2}$. This is possible if $N_s < (\varepsilon m c_v^2/4\pi e^2)^2$. The order of magnitude of k at the point of intersection is k_0, given in Sec. 2. A quantitative analysis of the problem shows that the electric potential and displacement in the wave have two terms which decrease exponentially with depth in a crystal at rates characterized by the decrements $\varkappa_1$ and $\varkappa_2$.

For $k \ll k_0$, we have

$$\left.\begin{aligned} &\varkappa_1 \approx k(1+2\gamma)^{1/2}, \quad \varkappa_2 \approx \frac{k^2}{(1+2\gamma)^{1/2}} \frac{\gamma(\varepsilon+1)c_v^2 m}{8\pi e^2 N_s}, \\ &\omega^2 = c_v^2(k^2 - \varkappa_1^2\varkappa_2^2/k^2). \end{aligned}\right\} \tag{39}$$

On increase of k this branch behaves analogously to $\omega_1(k)$ (see Fig. 1) and in the range $k \gg k_0$ it becomes close to a plasma branch, and we have $\varkappa_{1,2}^2 \approx k^2[1 + \gamma \pm [(1+\gamma)^2 - 1]^{1/2}$. For all values of k the group velocity of the waves of this branch is less than c_v, but there is no damping because of the emission of bulk sound by plasmons. It should be noted that for $k \ll k_0$ a wave of this kind is characterized by an anomalously large depth of penetration $\varkappa_2 \sim k^2/k_0$, whereas in the case of a crystal with the C_{6v} symmetry we may expect $\varkappa_2 \sim k$. The linear part of the dependence $\omega(k)$ is due to the presence of a plasma layer whose range of existence $k \ll k_0$ collapses to zero in the limit $N_s \to 0$ because $k_0 \propto N_s$.

CONCLUSIONS

We shall now consider the effectiveness of the damping mechanisms under discussion. In the absence of a plasma layer the surface waves in a piezoelectric insulator are damped out mainly by the lattice and dislocation absorption mechanisms. At frequencies of the order of 10^9 Hz at room temperature this damping may amount to a few decibels per centimeter. In the case of a piezoelectric semiconductor the electron damping is the principal process. The case of an MIS structure discussed here is characterized by spatial separation of the majority and minority carriers. Clearly, if the depth of penetration of a surface wave is less than the depletion layer thickness, the bulk electron absorption becomes unimportant and the damping considered above is the principal process. In the most likely experimental situation [Eq. (12a)] the damping is $30\gamma^2$ dB/cm for $\omega \sim 3\times10^9$ sec^{-1}, $N_s \sim 10^{12}$ cm^{-2}, $\tau \sim 10^{-12}$ sec, and $m \sim 10^{-28}$ g, i.e., in a typical piezoelectric crystal this damping may compete with other mechanisms.

We have thus considered the interaction of two-dimensional plasma waves with surface transverse acoustic waves in piezoelectric crystals. When the characteristic frequencies of these two types of oscillations are very different, their interaction results in an additional (to the Landau and collisional damping) plasmon damping because of the emission of piezoacoustic waves. A surface acoustic wave is damped out because of the electron mechanism and because of the renormalization of the velocity and depth of penetration, which are functions of the plasma properties. If the frequencies are similar (case of intersection of plasma and acoustic branches) there is a strong mixing of surface oscillations and a branch with the acoustic dependence $\omega(k)$ appears in a cubic crystal at low values of k and this branch is entirely due to the presence of a plasma. In all cases the frequency dependences of the damping and depth of penetration are specific to a given problem (for example, linear and then cubic dependences of the damping on ω at low frequencies defined by $\omega\tau \ll 1$, $kr_0 \ll 1$ or saturation of the damping in the $kr_0 \gg 1$, $kl \gg 1$ case). The most important feature from the point of view of applications is the considerable dependence of all the characteristic parameters of the investigated waves on the surface charge density, which should make it possible to control these parameters under experimental conditions.

[1)] For example, conduction electrons in a thin semimetal film evaporated on a piezoelectric substrate or electrons deposited on a helium film located on a piezoelectric crystal.

[2)] In this section we shall use the collisionless approximation.

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