$4\pi/H_e, \ldots$ should simultaneously go over into a nonstationary regime. On the other hand, junctions of width $\pi/H_e, 3\pi/H_e, \ldots$ are the last to become unstable and can withstand a maximum current $I=2/H_e$. This circumstance can probably be used for a width calibration of Josephson junctions.

[1)] All the solutions of Eq. (1) can be expressed in terms of Jacobi elliptic functions. The different types of solutions referred to here are characterized by different numbers of extremal points in the interval $0<x<L$ (different numbers of zeros of the current, see Sec. 3), and are determined by different initial values.

[2)] The onset of a topological factor $\sigma_N=\pm 1$ for the entire aggregate of the stable (or unstable) solutions is evidence that, from the point of view of stability, we are dealing not with different types of solutions, but with two branches—stable and unstable (see Fig. 2c). The subdivision, for example, of the stable branch into solutions of the type 0, 1, 3 is meaningful when it comes to classifying the solutions in accordance with the character of their coordinate dependences (in accordance with the number of extremal points, see Sec. 3).

[3)] The condition $H_e \gg 1$ means in dimensional units $H_e \gg H_J \sim 1G$, i.e., formulas (35) and (38) describe in fact a rather wide field interval.

[1] I. O. Kulik and I. K. Yanson, Effekt Dzhosefsona v sverkhprovodyashchikh tunnel'nykh strukturakh (Josephson Effect in Superconducting Tunnel Structures), Nauka, 1970.

[2] L. Solymar, Superconductive Tunneling and Applications, Halsted, 1972.

[3] C. S. Owen and D. J. Scalapino, Phys. Rev. **164**, 538 (1967).

[4] G. F. Zharkov, Zh. Eksp. Teor. Fiz. **71**, 1951 (1976) [Sov. Phys. JETP **44**, 1023 (1976)].

[5] G. F. Zharkov and S. A. Vasenko, Zh. Eksp. Teor. Fiz. **74**, 665 (1978) [Sov. Phys. JETP **47**, 350 (1978)].

[6] G. F. Zharkov and A. D. Zankin, Fiz. Nizk. Temp. **4**, 586 (1978) [Sov. J. Low Temp. Phys. **4**, 283 (1978)].

[7] S. A. Vasenko and G. Zharkov, Zh. Eksp. Teor. Fiz. **75**, 180 (1978) [Sov. Phys. JETP **48**, 89 (1978)].

[8] A. A. Andronov, A. A. Vitt, and S. E. Khaikin, Teoriya kolebanii (Theory of Oscillations), Fizmatgiz, 1959. [Addison-Wesley, 1966].

[9] P. Lebwohl and M. J. Stephen, Phys. Rev. **163**, 376 (1967).

[10] B. Sutherland, Phys. Rev. A **8**, 2514 (1973).

[11] J. Frenkel and T. Kontorova, Phys. Z Sowjetunion **13**, 1 (1938).

[12] F. C. Frank and J. H. Van der Merwe, Proc. R. Soc. London **198**, 203, 216 (1949).

[13] A. Seeger and A. Kochendorfer, Z. Phys. **127**, 531 (1950); 130, 321 (1951).

[14] A. Seeger, H. Dorth, and A. Kochendorfer, Z. Phys. **134**, 173 (1953).

[15] R. C. Jaklevic, J. Lambe, A. H. Silver, and J. E. Mercerau, Phys. Rev. Lett. **12**, 159 (1964).

[16] J. M. Rowell, Phys. Rev. Lett. **11**, 200 (1963).

Translated by J. G. Adashko

Accidental degeneracy of self-localized solutions of the Landau-Lifshitz equations

V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulagin

(Submitted 26 May 1978)
Zh. Eksp. Teor. Fiz. **75**, 2210–2219 (December 1978)

It is shown that the self-localized solutions of the Landau-Lifshitz equations for a uniaxial ferromagnet with anisotropy energy $K\sin^2\theta$ are degenerate. Specifically, for an arbitrary velocity of an isolated magnetic-moment wave there exists a continuous set of self-localized solutions, which correspond to a definite type of magnetic solitons. If one goes over to a more general expression for the uniaxial-anisotropy energy, such as $K(\sin^2\theta+\beta\sin^4\theta\ (\nu>0)$, or if one allows for an external field, the accidental degeneracy is removed; this leads to disintegration of the continous set of solutions of the soliton type and to formation of a countable set of self-localized solutions of the isolated-wave type, with a definite internal structure.

PACS numbers: 75.10.Jm, 75.30.Gw

1. Investigations of nonlinear magnetic-moment waves, carried out both by the method of analytic continuation of the spin-wave spectrum into the region of complex wave vectors[1] and by direct analysis of the asymptotic behavior of the magnetic-moment distribution in the region of establishment of a homogeneous state,[2] have made it possible to determine characteristic limiting velocities of "slow" and "fast" nonlinear waves, and also to separate the regions of existence of definite types of stationary-profile waves; for example, the moving-domain-wall type or the isolated-wave type (magnetic soliton).

0038-5646/78/1201113-06$02.40

In the present paper, in the example of a uniaxial ferromagnet and of stationary-profile waves propagated normal to the anisotropy axis, it is shown that a classification of types of magnetic-moment wave on the basis of analytic continuation of the spin-wave spectrum or of analysis of the asymptotic behavior of the solutions in the vicinity of singular points, without introduction of the results of qualitative and numerical analysis, is incomplete. Furthermore, when one goes over from a ferromagnet characterized by uniaxial anisotropy energy

$$\mathscr{E}=K\sin^2\theta \tag{1.1}$$

to a ferromagnet with a more general expression for the uniaxial anisotropy,

$$\mathscr{E}=K(\sin^2\theta+\beta\sin^4\theta),\ \beta>0, \tag{1.2}$$

there occurs a complete change of the structure of the self-localized solutions and of the types of stationary-profile waves. The essence of the problem consists in fact that the limiting velocities of slow and fast magnetic-moment waves, as determined by the asymptotic behavior of the solutions in the region of establishment of a homogeneous state, are independent of the second uniaxial-anisotropy constant βK. This result is correct; for when one goes over to the more general expression (1.2) for the uniaxial-anisotropy energy, the singular points of the Landau-Lifshitz equation, which correspond to a state with homogeneous magnetization, do not change their type.

But the system of Landau-Lifshitz equations for a ferromagnetic medium with the simplest uniaxial anisotropy (1.1) is highly degenerate. Specifically, for a given velocity of the wave there is a continuous set of solutions of a definite type of isolated waves. When one goes over to a ferromagnet characterized by the more general expression (1.2) for the uniaxial-anisotropy energy, the accidental degeneracy is removed, and to a given velocity of a simple wave there corresponds a countable set of solutions of a definite type of isolated waves. The solutions discussed earlier,[3] of the stationary isolated magnetic domain type with a turning of the plane of rotation of the magnetic moment, for a ferromagnet with the uniaxial-anisotropy energy (1.1), point to another possible cause of removal of accidental degeneracy, resulting from allowance for an external magnetic field.

The analysis made led us to the supposition that the reason for such peculiar behavior of the solutions of the Landau-Lifshitz equations is the following fact.

For a ferromagnet with the uniaxial-anisotropy energy (1.1) all solutions both of the isolated-wave (magnetic-soliton) type and of the moving-domain-wall type, for all allowable values of the velocities, are arranged on a two-dimensional surface embedded in some three-dimensional space. When one goes over to the more general expression (1.2) for the uniaxial-anisotropy energy or allows for an external field directed along the anisotropy axis, and also for a whole series of other changes of the system, there occurs a disintegration of the two-dimensional surface, and the carrier of the solutions of the type considered is a three-dimensional manifold. Since the problem of separating solutions of the isolated-wave type (separatrix solutions) can be reduced to the problem of finding the intersections of certain curves on the first-integral surface,[3] it is to be expected that going over from the case of a two-dimensional surface to a three-dimensional will lead to a decrease of the degeneracy of the solutions. By way of an analog, we point to the important difference between the probabilities of intersection of two random straight lines on a plane and in space.

We note that in the presence of accidental degeneracy, the continuous set of solutions of the isolated-wave type is characterized by a comparatively simple internal structure. On removal of the degeneracy, of the previously existing solutions there remain only symmetrical solutions with a simple internal structure, and new symmetrical solutions of the isolated-wave type, with a more complicated internal structure, are generated.

Thus the problem of self-localized states (magnetic solitons) for a uniaxial ferromagnet with the anisotropy energy (1.1) is structurally unstable, since perturbation of the uniaxial-anisotropy energy leads to disintegration of the complicated spectrum of soliton states and to the appearance of new types of nonlinear stationary-profile spin waves.

The physical importance of investigations of nonlinear stationary-profile waves in magnetic media is due, first, to the important practical problem of finding the limiting velocity of motion of domain walls and, second, to the search for fast carriers of information (magnetic solitons). The appearance of a possibility of existence of isolated waves (magnetic solitons) with a diverse internal structure may have independent practical value.

2. For nonlinear stationary-profile spin waves propagating orthogonally to the anisotropy axis, the dependence of the polar and azimuthal angles of the vector magnetic moment on the spatial and time variables has the form

$$\theta=\theta(x-ut),\ \varphi=\varphi(x-ut) \tag{2.1}$$

(the polar axis is directed along the anisotropy axis). Here u is the velocity of the wave divided by the characteristic velocity $2|\gamma|(AK)^{1/2}/M_s$ (γ is the gyromagnetic ratio, A and K are the exchange- and anisotropy-energy constants, M_s is the saturation magnetization). To the Landau-Lifshitz equations corresponds a system of Lagrangian equations, in the space of the angular coordinates and velocities ($\theta, \varphi, \theta', \varphi'$),

$$\begin{gathered}\theta''-(1+\varphi'^2+\varepsilon\cos^2\varphi+\beta\sin^2\theta)\sin\theta\cos\theta=u\varphi'\sin\theta,\\ (\varphi'\sin^2\theta)'+\varepsilon\cos\varphi\sin\varphi\sin^2\theta=-u\theta'\sin\theta.\end{gathered} \tag{2.2}$$

Here βK is the second anisotropy-energy constant, and the differentiations are carried out with respect to the argument of the functions (2.1).

We transform from the angular variable θ to the generalized coordinate

$$X=\ln\operatorname{tg}(\theta/2), \tag{2.3}$$

to which corresponds the generalized velocity

$$X'=\theta'/\sin\theta. \tag{2.4}$$

The Landau-Lifshitz equations take the form

$$\left(\frac{\partial L}{\partial X'}\right)'=\frac{\partial L}{\partial X}\qquad\left(\frac{\partial L}{\partial \varphi'}\right)'=\frac{\partial L}{\partial \varphi}\,. \tag{2.5}$$

Here the Lagrangian function for $\beta\equiv 0$ is determined by the relation

$$L=(X'^2+\varphi'^2+1+\varepsilon\cos^2\varphi-2uX'\varphi)\,\mathrm{ch}^{-2}X. \tag{2.6}$$

The first integral of the system (2.5),

$$\mathscr{H}=(X'^2+\varphi'^2-1-\varepsilon\cos^2\varphi)\,\mathrm{ch}^{-2}X \tag{2.7}$$

with a choice of the integration constant that corresponds to the asymptotic boundary conditions necessary for isolated waves, shows that all solutions of the separatrix type (solitons and domain walls) are located on the surface of the corrugated cylinder

$$X'^2+\varphi'^2=1+\varepsilon\cos^2\varphi \tag{2.8}$$

in three-dimensional (X', φ', φ) space.

For comparison we give the expression that determines the first integral in the case of the more complicated form (1.2) of the uniaxial-anisotropy energy:

$$X'^2+\varphi'^2=1+\varepsilon\cos^2\varphi+\beta/2\,\mathrm{ch}^2X. \tag{2.9}$$

In the latter case the solution of separatrix type belongs to a three-dimensional manifold, the surface (2.9) in the four-space $(X', \varphi', X, \varphi)$. We remark that going over to a more general expression for the uniaxial-anisotropy energy than that defined by the relation (1.2) does not lead to further qualitative change of the solutions.

According to an earlier paper,[2] in the case of a uniaxial ferromagnetic medium there are three characteristic velocities of magnetic-moment waves, on passage through which the type and structure of a stationary-profile wave change. On passage through the limiting velocity

$$U_-=2|\gamma|\frac{(AK)^{1/2}}{M_s}[(1+\varepsilon)^{1/2}-1]=2|\gamma|\frac{(AK)^{1/2}}{M}u_-(\varepsilon) \tag{2.10}$$

the solutions of the moving-domain-wall type disappear, and solutions of the isolated-wave (magnetic-soliton) type, characterized by precession of the magnetic moment, are excited. On attainment of the characteristic velocity

$$U_0=2|\gamma|\frac{(AK)^{1/2}}{M_s}(1+\varepsilon)^{1/2}=2|\gamma|\frac{(AK)^{1/2}}{M_s}u_0(\varepsilon) \tag{2.11}$$

along with precession, nutational motion of the magnetic moment is excited in an isolated wave. Finally, on attainment of the limiting velocity

$$U_+=2|\gamma|\frac{(AK)^{1/2}}{M_s}[(1+\varepsilon)^{1/2}+1]=2|\gamma|\frac{(AK)^{1/2}}{M_s}u_+(\varepsilon) \tag{2.12}$$

the self-localized solutions completely disappear.

By numerical analysis, the phenomenon of accidental degeneracy of solutions of the Landau-Lifshitz equations for uniaxial-anisotropy energy of the form (1.1) and the phenomenon of removal of this degeneracy on going over to the more general expression of the form (1.2) for the anisotropy energy have been investigated in all three regions of existence of isolated waves, defined by the limiting velocities (2.10)–(2.12).

In each of the regions, upon removal of the accidental degeneracy a disintegration of the continuous manifold of self-localized solutions was observed, and generation of a countable number of new self-localized solutions, characterized by a definite internal structure. In other words, removal of the accidental degeneracy upon going over from (1.1) to (1.2) occurs at all values of the velocities for which self-localized solutions exist.

3. We shall begin the discussion of the results of qualitative and numerical analysis of the problem with the case of zero velocities of stationary-profile waves. For $u=0$ and for the uniaxial-anisotropy energy (1.1), there are two types of known solutions of the Landau-Lifshitz equations, with

$$\varphi=\pi/2,\quad \varphi=0 \tag{3.1}$$

corresponding to Bloch and Néel domain walls. By numerical analysis it was established that there is a continuous manifold of solutions of the isolated-domain type, with one nodal point with respect to the polar angle θ and with turning of the aximuthal angle φ through π. One of the solutions of this type is symmetric (see Fig. 1).

In $(\varphi', \varphi, \theta)$ space we consider the limiting surface

$$\theta'(\varphi', \varphi, \theta)=0. \tag{3.2}$$

The singular point of saddle-point type corresponding to the homogeneous state

$$\theta'=\theta=\varphi'=0,\quad \varphi=\mp\pi/2 \tag{3.3}$$

is known[3] to be the origin of a one-parameter set of integral curves, forming on the limiting surface a curve of first contacts [a set of points $(\varphi', \varphi, \theta)$ of the limiting surface at which the integral curves mentioned first touch the limiting surface].

We introduce the concept of a curve of last contacts, as the set of points of the limiting surface that flow into the singular saddle point (3.3).

The assertion made above regarding the existence of a continuous set of solutions of the isolated-domain type,

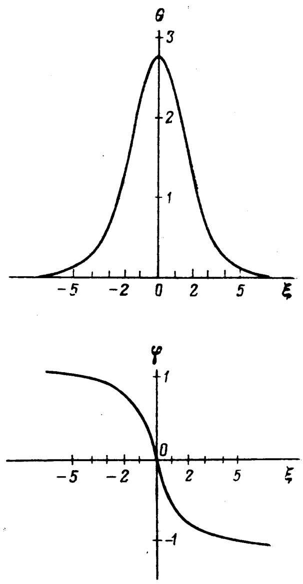

FIG. 1. Self-localized solution with one nodal point ($\varepsilon=0.2$; $\beta=0$; $u=0$).

with one nodal point with respect to θ and with turning of φ through π, is equivalent to the assertion that the curve of first contacts for the singular point

$$\theta'=\theta=\varphi'=0, \quad \varphi=-\pi/2 \tag{3.4}$$

coincides with the curve of last contacts for the singular point

$$\theta'=\theta=\varphi'=0, \quad \varphi=+\pi/2. \tag{3.5}$$

By virtue of the symmetry properties of the Landau-Lifshitz equations with respect to the transformation $\varphi\to-\varphi$, for verification of the conditions of coincidence of the curves of first and of last contacts it is sufficient to show that a curve of first, or equivalently of last, contacts is symmetric with respect to the plane $\varphi=0$, as was indeed observed in the numerical calculations. An independent numerical-analysis procedure is set forth in the Appendix.

On going over to the more general expression (1.2) for the uniaxial-anisotropy energy, numerical analysis showed that the curves of first and of last contacts do not coincide, and that they intersect at a single point belonging to the plane $\varphi=0$. Consequently there occurs a removal of the degeneracy, which leads to disintegration of all asymmetric solutions of the isolated-domain type with one nodal point with respect to θ and with turning of φ through π. Only the symmetric solutions survive. The noncoincidence of the curves of first and of last contacts leads to the appearance on the limiting surface (3.2) of curves of second, third, ..., n-th contacts. Their intersections with the curve of last contacts lead to a countable set of solutions of the isolated-domain type, with a definite number of nodal points with respect to the angle θ and to φ'. Figure 3 shows a solution with three nodes with respect to the polar angle θ.

Thus on going over from the exceptional situation that occurs for the simplest form (1.1) of the uniaxial-anisotropy energy to the more general expression (1.2) for the anisotropy energy, the limiting surface loses its property of "total reflection."

Specifically: For $\beta=0$, a continuous set of integral curves, starting from the singular point (3.4), returns, after a first contact with the limiting surface (3.2), to the singular point (3.5); but for $\beta\neq0$, only a countable set of integral curves, after a definite number of contacts, returns to the singular points

$$\theta'=\varphi'=\theta=0, \quad \varphi=\frac{\pi}{2}(2n+1), \quad n\geqslant0. \tag{3.6}$$

Self-localized solutions similar to those shown in Figs. 1 and 2 were obtained also for the case of isolated waves characterized by a turning of the plane of rotation of the vector magnetic moment, in the velocity range $u<u_-$.

Thus in the slow-wave range

$$0\leqslant u\leqslant u_-$$

the numerical analysis points to a single method of reorganization of the self-localized solutions when the accidental degeneracy is removed; by the appearance of clearly distinguishable intersections of the curves

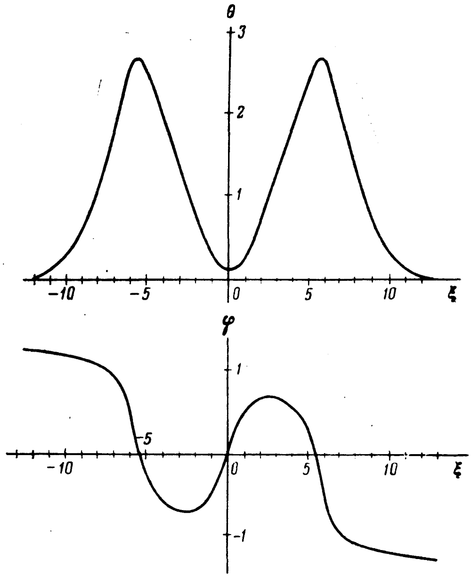

FIG. 2. Self-localized solution with three nodal points ($\varepsilon=0.2$; $\beta=0.5$; $u=0$).

of n-th order contact with the curves of last contact, and by the appearance of breaks on the curve of $(n+1)$th contact in the vicinity of the separatrix, with n nodal lines with respect to the polar angle θ.

Since for $\beta\neq0$ only a countable set of integral curves is reflected to the singular point (3.6) corresponding to uniform magnetization along the anisotropy axis, there arises a possibility of reaching, after a definite number of contacts, integral curves of the singular point

$$\theta'=\varphi'=0, \quad \theta=\pi, \quad \varphi={}^1/_2\pi(2n+1), \tag{3.7}$$

corresponding to uniform inverse magnetization. Such separatrix solutions correspond to moving domain walls with turning of the plane of rotation of the magnetic moment at a definite number of nodal lines with respect to the polar angle θ. The numerical analysis carried out confirms the existence of such magnetic-moment distributions.

4. For the velocity range of fast isolated waves

$$u_-\leqslant u\leqslant u_0$$

and for the uniaxial anisotropy energy (1.1), two types of symmetric magnetic solitons were determined earlier.[2] A more complete numerical analysis indicates the existence of a continuous set of asymmetric isolated waves with the same structure. The symmetric solu-

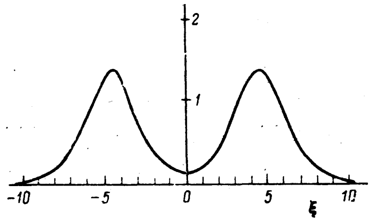

FIG. 3. Self-localized solution with three nodal points ($\varepsilon=0.2$; $\beta=0.5$; $u=1$).

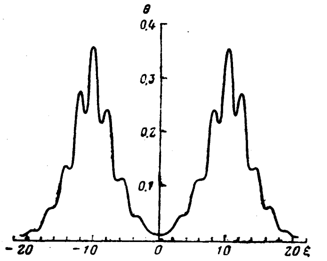

FIG. 4. Self-localized solution with three nodal points for the envelope of the polar angle $\theta(\varepsilon=3; \beta=1; u=2.9; u_0<u<u_+)$.

tions found earlier are limiting solutions; specifically, the value of the polar angle θ at a nodal point takes its largest and smallest values on these symmetric solutions. Removal of the degeneracy leads to disintegration of the whole continuous set of solutions with the exception of the symmetric solutions, and to generation of a countable set of new self-localized solutions; isolated waves with an internal structure determined by the number of nodal points with respect to the polar angle θ (see Fig. 3).

In the velocity range of fast isolated waves

$$u_0 \leqslant u \leqslant u_+ ,$$

solutions of the Landau-Lifshitz equations for the uniaxial-anisotropy energy (1.1) also lead to two types of symmetric self-localized solutions, distinguished by the fact that nutational oscillations in the polar angle θ lead to a local maximum or minimum on the plane of symmetry. Again, these solutions are limits for a continuous set of asymmetric self-localized solutions. On removal of the degeneracy, only the symmetric solutions are retained, and new self-localized solutions are generated; they can be classified according to the number of nodal points for the envelope of the distribution with respect to the polar angle θ. An example of such a solution is shown in Fig. 4.

APPENDIX

To an integral curve of the separatrix type, with n nodes with respect to the polar angle θ, connecting the singular points (3.4) and (3.5), correspond points of intersection of curves of n-th and of last contacts. We consider a separatrix with one tangency to the limiting surface $\theta'=0$ for the case $\beta>0$. In Fig. 5, AA_1 is a curve of first contacts of the singular point (3.4), CC_1 a curve of last contacts of the singular point (3.5) in $(\varphi', \varphi, \theta)$ space. By virtue of the symmetry of the Landau-Lifshitz equations, the curve AA_1 is symmetric to the curve CC_1 with respect to the plane $\varphi=0$. These curves intersect at the single point D, which corresponds to a symmetric self-localized solution with a single nodal point. As $\beta\to 0$, the curves AA_1 and CC_1 asymptotically approach the curve BB_1, and for $\beta=0$ they fuse. Therefore any integral curve (except the solutions $\varphi\equiv 0$ and $\varphi\equiv\pi/2$), after going out from the singular point (3.4), after a single tangency with the limiting surface $\theta'=0$, and after reflection, reaches the singular point (3.5). Such a situation, qualitatively, persists for velocities $0<u<u_0$. Furthermore, if we introduce the concept of curves of first (last) contacts with the limiting surface $\theta'=0$ for the envelope of the polar angle θ, then a similar situation persists also for velocities $u_0<u<u_+$.

The symmetry of the Landau-Lifshitz equations enables us to simplify the search for symmetric self-localized solutions with a $(2n-1)$th nodal point. For this purpose it is sufficient to find the point of intersection of the curve of n-th contacts with the plane of symmetry. On the limiting surface $\theta'=0$, this point is selected by the condition $\varphi''=0$, which was also used to find the symmetric self-localized solutions.

Another, independent method of finding the self-localized solutions is the following: the system of Landau-Lifshitz equations is of fourth order and has the first integral (2.8). Consequently, all solutions can be embedded in a three-dimensional space, in which the one-parameter family of solutions that go out from the singular point (3.4) and into the singular point (3.5) forms two-dimensional separatrix surfaces S_2^- and S_2^+. The intersections of the separatrix surfaces $S_2^\pm$ correspond to the self-localized solutions.[4] It is therefore to be expected that representative points located on opposite sides of the surface S_2^+ will have different behavior as $\xi\equiv x-ut\to+\infty$ (for the surface S_2^-, as $\xi\to-\infty$). This shows up in the fact that if the curve of n-th contacts intersects (generally) the curve of last contacts, then the curve of $(n+1)$th contacts experiences a break at this point. In particular, at the $(n+1)$th contact, φ' on opposite sides of the break takes values of opposite sign. This method was used to determine the self-localized solutions for the case $\beta>0$.

But all attempts to use this method in the case $\beta=0$ led to instability of the results of numerical analysis. Specifically, either we did not succeed in localizing a break point, or the break was not reproduced on slight change of the parameters of the problem. It was this fact that led to the hypothesis of coincidence of the curves of first and of last contacts when $\beta=0$. Nevertheless, the altered manner of breaking of the curves of contacts may be used for numerical substantiation of this hypothesis.

For some value

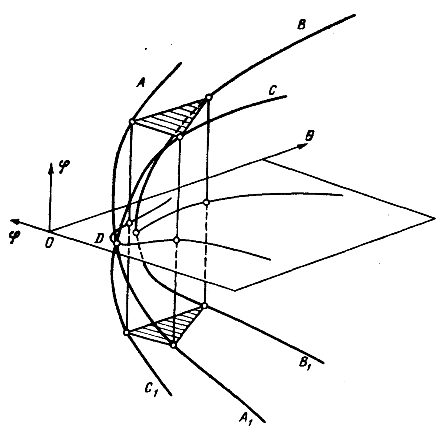

FIG. 5. Curves of first contacts AA_1 and of last contacts CC_1 for $\beta>0$, and fused curves of first and of last contacts BB_1 for $\beta=0$.

$$\varphi=\varphi_0, \quad -\pi/2<\varphi_0<\pi/2$$

we consider the solution that goes through a point of the interval

$$\theta'=0,\ \theta,\ \varphi_0',\ \varphi_0;\quad 0<\theta<\pi. \tag{A.1}$$

From the expression (2.7) for the first integral, we find that

$$\varphi_0'=\pm(1+\varepsilon\cos^2\varphi_0)^{1/2}. \tag{A.2}$$

For definiteness, we choose the lower sign in (A.2). From points of the interval (A.1), which lies on the limiting surface $\theta'=0$, we trace solutions of the Landau-Lifshitz equations to an intersection with the limiting surface. Thus we obtain a curve of first contacts Γ_1 for the interval (A.1). A curve of last contacts Γ_{-1} for the interval (A.1) is constructed similarly. To a point of intersection of the interval (A.1) with the curve of last contacts corresponds a break in the curve Γ_1. Let the break occur at $\theta=\theta_0(\varphi_0)$. Then if the hypothesis of coincidence of curves of first and of last contacts is correct, the curve Γ_{-1} must also experience a break at the point $\theta=\theta_0$. A break point of the curve Γ_1 was found and localized numerically; that is, values $\theta_\pm$ of the polar angle were found that lay on opposite sides of the break:

$$\theta_-<\theta_0<\theta_+,\quad \theta_+-\theta_-<\delta,$$

and it was verified that the coordinates φ' of the points on the curve Γ_{-1} corresponding to the values $\theta_\pm$ had different signs. Thus the curve Γ_{-1} also experiences a break. This procedure was carried out for various values of φ_0, with $\delta\sim10^{-5}$, and everywhere the same result was obtained. Introduction of a parameter $\beta>0$ or of an external field leads to the result that the breaks in the curves Γ_1 and Γ_2 occur at different points of the interval (A.1); that is, there occurs a disintegration of the continuous set of self-localized solutions with a single nodal point with respect to the polar angle θ.

[1] V. G. Bar'yakhtar, B. A. Ivanov, and A. L. Sukstanskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 226 (1978) [JETP Lett. **27**, 211 (1978)].

[2] V. M. Eleonskiĭ, N. N. Kirova, and N. E. Kulagin, Zh. Eksp. Teor. Fiz. **74**, 1814 (1978) [Sov. Phys. JETP **47**, 946 (1978)].

[3] V. M. Eleonskiĭ, N. N. Kirova, and V. M. Petrov, Zh. Eksp. Teor. Fiz. **68**, 1928 (1975) [Sov. Phys. JETP **41**, 966 (1975)].

[4] Yu. I. Neĭmark, Metod tochechnykh otobrazheniĭ v teorii nelineĭnykh kolebaniĭ (Method of Point Mapping in the Theory of Nonlinear Oscillations), Nauka, 1972.

Translated by W. F. Brown, Jr.

Calculation of particle mobility at high temperature

A. A. Ovchinnikov and N. S. Érikhman

L. V. Karpov Physicochemical Research Institute

(Submitted 29 May 1978)
Zh. Eksp. Teor. Fiz. **75**, 2220–2227 (December 1978)

The motion of a quantum particle in a stochastic time-dependent Gaussian potential $\xi(x,t)$ is considered. Assuming the correlator to be $\langle\xi(t),\xi(t')\rangle\sim\mu$ and the correlation time τ_c to be small, the particle mobility $\sigma(\omega)$ and the diffusion coefficient D are calculated and found to satisfy the Einstein relation. It is shown that $\sigma\propto T^{-2}$ in the high-temperature limit.

PACS numbers: 66.30.Dn

INTRODUCTION

The calculation of electron mobility in quasi-one-dimensional systems in the presence of impurities is a timely problem. The main reason is that violation, even slight, of the translational invariance leads in the one-dimensional case to strong qualitative changes in many properties of the system: the energy spectrum, the localization of all the eigenstates,[1,2] vanishing of the static conductivity,[3-6] and others. It was shown in a number of papers[3-6] that the mobility of noninteracting electrons in a random static potential of impurities is equal to zero. On the other hand, it has been noted[5] that allowance for the electron-phonon interaction leads to a mobility that differs from zero. Diagrams that give a nonzero contribution to the mobility of an electron interacting with the lattice phonons were obtained and estimated.[5] The calculation methods used in a number of studies[4-6] consist of summing an infinite chain of principal diagrams and are technically quite complicated. Interest attaches therefore to methods that make it possible to calculate the mobility without

0038-5646/78/1201118-05$02.40