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Translated by E. Brunner

Phonon retardation of a domain wall in a rare-earth orthoferrite

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(Submitted 18 May 1978; resubmitted 20 July 1978)
Zh. Eksp. Teor. Fiz. 75, 2183–2195 (December 1978)

The topic is interaction of a moving domain wall in a rare-earth orthoferrite with elastic vibrations of the lattice and with spin waves that are localized near the domain wall. It is shown that this interaction leads to a sharp increase of the retarding force acting on the domain wall at a velocity close to one of the velocities of sound. The retarding force due to one- and two-particle processes is calculated; consideration is also given to the effect of this phenomenon on the variation of the domain-wall velocity with external magnetic field.

PACS numbers: 75.60.Ch, 75.30.Ds

INTRODUCTION

In the motion of domain walls (DW) in perfect, magnetically ordered crystals, there is particular interest in the study of the dynamic retardation of DW that is caused by interaction of a DW with magnons and with lattice vibrations (phonons) and that exists even in an ideal crystal.

In calculation of the dynamic retardation of DW, usually only interaction of the DW with magnons is taken into account.[1] Actually, this is due to the fact that the coupling between the magnetic and elastic subsystems is small and manifests itself significantly only when definite resonance conditions are satisfied.[2] As we shall show below, in the present case this condition is coincidence of the domain-wall velocity with the phase velocity of an elastic wave; there is then the possibility of sound radiation, which leads to a significant contribution to the dynamic retarding force. This phenomenon has been observed in experiments[3,4] carried out on rare-earth orthoferrites (REO).

We note that such effects can in principle be observed only in magnetic materials in which the limiting velocity V_C of DW motion is larger than the velocity of sound. As an example, one may cite antiferromagnets or ferrites with equivalent magnetic sublattices (for example, REO),[1)] in which the limiting DW velocity is determined by exchange interaction alone[5] ($V_C \sim I_a/\hbar \sim 10^4$ m/sec; I is the exchange integral, a the lattice constant; the experimental value of V_C in REO, obtained by Konishi *et al.*[3] and by Chetkin *et al.*,[4] is of the order of $2\cdot 10^4$ m/sec).

The present paper treats radiation of sound during motion of a DW in REO. Characteristic of this problem is the fact that the DW is plane (a one-dimensional system). It is shown that this significantly changes the nature of the radiation as compared with the standard situation of Cerenkov radiation (a particle-like or linear system[6]); specifically: the condition for radiation of a phonon is satisfied only in a narrow range of DW velocity near the sound velocity s, and not for $V > s$ as in the standard situation. This fact leads to the necessity for considering processes of radiation of several particles (phonons or phonons and spin waves).

We shall consider processes of radiation of one and of two phonons, and also the process of radiation of a phonon and a spin wave (Sections 2 and 3). We shall calculate the contribution of these processes to the DW retarding force. It turns out that of all the two-particle processes, the one that makes the greatest contribution is the process of radiation of a volume phonon and of a spin wave localized near the DW. The closing section 4 of the paper discusses the effect of these processes on the character of the DW motion under the influence of an external magnetic field.

1. INTERACTION OF PHONONS WITH A DOMAIN WALL

We represent the energy of the REO as the sum of the energy (W_m of the magnetic subsystem and the energy) W_{me} of magnetoelastic interaction. An expression for W_m can be written in the form[7]

$$W_m\{\mathbf{m};\mathbf{l}\}=M_0^2\int w_m\,d\mathbf{r},$$
$$w_m={}^1/_2\delta\mathbf{m}^2+{}^1/_2\beta_1 l_x^2+{}^1/_2\beta_3 l_z^2+{}^1/_4\beta_1' l_x^4+{}^1/_4\beta_2' l_x^2 l_z^2+{}^1/_2\beta_3' l_z^4+d_1 m_x l_z-d_3 m_z l_x+{}^1/_2\alpha(\partial\mathbf{l}/\partial x_i)^2. \quad (1)$$

Here $\mathbf{m}=(\mathbf{M}_1+\mathbf{M}_2)/2M_0$, $\mathbf{l}=(\mathbf{M}_1-\mathbf{M}_2)/2M_0$; $\mathbf{m}^2+\mathbf{l}^2=1$, $\mathbf{m}\cdot\mathbf{l}=0$; $\mathbf{M}_1$ and $\mathbf{M}_2$ are the magnetic moments of the sublattices, M_0 the saturation magnetization of the sublattices ($|\mathbf{M}_1|=|\mathbf{M}_2|=M_0$). In the expression (1) we have neglected the energy of nonuniformity of the vector $\mathbf{m}$, since in REO $\mathbf{m}^2\ll\mathbf{l}^2\sim1$. The quantities β_1 and β_3 are anisotropy constants of the second order, β_1', β_2', and β_3' of the fourth order; d_1 and d_2 are Dzyaloshinskiĭ constants, δ the constant of uniform and α of nonuniform exchange. The z axis is chosen along the c axis of the crystal, the x axis along the a axis.

The magnetoelastic energy, in the linear approximation with respect to the strain tensor, has the form

$$W_{me}=M_0^2\int\Lambda_{ij}\{\mathbf{m};\mathbf{l}\}\frac{\partial u_i}{\partial x_j}d\mathbf{r}; \quad (2)$$

the expression for the tensor Λ_{ij} will be given later [see (12)].

We write the displacement vector $\mathbf{u}(\mathbf{r})$ of the elastic medium in standard form in terms of the phonon creation and annihilation operators $b^+_{\mathbf{q}\lambda}$ and $b_{\mathbf{q}\lambda}$:

$$\mathbf{u}(\mathbf{r})=\left(\frac{\hbar}{2\rho\Omega}\right)^{1/2}\sum_{\mathbf{q},\lambda}\frac{\mathbf{e}_\lambda(\mathbf{q})}{\omega_\lambda^{1/2}(\mathbf{q})}(b_{\mathbf{q}\lambda}e^{i\mathbf{q}\mathbf{r}}+b_{\mathbf{q}\lambda}{}^+e^{-i\mathbf{q}\mathbf{r}}), \quad (3)$$

where $\mathbf{q}$, $\omega_\lambda(\mathbf{q})$, and $\mathbf{e}_\lambda(\mathbf{q})$ are respectively the wave vector, the frequency, and the unit polarization vector of the phonon $(\mathbf{q},\lambda)$; ρ is the density of the material, Ω the volume of the crystal.

As a rule,[7] the plane of a DW in a REO coincides with the (XZ) plane; that is, for a plane wall moving with velocity V, $\mathbf{l}=\mathbf{l}(y-Vt)$ and $\mathbf{m}=\mathbf{m}(y-Vt)$. Hence also $\Lambda_{ij}=\Lambda_{ij}(y-Vt)$. Taking account of this fact and of the expression (3) for the vector $\mathbf{u}(\mathbf{r})$, we write the Hamiltonian of interaction of phonons with a moving DW in the form[2)]

$$W_{me}=\frac{M_0^2S}{\Omega^{1/2}}\sum_{\mathbf{q},\lambda}\{U_\lambda(q)e^{-iqVt}b_{\mathbf{q}\lambda}{}^++\text{H.c}\}, \quad (4)$$

$$U_\lambda(q)=iq\left(\frac{\hbar}{2\rho\omega_\lambda(q)}\right)^{1/2}\Lambda_\lambda(q),\quad \Lambda_\lambda(q)=\int_{-\infty}^{+\infty}d\xi\,e^{iq\xi}\Lambda_{iy}(\xi)e_\lambda{}^i(q), \quad (5)$$

where S is the area of the DW; $q\equiv q_y$; $\xi=y-Vt$.

Thus finding the Hamiltonian (4)–(5) reduces to calculation of the quantities $\Lambda_\lambda(q)$, expressed in terms of Fourier components of the distribution of magnetization in the moving DW.

As is known, the distribution of magnetization in a DW depends on its velocity V. The characteristic parameter that determines the change of the magnetization in a moving DW as compared with a stationary one is the ratio of the velocity to the critical velocity of motion V_C of the wall. When $(V/V_C)^2\ll1$, the change of structure of a DW because of its motion may be considered small.

As we shall show below, the interaction of a DW with phonons is most intense when the velocity V of the DW is close to the velocity s_λ of long-wave sound. If we suppose that $(s_\lambda/V_C)^2\ll1$ (for REO, $(s_\lambda/V_C)^2\sim10^{-1}$), we may neglect the change of structure of the DW and calculate the quantities $\Lambda_\lambda(q)$ by use of the known expressions for the magnetization distribution in a stationary DW, simply replacing the coordinate y in them by the quantity $\xi=y-Vt$.

It is known that in REO there exist two types of domain walls: DW I and DW II.[8,9] For DW I, the weak ferromagnetic vector $\mathbf{m}$ is parallel to $\mathbf{e}_z$ and changes only in magnitude, while the antiferromagnetism vector $\mathbf{l}$ rotates in the (XY) plane:

$$\mathbf{l}=(-\mathbf{e}_x\cos\theta+\mathbf{e}_y\sin\theta),\quad \mathbf{m}=\mathbf{e}_z\frac{d_3}{\delta}\cos\theta. \quad (6)$$

The characteristic of DW II is that both vectors $\mathbf{m}$ and $\mathbf{l}$ turn in the (XZ) plane:

$$\mathbf{l}=(-\mathbf{e}_x\cos\theta+\mathbf{e}_z\sin\theta),\quad \mathbf{m}=\frac{d_3+(d_1-d_3)\sin^2\theta}{\delta}(\mathbf{e}_z\cos\theta+\mathbf{e}_x\sin\theta). \quad (7)$$

The function $\theta=\theta(\xi)$ for DW I and DW II is determined by the single relation

$$\sin\theta(\xi)=2^{1/2}\left[(1+p)\operatorname{ch}\frac{2\xi}{y_0}+(1-p)\right]^{-1/2}, \quad (8)$$

where the quantities p and y_0 (the latter has the meaning of DW thickness) are determined by the crystal parameters and the type of DW:

$$y_0=\left(\frac{\alpha}{2|K_1+8K_2|}\right)^{1/2},\qquad p=-\frac{8K_2}{K_1+8K_2}; \quad (9)$$

$$K_1=\begin{cases}-d_3^2/2\delta+{}^1/_2\beta_1+{}^1/_4\beta_1' & \text{for DW I}\\ (d_1^2-d_3^2)/2\delta+{}^1/_2(\beta_1-\beta_3)+{}^1/_4(\beta_1'-\beta_3') & \text{for DW II}\end{cases}, \quad (10)$$

$$K_2=\begin{cases}{}^1/_{32}\beta_1' & \text{for DW I}\\ -(d_1-d_3)^2/2\delta+{}^1/_{32}(\beta_1'-\beta_2'+\beta_3') & \text{for DW II}\end{cases}. \quad (11)$$

The condition for existence of a 180-degree DW, separating domains with equilibrium orientation of the vectors $\mathbf{m}$ and $\mathbf{l}$ along the c and a axes respectively, bounds the values of the parameters K_1 and K_2 by the inequality $K_1+8|K_2|<0$. We note that when $K_2>0$ (then two second-order phase transitions can occur in the REO), the parameter $p>0$; but when $K_2<0$ (then a first-order phase transition can occur in the REO), the parameter p lies in the interval $(-\frac{1}{2},0)$.

For concrete calculation of the values of $\Lambda_\lambda(q)$ corresponding to DW of both types, it is necessary to write an explicit expression for the tensor $\Lambda_{ij}\{\mathbf{m};\mathbf{l}\}$ in the magnetoelastic energy (2). But there is no necessity for writing all the invariant combinations of the type $l_il_ju_{sp}$ and $m_il_ju_{sp}$, since, first, $\Lambda_\lambda(q)$ involves only the quantities Λ_{iy}, and, second, not all components of the vectors $\mathbf{m}$ and $\mathbf{l}$ differ from zero in the DW [see (6) and (7)]. Taking these facts into account and restricting ourselves to invariants of the second order in m_i and $\mathbf{l}_i$, we write the expression for the energy density of interaction of the DW with elastic strains in the form

$$A_{iy}u_{iy}=u_{xy}(b_1l_xl_y+f_3m_zl_y)+u_{yy}[b_2l_y^2+b_3l_z^2+f_1m_xl_z+f_2(m_zl_x-m_z^{(0)}l_x^{(0)})]. \quad (12)$$

Here b_1, b_2, and b_3 are relativistic magnetoelastic constants, due to the anisotropy energy ($b\sim\beta,\beta'$), and f_1, f_2, and f_3 are constants due to exchange-relativistic Dzyaloshinskiĭ interaction ($f\sim d$). The last term in (12) is prescribed by the consideration that at the equilibrium value ($\mathbf{m}^{(0)},\mathbf{l}^{(0)}$) of the magnetization, the magnetoelastic energy must vanish.

In the expression (12) invariants containing spatial derivatives of the vectors **m** and **l** have been omitted, since allowance for these terms, which make a contribution to the interaction of the DW with longitudinal sound, leads only to overdetermination of the corresponding magnetoelastic constants. In addition, invariants quadratic in the m_i have been omitted; they are small in comparison with (12).

From the expression (12) we see that a DW interacts both with longitudinal phonons ($\lambda=l$) and with transverse ($\lambda=t$) phonons polarized along the x axis. There is no interaction, however, with transverse phonons polarized along the z axis.

2. CONTRIBUTION OF SINGLE-PHONON PROCESSES TO RETARDATION OF DW

The Hamiltonian (4)–(5) describes processes of radiation and absorption by the domain wall of a phonon with $\mathbf{q}\|\mathbf{e}_y$. Here energy of DW motion is transferred to the phonon subsystem; that is, retardation of the DW occurs. The retarding force that acts on the DW can be found as the rate of transfer of momentum to the phonon subsystem; this can be calculated without difficulty by starting from standard thermodynamic perturbation theory (considering W_{me} as the perturbation, we use the smallness of the magnetoelastic coupling constant $\zeta=(bM_0^2/\rho s^2)\sim10^{-5}$). Using (4) and (5), we find the retarding force F_1 acting on unit area of the DW because of single-phonon processes:

$$F_1=\frac{2\pi SM_0^4}{\hbar\Omega}\sum_{q,\lambda}q|U_\lambda(q)|^2\delta(\omega_\lambda(q)-qV)$$

$$=\frac{M_0^4}{2\rho}\sum_\lambda\int_{-\infty}^{+\infty}dq\frac{q^3}{\omega_\lambda(q)}|\Lambda_\lambda(q)|^2\delta(\omega_\lambda(q)-qV). \quad (13)$$

It must be noted that formula (13) does not contain the occupation numbers of the phonons (the probability of radiation of a phonon is proportional to n_q+1, of the reverse process to $-n_q$). Consequently this mechanism of dissipation of DW energy, in contrast to the many-magnon mechanism, is independent of temperature and may become dominant at low temperatures.

It is important to note that in writing (13) we have not taken into account that phonons have a finite lifetime. Therefore this formula is strictly applicable only at $T=0$. The role at attenuation of phonons will be discussed at the end of this section.

It is easy to see that according to formula (13), $F_1\neq0$ only when the DW velocity V coincides with the phase velocity of some elastic wave propagating along the y axis; that is, only when for some phonon polarization λ there is a root of the equation

$$qV=\omega_\lambda(q). \quad (14)$$

The number of roots of Eq. (14) depends on the dispersion law of phonons with $\mathbf{q}\|\mathbf{e}_y$ over a whole Brillouin zone. We note, however, that for any form of the dispersion law, there exist velocity values $V_{\min}^{(\lambda)}$ and $V_{\max}^{(\lambda)}$ such that Eq. (14) has one or several roots for $V_{\min}^{(\lambda)}\leq V\leq V_{\max}^{(\lambda)}$ and has no roots outside this velocity range (see Fig. 1). As a rule, $V_{\max}^{(\lambda)}$ coincides with the velocity of long-wave sound of the prescribed polarization λ.

Taking account of this fact, we write the expression for the retarding force F_1 in the form

$$F_1=\sum_\lambda F_{1\lambda}$$

$$=\sum_\lambda\left\{\frac{M_0^4}{2\rho V}\sum_\alpha q_{\lambda\alpha}^2|\Lambda_\lambda(q_{\lambda\alpha})|^2\Big/\left|\frac{\partial}{\partial q}[\omega_\lambda(q)-qV]\right|_{q=q_{\lambda\alpha}}\right., \quad (15)$$

where $q_{\lambda\alpha}$ are the roots of Eq. (14) (the index α enumerates the roots of Eq. (14) for a given polarization λ of the phonon).

Using this relation and formulas (6), (8), and (12), we obtain the retarding force that acts on a DW of the first type (DW I) by virtue of interaction with transverse (F_{1t}) and longitudinal (F_{1l}) phonons:

$$F_{1t}^{(\mathrm{I})}(V)=\frac{M_0^4\pi^2}{4\rho Vs_t\sigma_t}\left(\frac{y_0}{a}\right)^2\frac{(b_1-d_3f_3/\delta)^2}{\mathrm{ch}^2(\pi q_0y_0/2)}\times\begin{cases}\frac{1}{p}\mathrm{sh}^2\left(\frac{q_0y_0}{2}\arccos\frac{1-p}{1+p}\right), & p>0\\ (q_0y_0)^2, & p=0\\ \frac{1}{|p|}\sin^2\left(\frac{q_0y_0}{2}\mathrm{arch}\frac{1-p}{1+p}\right), & p<0\end{cases}; \quad (16)$$

$$F_{1l}^{(\mathrm{I})}(V)=\frac{M_0^4\pi^2}{4\rho Vs_l\sigma_l}\left(\frac{y_0}{a}\right)^2\frac{(b_2+d_3f_2/\delta)^2}{\mathrm{sh}^2(\pi q_0y_0/2)}\times\begin{cases}\frac{1}{p}\mathrm{sh}^2\left(\frac{q_0y_0}{2}\arccos\frac{1-p}{1+p}\right), & p>0,\\ (q_0y_0)^2, & p=0\\ \frac{1}{|p|}\sin^2\left(\frac{q_0y_0}{2}\mathrm{arch}\frac{1-p}{1+p}\right), & p<0.\end{cases} \quad (17)$$

In the derivation of the expressions (16) and (17) we have used the fact that at large values of the vector q, the Fourier components of the magnetization distribution in the DW, $\Lambda_\lambda(q)$, are exponentially small; that is, it is small q that are of interest. Therefore we can take the phonon dispersion law approximately in the form

$$\omega_\lambda(q)=s_\lambda q[1-\sigma_\lambda(aq)^2], \quad (18)$$

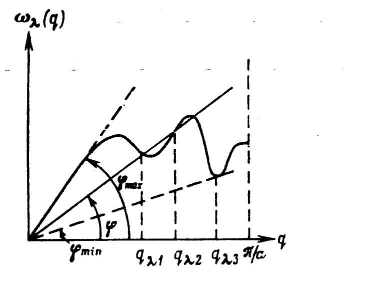

FIG. 1. Graphical solution of Eq. (14); $\omega_\lambda(q)$ is the dispersion law of phonons with polarization λ; $\tan\varphi=V$, $\tan\varphi_{\min}=V_{\min}^{(\lambda)}$, $\tan\varphi_{\max}=V_{\max}^{(\lambda)}$; $q_{\lambda\alpha}$ ($\alpha=1,2,3$) are the roots of Eq. (14).

where s_λ is the velocity of long-wave sound with polarization λ, and where σ_λ is a coefficient of order unity.

The parameter q_0 that occurs in (16) and (17) is the positive root of Eq. (14) for the dispersion law (18):

$$q_{0\lambda}=\frac{1}{a}\left(\frac{s_\lambda-V}{\sigma_\lambda s_\lambda}\right)^{1/2}. \tag{19}$$

We note that this root exists only when $V\leqslant s_\lambda$.

It must be remembered, however, that, as was mentioned earlier, the retarding force due to interaction of the DW with phonons itself shows up only when $V>V_{\min}^{(\lambda)}$. But in the model of the phonon dispersion law (18), there is formally no minimum velocity; this is due to the fact that formula (18) is valid only for $aq\ll 1$. A value of $V_{\min}^{(\lambda)}$ can nevertheless be determined by starting from the exact dispersion law. Thus, for example, if we take as the exact dispersion law

$$\omega_\lambda(q)=\frac{2s_\lambda}{a}\sin\frac{aq}{2},$$

then $V_{\min}^{(\lambda)}=2s_\lambda/\pi$, and radiation of phonons by a moving DW occurs in the velocity interval $2s_\lambda/\pi\leqslant V\leqslant s_\lambda$.

We see from formulas (16) and (17) that the retarding force as a function of the velocity V may have, depending on the sign of the parameter p, either oscillatory (for $p<0$) or nonoscillatory (for $p>0$) character. We note also that as $V\to s_l$, the value of $F_{1l}^{(I)}(V)$ approaches a finite limit; that is, at the point $V=s_l$ itself there is a discontinuity, equal to

$$F_{1l}^{(I)}(V=s_l)=\frac{M_0^4(b_2+f_2d_3/\delta)^2}{4\rho s_l^2\sigma_l}\left(\frac{y_0}{a}\right)^2 \times\begin{cases}\dfrac{1}{p}\left(\arccos\dfrac{1-p}{1+p}\right)^2, & p>0\\ 4, & p=0,\\ \dfrac{1}{|p|}\left(\operatorname{arch}\dfrac{1-p}{1+p}\right)^2, & p<0.\end{cases} \tag{20}$$

The value of $F_{1t}^{(I)}(V)$, however, approaches zero as $V\to s_t$, and its maximum value is attained at a velocity V such that $q_0y_0\sim 1$.

With increase of the value of q_0, the retardation force decreases exponentially. The parameter q_0y_0, which determines the function $F_1=F_1(V)$, is determined by (19); and since $y_0/a\gg 1$, therefore $q_0y_0\lesssim 1$ only in the velocity range $(s_\lambda-V)/s_\lambda\ll 1$. As soon as the velocity V becomes significantly different from s_λ, the quantity q_0y_0 becomes a large parameter, and the force $F_{1\lambda}(V)$ is exponentially small with respect to this parameter. In other words, without allowance for attenuation of phonons, the retardation force $F_{1\lambda}(V)$ is, in effect, nonzero only in a narrow (according to the parameter $(a/y_0)^2$ $\sim 10^{-4}$) velocity interval near the velocity of sound (see Fig. 2).

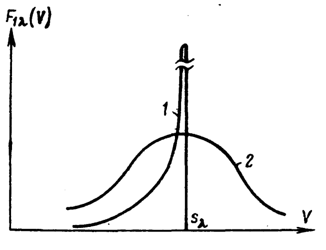

FIG. 2. Single-phonon retarding force $F_{1\lambda}(V)$ as a function of velocity (schematic); the peak shown corresponds to one of the polarizations λ of the phonons. Curve 1 corresponds to low temperatures ($T\ll 0.1\,\Theta_D$), Curve 2 to high ($T\gg 0.1\,\Theta_D$).

Formulas (16) and (17) determine the retardation force for DW I. By use of the relations (7), (8), and (12), we find similarly the retardation force for DW II:

$$F_{1t}^{(II)}=0,$$

$$F_{1l}^{(II)}(V)=\frac{M_0^4\pi^2}{4\rho Vs_l\sigma_l}\left(\frac{y_0}{a}\right)^2\operatorname{sh}^{-2}\left(\frac{\pi q_0y_0}{2}\right) \times\left[b_3+\frac{d_3(f_1+2f_2)-d_1f_2}{\delta}+\frac{2(f_1+f_2)(d_1-d_3)}{3\delta}\left(1+\frac{q_0^2y_0^2}{4}\right)\right]^2. \tag{21}$$

For brevity, we have given the formula for $F_{1l}^{(II)}$ only for $p=0$. The corresponding expression for $p\neq 0$ can be derived without difficulty, but it is not quoted here because of its unwieldiness.

Formula (21) has the same structure as formula (17): when $V\to s_l$ ($q_0\to 0$), the force $F_{1l}^{(II)}(V)$ approaches a finite limit, and it decreases exponentially with increase of q_0. Therefore the remark made above regarding the velocity interval in which the retardation force is significant for DW I is quite pertinent also to DW II.

We shall estimate the order of magnitude of the retardation force resulting from single-phonon processes (without allowance for attenuation of phonons). As follows from formulas (16), (17), and (21),

$$F_1\sim\frac{(bM_0^2)^2}{\rho s^2}\left(\frac{y_0}{a}\right)^2\infty\zeta(bM_0^2)\left(\frac{y_0}{a}\right)^2.$$

On setting $\zeta\sim 3\cdot 10^{-5}$,[6] $(bM_0^2)\sim 3\cdot 10^7$ erg/cm^3, and $y_0/a\sim 10^2$, we get $F_1\approx 10^6$ to 10^7 dyn/cm^2. This value is considerably larger than the retardation force corresponding to the linear section of the $V=V(H)$ relation,[4] where the main contribution to the retardation force is due to interaction of the DW with magnons[1] (see below).

Thus the maximum value of F_1 without allowance for attenuation of phonons is very large, but $F_1(V)$ drops rapidly (exponentially) outside a narrow range of DW velocity. This result is due to satisfaction of the very stringent condition (14) (jointly with $q_0y_0\lesssim 1$), which determines the law of conservation of energy in radiation of a phonon. We note that here allowance for the dispersion of phonons is very important, since the width of the peak in the $F_1(V)$ relation is completely determined by the dispersion, and when $\sigma_\lambda=0$ we have $F_{1\lambda}(V)\sim\delta(s_\lambda-V)$.

It is to be expected that the result will change substantially with allowance for attenuation of phonons. Since the Hamiltonian (4) corresponds to the Hamiltonian of linear-response theory, it can be easily shown, by standard procedures, that the retardation force is determined by the imaginary part of the single-time Green's function. Therefore allowance for attenuation of the phonons reduces to the following substitution for the δ function in formula (13):

$$\delta[\omega_\lambda(q)-qV]\to\frac{1}{\pi}\frac{\Gamma_\lambda(q)}{[\omega_\lambda(q)-qV]^2+\Gamma_\lambda^2(q)}, \tag{22}$$

where $\Gamma_\lambda(q)$ is the line-width of a phonon, which for $q\ll 1/a$ is determined by the expression[10]

$$\Gamma_\lambda(q)=\gamma_\lambda q^2=\begin{cases} q^2\left(\frac{\hbar s_\lambda^2}{k\Theta_D}\right)\left(\frac{T}{\Theta_D}\right)^3, & T\ll\Theta_D,\\ q^2\left(\frac{\hbar s_\lambda^2}{k\Theta_D}\right)\left(\frac{T}{\Theta_D}\right), & T\gg\Theta_D,\end{cases} \tag{23}$$

where Θ_D is the Debye temperature and k is Boltzmann's constant.

By means of expressions (13), (22), and (23), it is not difficult to show that the form of the function $F_{1\lambda}(V)$ is determined by the relation between the dispersion and the attenuation of sound; that is, allowance for attenuation is important under the condition

$$\gamma_\lambda>\sigma_\lambda(s_\lambda y_0)(a/y_0)^2 \quad \text{or} \quad (T/\Theta_D)^3>\sigma_\lambda(a/y_0). \tag{24}$$

Consequently, when $T\ll 0.1\,\Theta_D$ the function $F_{1\lambda}(V)$ is of the "low temperature" type (see Fig. 2). But if the inequality (24) is satisfied, *i.e.* if $T\gg 0.1\,\Theta_D$, then $F_{1\lambda}(V)$ is determined by the following interpolation formula:

$$F_{1\lambda}(V)\approx\zeta(bM_0^2)\cdot\frac{4}{3}\left(\frac{s_\lambda y_0}{\gamma_\lambda}\right)\left[1+\frac{7}{2}\left(\frac{y_0}{\gamma_\lambda}\right)(s_\lambda-V)^2\right]^{-1}. \tag{25}$$

[Here we have again restricted ourselves to the simplest case $p=0$, and in addition we have supposed that $\Lambda(\xi)\sim b\sin^2\theta(\xi)$.] In this case the function $F_{1\lambda}(V)$ has the form of a Lorentzian peak with a maximum at $V=s_\lambda$ (see Fig. 2); that is, in the "high-temperature" range the peak becomes symmetric, and the decrease of the function $F_{1\lambda}$ at the wings follows a power law.

The maximum value of $F_{1\lambda}(V)$ decreases with increase of γ_λ:

$$(F_{1\lambda})_{max}=F_{1\lambda}(V=s_\lambda)\approx\frac{4}{3}\zeta(bM_0^2)\left(\frac{s_\lambda y_0}{\gamma_\lambda}\right)\sim\zeta(bM_0^2)\frac{y_0}{a}\left(\frac{\Theta_D}{T}\right)^3.$$

But the width of the peak, which is determined by the relation

$$\left|\frac{V-s_\lambda}{s_\lambda}\right|\sim\frac{\gamma_\lambda}{y_0 s_\lambda}\sim\left(\frac{a}{y_0}\right)\left(\frac{T}{\Theta_D}\right)^3,$$

increases with increase of γ_λ. At room temperature ($T\approx 2$ to $3\,\Theta_D$) we get

$$(F_{1\lambda})_{max}\sim 10^3-10^4\,\frac{\text{dyn}}{\text{cm}^2},\qquad \left|\frac{V-s_\lambda}{s_\lambda}\right|\sim 10^{-1}-10^{-2}. \tag{26}$$

We note also that the presence of any mechanism of phonon relaxation in addition to the temperature mechanism leads to an additional decrease of $(F_{1\lambda})_{max}$ and broadening of the peak.

We recall that the Lorentzian peaks considered correspond to each of the polarizations of sound; and with sufficiently strong attenuation, it may turn out that the different peaks are indistinguishable (see below, Section 4). Nevertheless, when $V\gg s_l$ (s_l is the largest of the sound velocities), $F_1(V)\sim F_{1max}(s_l/V)^2$; that is, the single-phonon mechanism is turned off. Therefore in this velocity range, it is of interest to study processes of radiation of two quasiparticles.

3. TWO-PARTICLE RADIATION PROCESSES

As we showed in the preceding section, in the motion of a plane system of the DW type single-particle radiation is important only when $V\sim s_\lambda$. But when one allows for simultaneous radiation of several quasiparticles, the situation changes; in particular, the retardation force does not decrease with increase of the velocity above s_λ and may, on the contrary, increase.

In the REO under consideration, two-particle processes are possible in which phonons and spin waves (magnons) take part; in magnetic materials with a DW, there exist not only the usual volume magnons, but also unique ones localized near the DW.

In a paper of Abyzov and one of the authors[1] it was shown that the processes of volume spin-wave scattering are responsible for the linear section of the $V=V(H)$ relation (see below); and it is quite clear that this process is not of the threshold type. But processes involving simultaneous radiation (or absorption) of two quasiparticles are of threshold type; and for activationless quasiparticles, they begin only at velocities larger than the smallest phase velocity of the particles taking part in the process.[3)] For us, therefore, it is of interest to study processes of simultaneous radiation of two phonons, and also of a phonon and the above-mentioned spin wave localized near the DW, which has the linear dispersion law[11]

$$\tilde{\omega}(\varkappa)=c|\varkappa|, \tag{27}$$

where $\varkappa$ is a two-dimensional vector lying in the plane of the DW, and where c is the velocity of this wave. Analysis shows that it is this process, in which an acoustical phonon and a "parietal" magnon (PM) take part, that is most important; and we shall devote this section to investigation of it.

Spin waves localized near a DW can be described as elastic waves of DW bending.[12] In the long-wave limit, bending oscillations of a DW can be described by introducing the departure of the DW from its equilibrium value; that is, one supposes that the coordinate of the center of the DW is located at the point

$$\tilde{y}=Vt+f(x,z;t),\quad \mathbf{l}=\mathbf{l}_0(y-\tilde{y}),\quad \mathbf{m}=\mathbf{m}_0(y-\tilde{y}),$$

where the functions $\mathbf{l}_0$ and $\mathbf{m}_0$ describe the magnetization distribution in the DW plane. The energy of bending oscillations of the DW can be written in the form[12]

$$W_c=\int d\mathbf{r}_\perp\left\{\frac{m_*}{2}\left(\frac{\partial f}{\partial t}\right)^2+\frac{\sigma}{2}\left(\frac{\partial f}{\partial \mathbf{r}_\perp}\right)^2\right\}, \tag{28}$$

where m_* is the effective mass and σ is the energy of unit area of a plane DW. The integration in (28) extends over the plane of the DW.

By starting from the magnetoelastic energy (2) of the magnetic material, we can write the energy of interaction of the PM with sound:

$$W_{me}=\int d\mathbf{r}\,\Lambda_{ij}[y-Vt-f(\mathbf{r}_\perp;t)]\frac{\partial u_i}{\partial x_j}$$
$$\approx\int d\mathbf{r}\Lambda_{ij}(\xi)\frac{\partial u_i}{\partial x_j}+\int d\mathbf{r}\frac{\partial\Lambda_{i\alpha}(\xi)}{\partial\xi}\frac{\partial f}{\partial r_\alpha}u_i(\mathbf{r};t). \tag{29}$$

Here $\alpha=x,z$; the first term describes the single-phonon processes considered above, whereas the second describes processes in which an acoustic phonon and a PM take part.

Introducing the PM creation and annihilation operators according to the formula

$$f(\mathbf{r}_\perp;t)=\sum_\varkappa\left(\frac{\hbar}{2m_*S\tilde{\omega}(\varkappa)}\right)^{1/2}(a_\varkappa\exp(i\varkappa\mathbf{r}_\perp)+a_\varkappa^+\exp(-i\varkappa\mathbf{r}_\perp)),\qquad(30)$$

we write the Hamiltonian of PM, with allowance for their interaction with sound,

$$\mathscr{H}_c=\sum_\varkappa c|\varkappa|a_\varkappa^+a_\varkappa+\left(\frac{S}{\Omega}\right)^{1/2}\sum_{\mathbf{k}\lambda}\sum_\varkappa\{\mathcal{C}_\lambda(\mathbf{k};\varkappa)b_{\mathbf{k}\lambda}^+a_\varkappa^+e^{-iqVt}+\text{H.c.}\};\qquad(31)$$

here $c=(\sigma/m_x)^{1/2}$ is the velocity of a DW bending wave, which coincides, as is easily shown, with the smallest phase velocity of volume spin waves; $q\equiv k_y$;

$$\mathcal{C}_\lambda(\mathbf{k};\varkappa)=\frac{\Lambda_{i\alpha}(q)e_\lambda^i(\mathbf{k})\varkappa_\alpha q}{[\omega_\lambda(\mathbf{k})\tilde{\omega}(\varkappa)]^{1/2}}\Delta(\mathbf{k}_\perp+\varkappa),\qquad(32)$$

where $\alpha=x,z$; $\Delta(\mathbf{Q})$ is the Kronecker symbol.

The retardation force F_2 due to joint radiation (and absorption) of a phonon and a PM can be easily found from (31) and (32):

$$F_2=\sum_\lambda F_{2\lambda}=\frac{\hbar}{16\pi^2\rho m_*}\sum_\lambda\int d\mathbf{k}\frac{|\Lambda_{i\alpha}(q)e_\lambda^i(\mathbf{k})k_\alpha|^2q^3}{\omega_\lambda(\mathbf{k})\tilde{\omega}(\varkappa)}\times(n_{\mathbf{k}\lambda}+\tilde{n}_{\mathbf{k}_\perp}+1)\delta[\omega_\lambda(\mathbf{k})+\tilde{\omega}_{\mathbf{k}_\perp}-qV],\qquad(33)$$

where $n_{\mathbf{k}\lambda}$ and $\tilde{n}_{\mathbf{k}\perp}$ are the occupancy numbers of acoustic phonons and of PM respectively.

On setting, for an estimate, $p=0$ and $\Lambda(\xi)\sim b\sin^2\theta(\xi)$, we get from (33)

$$F_{2\lambda}\sim\begin{cases}\dfrac{8}{21}\zeta\tilde{\zeta}\left(\dfrac{V-s_\lambda}{c}\right)^2\left(\dfrac{\hbar s}{y_0^4}\right), & kT\ll\dfrac{\hbar|s_\lambda-V|}{y_0}\\[2ex] \dfrac{60}{\pi^5}\zeta\tilde{\zeta}\dfrac{V(V-s_\lambda)}{c^2}\left(\dfrac{kT}{y_0^3}\right), & kT\gg\dfrac{\hbar(s_\lambda-V)}{y_0},\end{cases}\qquad(34)$$

where $\tilde{\zeta}=bM_0^2y_0/m_*c^2\sim1$. At room temperature and for $V\lesssim c$ we get

$$F_2\sim10^{-1}-1\ \text{dyn/cm}^2,$$

that is, the contribution of the process considered may be considerable both in comparison with the single-phonon (for $V\gg s_\lambda$) and in comparison with the magnon contribution.[1]

As regards processes in which two acoustic phonons take part, calculation shows that the corresponding contribution to the retardation force for $kT\gg\hbar|V-s|/y_0$ has the form

$$(F_2)_{ph}\approx\frac{8}{21\pi}\zeta^2\left(\frac{kT}{y_0^3}\right)\varphi\left(\frac{V^2}{s^2}\right),\qquad(35)$$

where $\varphi(V^2/s^2)$ is a function of the velocity that for an estimate may be considered to be of order unity. From formula (35) it is easy to see that at room temperature, $(F_2)_{ph}\sim10^{-4}$ to 10^{-5} dyn/cm^2, which is several orders of magnitude smaller than the contribution, considered above, due to the process of radiation of an acoustic phonon and a PM. At low temperatures ($kT\ll\hbar|V-s|/y_0$), the value of $(F_2)_{ph}$ is still smaller, and therefore this mechanism of retardation may practically always be disregarded.

4. EFFECT OF PHONON-RADIATION PROCESSES ON THE CHARACTER OF DW MOTION UNDER THE INFLUENCE OF AN EXTERNAL FIELD

The formulas obtained in the preceding sections determine the variation of the force of retardation that acts on a DW with the velocity of motion of the DW. But it is of interest to know the dependence of the DW velocity on the value of the external force F_{ext} that causes this motion. Usually a magnetic field is used as the external driving force. For the simplest case of a plane DW separating domains with a magnetization discontinuity $2m^{(0)}$, the force acting on unit area of the DW is $F_{ext}=2m^{(0)}H$. By equating this force to the retardation force, one can find the velocity of stationary motion of a DW as a function of the magnetic field H:

$$2m^{(0)}(H-H_c)=F_{ret}=\frac{1}{B}V+F_{ph}(V).\qquad(36)$$

Here H_c is the coercive force, B is a mobility coefficient due to interaction of the DW with thermal magnons (see Ref. 1), and F_{ph} is due to the above-considered processes of phonon radiation. In contrast to the magnon force of retardation, which for $V\ll c$ is linear in the DW velocity V, the function $F_{ph}(V)$ is sharply nonlinear, and this may lead to noticeable anomalies in the function $V=V(H)$.

According to the analysis presented above, the function $F_{ph}(V)$ has the form of several peaks, caused by single-phonon processes at $V\sim s_\lambda$ ($\lambda=1,2,3$), whose height and shape are determined by attenuation of phonons, plus two-particle corrections, whose contribution is relatively small but exists at all $V>s_\lambda$. The general character of the $F_{ph}(V)$ variation is determined by the relation between the attenuation and the dispersion of the phonons.

At high temperatures, the widths of the phonon peaks may prove comparable with the distance between them; that is, individual peaks may prove indistinguishable. In this limiting case of large attenuation, the single-phonon contribution is nonzero over the whole interval $(s_\lambda)_{min}\leq V\leq(s_\lambda)_{max}$ (see Fig. 3a), and within this interval it may be considered only slightly dependent on the velocity; here $F_{ph}\sim10^3$ to 10^4 dyn/cm^2 [see (26)]. Outside this interval, only the two-particle contribution is important, and it is much smaller ($F_2\leq1$ dyn/cm^2).

The function $V(H)$ corresponding to such behavior of $F_{ph}(V)$ can be easily constructed (see Fig. 3b). Characteristic is the presence of a plateau in the $V(H)$ relation; that is, a range of field values (and consequently also of the force) to which corresponds a very slow change of the DW velocity. The width ΔH of this interval is determined by the relation

$$\Delta H\approx F_{ph}/2m^{(0)}\approx10^2-10^3\ \text{Oe}$$

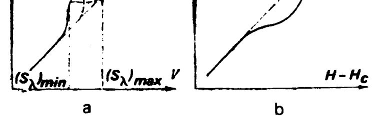

FIG. 3. Total retardation force $F_{ph}(V)$ (a) and variation of DW velocity with external magnetic field (b) at high temperatures ($T\gg0.1\,\Theta_D$). In Fig. 3a, $(s_\lambda)_{min}$ and $(s_\lambda)_{max}$ are, respectively, the smallest and largest of the sound velocities.

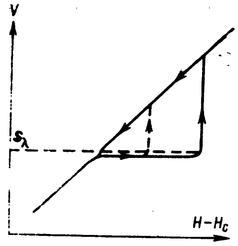

FIG. 4. Variation of DW velocity with external magnetic field at low temperatures ($T \ll 0.1\ \Theta_D$); the hysteresis corresponding to one of the phonon polarizations λ is shown. The dotted curve corresponds to a possible decrease of the amount of hysteresis because, for example, of a disturbance of the homogeneity of the DW.

(we have supposed that $2m^0 \sim 10\ G$). We note that approximately this variation has been observed experimentally;[4] for yttrium orthoferrite $\Delta H \approx 30$ Oe, for thulium orthoferrite $\Delta H \approx 100$ Oe. On the $V = V(H)$ graph for $YFeO_3$ it is also possible to detect the turning off of the additional retardation at $V \sim 8$ km/sec.[4]

At low temperatures the behavior of $F_{ph}(V)$, and consequently of $V(H)$, is substantially different. The widths of the phonon peaks are now much smaller than the distance between them ($|s_\lambda - V|/s_\lambda \sim 10^{-4}$), and the height of the peaks is of the order 10^6 dyn/cm^2, which corresponds to very large fields ($H \approx 10^5$ Oe). Thus at low temperatures ($T \ll 0.1\ \Theta_D$) there is again a plateau in the $V = V(H)$ relation; but, first, its width is much greater, and, second, it exists only on increase of the magnetic field. On decrease of the magnetic field, this plateau is absent; that is, there is a peculiar hysteresis on the $V = V(H)$ curve (see Fig. 4).

We note that because of the small width of the velocity interval, it is not ruled out that the DW might surmount this interval in an external field less than $(F_{ph}/2m^{(0)})$; for example, because of disturbance of the homogeneity of the DW, *i.e.* as a result of the fact that different sections of the DW can move with different velocities. In this case the width of the hysteresis in $V = V(H)$ may be considerably less than $F_{ph}/2m^{(0)}$, but the effect in question will show up in poor reproducibility of the $V = V(H)$ curve at $V \approx s_\lambda$.

In conclusion we remark that if the sound velocity s is close to the limiting velocity of motion V_C of the DW, then for $V \sim s \sim V_C$ it is necessary to allow for the decrease of the DW thickness to microscopic dimensions (of the order of the lattice constant).[5] Then the Fourier components of the magnetization distribution no longer decrease exponentially with increase of q, and therefore there may be efficient excitation of phonons over a while Brillouin zone, and not just of ones with small wave vectors. In order to calculate the resistive force in this case, it is necessary to know the exact dispersion law of the phonons; but it obvious in advance that the function $F_{ph}(V)$ will no longer have sharp maxima near the sound velocities, and that phonon retardation of the DW will manifest itself over a considerably wider range of velocities.

1) We recall that in uncompensated magnetic materials of the iron-garnet type, V_C is determined by relativistic interactions and as a rule does not exceed 10^2 m/sec, which is less than the velocity of sound.

2) We note that in an analogous manner one can obtain a Hamiltonian that describes radiation (and absorption) of two or more phonons. The corresponding estimate of the contribution of two-phonon processes to the DW retarding force is given in Section 3. Allowance for processes in which three or more phonons take part, at arbitrary DW velocities, can be shown to give small corrections, and we shall ignore them.

3) When particles with activation take part in the process, the radiation condition becomes considerably more stringent. For example, the contribution of processes of radiation of a phonon and of a volume spin wave is small in proportion to the smallness of the parameter $\exp(-\pi c/s_\lambda)$, where c is the velocity of the spin wave.

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Translated by W. F. Brown, Jr.