and disappear from the final expression for the matrices R_k; we therefore set them equal to one.

[1]A. Einstein and N. Rosen, J. Franklin Inst. 223, 43 (1937).
[2]A. S. Kompaneets, Zh. Eksp. Teor. Fiz. 34, 953 (1958) [Sov. Phys. JETP 7, 659 (1958)].
[3]V. P. Frolov, Trudy FIAN 96, 72 (1977).
[4]G. A. Alekseev and V. I. Khlebnikov, Preprint No. 79, Inst. Appl. Math. USSR Acad. Sci., 1977.
[5]V. A. Belinskiĭ and I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. 57, 2163 (1969) [Sov. Phys. JETP 30, 1174 (1970)].
[6]R. H. Gowdy, Ann. Phys. (N.Y.) 83, 203 (1974).
[7]K. Tomita, Preprint RRK 77-8, Inst. Theor. Phys. Hiroshima University, 1977.
[8]V. E. Zakharov and A. B. Shabat, Fukts. Analiz i ego Prilozh. 12, No. 3 (1978) [Functional Analysis and Appl. 12, No. 3 (1978)].
[9]V. E. Zakharov and A. V. Mikhaĭlov, Zh. Eksp. Teor. Fiz. 74, 1953 (1978) [Sov. Phys. JETP 47, 1017 (1978)].

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Quasiparticle excitations in a rotating nucleus

I. M. Pavlichenkov

I. V. Kurchatov Institute of Atomic Energy

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A method is developed for solving the Hartree-Fock-Bogolyubov equations for the rotational states of an axially deformed nucleus with large angular momentum. The method is based on the quasiclassical approximation and uses a one-dimensional realization of the group $SU(2)$. Rotational states of two intersecting bands (the ground-state band and a band based on a two-quasiparticle excitation from the subshell with maximal j on the Fermi surface) are found in the zeroth approximation in the interaction between them. The point of intersection of these bands corresponds to vanishing of the energy of the two-quasiparticle excitation. The energies of neutron quasiparticle excitations in the $i_{13/2}$ subshell are calculated in the model with rectangular potential well. The results of the calculations agree with the experiments.

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§1. INTRODUCTION

Investigation of rotational excitations is an effective method for studying the structure of nuclei. For example, the existence of pairing correlations is most clearly manifested in the value of the moment of inertia of the nucleus.[1] Investigation of the lowest states (up to spin $I=10$) of rotational bands made it possible to establish the degree of adiabaticity of the rotational motion. It was shown that the distortion of the rotational spectrum in even–even nuclei is due to the interaction of the rotation with the quasiparticle degrees of freedom.[2] The parameter of this interaction is the ratio $\alpha=j_F\Omega/\Delta$ of the energy of the Coriolis interaction of a pair to the correlation energy Δ (Ω is the rotation frequency of the nucleus and j_F is the single-particle angular momentum of a nucleon on the Fermi surface). The parameter α is $A^{1/3}$ (A is the number of nucleons in the nucleus) times greater than the parameter of the interaction of the rotational motion with the vibrational motion.[3]

In experiments in recent years on the excitation of rotational levels in reactions with heavy ions in rotational bands there has been discovered an S-shaped dependence of the moment of inertia on Ω^2, this being observed at spins $I\sim 12$–16. This anomaly of the rotational spectrum is known in the English literature as backbending. The numerous attempts to explain this phenomenon reduce ultimately to two alternative hypotheses: 1) The backbending arises as a result of a phase transition at large angular momenta due to the vanishing of the pairing correlation[4] of the anisotropy along the directions of the symmetry axis of the axially deformed nucleus[5]; 2) the backbending is due to the intersection of the ground-state band with a band based on a two-quasiparticle excitation whose angular momentum is aligned along the rotation axis of the nucleus. In the literature, this band has been called the superband. The model was proposed by Stephens and Simon.[6]

Intersection of bands belonging to different phases also occurs in a phase transition. However, the upper parts of the intersecting bands are absolutely unstable and cannot exist in nuclei. Upper and lower levels of intersecting bands on both sides of the intersection point have now been found experimentally[7] in the nuclei Gd^{154}, Dy^{156}, and Er^{164}. The difficulty of detecting upper levels due to their being weakly populated in electromagnetic $E2$ transitions can be successfully overcome if the method of direct Coulomb excitation is used. As a result, it can now be regarded as a reliably established fact that there is no phase transition in the backbending region.

On the other hand, the nature of the superband has not yet been sufficiently well established. In the model of Stephens and Simon, it is a band based on an excitation whose angular momentum is completely decoupled from the deformation. Such bands really are observed in transition nuclei with small deformation. However, backbending also exists in strongly deformed nuclei. This forces us to look for a more general explanation of the phenomenon. We see such an explanation in the

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vanishing of the energy of a two-quasiparticle excitation in a rotating nucleus.

This phenomenon recalls gapless superconductivity, which has been discussed on a number of occasions[8] in connection with nuclei. More detailed investigations have recently been made by Hamamoto and Mottelson[9] in a model of one j level by means of numerical diagonalization of the Hartree–Fock–Bogolyubov equations.

Below, we develop an analytic method of solving the Hartree–Fock–Bogolyubov equations in the quasiclassical approximation using a one-dimensional realization of the group $SU(2)$. We show that the quasiclassical treatment in the cranking model does not lead to hybridization of the intersecting bands depending on the rotational frequency, as in Ref. 9. In the considered approximation, the ground-state band intersects the two-quasiparticle band, and the point of intersection corresponds to vanishing of the energy of the lowest two-quasiparticle excitation.

The absence of hybridization of the bands eliminates strong fluctuation of the angular momentum. On the other hand, the solution found here must be regarded as a zeroth approximation, which does not take into account the interaction between the bands. Although this interaction is small (the region of hybridization of the bands in the nuclei mentioned above includes not more than one rotational state on each side of the point of intersection), it is extremely important for obtaining the condition for the existence of backbending.[10] However, this last problem cannot be solved in the framework of the cranking model.

In the spirit of the considered mechanism for the explanation of backbending, we compare with the experiments the energies of the quasiparticle excitations, which, as is shown in §6, can be deduced from the energies of the rotational bands. The lowest neutron quasiparticle excitations in the $i_{13/2}$ subshell found in §3 agree satisfactorily with the corresponding experimental values for nuclei of the rare earth elements.

It should be noted that the dependence of the energy of the quasiparticles on the rotational frequency of the nucleus is very sensitive to the position of the levels of the self-consistent field. This circumstance may be used to deduce more precisely the parameters of this field in deformed nuclei.

§2. METHOD OF SOLUTION OF THE HARTREE–FOCK–BOGOLYUBOV EQUATIONS

1. To describe the rotational states of an axially deformed nucleus, we use the cranking model, which, as Belyaev[11] has shown by means of a generalized density matrix, corresponds to the quasiclassical approximation at large angular momenta. The Hamiltonian of the nucleons in the rotating coordinate system has the form

$$H'=\sum_{\lambda\lambda'}h_{\lambda\lambda'}(\Omega)a_\lambda^+a_{\lambda'}-\frac{G}{4}\sum_{\lambda\lambda'}\gamma_\lambda\gamma_{\lambda'}a_\lambda^+a_{\bar\lambda}^+a_{\bar\lambda'}a_{\lambda'},$$

$$|\gamma_\lambda|=1,\ \gamma_{\bar\lambda}=-\gamma_\lambda,\ h(\Omega)=h_0-\beta\varkappa(r)Y_{20}(\theta,\varphi)-\Omega j_x-\varepsilon_F.$$

Here, a_λ^+ and a_λ are creation and annihilation operators of nucleons in the state λ ($\bar\lambda$ is the state conjugate with respect to the time), h_0 is the Hamiltonian of the spherical average field, β is the deformation of the nucleus, j_x is a component of the angular momentum operator, G is the coupling constant of the pairing interaction, and ε_F is the Fermi energy.

We consider the simplest type of pairing interaction for which the correlation energy Δ is constant over the volume of the nucleus (uniform pairing). For deformed nuclei this is a good approximation, since $A^{1/3}$ single-particle levels are distributed over the energy interval Δ.[1)] With increasing angular momentum, the importance of nonuniform pairing increases. However, the approximation of uniform pairing does not lead to serious errors for spins $I\leqslant 20$.

We introduce the quasiparticle operators α_ν by means of the transformation

$$\psi(\mathbf{r})=\sum_\nu\{u_\nu(\mathbf{r})\alpha_\nu-R_yv_\nu(\mathbf{r})\alpha_\nu^+\},$$

where $\psi(\mathbf{r})$ is the second-quantization operator of annihilation of nucleons, and $R_y=\exp(-i\pi j_y)$ is the operator of rotation of the coordinate system attached to the nucleus. In the Hartree–Fock–Bogolyubov approximation, the amplitudes u and v of the quasiparticles satisfy the equations (E_ν is the energy of the quasiparticles)

$$\begin{aligned}h(\Omega)u_\nu(\mathbf{r})-\Delta v_\nu(\mathbf{r})&=E_\nu u_\nu(\mathbf{r}),\\ h(-\Omega)v_\nu(\mathbf{r})+\Delta u_\nu(\mathbf{r})&=-E_\nu v_\nu(\mathbf{r}),\end{aligned}\tag{1}$$

and the orthogonality conditions

$$\begin{aligned}\int\{u_\nu^*(\mathbf{r})u_{\nu'}(\mathbf{r})+v_\nu^*(\mathbf{r})v_{\nu'}(\mathbf{r})\}d\mathbf{r}&=\delta_{\nu\nu'},\\ \int\{u_\nu^*(\mathbf{r})R_yv_{\nu'}(\mathbf{r})-v_\nu(\mathbf{r})R_yu_{\nu'}(\mathbf{r})\}d\mathbf{r}&=0.\end{aligned}\tag{2}$$

The equation for Δ has the form[2)]

$$\Delta=-\frac{G}{2}\sum_\nu(1-2n_\nu)\int v_\nu^*(\mathbf{r})u_\nu(\mathbf{r})d\mathbf{r},$$

where n_ν are the quasiparticle population numbers. The integral in this expression can be expressed in terms of the energy of the quasiparticles if we use Eqs. (1) and (2). We obtain the equation

$$\Delta=\frac{G}{4}\sum_\nu(1-2n_\nu)\frac{\partial E_\nu}{\partial\Delta}.\tag{3}$$

From this, using the realtion known from superconductivity theory for the difference between the expectation values of the Hamiltonian in the superfluid state and the normal state,[12] we can find the energy of the nucleus in the rotating coordinate system:

$$\mathscr{E}'(\Omega)=\frac{1}{2}\sum_\lambda\varepsilon_\lambda^0+\frac{\Delta^2}{G}-\frac{1}{2}\sum_\nu(1-2n_\nu)E_\nu.\tag{4}$$

Here, ε_λ^0 is the energy of a nucleon in the self-consistent field of the nucleus for $\Omega=0$.

Equations (1) are invariant under rotation of the coordinate system through angle π about the x axis (R_x) and under the transformation TR_y (T is the operator of time reversal).

2. The Coriolis force acting on a nucleon in the rotating nucleus is proportional to the single-particle angular

momentum j. Therefore, nucleons in levels with $j \sim A^{1/3}$ near the Fermi surface interact the most strongly with the rotation. For the rare earths, these are levels of the $i_{13/2}$ subshell for neutrons and $h_{11/2}$ subshell for protons. These levels are distinguished by their parity from the other states of the closed shell. Therefore, j for them is a good quantum number, since admixture of states with different j due to deformation and rotation corresponds to transitions to a neighboring shell. Therefore, states with maximal j at the Fermi surface can be assumed to be isolated. At rotational frequency $\Omega \sim \varepsilon_F/A$, corresponding to the backbending region, the parameter α is of order unity, $\alpha \sim 1$, and perturbation theory cannot be used for these states. On the other hand, the interaction of the rotation with the nucleons in the remaining levels can be treated by perturbation theory, since they either have small j or are situated far from the Fermi surface.

For the isolated j level, Eqs. (1) can be written in the form

$$\begin{aligned}&[3\delta(^1/_4\epsilon+j_z^2)-\Omega j_x]u_\nu-\Delta v_\nu=E_\nu u_\nu,\\&[3\delta(^1/_4\epsilon+j_z^2)+\Omega j_x]v_\nu+\Delta u_\nu=-E_\nu v_\nu,\end{aligned}\tag{5}$$

where δ determines the splitting of the levels under the influence of the quadrupole deformation, and

$$\epsilon=^4/_3(\varepsilon_j-\varepsilon_F)/\delta-^4/_3 j(j+1)\tag{6}$$

is determined by the distance (ε_j) of the j level in the spherical nucleus from the Fermi surface.

3. To solve the system (5), we use a one-dimensional complex realization of the group $SU(2)$ (see Ref. 13), in which the operators of the angular momentum have the form (the complex variable $z=x+iy$ is defined in the strip $0 \leqslant x < \pi$)

$$j_x=j\cos 2z-\frac{1}{2}\sin(2z)\frac{d}{dz},\quad j_y=j\sin 2z+\frac{1}{2}\cos(2z)\frac{d}{dz},\quad j_z=\frac{1}{2i}\frac{d}{dz}.\tag{7}$$

The operators of finite rotations are given by

$$R_x\psi(z)=(-1)^{j+1}\psi(-z),\quad R_y\psi(z)=(-1)^{j+1}\psi\left(\frac{\pi}{2}-z\right).\tag{8}$$

All the operators of this representation act on the space of functions

$$\psi(z)=\sum_{m=-j}^{j}a_m e^{2imz}.\tag{9}$$

The chosen representation makes it possible to reduce the system of partial differential equations (5) in two variables to a system of ordinary differential equations in the complex variable z. These equations contain the large parameter j, and for their solution we can therefore use the quasiclassical approximation. Quasiclassical solutions must be found for the complete range of the complex variable z and must satisfy the requirement that the scalar product be finite. This requirement determines the quantization conditions for E_ν.

The invariance of Eqs. (5) under the transformation R_x (8) means that there exist even (e) and odd (o) solutions of these equations. The symmetry of the system (5) leads to the relation

$$E_{e,o}(\Omega)=-E_{o,e}(\Omega).\tag{10}$$

The quantization conditions can be obtained from the quasiclassical solutions on the real axis if we use the invariance of the equations under R_x. In accordance with (8) and (9), the solutions of the system (5) have the form

$$\Psi_{e,o}(z)=\begin{pmatrix}u\\v\end{pmatrix}=\sum_{k=0}^{j-1/2}A_k\cdot\begin{cases}\cos(2k+1)z,\\\sin(2k+1)z.\end{cases}$$

Therefore, the relations

$$\Psi_e(\pi/2)=\Psi_o'(\pi/2)=0\tag{11}$$

are boundary conditions for the solution of Eqs. (5) on the real axis.

§3. CALCULATION OF THE ENERGIES OF THE QUASIPARTICLE EXCITATIONS

1. We write out the system of equations (5) on the real axis:

$$\begin{aligned}&\left[\frac{d^2}{dx^2}-f(x)\frac{d}{dx}+g(x)-\epsilon\right]u(x)+\nu v(x)=0,\\&\left[\frac{d^2}{dx^2}+f(x)\frac{d}{dx}-g(x)-\epsilon\right]v(x)-\nu u(x)=0.\end{aligned}\tag{12}$$

Here, we have introduced the notation

$$f(x)=^1/_2\xi\sin 2x,\quad g(x)=\eta+j\xi\cos 2x,\quad \xi=4\Omega/3\delta,\quad \nu=4\Delta/3\delta,\quad \eta=4E/3\delta.$$

We transform (12) into a fourth-order equation and seek a solution of it in the form e^S, where

$$S=S_0+\rho S_1+\rho^2 S_2+\dots.\tag{13}$$

When $|\eta|\sim j$, the parameter of the expansion (13) is $\rho=j^{-1/2}$; when $|\eta|\sim j^2$, it is $\rho=j^{-1}$. If $\Omega\sim\varepsilon_F/A$, then $\nu\sim j$ and $\xi\sim 1$.

The parameter ϵ (6) varies in a wide interval. Depending on its value, solutions of the system (12) will be different. If $|\epsilon|\ll j$, then in the zeroth approximation the phase S' can be found from the equation

$$S_0'^4-g^2(x)+\nu^2=0.$$

For large ϵ, one must use the equation

$$S_0'^4-8\epsilon S_0'^2-g^2(x)+\nu^2+\epsilon^2=0.$$

These two cases differ in the turning points on the real axis and the rules for going round them. Let us consider these cases separately.

2. For small ϵ, the four linearly independent solutions of the system (12) in the quasiclassical approximation have the form

$$\begin{aligned}u_p(x)&=\frac{\exp\{e^{ip\pi/2}\phi_s(x)\}}{[g+e^{ip\pi}(g^2-\nu^2)^{1/2}]^{1/4}(g^2-\nu^2)^{1/8}},\\v_p(x)&=-\frac{1}{\nu}[g+e^{ip\pi}(g^2-\nu^2)^{1/2}]u_p(x),\end{aligned}\tag{14}$$

where $p=0,1,2,3$. In these functions, the phase ϕ is measured from the turning point x_s:

$$\phi_s(x)=\int_{x_s}^{x}(g^2(t)-\nu^2)^{1/2}dt.$$

On the real axis, there are not more two turning points x_1 and x_2:

$$\cos 2x_{1,2}=(\pm\nu-\eta)/j\xi,\quad \pm\nu-j\xi<\eta<\pm\nu+j\xi. \tag{15}$$

The comparison equation near these points has the form

$$u^{IV}\pm bj^2(x-x_{1,2})u=0,\quad 0<b\sim 1, \tag{16}$$

if $|\epsilon|<j$. This last inequality determines the region of small ϵ. Equation (16) can be solved exactly by Laplace's method. Comparing the exact solutions with the approximate solutions (14), we find the formulas for connecting the quasiclassical solutions on the two sides of the turning point x_1 or x_2.

From the functions (14), we form even and odd linear combinations and continue them from the coordinate origin through the turning points in such a way as to satisfy the boundary conditions (11). Ignoring exponentially small terms, we obtain the quantization conditions, which are determined solely by the turning point. If there are not turning points on the real axis, then $\eta_e=\eta_o$ are found from the equation

$$\int_0^{\pi/2}(g^2(x)-\nu^2)^{1/4}dx=\pi(n-1/2),\quad n=n_0+1,\ n_0+2,\ldots,j+1/2. \tag{17}$$

If there is one turning point, x_1, on the real axis, then the quantization conditions are

$$\int_0^{x_{1\sigma}}(g_\sigma^2(x)-\nu^2)^{1/4}dx=\pi[n-1/4(1+2\delta_{\sigma,e})],\quad n=1,2,\ldots,n_0. \tag{18}$$

The index σ is used to denote the even (e) and odd (o) solutions, and the integer n_0 is determined from (18) for $\eta=\nu+j\xi$.

The quantization conditions with the turning point x_2 are obtained from (18) by replacing η by $-\eta$ and x by $\pi/2-x$ in accordnace with Eq. (10). If $\Delta<j\Omega$, then on the real axis there may be two turning points x_1 and x_2. In this case, the spectrum of quasiparticles is found from the quantization condition for the point x_1. In this region, the lowest quasiparticle excitations change sign with increasing Ω.

2. In the case of large ϵ, the linearly independent solutions are

$$u_p(x)=\frac{[g+e^{ip\pi}(g^2-\nu^2)^{1/2}]^{-1/2}\exp\{e^{ip\pi/2}\phi_{p\epsilon}(x)\}}{[(g^2-\nu^2)(\epsilon+e^{ip\pi}(g^2-\nu^2)^{1/2})]^{1/4}},$$
$$v_p(x)=-\frac{1}{\nu}[g+e^{ip\pi}(g^2-\nu^2)^{1/2}]u_p(x), \tag{19}$$
$$\phi_{p\epsilon}(x)=\int_{x_\epsilon}^{x}[(g^2(t)-\nu^2)^{1/2}+\epsilon e^{ip\pi}]^{1/2}dt.$$

In the interval $0\leqslant x\leqslant\pi/2$ there are not more than four turning points. Besides x_1 and x_2 (15), there are two further points corresponding to the vanishing of the factor in the denominator of u_p (19); these are determined by the equation

$$\cos 2x_{3,4}=\frac{1}{j\xi}[\pm(\nu^2+\epsilon^2)^{1/2}-\eta],\quad \pm\eta_-<\eta<\pm\eta_+,$$
$$\eta_\pm=(\nu^2+\epsilon^2)^{1/2}\pm j\xi. \tag{20}$$

The number of turning points on the interval $0\leqslant x\leqslant\pi/2$ depends on the value of the parameters $j\xi$, ν, and ϵ and the eigenvalue η. The corresponding regions are shown in Fig. 1.

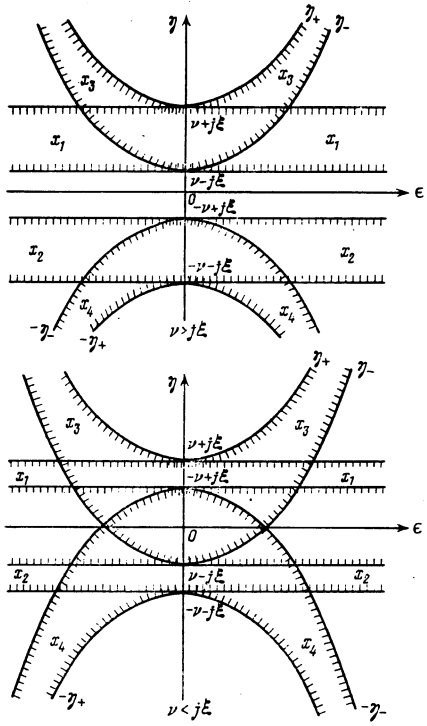

FIG. 1. Positions of the turning points on the real axis as functions of ξ, ν, and η.

The comparison equation for the turning points x_1 and x_2 is obtained from (12) under the condition

$$j^{1/3}<|\epsilon|<j^2, \tag{21}$$

which is the definition of the region of large ϵ. It decomposes into two second-order equations for the functions u_0, u_1 and u_2, u_3. Therefore, the formulas for connecting the solutions on the two sides of the turning points have the usual form.[14] The comparison equation for the points x_3 and x_4 also reduces to a second-order equation for the functions u_1 and u_3 for $\epsilon>0$ and for u_0 and u_2 for $\epsilon<0$ if the inequality (21) is satisfied. It should be noted that this inequality also enables us to assume that all the points are isolated if there is more than one turning point in the interval $0<x<\pi/2$.

Satisfying the boundary conditions (11) and ignoring the exponentially small terms, we obtain the quantization conditions, which are again determined by the turning points on the real axis. If there are no turning points on the interval $0<x<\pi/2$, then $\eta_e=\eta_o$ can be found from the equations

$$\int_0^{\pi/2}[\pm(g^2(x)-\nu^2)^{1/2}-\epsilon]^{1/2}dx=\pi(n-1/2). \tag{22}$$

The quantization condition with the upper sign is used to determine the eigenvalues in the region $\eta>\eta_+$ for $\epsilon\gtrless 0$, and the condition with the lower sign is used in the region $\nu+j\xi<\eta<\eta_-$ for $\epsilon<0$.

For one turning point x_1, the solutions $\eta_e=\eta_o$ exist only for negative ϵ and are determined by the equation

$$\int_0^{x_1}\{[|\epsilon|+(g^2(x)-\nu^2)^{1/2}]^{1/2}-[|\epsilon|-(g^2(x)-\nu^2)^{1/2}]^{1/2}\}dx=\pi(n-1/2). \tag{23}$$

For one turning point x_3, the cases $\epsilon > 0$ and $\epsilon < 0$ are different. If $\epsilon > 0$, then the quantization conditions are

$$\int_0^{x_{3\sigma}} [(g_\sigma^2(x)-\nu^2)^{1/2}-\epsilon]^{1/2}dx=\pi(n-{}^1/_4(1+2\delta_{\sigma,e})). \quad (24)$$

For $\epsilon < 0$, there exist two possibilities:

$$\int_{x_{3\sigma}}^{\pi/2} [|\epsilon|-(g_\sigma^2(x)-\nu^2)^{1/2}]^{1/2}\,dx=\pi(n-{}^1/_4(1+2\delta_{\sigma,e})), \quad (25)$$

$$\int_0^{\pi/2} [|\epsilon|+(g_\sigma^2(x)-\nu^2)^{1/2}]^{1/2}dx=\pi(n-{}^1/_2). \quad (26)$$

If there are two turning points x_1 and x_3 in the interval $0 < x < \pi/2$, then for $\epsilon > 0$ the quantization condition is determined by Eq. (24), and for $\epsilon < 0$ it has the form

$$\int_0^{x_{1\sigma}} [|\epsilon|+(g_\sigma^2(x)-\nu^2)^{1/2}]^{1/2}dx-\int_{x_{3\sigma}}^{x_{1\sigma}} [|\epsilon|-(g_\sigma^2(x)-\nu^2)^{1/2}]^{1/2}\,dx$$
$$=\pi[n-{}^1/_4(1+2\delta_{\sigma,e})]. \quad (27)$$

The ranges of variation of η in Eqs. (23)–(27) can be found in Fig. 1.

The quantization conditions with the turning points x_2, x_4 or x_2 and x_4 are obtained from the conditions given above by replacing η by $-\eta$ and x by $\pi/2 - x$, so that the symmetry property (10) is satisfied. If $\Delta < j\Omega$, then to the points x_1 or (and) x_3 there are added the points x_2 or (and) x_4 (see Fig. 1). The quantization conditions with the points x_2 or (and) x_4 contain nothing new compared with the property (10).

3. The quantization conditions (22)–(27) go over into the conditions (17) and (18) then ϵ tends to zero. This makes it possible to use Eqs. (22)–(27) to determine the quasiparticle energies in the interval $|\epsilon| < j^2$. Note that the dependence of the energies of the quasiparticle excitations on the rotational frequency is determined by $j\xi$.

4. At low rotational frequencies, the quasiparticle energies can be found from Eqs. (22) and (26). The remaining conditions become inoperative, since their left-hand sides become less than unity. To terms quadratic in ξ, we have

$$\eta_m=\eta_m^{(0)}+\frac{j^2\xi^2}{4(\epsilon+4m^2)\eta_m^{(0)}}\left\{\frac{\nu^2}{\epsilon+4m^2}+\frac{1}{2}\left(\frac{\eta_m^{(0)}}{2m}\right)^2\right\},$$
$$\eta_m^{(0)}=[(\epsilon+4m^2)^2+\nu^2]^{1/2}.$$

Here, m is the projection of the angular momentum of the nucleon onto the symmetry axis of the nucleus. The quantity η_m is equal to the quasiparticle energy obtained by perturbation theory from Eqs. (5) for $|m| < j$. The restriction at large m is due to the fact that for $|m| \sim j$ and small ξ terms that for $\xi \sim 1$ are small become important in the equation for the phase S_0'. For the same reason, one cannot obtain from Eqs. (22)–(27) the other limiting case for $\xi \gg 1$.

5. When $\xi \sim 1$, the obtained approximation is a good one except for the case when two closely spaced turning points are situated on either side of $x = \pi/2$. This occurs when one of the turning points is near $x = \pi/2$, since Eqs. (12) are symmetric about $\pi/2$.

Another restriction arises in the region of intersection of the quasiparticle "terms." The quantization conditions (22)–(27) do not hold near the point of intersection of quasiparticle levels of the same parity. In the quasiclassical approximation, each level is characterized by a quantum number that does not depend on ξ or another parameter because the left-hand side of the quantization conditions is an adiabatic invariant. On the other hand, in the case of repulsion of the levels the quantum number changes on the transition through the point of intersection. Therefore, in the quasiclassical treatment the terms can only intersect.

The exact and approximate dependence of the quasiparticle energies on the rotational frequency for constant Δ are shown in Fig. 2 for $j = 13/2$. In Fig. 2, the index n is used to designate the exact values of the quasiparticle energies in the order in which their energy increases. It can be seen from Fig. 2 that the obtained approximation can be used up to values $\xi = 10$.

§4. EQUATION FOR Δ

1. We now turn to the determination of the pairing energy Δ. To this end, we find the contribution of a subshell with large j to the right-hand side of Eq. (3) for $n_\nu = 0$. Using the quantization conditions (22)–(27) and replacing the summation over the quantum number n by integration, we obtain

$$\sum_{\sigma,n=1}^{j+1/2}\frac{\partial E_{jn\sigma}}{\partial\Delta}=\frac{4\nu}{\pi}\int_0^{\pi/2}dx\int_0^{t_j(x)}[(4t^2+\epsilon)^2+\nu^2]^{-1/2}\,dt, \quad (28)$$
$$2t_j(x)=[[(\eta_j+j\xi\cos 2x)^2-\nu^2]^{1/2}-\epsilon]^{1/2},$$

where η_j is the quasiparticle energy determined by Eq. (22) for $n = j + \frac{1}{2}$. This energy depends weakly on the

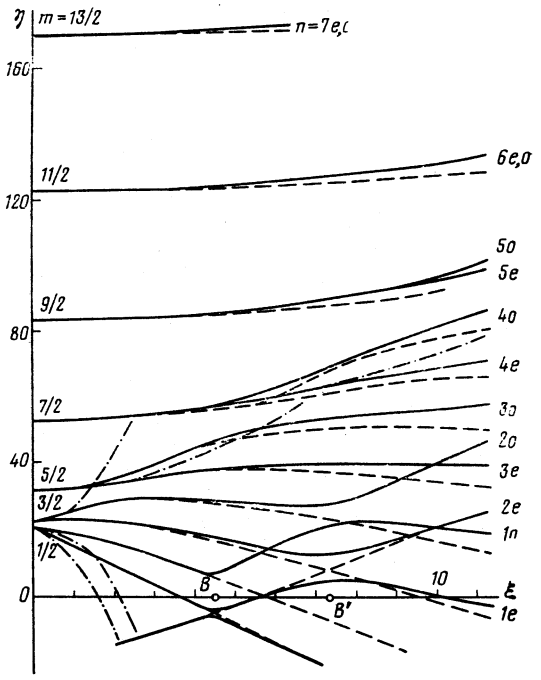

FIG. 2. Quasiparticle levels in rotating nucleus ($\epsilon = 0$, $\nu = 20$). The continuous curves are exact solutions, the dashed curves are solutions obtained quasiclassically, and the chain curves are solutions obtained by perturbation theory.

rotation. Calculating it by perturbation theory, we find

$$\sum_{\sigma,n=1}^{j+1/2}\frac{\partial E_{jn\sigma}}{\partial\Delta}=2\nu\left\{\int_0^j[(4t^2+\epsilon)^2+\nu^2]^{-1/2}\,dt-\frac{j\xi^2}{32(4j^2+\epsilon)}[(4j^2+\epsilon)^2+\nu^2]^{-1/2}\right\}.$$

Thus, rotation, which strongly influences the spectrum of quasiparticle excitations of the subshell with large j, hardly changes the sum (28). The weak dependence on the rotational frequency [the parameter $(j\Omega/\Delta)^2/j^3$] is explained by the quasiclassical canceling of the terms that depend on Ω.

2. Equation (3) contains the sum of the expressions (28) over all j levels. If the rotational frequency of the nucleus does not exceed ε_F/A, then to calculate this sum we can use perturbation theory; for at small j perturbation theory is valid because of the smallness of the adiabaticity parameter α, while for large j it is valid because of the weak dependence of the quantity (28) on Ω.

The correction to Δ in the second order of perturbation theory was calculated by Grin' in Ref. 15. Using the results of Ref. 15, we can obtain the following expression for the dependence $\Delta(\Omega)$ in the model of a rectangular well:

$$\Delta(\Omega)=\Delta_0\{1-(\Omega/\Delta_0)^2\tau\},$$
$$\tau=2.707N^{1/3}(1+0.31\beta)\int_0^1 ds\,s^3(1-s^2)^{1/2}\int_0^1 dt(1-3t^2)\,g\left(\varkappa\frac{t}{s}\right), \qquad (29)$$
$$\varkappa=\frac{\beta}{1+\beta/3}\frac{27}{\Delta_0A^{1/3}}\left(\frac{N}{A}\right)^{1/3}.$$

Here, Δ_0 is the pairing energy in the ground state of a nucleus with N neutrons (or protons for proton quasiparticles). The function g is determined in Migdal's paper (Ref. 1):

$$g(x)=\frac{\operatorname{arg\,sh}x}{x(1+x^2)^{1/2}}.$$

To understand how good perturbation theory is to calculate $\Delta(\Omega)$, we use the results of §7. The maximal rotational frequency $\Omega=360$ keV ($I=22$) is observed in the nuclei Dy^{158} and Er^{164}. At this frequency, the value of Δ is reduced by 18 and 16%, respectively. On the average for eight rare earth elements, we have the maximal values $(\Delta_0-\Delta)/\Delta_0=0.11$.

§ 5. INTERSECTION OF ROTATIONAL BANDS

The dependence of the quasiparticle energies on the rotational frequency of the nucleus enables us to understand how the backbending arises.

1. We consider first an even–even nucleus in which one of the levels of a subshell with large j is near the Fermi surface. In the expression (4) for the energy of the nucleus in the rotating coordinate system the term $-\frac{1}{2}\sum_\nu E_\nu$ can be conveniently regarded as the "vacuum" of the quasiparticles, corresponding to filling of all quasiparticle levels with negative energy in the ground state of the even–even nucleus. The lowest two-quasiparticle excitation in the subshell j can be formed by transferring two quasiparticles from the highest vacuum levels to the lowest quasiparticle levels. The excitation energy of the nucleus will differ from the ground-state energy by the amount $E_{j1e}+E_{j1o}$. For large j, this quantity decreases with increasing Ω, becoming zero[3)] (the point B in Fig. 2). At this point, the ground-state band intersects the band based on the two-quasiparticle excitation of the subshell j. For appropriate interaction between these bands, backbending occurs.[10]

For every two-quasiparticle excitation in subshell j or superposition of excitations with different j in an even nucleus there is corresponding superband. It contains only even spins I if the excitations have different parity under the transformation R_x. For the same parity, the sequence of spins is odd.[16] The parity of the state of the superband is determined by the parity of the j level.

In the considered approximation, the rotational bands can only intersect, since the quasiclassical treatment does not describe the phenomenon of repulsion of the levels. Therefore, our approximation corresponds to noninteracting bands. Since there is no hybridization of the bands at fixed rotational frequency, there is no strong fluctuation of the angular momentum in each separately taken band.

2. We consider the lowest state of an odd nucleus in which the odd particle is in the lowest quasiparticle state of subshell j. The energy of the lowest three-quasiparticle state in this subshell differs from the energy of the single-quasiparticle state $j1e$ by the amount $E_{j1o}+E_{j2e}$, which vanishes at values of Ω greater than in an even nucleus (the point B' in Fig. 2). Therefore, the point of intersection of the bands in the odd nucleus is shifted to the region of higher rotational frequencies. Experimental investigation of the bands in odd nuclei based on states of the subshells $i_{13/2}$ (neutrons) and $h_{9/2}$ and $h_{11/12}$ (protons) shows that backbending is absent in these bands at values of the rotational frequencies for which it is observed in the neighboring even–even nuclei.

3. Finally, we consider a rotational band based on a vibrational state of an even–even nucleus. In the rotating nucleus, the energy of the vibrational excitation, which is a superposition of two-quasiparticle states with different j, will decrease with increasing Ω slower than the energy of the lowest two-quasiparticle excitation with the maximal j. To the intersection of these two excitations there will correspond intersection of the bands based on the vibrational level and the two-quasiparticle level.

It is clear from what we have said above that quasiparticle excitations from the subshell with maximal j at the Fermi surface play a decisive part in the backbending phenomenon. It is of interest to compare the theoretical and experimental values of these quantities.

§ 6. DETERMINATION OF THE ENERGIES OF QUASIPARTICLE EXCITATIONS FROM THE ROTATIONAL SPECTRA

The dependence of the energy of a quasiparticle excitation on the rotational frequency can be deduced from the observed energies of rotational bands.[4)]

1. If in an even–even nucleus we know the energies of two intersecting bands, the ground-state band $\mathscr{E}_0$ and the superband $\mathscr{E}_s$, then the energy of the two-quasiparticle excitation in the subshell with maximal j is

$$E_{2qp}=\mathscr{E}_s'(\Omega)-\mathscr{E}_0'(\Omega). \quad (30)$$

The energy of the nucleus in the rotating coordinate system is determined by

$$\mathscr{E}'(I)=\mathscr{E}(I)-\Omega R(I), \quad R(I)=[I(I+1)-\langle J_z\rangle^2]^{1/2}, \quad (31)$$

and the rotational frequency can be found from the expression

$$\Omega(I)=[\mathscr{E}(I)-\mathscr{E}(I-2)]/[R(I)-R(I-2)]. \quad (32)$$

Note that to the same value of I there correspond different Ω values in different bands.

2. We consider an odd nucleus formed from an even–even nucleus with N neutrons (protons) by the addition of one particle to the state λ in subshell j, and the rotational band based on this state. One can show that the even (e) states in the coordinate system attached to the nucleus are projected onto the state of the band with even $I-\frac{1}{2}$, and that the odd (o) states are projected onto states with odd $I-\frac{1}{2}$ (see Ref. 16). If $\mathscr{E}_e$ and $\mathscr{E}_o$ are the energies of the levels of the band with corresponding sequences of spins measured from the energy of the state λ, the quasiparticle excitations are found as follows:

$$E^e_{1qp}=E_\lambda+\mathscr{E}_e'(\Omega)-\mathscr{E}_0'(\Omega), \quad E^o_{1qp}=E_\lambda+\mathscr{E}_o'(\Omega)-\mathscr{E}_0'(\Omega). \quad (33)$$

The values of $\mathscr{E}'$ and Ω are determined in accordance with Eqs. (31) and (32).

In the expression (33), E_λ is the energy of the quasiparticle at $\Omega=0$. If the band is based on the excited state of an odd nucleus with energy ε_λ, then $E_\lambda=E_{\lambda 0}+\varepsilon_\lambda$. The quasiparticle energy corresponding to the ground state can be deduced from the binding energies of the nuclei[18]:

$$E_\lambda=E_0(N+1)-E_0(N)-\varepsilon_F(N), \quad \varepsilon_F(N)=\tfrac{1}{2}[E_0(N+2)-E_0(N)]. \quad (34)$$

In concrete calculations, as ε_F it is best to use the value averaged over the neighboring nuclei with $\Delta N=2$.

§ 7. COMPARISON WITH EXPERIMENT

To compare the quasiparticle energies calculated in accordance with Eqs. (22)–(27) with the experimental values, it is necessary to know the dependence $\Delta(\Omega)$ and the parameters δ and ϵ. The value of $\Delta(\Omega)$ was calculated by means of the expression (29), and Δ_0 was found from the binding energies of the nuclei. The parameter δ determines the splitting of the levels in subshell j due to the quadrupole deformation. In the model of an infinite rectangular well, it is equal to

$$\delta=(5/16\pi)^{1/2}\beta\varepsilon_{nl}/j(j+1),$$

where ε_{nl} is the energy of the j level in the spherical well without allowance for the spin–orbit interaction:

$$\varepsilon_{nl}=14.39\chi_{nl}/A^{1/3}.$$

Here, χ_{nl} is a root of the corresponding Bessel function. For the Woods–Saxon potential, the parameter δ is on the average 30% smaller than for the rectangular well.[5)]

The parameter ϵ (6) depends on the population in the singleparticle levels of the deformed average field for $\Omega=0$. It can be determined from the energy E_λ of the quasiparticle excitation:

$$\epsilon=-4K^2\pm 4(E_\lambda^2-\Delta_0^2)^{1/2}/3\delta, \quad (35)$$

where K is the projection of the angular momentum of the nucleon onto the symmetry axis of the nucleus, and the plus and minus signs correspond to particle and hole excitations, respectively. The uncertainty due to the Coriolis interaction does not lead to an appreciable error if the band is not strongly distorted, since the quasiparticle energies depend weakly on ϵ. For the same reason, the blocking effect is not taken into account. Therefore, the expression (35) contains Δ_0 of the neighboring even nucleus.[18] In the cases when the rotational band in an odd nucleus is strongly distorted at the bottom (for example, Dy^{157}) or is unknown, the parameter ϵ can be determined approximately from the population of the single-particle states.

The quantity $\langle J_z^2\rangle$, which is needed to determine the quasiparticle energies from the experiment, was taken equal to K^2 in all calculations. This is a reasonable approximation for bands that are weakly distorted at the bottom.

The experimental and theoretical values of the neutron

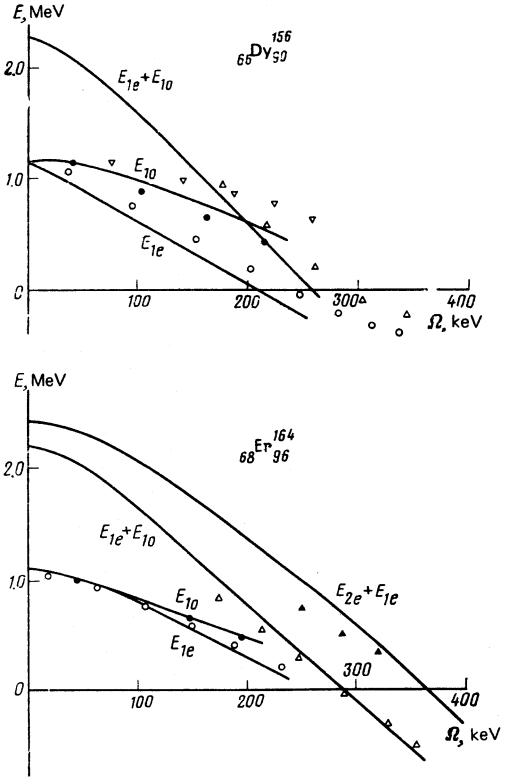

FIG. 3. Quasiparticle excitations in the nuclei Dy^{156} and Er^{164}. The experimental points are determined from the bands: a) in odd nucleus: open circles for $I-\frac{1}{2}$ even and black circles for $I-\frac{1}{2}$ odd; b) in even nucleus; the open triangles are the lowest superband (I even), the black triangles are for superbands with odd I^+, and the inverted open triangles are for superbands with odd I^-.

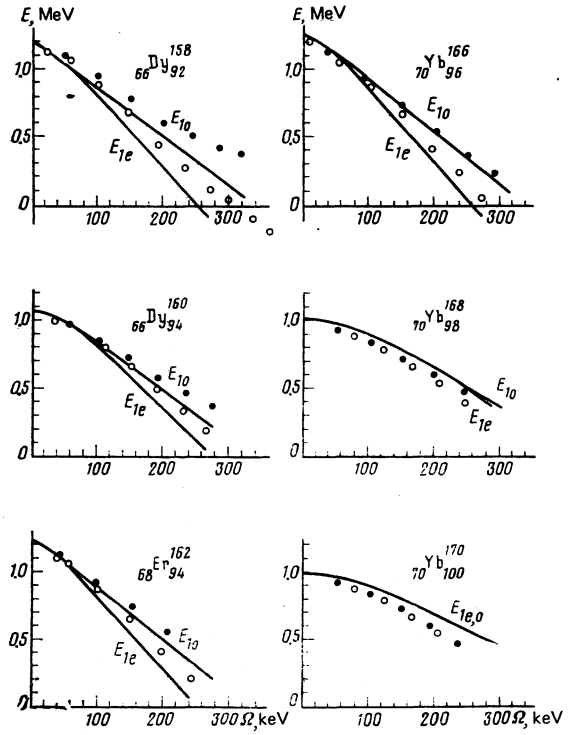

FIG. 4. Single-quasiparticle excitations of the subshell $i_{13/2}$ in nuclei of the rare earth elements. The experimental points are determined from a band of the odd nucleus; the open circles are for $I-\frac{1}{2}$ even and the black circles for $I-\frac{1}{2}$ odd.

quasiparticle excitations in the subshell $i_{13/2}$ for nuclei of the rare earth elements are compared in Figs. 3 and 4. The parameters used in the calculation are given in Table I, in which we also give the quantum numbers of the base level of the rotational band used to define the quasiparticle energies. The letter S stands for the superband in an even–even nucleus, and there then follow the spin, parity, and energy of the first known level in this band.

The intersection of bands in the nuclei Dy^{156} and Er^{164} has been studied in the greatest detail.[7] Three superbands have been found in the second of these nuclei. In determining the quasiparticle energies from the data of Ref. 7, a correction for the interaction of the bands was introduced, so that the experimental points in Fig. 3 correspond to noninteracting bands.

Let us consider the energies of the single-quasiparticle excitations as a function of the position of the Fermi surface. With increasing ε_F (which corresponds to decreasing ϵ) the splitting of the lowest quasiparticle levels of opposite parity must decrease, and the rotational frequency corresponding to vanishing of the energy of the first level is shifted to the right. Qualitatively, this picture is reproduced for the isotopes Er and Yb. In the Dy isotopes, the agreement is less good because of the admixture of states from the $N=4$ shell.[21] These states have smaller j, and therefore their admixture must decrease the slope of the curve $E_{1q\beta}(\Omega)$. In the Yb^{170} nucleus, a deviation of theory from experiment in the opposite direction is observed. It can be explained by the influence of hexadecapole deformation, which increases the part played by states with small K.

TABLE I.

Nucleus	Δ_0, MeV	β (Ref. 19)	ϵ	Band
Dy^{156}	1.14	0.30	−1.2*	S 12+ 2707 keV, 3− 1368 keV, $^3/_2$+ 235 keV Dy^{157}
Dy^{158}	1.00	0.33	−17.2	$^5/_2$+ [642] 178 keV Dy^{159}
Dy^{160}	0.97	0.33	−20.0	$^5/_2$+ [642] *gs* Dy^{161}
Er^{162}	1.06	0.32	−16.8	$^5/_2$+ [642] 69 keV Er^{163}
Er^{164}	1.01	0.31	−19.6	S 12− 2519 keV, S 15+ 3518 keV, $^5/_2$+ [642] 47 keV Er^{165}
Yb^{166}	1.12	0.30 Ref. 20	−18.0	$^5/_2$− [642] 30 keV Yb^{167}
Yb^{168}	0.95	0.28	−44.8	$^7/_2$− [633] *gs* Yb^{169}
Yb^{170}	0.81	0.28	−56.4	$^7/_2$− [633] 95 keV Yb^{171}

*Found from population of single-particle levels.

In Dy^{156}, a band with negative parity was investigated. The experimental dependence of the excitation energy in this band on Ω is shown in Fig. 3. If this excitation is described by two quasiparticles in the subshells $i_{13/2}$ and $h_{9/2}$ ($E_{13/2\,1e}+E_{9/2\,1e}$, since the spins I are odd), the theoretical points lie appreciably higher than the experimental ones, particularly at small Ω. This is evidently an octupole band, which must intersect the two-quasiparticle band at large angular momenta.[22]

[1)]Numerical calculations show that the extent of nonuniformity of the pairing in the ground state of deformed nuclei does not exceed 5–7%.

[2)]The remaining self-consistency conditions are not considered, since at frequencies $\Omega \lesssim \varepsilon_F/A$ the change in the Fermi energy and the deformation β with the rotation can be ignored.

[3)]The vanishing of E_{j1e} does not lead to any physical consequences because the parity of the number of particles is conserved.[16]

[4)]Such an attempt was undertaken by Bengtsson and Frauendorf.[17] However, this attempt is of a model nature, since theoretical values of the moment of inertia were used in the determination of the quasiparticle energies.

[5)]The author is grateful to É. E. Sapershtein, who kindly provided the numerical data needed for this calculation.

[1]S. T. Belyaev, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 31, 11 (1959); A. B. Migdal, Zh. Eksp. Teor. Fiz. 37, 249 (1959) [Sov. Phys. JETP 10, 176 (1960)].

[2]Yu. T. Grin' and I. P. Pavlichenkov, Zh. Eksp. Teor. Fiz. 43, 465 (1962) [Sov. Phys. JETP 16, 333 (1963)]; E. R. Marshalek, Phys. Rev. 158, 993 (1967).

[3]I. M. Pavlichenkov, Nucl. Phys. 55, 225 (1964).

[4]B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5, 511 (1960).

[5]V. G. Nosov and A. M. Kamchatnov, Zh. Eksp. Teor. Fiz. 73, 785 (1977) [Sov. Phys. JETP 46, 411 (1977)].

[6]F. S. Stephens and R. S. Simon, Nucl. Phys. A183, 257 (1972).

[7]T. L. Khoo, F. M. Bernthal, J. S. Boyno, and R. A. Warner, Phys. Rev. Lett. 31, 1146 (1973); F. W. N. De Boer, P. Koldewijn, R. Beetz *et al.*, Nucl. Phys. A290, 173 (1977); N. R. Johnson, D. Cline, S. W. Yates *et al.*, Phys. Rev. Lett. 40, 151 (1978).

[8]A. Goswami, L. Lin, and G. L. Struble, Phys. Lett. 25B, 451 (1967); Yu. T. Grin', Phys. Lett. 52B, 135 (1974).

[9]I. Hamamoto. Nucl. Phys. A271, 15 (1976); R. Bengtsson, I. Hamamoto, and B. Mottelson, Phys. Lett. 73B, 259 (1978).

[10]I. M. Pavlichenkov, Phys. Lett. 53B, 35 (1974).

[11]S. T. Belyaev, Struktura yadra. Lektsii. Mezhdunarodnaya shkola po strukture yadra, Alushta 1972 (Nuclear Structure. Lectures, International School on Nuclear Structure,

Alushta, 1972), published by JINR, Dubna, p. 491.
[12]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike. Fizmatgiz (1962), p. 395; English translation: [Methods of Quantum Field Theory in Statistical Physics, Englewood Cliffs (1963)].
[13]C. P. Bouer, E. G. Kalnis, and W. J. Miller, J. Math. Phys. 16, 512 (1975); I. M. Pavlichenkov, Preprint IAÉ-2769, Moscow (1977).
[14]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Gostekhizdat (1948); (English translation: Quantum Mechanics, Pergamon Press, Oxford (1965)).
[15]Yu. T. Grin', Zh. Eksp. Teor. Fiz. 41, 445 (1961) [Sov. Phys. JETP 14, 320 (1962)].
[16]H. J. Mang, Phys. Rep. Phys. Lett. 18C, No. 6 (1975).
[17]B. Bengtsson and S. Frauendorf, Contribution 819 to the Intern. Conf. on Nuclear Structure, Tokyo (1977).
[18]Yu. T. Grin', S. I. Drozdov, and D. F. Zaretskiĭ, Zh. Eksp. Teor. Fiz. 38, 222 (1960) [Sov. Phys. JETP 11, 162 (1960)].
[19]P. H. Stelson and L. Grodzins, Nucl. Data 1A, 1 (1965).
[20]B. Bochev, S. Iliev, R. Kalpakchieva, S. A. Karamian, T. Kutsarova, E. Nadjakov, and Ts. Venkova, Nucl. Phys. A282, 159 (1977).
[21]M. I. Baznat, N. I. Pyatov, and M. I. Cherneĭ, Fiz. Elem. Chastits At. Yadra 4, 941 (1973) [Sov. J. Part. Nucl. 4, 384 (1974)].
[22]P. Vogel, Phys. Lett. 60B, 431 (1976).

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Relaxation of μ^+-meson spin in the crystal lattice of copper, vanadium, or niobium in weak magnetic fields

V. G. Grebinnik, I. I. Gurevich, V. A. Zhukov, I. G. Ivanter, A. I. Klimov, V. N. Maĭorov, A. P. Manych, B. A. Nikol'skiĭ, A. V. Pirogov, A. N. Ponomarev, V. I. Selivanov, and V. A. Suetin

I. V. Kurchatov Institute of Atomic Energy
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We compare the experimental and calculated values of the relaxation rates Λ of the spin of a non-diffusing μ^+ meson in copper, vanadium, and niobium in the absence of an external magnetic field. The enlargement of the interstitial pores of the crystal lattices of these metals by the localization of the μ^+ meson in the pores is estimated. The $\Lambda(B)$ dependence is investigated for weak transverse magnetic fields B.

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1. INTRODUCTION

The spin of a μ^+ meson in a crystal lattice relaxes because of the magnetic dipole interactions with the magnetic moments of the surrounding nuclei.[1] Measurement of the relaxation rate Λ of the spin of the μ^+ meson makes it possible to determine the type of the interstitial pore in which the μ^+ meson is localized, and also find the deformation of this pore. The determination type of the pore and the degree of its deformation are determined by comparing the experimental and calculated values of Λ. This method is quite sensitive, since $\Lambda \sim r_i^{-3}$, where r_i is the distance between the μ^+ meson and the nuclei of the neighboring atoms of the metal. This comparison is possible only in the limiting cases of strong and weak (zero) external magnetic fields B, since the calculation of the values of Λ for an arbitrary field B entails very great difficulties.

The values of Λ should be measured at sufficiently low temperature, when the μ^+ meson hardly diffuses in the crystal. The diffusion of the μ^+ meson causes the local dipole magnetic field at the meson to become alternating in time, as a result of which the relaxation rate Λ decreases. In the present study we have measured the relaxation rate of the spin of a non-diffusing μ^+ meson in a crystal lattice of copper, vanadium, or niobium in weak transverse magnetic fields B and at $B=0$. The measurement of the relaxation rate Λ at $B=0$ is in certain respects more convenient than at $B \to \infty$, inasmuch as to obtain inpractice the asymptotic values of $\Lambda(B \to \infty)$ in these metals it is necessary to produce very strong magnetic fields. The magnetic fields B needed to measure the values of $\Lambda(B \to \infty)$ must lead to a practically complete suppression of the influence of the electric quadrupole interactions of the μ^+ meson with the neighboring metal-atom nuclei. Camani *et al.*[2] have shown experimentally that in copper this requires a field B ~10 kOe. Allowance for the quadrupole interactions at $B=0$, in the case of sufficiently large quadrupole moments Q of the nuclei, reduces to simple corrections to the calculated values of $\Lambda(B=0)$, and when these are calculated one can assume the limiting value $Q \to \infty$.

2. THEORY

The calculated value Λ_{calc} of the relaxation rate of the spin of a μ^+ meson in a metal is determined from the dependence of the polarization $P(t)$ of the μ^+ meson on the time. In the calculation of the $P(t)$ we represent this function as a series in even powers of t:

$$P(t)=1-{}^1/_2 t^2 M_2+{}^1/_4 t^4 M_4-\dots \qquad (1)$$

There are no odd powers of t in the expression (1), in-

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