Polarization operator of electron-positron gas in a constant external magnetic field

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The polarization operator of an electron-positron gas in a constant external magnetic field is calculated in the one-loop approximation, using the Green's function temperature technique to the exact account of the field. All the components of the tensor are evaluated separately and the hermiticity in the quantum kinetical case and the behavior under charge conjugation and space-time inversion are studied. The requirement of four-dimensional transversality, demanded by gauge invariance, is seen to be fulfilled if standard renormalization is carried out. Expressions are given for the dielectric constants for wave propagation parallel and perpendicular to the external magnetic field. The $k_\mu \to 0$ and the $eB \to 0$ limits as well as the singularities of the polarization operator connected with electromagnetic wave absorption are briefly considered in the conclusion.

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INTRODUCTION

In the last two decades there was a great advance in quantum statistics after Matsubara[1] had introduced the temperature Green functions, with which Fradkin (see Ref. 2 and earlier papers quoted therein), Abrikosov, Gor'kov and Dzyaloshinskiĭ[3] and Martin and Schwinger[4] built the calculational apparatus for the quantum statistics. The formulation given by Fradkin[2] includes the renormalized set of equations for the temperature Green functions of interacting Bose and Fermi fields, which describes both nonrelativistic and relativistic systems. As the limiting case of vanishing temperature $\beta^{-1}=0$ and chemical potential $\mu=0$, it contains the Euclidean formulation of the quantum field theory,[5] which was also found by Schwinger.[6] By analytical continuation into the domain of imaginary values of the Euclidean "momentum" $k_4 \to i\omega$ within this method one gets[2] (for nonrelativistic systems see also Ref. 3) the time-dependent Green functions, which, in turn, are described by a set of equations of quantum kinetics[7] if $\mu \neq 0, \beta^{-1} \neq 0$, or the quantum field theory in Minkowski space-time if $\mu = \beta^{-1} = 0$.[2,5]

A very important place in this theory is occupied by the polarization operator tensor, the analog of the photon self-energy tensor in quantum electrodynamics, which is defined in statistical quantum electrodynamics by the expression

$$\Pi_{\mu\nu}(x,y)=e^2\,\mathrm{Sp}\int \gamma_\mu G(xz)\Gamma_\nu(zy'y)G(y'x)\,d^4z\,d^4y', \qquad (1)$$

$G(x,z)$ being the temperature-dependent Green's function for the electron-positron field. Transformed to momentum space, the polarization operator can be used to calculate the radiative corrections to the thermodynamic potential and to the photon Green's function. The spatial part of the polarization operator tensor is simply related to the three-dimensional dielectric tensor of the medium. Singularities of the polarization operator for some values of the photon four-momentum vector are connected in the case of quantum kinetics to the elementary absorptive processes which take place in the medium.

In the one-loop approximation $\Gamma_\nu=\gamma_\nu\delta(z-y')\delta(z-y)$, the polarization operator in statistics was first calculated by Fradkin,[2] by Akhiezer and Peletminskiĭ,[8] by Tsytovich,[9] and by Vologodskiĭ.[10] Recently the calculations for the Yang-Mills field were also performed.[11]

The polarization operator in an external magnetic field in quantum field theory has been obtained, by the proper time method, by Batalin and Shabad[12] and by Tsai, Bayer *et al.*,[13] and its analytic properties were extensively discussed by Shabad.[14] More recently Bakshi, Cover, and Kalman[15] obtained some expressions for the spatial components of the polarization tensor in an external magnetic field in quantum field theory and quantum statistics. In the latter case their results may claim to be correct only for zero temperature (see the criticism concerning their results in Ref. 16 and in the present paper).

In the previous paper by Shabad and the present author,[16] which can be regarded as the first part of the present paper, we have considered the general structure of the polarization tensor of a relativistic plasma in a magnetic field, a structure that can be expected from pure kinematical considerations, so long as the microscopic theory underlying it (in this case—quantum electrodynamics) satisfies the requirements of gauge, charge, P, and T invariance, relativistic covariance, unitarity and the Onsager principle of the symmetry of kinetical coefficients. On this kinematical level we succeeded in progressing quite far in the investigation of the electromagnetic eigenmodes in a medium and conclude, in particular, that one of the principal axes of the ellipse of polarization of the electric field in the eigenmode lies in the plane that contains the direction of the external magnetic field and the direction of propagation of the wave, while the other axis is perpendicular to this

plane.

In the present paper we calculate the components of the polarization tensor $\Pi_{\mu\nu}$ by the temperature technique for an electron-positron gas in a magnetic field B with $\beta^{-1} \neq 0$ and $\mu \neq 0$ (β^{-1} and μ are the temperature and chemical potential) and find explicitly, from the obtained expressions, that the properties predicted by our earlier analysis[16] are attainable. In particular, as expected, it is seen that (unlike in an isotropic plasma) terms appear that are antisymmetric in the tensor indices, are connected with charge asymmetry, and are therefore odd in the chemical potential (they contain linearly the difference between the Fermi distributions of the electrons and positrons). This antisymmetric part is responsible for the elliptical polarization of the eigenmodes, and disappears in the hot vacuum case ($\mu = 0$), when the total charge of the electron-positron system is zero. (From the point of view of thermodynamic equilibrium, an electron-positron gas with zero charge demands only the presence of some photon gas; this corresponds to blackbody radiation at very high temperature. If the total charge is nonzero ($\mu \neq 0$), it is required also that some additional compensating charge, say an "ion core," be present to ensure the charge neutrality of the total system. In the present work we do not consider the contribution of the ion core to $\Pi_{\mu\nu}$.)

We consider next, by investigating the analyticity properties of the calculated expressions, the mechanisms whereby the medium absorbs an electromagnetic wave, and establish the transparency regions. We present also relatively simpler expressions corresponding to normal waves propagating across and along a magnetic field. The limit of a zero 4-momentum of the wave, which gives the Debye radius, and the limit of an infinitely strong external field are considered.

1. UNRENORMALIZED COMPONENTS OF THE POLARIZATION OPERATOR

For the calculation of $\Pi_{\mu\nu}$ we take (1) in the one-loop approximation with the temperature Green's function $G(x, y)$ being the solution of the Dirac equation in a constant homogeneous magnetic field $A_\nu = Bx_1\delta_{\nu 2}$, directed along x_3 axis:

$$[\gamma_\nu(\partial_\nu + ieA_\nu) + m]G(xy|A) = \delta(x-y). \tag{2}$$

Here $\partial_4 = \partial/\partial x_4 - \mu$, e is the modulus of the electron charge. Equation (2) determines the temperature-dependent Green's function $G(xy|A)$ in the interval $-\beta < x_4 < \beta$.

In the formulas that follow, q denotes the aggregate of the quantum numbers (p_2, p_3, n), and $\varepsilon_q = (p_3^2 + m^2 + 2eBn)^{1/2}$ is the energy of the electron as a function of its momentum p_3 along the external magnetic field and of the discrete Landau quantum number $n, 0 < n < \infty$. The electron energy is degenerate with respect to the transverse momentum p_2 of the electron momentum. In addition,

$$p_i' = p_i + i\mu, \quad \varepsilon_{q'} = [(p_3+k_3)^2 + m^2 + 2eBn']^{1/2},$$

where k_3 is the photon momentum component along the magnetic field. The symbols $z_2 = k_1^2 + k_2^2$ and $z_1 = k_3^2 + k_4^2$ for the relativistically invariant combinations of the photon momentum components will also be encountered.

The expressions for $G(xy|A)$ were written out by Shabad and the present author in a preceding paper[17] (where there is a misprint: the coefficient p_2 in the definition of $D_{kk'}$ must be replaced by p_3).

To calculate the Fourier transform of the polarization operator $\Pi_{\nu\rho}(k_4, \mathbf{k}|A, \mu, \beta)$ from the coordinate difference $(x_\mu - y_\mu)$ we must find the trace of the corresponding matrix and sum over the discrete fourth component $p_4 = (2s+1)\pi/\beta (s = 0, \pm 1, \pm 2\ldots)$ of the electron "momentum" in the loop. These operations, as well as a number of trivial ones, can be carried through to conclusion (for details see the author's preprint[18]), and as a result we get expressions containing sums over the Landau-level numbers $n, n' = 0, 1, 2, \ldots$, pertaining to states of the virtual pair and integrals over the intermediate momentum components p_3 along the magnetic field. A specific feature of our problem compared with the case of a nongyrotropic plasma is the onset of polarization-operator terms odd in the chemical potential when summed over p_4. This summation is effected by integrating along a circle of infinite radius with additional multiplication by any of the two auxiliary functions $f^{\pm}(p_4) = \pm i\beta(1 + \exp(\pm i\beta p_4))^{-1}$. In the expression (7) below one can see the structure of the terms obtained after the summation. From among the elements of the matrix $C_{\nu\rho}$, in (7), we shall need hereafter only four:

$$C_{33,44} = [\mp p_4'(p_4'+k_4) \pm p_3(p_3+k_3) + m^2]F^{(1)}_{nn'} + 2eB(nn')^{1/2}G^{(1)}_{nn'}, \tag{3}$$

$$C_{43} = C_{34} = -[2p_3p_4' + p_3k_4 + p_4'k_3]F^{(1)}_{nn'}. \tag{4}$$

The + and − signs should be taken for the first and second pairs of the indices, respectively. Here

$$F^{(1)}_{nn'}(\chi) = \left\{[L^{n-n'}_{n'-1}(\chi)]^2 + \frac{n'}{n}[L^{n-n'}_{n'}(\chi)]^2\right\}\frac{(n'-1)!}{(n-1)!}\chi^{n-n'}e^{-\chi}, \tag{5}$$

$$G^{(1)}_{nn'}(\chi) = 2\left(\frac{n'}{n}\right)^{1/2}\frac{(n'-1)!}{(n-1)!}\chi^{n-n'}L^{n-n'}_{n'-1}(\chi)L^{n-n'}_{n'}(\chi)e^{-\chi}, \tag{6}$$

where $L^{\alpha}_{n}(\chi)$ are generalized Laguerre polynomials, and $\chi = z_2/2eB$.

Thus, summation over p_4 in the loop diagram for $\Pi_{\nu\rho}(k_4, \mathbf{k}|A, \mu, \beta)$ yields, apart from a factor,

$$\begin{aligned}
&\sum_{n,n'}\sum_{p_4}\int dp_3 \frac{C_{\nu\rho}(p_4', p_3)}{[(p_4')^2+\varepsilon_q^2][(p_4'+k_4)^2+\varepsilon_{q'}^2]} \\
&= -\sum_{n,n'}\int dp_3 \Bigg\{\frac{1}{2\varepsilon_q}\Bigg[\mathrm{Ev}\left(\frac{C_{\nu\rho}(p_4', p_3)}{(p_4'+k_4)^2+\varepsilon_{q'}^2}\right)(n_e+n_p-1) \\
&\quad + i\,\mathrm{Od}\left(\frac{C_{\nu\rho}(p_4', p_3)}{(p_4'+k_4)^2+\varepsilon_{q'}^2}\right)(n_e-n_p)\Bigg]_{p_4'=i\varepsilon_q} \\
&\quad + \frac{1}{2\varepsilon_{q'}}\Bigg[\mathrm{Ev}\left(\frac{C_{\nu\rho}(p_4'-k_4, p_3)}{(p_4'-k_4)^2+\varepsilon_q^2}\right)(n_e'+n_p'-1) \\
&\quad + i\,\mathrm{Od}\left(\frac{C_{\nu\rho}(p_4'-k_4, p_3)}{(p_4'-k_4)^2+\varepsilon_q^2}\right)(n_e'-n_p')\Bigg]_{p_4'=ie_{q'}}\Bigg\},
\end{aligned} \tag{7}$$

where $n_{e,p} = [1+\exp(\varepsilon_q \mp \mu)]^{-1}$ are respectively the mean number of electrons and positrons in the state $q(p_3, n)$ and n'_e, and n'_p have the same meaning for the state $q'(p_3+k_3 mb')$. We have indicated by the symbols Ev(...) and iOd(...) even and odd parts of functions of p'_4 (i.e., in fact the parts even and odd in $i\varepsilon_q$ and $i\varepsilon_{q'}$). This notation is preferred to the traditional symbols Re and iIm because sources of imaginary quantities other than the $\sqrt{-1}$ connected with $i\varepsilon_q$, appear after the transition to quantum kinetics.

The terms odd in $i\varepsilon_q$ enter the integrands as the coefficients of the differences of the mean numbers of the electrons and positrons, they are odd in the chemical potential [which enters into (7) only through $n_{e,p}$]. These terms vanish for electrically neutral systems. If we change the integration variable $p_3 \to -(p_3+k_3)$ and the summation index $n \to n'$ in the second square bracket in (7) (this leads to the change $n'_{e,p} \to n_{e,p}$), we see that for some of the $C_{\nu\rho}$ components the terms odd in μ cancel when the second square bracket is summed with the first one, while for other $C_{\nu\rho}$ components these terms survive, depending on how the $C_{\nu\rho}$ components behave under the above change. For example, the diagonal components, $C_{\nu\nu}$ are invariant to such a change the even parts of both halves of (7) become identical, and the odd parts cancel. The quantities $C_{34} = C_{43}$ also give rise to terms even in μ. In Π_{12}, on the contrary, both the Ev and Od parts remain. For the components (1,3; 2,3; 1,4; 2,4) there are also parts even and odd in μ. We can verify ultimately in this manner that the calculated polarization operator is the sum of a symmetric (to the interchange of indices $\nu \leftrightarrow p$) tensor even in μ and an antisymmetric tensor odd in μ, as expected from the general analysis.[16] The Euclidean quantum field theory limit is contained in the symmetrical even part.

2. THE RENORMALIZED COMPONENTS OF THE POLARIZATION OPERATOR. INVERSION AND CONJUGATION PROPERTIES

To eliminate the polarization operator divergences by the general procedure of Ref. 2 we must take as the renormalized polarization operator the quantity

$$\Pi_{\nu\rho}^{R}(k|A, \mu, \beta^{-1}) = \Pi_{\nu\rho}^{st}(k|A, \mu, \beta^{-1}) + \Pi_{\nu\rho}^{R}(k|A, 0, 0), \tag{8}$$

where the statistical part is

$$\Pi_{\nu\rho}^{st}(k|A, \mu, \beta^{-1}) = \Pi_{\nu\rho}(k|A, \mu, \beta^{-1}) - \Pi_{\nu\rho}(k|A, 0, 0), \tag{9}$$

and $\Pi^{R}_{\nu\rho}(k|A,0,0)$ is the renormalized polarization operator in the external field A in vacuum:

$$\Pi_{\nu\rho}^{R}(k|A, 0, 0) = \Pi_{\nu\rho}(k|A, 0, 0) - \Pi_{\nu\rho}(k|0, 0, 0) + \Pi_{\nu\rho}^{R}(k); \tag{10}$$

$\Pi^{R}_{\nu\rho}(k)$ is the same without the external field. The only divergencies which can remain in (9) [and in (10) below], are gauge noninvariant (i.e. not transverse in the indices ν and ρ). If the calculations are made in a gauge-invariant way, they are absent. We shall see below that (9) [together with (10)] as calculated by us is transverse, i.e., gauge invariant. This guarantees its finiteness.

Formally, the statistical part of (9) can be obviously obtained by dropping -1 from the factors (n_e+n_p-1) inside the integrals. The field theoretical part (10) in one-loop approximation is known[12,13,14]. Now we are going to write explicit final expressions for the statistical part of $\Pi^{st}_{\nu\rho}$.

We introduce

$$Q = 2p_4'k_4 + 2p_3k_3 + z_1 + 2eB(n'-n) \tag{11}$$

(we recall that $z_1 = k_3^2 + k_4^2$). The diagonal terms and the terms $\Pi_{34} = \Pi_{43}$ have the following structure

$$\Pi_{\nu\rho}^{st} = -\frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\,\mathrm{Ev}\left(\frac{C_{\nu\rho}}{Q}\right)_{p_4'=i\varepsilon_q}(n_e+n_p). \tag{12}$$

In more explicit form, the first two diagonal terms are

$$\Pi_{11,22}^{st} = h \pm (k_1^2 - k_2^2)\,g, \tag{13}$$

where

$$h = -\frac{e^3B}{4\pi^2}\sum_{n,n'}F^{(2)}_{nn'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\left\{1-[z_1+2eB(n+n')]\times\frac{2p_3k_3+z_1+2eB(n'-n)}{|Q|^2}\right\}(n_e+n_p), \tag{14}$$

$$g = -\frac{e^3B}{\pi^2}\sum_{n,n'}N^{(1)}_{nn'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\left\{\frac{2p_3k_3+z_1+2eB(n'-n)}{|Q|^2}\right\}(n_e+n_p). \tag{15}$$

We have used here [and in (20) below] the definitions

$$F^{(2,3)}_{nn'}(\chi) = \left\{\frac{\chi}{n}[L^{n-n'+1}_{n'-1}(\chi)]^2 \pm \frac{n'}{\chi}[L^{n-n'-1}_{n'}(\chi)]^3\right\}\frac{(n'-1)!}{(n-1)!}\chi^{n-n'}e^{-\chi}, \tag{16}$$

$$N^{(1)}_{nn'}(\chi) = \frac{-n'!}{(n-1)!}\chi^{n-n'-1}L^{n-n'+1}_{n'-1}(\chi)\,L^{n-n'-1}_{n'}(\chi)\,e^{-\chi}, \tag{17}$$

and $|Q|^2$ is defined as

$$|Q|^2 = [2p_3k_3 + z_1 + 2eB(n'-n)]^2 + 4k_4^2\varepsilon_q^2. \tag{18}$$

After going to quantum kinetics by the analytical continuation $k_4 \to i\omega$, the denominator $|Q|^2$ can vanish and then it must acquire an infinitesimal imaginary increment. The polarization operator should be Hermitean only as long as the absorption processes due to the vanishing of $|Q|^2$ are taken into account, i.e., only so long as the complexity coming from this infinitesimal increment is disregarded. Therefore, once we make sure below that the polarization operator calculated by us is Hermitean in this sense, we consider $i\varepsilon_q$ and k_4 as the only sources of the complexity. In other words, we must check the hermiticity of Eqs. (7) with the zeros of the denominators ($|Q|^2$) integrated in the principal value sense or, alternatively, the hermiticity in the domain of the momentum variables k_μ, where $|Q|^2 \neq 0$. (It can be shown that in our case such a transparency region does exist for every pair of values n, n' see the last section of the article.)

All five terms in (12) are real for the temperature case (i.e., at real k_4). After the analytic continuation $k_4 \to i\omega$, the diagonal terms remain real, but $\Pi^{st}_{34} = \Pi^{st}_{43} = -\Pi^{st}_{34}{}^{*}$ becomes antihermitean, while $\Pi^{st}_{03} = \Pi^{st}_{30}$

$= \Pi_{30}^{st*}$ is Hermitean, as it should. In the same way we get

$$\Pi_{12}^{st} = -f + 2k_1k_2g, \quad \Pi_{21}^{st} = f + 2k_1k_2g, \tag{19}$$

where

$$f = -\frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Od}\left\{\frac{[p_4'(p_4'+k_4)+p_3(p_3+k_3)+m^2]F_{nn'}^{(3)}}{Q}\right\}_{p_4'=i\varepsilon_q}$$

$$\times(n_e-n_p) = -\frac{e^3Bk_4}{2\pi^2}\sum_{n,n'}F_{nn'}^{(3)}\int_{-\infty}^{\infty}dp_3\left\{\frac{z_4+2eB(n+n')}{|Q|^2}\right\}(n_e-n_p), \tag{20}$$

and $F_n^{(3)}$ is given by (16). After the analytical continuation g and h are real, while f is pure imaginary. Equations (13) and (14) can be represented jointly as a Hermitean tensor in the subspace $i,j=1,2$

$$\Pi_{ij}^{st} = -f\varepsilon_{ij} + (h-(k_1^2+k_2^2)g)\delta_{ij} + 2k_ik_jg,$$

where ε_{ij} is the unit antisymmetrical (2×2) tensor $\varepsilon_{12} = -\varepsilon_{21} = 1$, $\varepsilon_{11} = \varepsilon_{22} = 0$, and the functions f, g, h, (14), (15), (20) depend only on the combination $k_1^2+k_2^2$, which remains invariant under rotations in the $(1,2)$ plane. The components k_3 and k_4 and the external magnetic field B are not affected by these rotations and this is why the results of the calculations admit of such an explicit tensor structure. The tensor ε_{ij} stems from the electromagnetic-field intensity tensor $F_{\mu\nu}$ which is different from zero only in the sector $(1,2)$, where it is equal to $F_{ij} = B\varepsilon_{ij}$. According to Ref. 16, the antisymmetrical part of the (4×4) tensor $\Pi_{\nu\rho}^{st}$ is formed by the two matrices

$$\psi_{\nu\rho}^{(5)} = (uk)[k_\nu(Fk)_\rho - k_\rho(Fk)_\nu + k^2F_{\nu\rho}],$$
$$\psi_{\nu\rho}^{(6)} = u_\nu(Fk)_\rho - u_\rho(Fk)_\nu + (uk)F_{\nu\rho}, \tag{21}$$

where u_ν is the 4-velocity vector of the medium and has in the rest frame (to which the present calculations pertain) only one nonvanishing component $u_0 = 1$. It is seen therefore that at ν, $\rho = i,j = 1,2$ the antisymmetrical part takes the form

$$f\varepsilon_{ij} = \pi_6k_ik_jF_{ij} + \pi_5k_i(k^2F_{ij} + k_iF_{jm}k_m - k_jF_{im}k_m)$$
$$= k_iB\varepsilon_{ij}(\pi_6 + k^2\pi_5 - \pi_5(k_1^2+k_2^2)),$$

where $\Pi_{5,6}$ are the purely imaginary coefficients that are even with respect to k_μ of the matrices $\psi_{\nu\rho}^{(5,6)}$ that enter into the expansion of $\Pi_{\nu\rho}^{st}$. In full agreement with this formula, the function f (20) contains the factor k_4 raised to an odd power.

To write the other components of $\Pi_{\nu\rho}$, we introduce the notation

$$M_{nn'}^{(1),(3)}(\chi) = \left[\frac{(n'-1)!}{(n-1)!}\chi^{n-n'}L_{n'-1}^{n-n'+1}(\chi)L_{n'-1}^{n-n'}(\chi) \mp \frac{n'!\chi^{n-n'-1}}{(n-1)!}L_{n'}^{n-n'-1}(\chi)L_{n'}^{n-n'}(\chi)\right]e^{-\chi},$$
$$M_{nn'}^{(3),(4)}(\chi) = \left[\frac{n'!}{n!}\chi^{n-n'}L_{n'-1}^{n-n'+1}(\chi)L_{n'}^{n-n'}(\chi) \mp \frac{n'!\chi^{n-n'-1}}{(n-1)!}L_{n'}^{n-n'-1}(\chi)L_{n'-1}^{n-n'}(\chi)\right]e^{-\chi} \tag{22}$$

and

$$a_m = \frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Ev}\left\{\frac{p_m'M_{nn'}^{(4)} + (p_m'+k_m)M_{nn'}^{(2)}}{Q}\right\}_{p_4'=i\varepsilon_q}(n_e+n_p),$$
$$b_m = \frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Od}\left\{\frac{p_m'M_{nn'}^{(3)} + (p_m'+k_m)M_{nn'}^{(1)}}{Q}\right\}_{p_4'=i\varepsilon_q}(n_e-n_p), \tag{23}$$

where $m = 3,4$ and $p_3' = p_3$, $p_4' = p_4 + i\mu$. Then

$$\Pi_{im,mi}^{st} = -a_mk_i \pm \varepsilon_{ij}k_jb_m, \quad i,j=1,2; \quad m=3,4. \tag{24}$$

It is seen from (23) that at $k_4 = i\omega$ the quantities a_3 and b_4 are real and a_4 and b_3 are imaginary, so that in Minkowski metric the matrices (28) are Hermitean: $\Pi_{is}^{st} = \Pi_{si}^{st*}$, $s = 3,0$. This completes the establishing of the hermiticity of the entire tensor $\Pi_{\mu\nu}^{st}$ calculated in the one-loop approximation in the transparency region.

In Eq. (28) one can also plainly see the tensor structure of Π_{im}^{st} under rotations in the $(1,2)$ plane. From (23) it also follows that in Eq. (24) the part a_m which is symmetrical under the inversion $m \leftrightarrow i$ is even with respect to the chemical potential, while the antisymmetrical part b_m is odd. An analogous property is possessed also by the components (12) and (23): the components (12) are even in μ and symmetrical under the inversion $\nu \leftrightarrow \rho$. The only antisymmetrical part in (19) is $\varepsilon_{ij}f$ and it is odd in the chemical potential according to (20) since it contains the difference $(n_e - n_p)$. As for the functions g and h, which form the symmetrical parts of (20) and (23), they include only the sum (n_e+n_p). Therefore the calculated polarization operator is indeed invariant, as predicted in Ref. 16, to simultaneous reversal of the sign of the chemical potential and the indices $\nu \leftrightarrow \rho$, $\nu,\rho = 0,1,2,3$:

$$\Pi_{\nu\rho}^{st}(k|A,\mu,\beta^{-1}) = \Pi_{\rho\nu}^{st}(k|A,-\mu,\beta^{-1}). \tag{25}$$

This equation incorporates the Furry theorem as well as the Onsager relations (see below).

It is also readily seen from (12), (19) and (24), that the property

$$\Pi_{\nu\rho}^{st}(k|A,\mu,\beta^{-1}) = \Pi_{\rho\nu}^{st}(-k|A,\mu,\beta^{-1}) \tag{26}$$

holds. This property follows in the general case from the representation of $\Pi_{\rho\nu}$ as the second variational derivative of a certain effective action,[16] and is a direct consequence of (1) in the one-loop approximation. Unfortunately our results do not make it possible to verify the relations connected with the reversal of the sign of the external field, inasmuch as in the formulas presented above, in accord with the manner of their derivation, B is essentially a positive quantity $(B = |\mathbf{B}|)$. We have in mind the formula

$$\Pi_{\rho\nu}(k|A,\mu,\beta^{-1}) = \Pi_{\rho\nu}(k|-A,-\mu,\beta^{-1}), \tag{27}$$

which is the generalized Furry theorem[2] or an expression of the invariance of the polarization operator to total charge conjugation. In conjunction with (25) this means that the asymmetrical part of $\Pi_{\mu\nu}^{st}$ is odd in the external field, i.e., the Onsager relation[10]:

$$\Pi_{\rho\nu}(k|A,\mu,\beta^{-1}) = \Pi_{\nu\rho}(k|-A,\mu,\beta^{-1}).$$

3. FOUR DIMENSIONAL TRANSVERSALITY OF THE POLARIZATION

Using the relations written in the Appendix, it is possible to verify that our polarization operator satisfies the transversality condition demanded by gauge

invariance $\Pi_{\mu\nu}k_\nu = k_\rho\Pi_{\mu\nu} = 0$. It is easy to see that the transversality condition must be satisfied separately by the symmetrical and antisymmetrical parts of $\Pi^{st}_{\nu\rho}$. For the antisymmetrical part, the last two rows and two columns of the matrix $\Pi^{A}_{\nu\rho}$ satisfy the relations

$$k_\nu\Pi_{vm}{}^{A} = \Pi_{m\lambda}{}^{A}k_\lambda = 0, \quad m = 3, 4,$$

as can be seen by direct substitution from (24). The first two pairs of rows and columns give, after substituting (14)–(19) and (23), (24) and gathering like terms in a common integral

$$-\Pi_{1\rho}{}^{A}k_\rho = k_\lambda\Pi_{\lambda 1}{}^{A} = k_2 T, \quad \Pi_{2\rho}{}^{A}k_\rho = -k_\rho\Pi_{\rho 2}{}^{A} = k_1 T,$$

$$T = -\frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Od}\left\{\left[-\left(\frac{1}{2}z_1 + eB(n+n')\right)F^{(3)}_{nn'} - \left(\frac{z_1}{2} + eB(n'-n)\right)(M^{(1)}_{nn'} + M^{(3)}_{nn'}) + z_1 M^{(1)}_{nn'}\right]Q^{-1}\right\}_{p_4' = i\varepsilon_q}(n_e - n_p) = 0, \tag{28}$$

because the numerator inside the curly bracket vanishes identically by virtue of the relations (A.1) in the Appendix.

For the last pair of rows and columns of the symmetrical part $\Pi^{s}_{\nu\rho}$ we get, writing

$$C_{44} = [-2p_4'^2 - p_4'k_4 + p_3k_3 - 2eBn]F^{(1)}_{nn'} + 2eB(nn')^{1/2}G^{(1)}_{nn'}$$

and making some simplifications,

$$\Pi_{4\rho}{}^{s}k_\rho = k_\nu\Pi_{\nu 4}{}^{s} = -\frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Ev}\{-p_4'+p_4'[z_2(M^{(2)}_{nn'} + M^{(4)}_{nn'}) + 2eB(n'-n)F^{(1)}_{nn'}]Q^{-1}\}_{p_4' = i\varepsilon_q}(n_e + n_p) = 0 \tag{29}$$

[where the relations (A.4) of the Appendix were used]. In the same way, for the third row and column we use the form

$$C_{33} = [-2p_3^2 + p_4k_4 - p_3k_3 - 2eBn]F^{(1)}_{nn'} + 2eB(nn')^{1/2}G^{(1)}_{nn'}.$$

The common integral gives

$$\Pi_{3\rho}{}^{s}k_\rho = k_\lambda\Pi_{\lambda 3}{}^{s} = -\frac{e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Ev}\{p_3 + p_3[z_2(M^{(2)}_{nn'} + M^{(4)}_{nn'}) + 2eB(n'-n)F^{(1)}_{nn'}]Q^{-1}\}_{p_4' = i\varepsilon_q}(n_e + n_p) = 0. \tag{30}$$

The first term in the curly bracket gives a vanishing contribution after integration because it is odd in p_3, and the numerator in the second term is zero by virtue of the relation (A.4). For the first two rows and columns of the symmetrical part we have

$$\Pi_{1\rho}{}^{s}k_\rho = k_\nu\Pi_{\nu 1}{}^{s} = -\frac{k_1e^3B}{2\pi^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\mathrm{Ev}\left\{\frac{1}{2}(F^{(2)}_{nn'} + M^{(2)}_{nn'} + M^{(4)}_{nn'}) - \left[\left(\frac{z_1}{2} + eB(n+n')\right)F^{(2)}_{nn'} - 2N^{(1)}_{nn'}z_2 + \left(\frac{z_1}{2} + eB(n'-n)\right)\times(M^{(2)}_{nn'} + M^{(4)}_{nn'}) - z_1M^{(2)}_{nn'}\right]Q^{-1}\right\}_{p_4' = i\varepsilon_q}(n_e + n_p) = 0, \tag{31}$$

where (A.2)–(A.5) and (A.7) were used. This completes the proof of the transversality.

We see that the calculations for the test of transversality do not involve the factor $(n_e + n_p)$ inside the integrals. If this term is replaced by -1, we find that transversality is formally fulfilled by the unrenormalized polarization operators in vacuum in a magnetic field. This means that in the assumed calculation method this quantity contains only a gauge-invariant (i.e. logarithmic) divergence.

In concluding this section, we must refer to the work by Bakshi, Cover and Kalman,[15] where the calculations were made in a magnetic field at $\beta^{-1} = 0$ and $\mu \neq 0$. Since they calculated only the spatial components at $k_2 = 0$, it is not possible to check the transversality or the tensor properties with respect to rotations in the (1,2) plane. Although the hermiticity of their results seems to hold [apart from a possible misprint (?)], the equivalence of the interchange of the tensor indices $\rho \rightarrow \nu$ of the 4-momentum inversion $k_\lambda \rightarrow -k_\lambda$ (25) is violated. As for the attempt of the authors of Ref. 15 to extend results of their calculations at $\beta^{-1} = 0$ to nonzero temperature by introducing the Fermi distribution into the integrand, it is absolutely untenable, since they apparently did not know that the antisymmetrical part of $\Pi^{st}_{\mu\nu}$ contains the difference of Fermi-distributions of the electrons and positrons.

4. THE DIELECTRIC CONSTANTS FOR PARALLEL AND PERPENDICULAR PROPAGATION

For wave propagation parallel to the external magnetic field B, the polarization operator has two eigenvalues corresponding to the transverse circularly polarized Faraday normal modes[16] and one eigenvalue $\varkappa_2$ corresponding to a longitudinal Coulomb wave. The dielectric tensor $\varepsilon_{ij} = \delta_{ij} + \Pi^{R}_{ij}/\omega^2$, $i,j = 1,2$, also turns out to be diagonalized, two of its eigenvalues being the transverse dielectric constants

$$\varepsilon_{1,2} = 1 + \varkappa_{1,2}/\omega^2. \tag{32}$$

The quantity

$$\varepsilon_{33} = 1 - \Pi_{33}/k_4^2 \tag{33}$$

corresponds to the longitudinal dielectric constant, because ε_{33} coincides in our case $k_1 = k_2 = 0$ with the definition

$$\varepsilon^l = k_i\varepsilon_{ij}k_j/k_i^2 = 1 - \Pi_{44}/k_4^2. \tag{34}$$

We can then write therefore

$$\varepsilon^l = \varepsilon_{33} = 1 - \varkappa_2/z_1. \tag{35}$$

For (32) and (35) we get

$$\varepsilon_{1,2} = 1 - \frac{e^3B}{4\pi^2\omega^2}\sum_{n,n'}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\left\{\left[1 - \frac{(z_1 + 2eB(n+n'))(2p_3k_3 + z_1 + 2eB(n'-n))}{D}\right]\times(\delta_{n-1,n'} + \delta_{n,n'-1})(n_e + n_p) \pm \frac{2\omega\varepsilon_q(z_1 + 2eB(n+n'))}{D}(\delta_{n,n'-1} - \delta_{n-1,n'})\times(n_e - n_p)\right\} + \frac{p_0}{\omega^2}, \tag{36}$$

$$\varepsilon^l = 1 - \frac{e^3B}{\pi^2 z_1}\sum_{n}\alpha_n\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}\frac{(2eBn + m^2)(2p_3k_3 + z_1)}{(2p_3k_3 + z_1)^2 - 4\varepsilon_q^2\omega^2}(n_e + n_p) + \frac{s_0}{z_1}, \tag{37}$$

where

$$D=|Q|^2|_{k_4=i\omega},\quad \alpha_n=2-\delta_{0n},\quad z_1=k_3^2-\omega^2,$$

and p_0 and s_0 are the corresponding values at $\mu=\beta^{-1}=0$.

For propagation perpendicular to the magnetic field, $k_3=0$, there is an eigenvalue related to the ordinary wave with its electric vector polarized linearly, parallel to the external magnetic field **B**.

$$\varepsilon^1=\varepsilon_{33}=1-\Pi_{33}/k_4^2. \tag{38}$$

There are also two normal waves elliptically polarized in a plane perpendicular to **B**. The dielectric constrants for these (extraordinary) waves are

$$\varepsilon_{1,2}=1-\varkappa_{1,2}/k_4^2,\quad \varkappa_{1,2}=\{p+t\pm[(p-t)^2-4r^2]^{1/2}\}/2,$$
$$p=\frac{\omega^2-z_2}{\omega^2}(h+z_2g)+p_0,\quad t=h-z_2g+t_0,\quad r=-\left(\frac{\omega^2-z_2}{\omega^2}\right)^{1/2}f, \tag{39}$$

where f, g, and h must be taken from (14), (15), and (20) with $k_3=0$ and $k_4=-i\omega$, while p_0 and t_0 are the corresponding values at $\mu=\beta^{-1}=0$.

5. THE LIMITS $k_\mu \to 0$ AND $eB \to \infty$

We consider briefly these two limits for our polarization operator. The first limit is meaningful only in the order $k_4 < |\mathbf{k}| \to 0$ in the temperature case.[2] Taking into account the properties of the functions F, G, and N given in (A.8) it is not difficult to obtain

$$\lim_{k_4<|\mathbf{k}|\to 0}\Pi_{\nu\rho}(k|A,\mu,\beta^{-1})=0, \tag{40}$$

except for the component $\nu=\rho=4$, for which

$$\lambda^{-2}=\lim_{k_4<|\mathbf{k}|\to 0}\Pi_{44}(k|A,\mu,\beta^{-1})=\frac{e^3B}{4\pi^2}\sum_n\alpha_n\int_{-\infty}^{\infty}dp\,\frac{d}{d\mu}(n_e-n_p), \tag{41}$$

where λ is the Debye radius of the electron-positron plasma.

In the $eB\to\infty$ limit, it can be proved that the dominant terms are Π_{33}, $\Pi_{34}=\Pi_{43}$, Π_{44}, to which only the term $n=n'=0$ in the sums over and n' contributes. The dominant scalar is $s=-a_\nu^{(2)}\Pi_{\nu\rho}a_\rho^{(2)}$:

$$s=-\frac{m^2e^3B}{\pi^2}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_0}\frac{(2p_3k_3+z_1)(n_e(\varepsilon_0)+n_p(\varepsilon_0))}{(2p_3k_3+z_1)^2+4\varepsilon_0^2k_4^2}+s_{0,\infty}, \tag{42}$$

where

$$s_{0,\infty}=\lim_{eB\to\infty}s_0.$$

For a vacuum with a magnetic field this limit was considered in Ref. 20. The scalar s is at the same time an eigenvalue of the polarization operator with eigenvector $a_\nu^{(2)}$:

$$a_\nu^{(3)}=-F_{\nu\lambda}^*k_\lambda(kF^{*2}k)^{-1/2},$$

where $F_{\nu\lambda}^*$ is a tensor dual to the external field intensity tensor. The corresponding normal wave is a longitudinal Coulomb wave when propagating parallel to the external field and an ordinary transverse wave polarized along **B** when propagating perpendicular to **B**.

6. ABSORPTION MECHANISMS AND TRANSPARENCY REGIONS

To study the mechanisms of propagation of an electromagnetic wave by a plasma it is sufficient to consider the regions where the calculated expressions for the polarization operator are analytic in $z_1=k_3^2-\omega^2$ at fixed real variables $z_2=k_1^2+k_2^2$ and k_3. These analyticity regions are independent of z_2 and k_3 because of the degeneracy of the electronic levels in the magnetic field (so long as $T\neq 0$, see below).

The analytic properties with respect to z_1 are determined by the zeros of expression (18), which enters as a denominator in the integrands of (12), (14), (15), (20), and (23). It is easy to show that the denominator (18) vanishes only at real z_1 when p_3 is varied along the integration path $-\infty<p_3<\infty$. Therefore the integrals with respect to p_3, being convergent, are analytic functions in the z_1 plane, with the possible exception of singularities located on the real axis. To determine their location, we obtain the roots of the denominator (18). There are two of them:

$$p^{(1,2)}=-(k_3\alpha\pm\omega\Lambda)/2z_1,$$
$$\alpha=z_1+2eB(n'-n),\quad \Lambda=[\alpha^2+4(m^2+2eBn)z_1]^{1/2}, \tag{43}$$

and the frequency at fixed k_3 is $\omega=(k_3^2-z_1)^{1/2}$. Therefore the denominator can be expressed in the form

$$D=4z_1(p_3-p^{(1)})(p_3-p^{(2)}). \tag{44}$$

In the z_1-plane region where the roots $p^{(1,2)}$ are complex, they do not lie on the integration paths and the integrals are analytic in z_1. When $p^{(1,2)}$ are real, the integrals are not uniquely defined—the corresponding real values of z_1 belong to a cut. The branch points correspond to those values of z_1 for which the imaginary parts of $p^{(1,2)}$ first vanish. In this case the poles $p_3=p^{(1,2)}$ of the integrand pinch the integration path (the only possible exception is at a temperature $T=0$ in some regions of the variable k_3, for in this case the Fermi distribution makes the region of integration with respect to p_3 bounded), causing a singular threshold-dependent behavior of the reciprocal square root type $(z_1-(z_1)_{\text{thr}})^{-1/2}$, which is characteristic of systems with magnetic fields. In region where the real axis of the z_1 plane is analytic the roots $p^{(1,2)}$ are complex and mutually conjugate, $p^{(1)}=p^{(2)*}$; therefore the denominator (44) is real and the calculated tensor $\Pi_{\mu\nu}$, as shown in Sec. 3, is Hermitean. Thus, the non-Hermitean part of $\Pi_{\nu\rho}$ exists only on the cut due to the imaginary increment of the denominator (44), when the latter vanishes, and assumes a value jumpwise on the cut. This statement is the relaization of the optical theorem in our case. The jump on the cut, as usual, is formed by absorption of an electromagnetic wave. We shall therefore call the region occupied by the cut the absorption region, and the region free of the cut will be called the transparency region. It follows from (43) that $p^{(1,2)}$ are complex if $k_3^2>z_1$, $\alpha^2+4(m^2+2eBn)z_1<0$ or $k_3^2<z_1$, $\alpha^2+4(m^2+2eBn)z_1>0$.

Thus, representing the discriminant of Λ^2 in the form

$$\alpha^2+4(m^2+2eBn)z_1=(z_1-z_1')(z_1-z_1''),$$

where

$$z_1'=-[(m^2+2eBn)^{1/2}+(m^2+2eBn')^{1/2}]^2,$$
$$z_1''=-[(m^2+2eBn)^{1/2}-(m^2+2eBn')^{1/2}]^2,$$

we see that the transparency region for each pair n, n' is located between branch points z_1' and z_1'' ($z_1'<z_1<z_1''$), and also in the region $z_1>k_3^2$. The last inequality is simply the condition under which the frequency ω is real. The inequality $z_1'<z_1''$ may be violated if z_1' and z_1'' are taken for different n and n', so that there is no universal transparency region on the real ω axis.

A purely kinematic analysis based on the laws of conservation of the energy and of the momentum component along the magnetic field makes it possible to identify the region of the cut $z_1''\leq z_1\leq k_3^2$ with the region in which a (virtual or real) process takes place wherein the incident photon excites an electron or positron of the medium on the level n' with momentum $p_3'=p_3+k_3$. (At $z_1>0$ it is also possible to have excitation with transition from a higher Landau level to a lower one $n>n'$, $|p_3'|>|p_3|$, $\varepsilon_{q'}>\varepsilon_q$.) In other words, it can be shown that this is the region of z_1 in which the inequalities

$$\omega+\varepsilon_q=\varepsilon_{q'},\quad k_3+p_3=p_3'$$

can be satisfied for all real p_3 and p_3'. The value $z_1=z_1''$ corresponds to

$$p_3=k_3\frac{(m^2+2eBn)^{1/2}}{(m^2+2eBn')^{1/2}-(m^2+2eBn)^{1/2}}. \tag{45}$$

This is the value of the electron (positron) momentum that serves as the threshold for the excitation process. Apart from the sign[1)] this expression agrees with (43) at $z_1=z_1''(\Lambda=0)$. At $n=n'$ (the particle changes its momentum under the influence of the wave, but remains on the same Landau level), Eq. (45) becomes infinite. Since the Fermi distribution has no particle with infinite momentum, the process of excitation at the threshold is suppressed. As a result the threshold value $z_1''=0$ is not subject to the aforementioned root singularity, as can be seen directly by substituting $n=n'$ and $z_1=0$ in $\Pi_{\nu\rho}$: they are finite.

It can be analogously established that the cut region $z_1<z_1'$ corresponds to virtual or real production of an electron-positron pair by a photon. This process exists even in vacuum. Therefore the sign between the two radicals in the denominator in the formula analogous to (45) is plus, p_3 is finite, and all the thresholds are singular, with exception of the degenerate case $T=0$, when the integral with respect to p_3 is cut off by the Fermi distribution. The singularity of the threshold behavior exerts a substantial influence on the dispersion curves[14,21] and leads to peaks on the absorption curves in the threshold regions.

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APPENDIX

We state here the relations between the functions of the Laguerre polynomials introduced in the body of the paper. These relations can be obtained by using trivial recurrence relations as well as the completeness propertiess of the Hermite functions, whose integration leads to the combinations of Laguerre polynomials in our problem. The relations we need are

$$M_{nn'}^{(1)}=\frac{n}{n-n'}F_{nn'}^{(3)},\quad M_{nn'}^{(3)}=\frac{n'}{n-n'}F_{nn'}^{(3)}; \tag{A.1}$$

$$M_{nn'}^{(2)}=\frac{nF_{nn'}^{(1)}-(nn')^{1/2}G_{nn'}^{(1)}}{\chi}=\frac{nF_{nn'}^{(2)}-2\chi N_{nn'}^{(1)}}{n-n'}; \tag{A.2}$$

$$M_{nn'}^{(2)}-M_{nn'}^{(4)}=F_{nn'}^{(2)}=\frac{(n+n')F_{nn'}^{(1)}-2(nn')^{1/2}G_{nn'}^{(1)}}{\chi}; \tag{A.3}$$

$$M_{nn'}^{(2)}+M_{nn'}^{(4)}=\frac{(n-n')F_{nn'}^{(1)}}{\chi}; \tag{A.4}$$

$$M_{nn'}^{(4)}=-\frac{n'F_{nn'}^{(1)}-(nn')^{1/2}G_{nn'}^{(1)}}{\chi}=\frac{n'F_{nn'}^{(3)}-2\chi N_{nn'}^{(1)}}{n-n'}; \tag{A.5}$$

$$\sum_{n'=0}^{\infty}M_{nn'}^{(1,3)}(0)=0,\quad \sum_{n=0}^{\infty}M_{nn'}^{(3,4)}(0)=0; \tag{A.6}$$

$$\sum_{n'=0}^{\infty}M_{nn'}^{(2)}=\sum_{n=0}^{\infty}M_{nn'}^{(4)}=\sum_{n'=0}^{\infty}M_{nn'}^{(1)}=\sum_{n=0}^{\infty}M_{nn'}^{(3)}=0; \tag{A.7}$$

$$F_{nn'}^{(1)}(0)=(2\delta_{n,n'}-\delta_{n,0}\delta_{n',0}),\quad F_{nn'}^{(2,3)}(0)=(\delta_{n,n'-1}\pm\delta_{n-1,n'}),$$
$$G_{nn'}^{(1)}(0)=2(\delta_{n,n'}-\delta_{0,n}),\quad N_{nn'}^{(1)}(0)=0. \tag{A.8}$$

[1)]The sign in (43) has in itself no meaning, since it is always possible to make the change of variable $p_3\to -p_3$, i.e., reverse the electron direction in the loop.

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Kinetics of infrared absorption spectra of SF_6 molecules vibrationally excited by a high-power CO_2 laser pulse

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The infrared linear absorption spectra of the SF_6 molecule is studied experimentally with time resolution after multiphoton excitation in the strong field of a CO_2 laser. A comparison is made with SF_6 spectra in equilibrium heating and the existence is shown of two ensembles of molecules—cold molecules which do not interact with the field and hot, vibrationally excited molecules. Measurements are made of the fraction of molecules interacting with the field q, the average level of excitation $\langle n_q \rangle$, and the time of establishment of vibrational equilibrium as a function of the level of excitation.

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1. INTRODUCTION

In recent years a large number of studies have been devoted to the multiphoton excitation and istotopically selective dissociation of polyatomic molecules in strong infrared laser fields (see the review by Ambartzmian and Letokhov[1]). The molecule most studied in this area is SF_6, for which detailed investigations have been made of the characteristics of multiphoton absorption, dissociation, and laser isotope separation and for which a real possibility has been shown of isotope separation in large quantities.[2] An important refinement of the process is the use of the two-frequency method,[3] in which the first field excites a significant fraction of the molecules and the second, with some delay in time and detuning in frequency, dissociates the vibrationally excited molecules.

At the same time there remain unanswered many questions relating both to the excitation of the lower and higher vibrational levels and to the dissociation process.

The interpretation is particularly difficult in the case of two-frequency excitation. While the absorption spectrum and distribution of molecules in vibrational levels are known for single-frequency excitation, in two-frequency excitation the second field interacts already with a substantially nonequilibrium system, for which neither the absorption spectrum nor the vibrational distribution function $N(E_{v_t})$ are known.

Of exceptional importance is the question of the fraction of molecules q which interact with the field and of the average level of excitation of the molecules $\langle n_q \rangle$, and also the dependence of these quantities on the conditions of excitation. In experiments on measurement of absorption in a strong field[4,5] one usually determines $\langle n \rangle$—the average number of absorbed photons for all molecules in the region of the strong field, i.e., the quantity

$$\langle n \rangle = \langle n_q \rangle q.$$

However, obtaining information individually on q and $\langle n_q \rangle$ from experiments on dissociation of molecules is extremely difficult as a consequence of possible processes of successive excitation of dissociation products, and also possible recombination of dissociation products with formation of the initial molecules.

Another important question is the lifetime of molecules excited in a strong field to high vibrational states, i.e., the time of their de-excitation in collisions with unexcited molecules.

The present work is devoted to investigation with time resolution of the linear absorption spectra of SF_6 molecules after their multiphoton excitation in the strong infrared field of a CO_2 laser. This permits determination of important characteristics both of the multiphoton excitation process itself, and of the molecules excited to high vibrational states.

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