Investigation of the temperature dependence of the density of the Bose condensate in helium-4 in connection with the superfluidity phenomenon

E. V. Dokukin, Zh. A. Kozlov, V. A. Parfenov,[1)] and A. V. Puchkov[1)]

Joint Institute for Nuclear Research
(Submitted 22 June 1978)
Zh. Eksp. Teor. Fiz. **75**, 2273–2279 (December 1978)

Results are presented of experimental investigations of the temperature dependence of the relative density of the Bose condensate and of the average kinetic energy per atom in liquid helium at temperatures 1.2–4.2 K. It is found that the relative density of the Bose condensate as $T \to 0$ is 0.022 ± 0.002, and the Bose-condensation temperature coincides with the λ-transition temperature.

PACS numbers: 67.40.Kh

The existence of Bose condensate (BC) as a necessary condition for the onset of macroscopic superfluidity and superconductivity has, strictly spearking, not been proved.[1] In quasi-one-dimensional and quasi-two-dimensional systems (helium-II films or superconducting wires) there is no BC.[2] At the same time in helium-II and in superconductors the lowest energy level is occupied by a finite number of particles.

Under conditions when there is still no microscopic theory of liquid helium, and quantum phenomena in condensed media are not only of fundamental but also of practical interest, the neutron methods of measuring the BC, developed in the last decade, are of particular importance.

Several methods have by now been proposed for the estimate of the relative density of the BC in liquid helium-4.[7–13] In experiments on BC by inelastic neutron scattering, two trends have been followed. The first is to obtain the momentum distribution $pn(p)$ of the helium atoms by measuring the spectra of the neutrons scattered by liquid helium using a spectrometer of high resolution (1%).[14] The distribution $pn(p)$ is calculated by using the impulse approximation for the scattering law. The estimate of the BC is determined in this case from the area of the peak attribed to the atoms with $p \to 0$. The other trend is to estimate the BC density by investigating the spectra of the neutrons scattered by liquid helium. In this case the neutron spectra are mathematically resolved with the aid of two Gaussian curves into a BC part and a supercondensate (SC) part.[11] The amount of the BC is estimated by comparing the results obtained above and below the λ-transition temperature, assuming that there is no BC at $T > T_\lambda$. Experiments[4–6] have shown that at $T \sim 1.2$ K the relative BC density is $n(0)/n = (0.024 - 0.074)$. It should be noted that a revision of the results of Ref. 5, reported in Ref. 15, has shown that if the BC does exist, its relative density is $n(0)/n < 0.01$.

It was natural under these conditions to introduce into the experiment an additional physical parameter that influences the BC density and the superfluid component. It seemed advantageous to start with the temperature dependence of the BC density, since the temperature dependence of the superfluid component of helium-II was obtained in Andronikashvili's classical work.[16]

In this paper we discuss the results of a continuation of the researches[7] on the temperature dependence of the relative density of the BC in liquid helium-4 by studying the spectra of inelastic neutron scattering at momentum transfers $k = 12 - 14$ Å^{-1} and temperatures $T = 1.2 - 4.2$ K.

EXPERIMENTAL TECHNIQUE

The measurements were made by the time-of-flight method with a DIN-1M spectrometer in the booster regime of the IBR-30 reactor.[18] Figure 1 shows schematically the main units of the experimental equipment. The neutrons were made monochromatic with a mechanical chopper 3 phased with the reactor. The neutrons scattered by sample 4 were registered by detectors 5, which were filled with helium-3.

The spectra of the neutrons scattered by the liquid helium were measured simultaneously at three scattering angles $\theta = 122.6°$, $109.5°$, and $96.5°$ at an initial neutron energy $E_0 = 190$ meV.

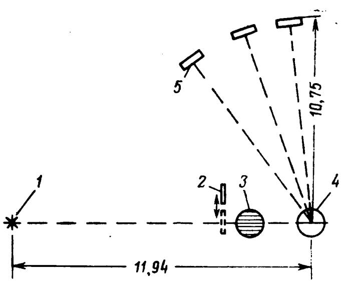

FIG. 1. Principal units of the spectrometer: 1-active zone of the IBR-30 reactor, 2-cadmium shutter, 3-chopper, 4-sample, 5-detectors.

 0038-5646/78/1201146-04$02.40

We used in the measurement a cryostat with helium-4 vapor pumped on. The helium temperature was determined by measuring the vapor pressure over the liquid and with the aid of a carbon resistor. The temperatures in the intervals 1.2 – 1.8 K and 1.95 – 4.2 K were maintained accurate to ~0.02 and 0.01 K, respectively.

The backgrounds of the empty cryostat and of the cryostat filled with liquid helium were determined by performing the measurements alternately using a cadmium shutter 2 to block the neutron beam and without the shutter.[17]

MATHEMATICAL MODEL

The analysis of the experimental data was carried out with the aid of a two-Gaussian model with a non-Gaussian increment, as proposed in Ref. 6:

$$\varphi(t)=E^2A_1\exp(P_1)+E^2A_4\exp(P_2)+(A_7+A_8t), \qquad (1)$$

where

$$P_{1,2}=-(E-E_r+A_{3,6})^2(A_{2,5}k^2)^{-1},$$

A_1-A_8 are free parameters, t is the number of the time-analyzer channel, and E_r is the excitation energy for the free helium atom. The first Gaussian describes the scattering by the BC. The introduction of the third term is necessitated by the fact that the two-Gaussian model does not account for the experimentally observed spectrum at large energy transfers.

The experimental data were reduced also without allowance for the BC part ($A_4=0$), i.e., by using the single-Gaussian model. An analysis of the experimental results with consideration of a statistical criterion of the quality of the approximation is given in Ref. 6, for both the one-Gaussian and two-Gaussian models.

The calculations were made with the aid of the library program "COMPIL," c-401, Dubna. The statistical errors of the free parameters A_1-A_8 ($A_3=A_6$) were calculated by using the theory the errors of the of least-squares method.

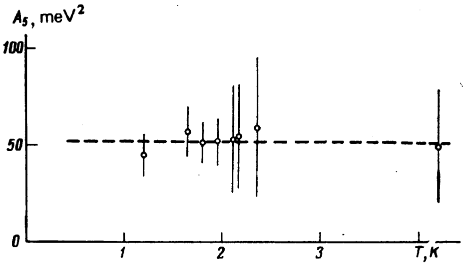

FIG. 3. Temperature dependence of the square of the width A_5 of the Bose-condensate Gaussian, in meV^2. The dashed line shows the quantity corresponding to the square of the width of the spectrometer resolution function.

ANALYSIS OF THE EXPERIMENTAL DATA

Figure 2 shows the spectra of neutrons scattered by liquid He-4 at $T=1.8$ and 2.35 K for $\theta=109.5°$. The points were calculated by formula (1), and the dashed curves pertain to the BC part (curve 1) and to the supercondensate part (curve 2).

The width of the Gaussian curve for the Bose-condensate fraction depends on the spectrometer resolution and on a quantity that characterizes the interaction in the final state. In this case the width is determined by the resolution, which is independent of the neutron-scattering angle within several per cent. Figure 3 shows the parameter A_5 as a function of the temperature, averaged for the three scattering angles. The dashed line denotes the value corresponding to the spectrometer resolution. In the entire temperature interval, the parameter A_5 is equal, within the limit of the statistical errors, to the square of the width of the spectrometer resolution function.

Figure 4 shows the values of the square of the width of the supercritical Gaussian (A_2) for the scattering angle $\theta=109.5°$ when the experimental data are analyzed in accord with the one-Gaussian (a) and two-Gaussian (b) models without allowance for the width of the resolution function.[2)]

Using the results of the one-Gaussian and two-Gauss-

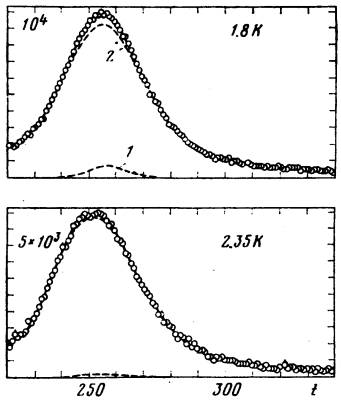

FIG. 2. Experimental spectra of neutrons scattered by liquid helium at $k=1.34$ Å^{-1} and $T=1.8$ and 2.35 K. The calculated curves for the BC part (curve 1) and the SC part (curve 2) are shown dashed; t is the number of the analyzer channel.

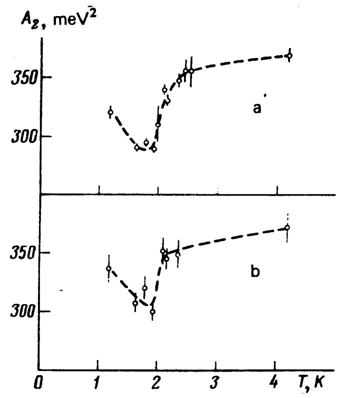

FIG. 4. Temperature dependence of the width A_2 of the SC Gaussian, in meV^2, obtained by using the one-Gaussian (a) and two-Gaussian (b) models.

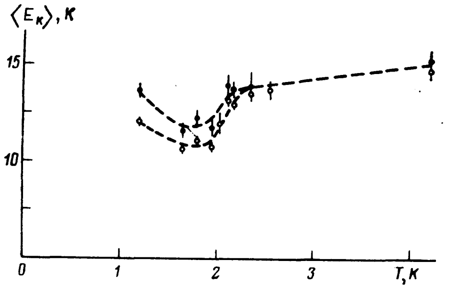

FIG. 5. Temperature dependence of the average kinetic energy per helium atom, calculated using the one-Gaussian (light circles) and two-Gaussian (dark) models.

ian approaches, we can estimate the average kinetic energy per helium atom in the liquid.[2)] Figure 5 shows the calculated values of $\langle E_k \rangle$ for the cases of the one-Gaussian (light circles) and two-Gaussian (dark) models [see formula (10) of Ref. 6]. We note that at liquid-helium temperatures $T > T$ the values of $\langle E_k \rangle$ coincide for both models within the limits of errors, whereas at $T < T_\lambda$ the values of $\langle E_k \rangle$ obtained from the one-Gaussian model lie considerably lower. This serves as an additional confirmation that at $T < T_\lambda$ the spectrum of the neutrons scattered by the liquid helium is better described by the two-Gaussian model, and the introduction of the BC part into the mathematical model is essential, whereas at $T > T_\lambda$ the BC Gaussian is not necessary for the description of the experimental spectrum.

The relative density of the BC was estimated from the formula

$$\frac{n(0)}{n} = \frac{S_{BC}}{S_{SC} + S_{SC}}, \tag{2}$$

where S_{BC} and S_{SC} are the areas of the spectra for the BC and SC spectra for the BC and SC parts, respectively.

Figure 6 shows the values of $n(0)/n$ obtained at temperatures from 1.2 to 4.2 K. At $T > T_\lambda$, within the limits of statistical errors, the values of $n(0)/n$ remain unchanged. It can be assumed that the obtained relative BC density at $T > T_\lambda$ is a systematic error due to the employed experimental procedure and to the method of the two-Gaussian mathematical resolution of the spectra.

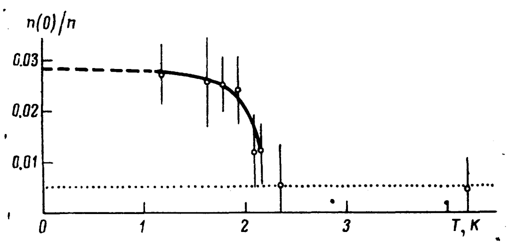

FIG. 6. Temperature dependence of the relative density of the Bose condensate in liquid helium. The dotted line is the systematic error, and the solid line is a plot of (3) with the parameters (4).

At $T < T_\lambda$, the calculated values of the relative BC density were described by the empirical formula

$$\frac{n(0)}{n} = \xi_0 [1 - (T/T_0)^m], \tag{3}$$

where ξ_0 is the relative BC density at $T = 0$; T_0 is the Bose condensation temperature. The values of the free parameters ξ_0, T_0, and m were determined by least squares:

$$\xi_0 = 0.022 \pm 0.002; \quad T_0 = 2.24 \pm 0.04; \quad m = 9 \pm 4. \tag{4}$$

CONCLUSION

The relative density of the BC was calculated by the method of two-Gaussian resolution of the spectra of the neutrons scattered by liquid helium in the temperature interval $T = 1.2 - 4.2$ K. The temperature dependence of $n(0)/n$ has a singularity at the temperature T_0. The BC is observed at $T < T_0$, whereas at $T > T_0$, within the limits of the accuracy of the experiment and of the mathematical reduction, no BC was observed. The Bose-condensation temperature T_0 coincides with the temperature of the transition of the liquid helium into the superfluid state: $T_0 = T_\lambda$.[20] The character of the temperature dependence of the BC density coincides with the temperature dependence of the superfluid-component density.

Further investigations of the temperature dependence of the BC density at temperatures $T < 1$ K are of interest in view of the indication, obtained in the theoretical paper of Hyland and Rowlands,[21] that the BC density increases strongly at $T \sim 0.6$ K.

One of the possible ways of measuring the BC density and of investigating the connection between the Bose condensation and superfluidity is to perform experiments on inelastic scattering of neutrons in liquid helium-4 with small admixtures of helium-3.[13] The neutron flux density in these experiments must be smaller by approximately two orders of magnitude than the existing flux, in view of the large cross section for neutron capture in helium-3. Such experiments will become possible when the DIN-2 spectrometer goes into operation on the IBR-2 reactor.

Using the DIN-1M spectrometer, we initiated the first experiments on elastic scattering of neutrons by turbulently moving helium-4 at temperatures above and below T_λ. The preliminary results, however, still yield no unequivocal conclusions concerning the influence of the liquid motion on the BC density.

One can hope that further progress along the trends indicated above will provide the answer to the question of the connection between Bose condensation and superfluidity.

In conclusion, the authors thank O. D. Kazachkovskiĭ, Yu. M. Ostanevich, L. P. Pitaevskiĭ, I. M. Frank, and N. A. Chernoplekov for constant interest in the work and for helpful discussions.

The authors are sincerely grateful to V. A. Zagrebnov, and V. B. Priezzhev for help with the work and for discussions. The authors are indebted to the DIN-1M crew for technical help with the experiments.

[1)] Physics and Power Institute, Obninsk.

[2)] The experimental data obtained at temperatures $T = 2.02$ and 2.55 K were analyzed only in the one-Gaussian approximations, inasmuch as the measurements were made with lower statistical accuracy.

[3)] The temperature dependence of $\langle E_k \rangle$ was calculated also in Ref. 19.

[1] L. D. Landau, Zh. Eksp. Teor. Fiz. **11**, 592 (1941).

[2] D. R. Tilley and J. Tilley, Superfluidity and Superconductivity, Halsted, 1975.

[3] R. Cowley and A. D. B. Woods, Phys. Rev. Lett. **21**, 787 (1969).

[4] O. Harling, Phys. Rev. A **3**, 1073 (1971).

[5] H. A. Mook, R. Sherm, and M. K. Vilkinson, Phys. Rev. A **6**, 2268 (1972); L. J. Rodrigues, H. A. Gersch, and H. A. Mook, Phys. Rev. A **9**, 2085 (1974).

[6] L. Aleksandrov, V. A. Zagrebnov, Zh. A. Kozlov, V. A. Parfenov, and V. B. Priezzhev, Zh. Eksp. Teor. Fiz. **68**, 1825 (1975) [Sov. Phys. JETP **41**, 915 (1975)].

[7] J. Gavoret and P. Nozieres, Ann. Phys. (Paris) **28**, 349 (1964).

[8] P. Hohenberg and P. Platzman, Phys. Rev. **152**, 199 (1966).

[9] R. A. Ferrell, N. Nenyhard, and H. Schmidt, Ann. Phys. (N.Y.) **47**, 595 (1968).

[10] W. P. Francis, G. V. Chester, and L. Reatto, Phys. Rev. A **1**, 86 (1970).

[11] R. Puff and J. Tenn, Phys. Rev. A **1**, 125 (197).

[12] G. J. Hyland, G. Rowlands, and F. W. Comming, Phys. Lett. A **31**, 465 (1970).

[13] V. A. Zagrebnov and V. B. Priezzhev, JINR R17-3634, Dubna, 1976.

[14] H. A. Mook, Phys. Rev. Lett. **32**, 1167 (1974).

[15] H. W. Jackson, Phys. Rev. A **10**, 278 (1974).

[16] E. Andronikashvili, J. Phys. USSR **10**, 201 (1964).

[17] E. B. Dokukin, Zh. A. Kozlov, V. A. Parfenov, and A. V. Puchkov, Pis'ma Zh. Eksp. Teor. Fiz. **23**, 459 (1976)], Preprint FEN-710, Obninski, 1976.

[18] V. V. Galikov, Zh. A. Kozlov, L. K. Kul'kin, L. B. Pikel'ner, V. T. Rudenko, and E. I. Sharapov, JINR R3-5736, Dubna, 1971.

[19] A. G. Gibbs and O. Harling, Phys. Lett. A **36**, 203 (1971); Phys. Rev. A **3**, 1713 (1971).

[20] N. N. Borolyubov, Izbrannye trudy, Vol. 2, Naukova Dumka, 1970.

[21] G. J. Hyland and G. Rowlands, Phys. Lett. A **62**, 154 (1977).

Translated by J. G. Adashko

Microscopic mechanism of martensitic transformation in the Fe–Ni system

M. P. Kashchenko and R. I. Mints

S. M. Korov Ural Polytechnic Institute, Sverdlovsk

(Submitted 27 June 1978)
Zh. Eksp. Teor. Fiz. **75**, 2280–2289 (December 1978)

A martensitic transformation is assumed to be due to phonon generation by narrow-band electrons. The threshold population inversion and amplitude of the generated vibrations are found in the single-mode approximation. Two basically different ways of population inversion are considered. In both cases flat constant-energy parts of the electron spectra of the transforming phases are of fundamental importance. An analysis of the electron spectra with the aim of revealing such parts is made for the specific examples of the fcc and bcc modifications of iron. It is pointed out that different martensitic transformations can occur for the same type of flat parts of constant-energy surfaces.

PACS numbers: 64.80.Gd, 81.30.Kf

§1. INTRODUCTION. POPULATION INVERSION. RESONATOR

A martensitic transformation is a diffusionless change in the lattice structure occurring in steels, many transition metals, and their alloys with pronounced features of first-order phase transitions.[1] For example, in the case of the Fe–Ni system (0–34% Ni) in which cooling (forward transformation, beginning at a temperature M_s) or heating (reverse transformation) produces fcc → bcc ($\gamma \to \alpha$) or bcc → fcc ($\alpha \to \gamma$) changes in the lattice,[1)] the relative change in volume amounts to 2.4%. This significant change suggests that longitudinal displacement waves play a leading role in the transformation. In an earlier paper[2] we suggested a mechanism of generation of longitudinal acoustic waves by electrons in narrow energy bands in the presence of a temperature gradient ∇T. The aim of the present paper is to give a description of a martensitic transformation in the theoretical framework developed for lasers (see the lectures of Haken and Weidlich in Ref. 3). The idea of describing a martensitic transformation in the phonon maser model without specifying the mechanism of its action was put forward earlier by Kayser.[4]

A necessary condition for stimulated emission is a

 0038-5646/78/1201149-06$02.40