Carroll[9] used the Helfrich–Orsay theory[3] to calculate the dependence of the velocity v and angle Θ_0 for a small excess over the threshold in a static field. Expanding the moments of the forces as a series in Θ^2, he obtained the following relationships in a simple nonlinear approximation:

$$\Theta_0^2 \propto (1 - V_{th}^2/V^2), \quad v \propto \Theta_0 E^2.$$

Clearly, the approach to the threshold should give near-zero values of the velocity and angle in the immediate vicinity of V_{th}. The experimental dependence $\Theta_0(V)$ was in good agreement with the calculations. In all these investigations[7-9] the measured or calculated values of v and Θ_0 tended to zero on approach of V to V_{th}.

Our experiments showed that the quadratic dependence of the velocity on the voltage was retained in the case of an alternating electric field but was more complex. The velocity v_c at V_{th} was a constant independent of the field frequency. In the case of PAA this velocity was $v_c \approx 20~\mu/\text{sec}$; in the case of MBAA, we obtained $v_c \approx 3~\mu/\text{sec}$. The finite velocity v_c correspond to finite and fairly considerable deformation angles Θ_0 (from 13 to 26°) for values of V_{th} in the investigated frequency range. Thus, the results of experimental determinations of the velocities and deformation angles were in mutual agreement and indicated considerable nonlinear

effects even at the threshold of the electrohydrodynamic instability in the conduction regime.

These effects, ignored in most investigations, may be responsible for the growth of the initially small perturbation to the relatively large values of the velocity and deformation found by us. Our results differ considerably from those published earlier and it is not yet clear whether they can be explained within the framework of the existing theories of the electrohydrodynamic instability.

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Phase transitions in a cubic crystal with dipolar forces and an anisotropic correlation function

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The static critical behavior of a cubic ferroelectric (ferromagnet) is studied taking into account both the dipolar and the crystal anisotropy of the correlation function of the order-parameter fluctuations. It is shown that the interaction of the critical fluctuations can lead in this case to a curious effect—a phase transition to an ordered phase that is not energetically the most favorable from the standpoint of the Landau theory. The form of the diagram of states in the coordinates of inverse correlation length, anisotropy of the correlator, and anisotropy of the anharmonicity is established in the critical region. The features that arise in the diagrams of states of the system as a result of the interaction of the fluctuations with the anisotropic spectrum are discussed.

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1. INTRODUCTION

The problem of phase transitions in cubic crystals can be regarded at the present time as classical for the theory of static critical phenomena. Indeed, several dozen papers have been devoted to the critical thermodynamics of systems with cubic and hypercubic symmetry,[1] and the most important of the results obtained have already appeared in reviews and books.[1-4] While studying the simplest model of this type, Wilson and

Fisher[5] encountered for the first time two very interesting phenomena, having, as was made clear later, an extremely general character—the change of the order of the phase transition in an anisotropic system under the influence of the critical fluctuations, and what it is now customary to call asymptotic symmetry or isotropization. Models with generalized cubic symmetry have been investigated in detail by various methods based on the ideas of the renormalization group, and the corresponding problem has succeeded in becoming something

of the nature of a test for checking the effectiveness of these methods. To data the critical behavior of $(4-\varepsilon)$-dimensional models with the point group of a cube (hypercube) has been analyzed in first, second, and third order in ε.[6-8] The Gell-Mann-Low (GML) functions of these models, which determine the critical thermodynamics of the system in the framework of the philosophy of the renormalization group, are known to fourth order in the invariant charges, both for four-dimensional space[9] and for physical, three-dimensional space.[10] Certain exact relations between the GML functions have been found in the case when the field of the fluctuations is a two-component field.[11] The citical behavior of many-component hypercubic models has been studied by means of the $1/n$ expansion.[12] Finally, the variational principle and approximate recursion formulas of Kadanoff[13] have recently been applied to solve the problem under discussion.

Actually, however, we do not know too much about the behavior of a cubic crystal in the region of strongly developed critical fluctuations. The fact is that all this abundance of theoretical information pertains principally to systems having an openly model character. To be specific, what was investigated in the papers enumerated above was strictly not so much a cubic crystal as what is called, in the language of quantum field theory, a two-charge model with an n-component isospin and an isotropic propagator. The anisotropic invariants characteristic for a cubic crystal and constructed from the projections of the gradients of the isospin variables[14] are absent in the Hamiltonian of this model. But we now know[15] that taking these invariants into account alters substantially the predictions of the theory concerning the character of the phase transition. The same is also true with regard to taking account of the long-range dipole-dipole forces, which lift the degeneracy of the spectra of the longitudinal and transverse long-wavelength fluctuations of the order parameter in ferroelectrics and ferromagnets.

Apparently, the critical thermodynamics of cubic crystals was first studied on the basis of a realistic model by Aharony and Fisher.[16] They obtained a system of Wilson recursion equations for all six parameters of the Landau effective fluctuation Hamiltonian and found the asymptotic forms of some of the solutions of the system (the fixed points). In Ref. 16 the so-called isotropic dipolar limit was investigated in detail and the corresponding critical indices were calculated. In the general case, however, it was not possible to reach a definite conclusion concerning the character of the critical behavior of a cubic ferromagnet. One of the reasons for this was the use of approximate recursion equations, linearized in the crystal-anisotropy parameter of the propagator. At the same time, in Ref. 16 a number of important results were obtained, stating unambiguously that the crystal and dipolar anisotropies of the correlation function can play a decisive role in shaping the critical thermodynamics of a cubic crystal.

Following Aharony and Fisher, other authors considered simplified models, ignoring either the dipolar[15] or the crystal[17] anisotropy of the propagator of the criti-

cal fluctuations. It was found that in both cases there is a strong tendency to change a phase transition that is continuous (in the framework of the Landau theory) into a first-order transition, and in the second case this change occurs in the presence of arbitrarily small cubic anisotropy of the bare anharmonic potential. These papers enlarged our store of information about model systems, but they left open the question of the critical behavior of a cubic ferromagnet (ferroelectric). The present paper is devoted to the elucidation of this question.

Below we shall study the phase transitions in a cubic crystal with dipolar forces on the basis of the full Aharony-Fisher Hamiltonian,[16] i.e., without assuming the absence or smallness of the crystal or dipolar anisotropy of the correlator of the critical fluctuations. The system of GML equations for a three-dimensional cubic ferromagnet will be obtained in the quadratic approximation in the invariant charges and investigated. We shall show that, when the crystal anisotropy of the correlator is taken into account, phase transitions to an ordered phase that is not energetically the most favorable from the standpoint of the Landau theory turn out to be possible in our system. This leads to a substantial change in the form of the diagram of states of the crystal in the critical region as compared with that predicted by the phenomenological theory. In the concluding section of the article we discuss the experimental situation and refine the limits of applicability of the results obtained.

2. THE HAMILTONIAN, PROPAGATOR, AND GML EQUATIONS

We shall start, then, from the following effective Hamiltonian[16]:

$$H = \frac{1}{2} \sum_{\alpha,\beta=1}^{3} \sum_{q} \left\{ \left[(x_0^2 + q^2 + fq_\alpha^2)\delta_{\alpha\beta} + (\Delta^2 + hq^2)n_\alpha n_\beta \right] \varphi_\alpha(q)\varphi_\beta(-q) \right.$$
$$+ \frac{1}{4} \sum_{q'+q''+q'''=-q} [\gamma_2^{(0)} + (\gamma_1^{(0)} - \gamma_2^{(0)})\delta_{\alpha\beta}]$$
$$\left. \times \varphi_\alpha(q)\varphi_\alpha(q')\varphi_\beta(q'')\varphi_\beta(q''') \right\}. \tag{1}$$

Here φ_α is the vector field of the fluctuations of the order parameter, q is the three-dimensional wave vector, $n_\alpha = q_\alpha/q$, and $\gamma_1^{(0)}$ and $\gamma_2^{(0)}$ are the bare coupling constants. The bare "mass" x_0^2 is a linear function of the distance from the line or surface of phase transitions in the phase diagram, and the dipolar gap Δ and parameters f and h characterize the bare anisotropy of the spectrum of the fluctuations. The case of absence of crystal anisotropy corresponds to $f=0$ and $\gamma_1^{(0)} = \gamma_2^{(0)}$.

As is well known, as the system approaches the region of strongly developed critical fluctuations the parameters of the effective Hamiltonian (1) begin to change and are renormalized, their evolution inside this region being described by the equations of the renormalization group. In principle, to study the critical behavior of the crystal it ought to be necessary to derive and analyze the renormalization-group equations for all the parameters appearing in (1). In practice, however, in view of its extreme complexity such an analysis can hardly be performed. Therefore, the problem arises of seeking approximations of a kind such that

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using them would not impose restrictions on the size of the anisotropy constants.

We know very well that in the critical region the constants characterizing the dispersion of the spectrum of the fluctuations are renormalized very slowly. To be specific, the rate of change of these parameters with variation of $\varkappa_0$ is of the order of magnitude of the critical index η, as is not difficult to see by considering the diagrammatic expansion of the mass operator. In three-dimensional systems with a scalar or vector order parameter, $\eta \sim 0.05$. Therefore, only in a very narrow region about the line of phase transitions, which, according to numerical estimates, it is far beyond the capabilities of any conceivable experiment to reach,[18] will the effective, "dressed" values of the dispersion constants differ at all significantly from their bare values. This permits us to neglect the renormalizations of f, h, and the coefficient of q^2 in the critical region and assume these to be equal to their bare values.

A second substantial simplification arises if we assume that the dipolar gap Δ is sufficiently large, viz., that $\Delta^2 \gg |\varkappa_0^2|$. In fact, as can be seen from the Dyson equation, the additive renormalizations of Δ^2 and $\varkappa_0^2$ should have the same order of magnitude. In the critical region the physical "mass" $\varkappa^2 = \varkappa_0^2 + \delta\varkappa_0^2 \approx 0$, and, consequently the "mass" renormalization $\delta\varkappa_0^2 \sim |\varkappa_0^2|$. Hence, the gap renormalization $\delta\Delta^2 \sim \delta\varkappa_0^2 \ll \Delta^2$ and can be neglected. There is, however, another reason for neglecting the renormalization of the gap, weightier than its relative smallness. The point is that, when the condition $\Delta^2 \gg |\varkappa_0^2|$ is fulfilled, the inequality $\Delta^2 \gg \varkappa^2$, q_c^2 is valid, where q_c is the scale of the momenta that are characteristic for the critical region. Therefore, the gap Δ is found here to be an irrelevant variable in the renormalization-group sense. The reason for this is simple. As is well known, the dipolar forces lead to the appearance of extra "stiffness" of the long-wavelength longitudinal fluctuations of the order parameter. As a result, this branch of the spectrum becomes noncritical, and in each order of perturbation theory the graphs containing one or more lines of longitudinal "fluctuons" are found to be small compared with the "purely transverse" diagrams, to the extent that the ratio $\varkappa/\Delta$ is small. The role of the dipolar interaction here reduces simply to suppression of the longitudinal fluctuations, and, for so long as the gap Δ is still sufficiently large, the dipolar interaction copes with this problem successfully, irrespective of the actual value of Δ. It is clear that, in this situation, even appreciable renormalizations of Δ cannot affect the critical thermodynamics of the system.

To what does the restriction $\Delta^2 \gg \varkappa^2$ correspond when applied to particular types of crystals? In ferromagnets this inequality defines the dipolar fluctuation region, which it is our main interest to investigate, since for $\Delta^2 \lesssim \varkappa^2$ our system behaves like a cubic crystal with no dipolar forces, whose critical behavior has by now been thoroughly studied. In ferroelectrics the dipolar interaction is always strong, and the gap Δ has the same order of magnitude as the Debye momentum q_D. Therefore, the inequality $\Delta^2 \gg \varkappa^2$ here does not impose any restrictions at all on the quantity $\varkappa$ beyond those which

already obtain in the critical region.

Thus, we have established that, in the dipolar region, only the coupling constants $\gamma_1^{(0)}$ and $\gamma_2^{(0)}$ and the inverse correlation length $\varkappa_0$ are subject to substantial critical renormalizations. Going over from $\varkappa_0$ to the dressed inverse correlation length $\varkappa$ and choosing this as the independent variable, we can obtain, for the effective coupling constants γ_1 and γ_2, GML equations that are equivalent, in our case, to the complete system of renormalization-group equations. To derive the GML equations, however, we need to know the structure of the dressed propagator $G_{\alpha\beta}(q)$. This is not difficult to find, if we turn to the well known expression for the bare Green function[16]:

$$G_{\alpha\beta}^{(0)}(q) = \frac{1}{\varkappa_0^2 + q^2 + fq_\alpha^2} \left\{ \delta_{\alpha\beta} - \frac{(\Delta^2 + hq^2)q_\alpha q_\beta}{\varkappa_0^2 + q^2 + fq_\beta^2} \right.$$
$$\times \left[q^2 + (\Delta^2 + hq^2) \sum_{\tau=1}^{3} \frac{q_\tau^2}{\varkappa_0^2 + q^2 + fq_\tau^2} \right]^{-1} \right\}. \tag{2}$$

This expression has the most general form permitted by the symmetry of the problem. Obviously, the exact propagator should also have this form, with the sole difference that in the expression for $G_{\alpha\beta}(q)$ the bare constants will be replaced by their effective, renormalization values.[2] We recall, however, that the constants f, Δ, and h are practically unchanged in the critical region, and the gap Δ is always much greater than the important momenta. Therefore, for $G_{\alpha\beta}(q)$ the following approximate formula will be very accurate:

$$G_{\alpha\beta}(q) \approx \frac{1}{\varkappa^2 + q^2 + fq_\alpha^2} \left\{ \delta_{\alpha\beta} - \frac{q_\alpha q_\beta}{\varkappa^2 + q^2 + fq_\beta^2} \right.$$
$$\times \left[\sum_{\tau=1}^{3} \frac{q_\tau^2}{\varkappa^2 + q^2 + fq_\tau^2} \right]^{-1} \right\}. \tag{3}$$

As we should expect, the dipolar gap Δ has dropped out of (3), and the propagator in this limit has turned out to be purely transverse: $n_\alpha G_{\alpha\beta}(q) = 0$.

We proceed now to the derivation of the GML equations. Since the technique for obtaining these is well known,[19,20,10] we shall discuss only some of the most characteristic details. The GML equation will be derived directly for three-dimensional space in the lowest approximation in the invariant charges, which, for $d = 3$, is evidently the optimal approximation.[10] In this approximation terms associated with the renormalized mass operator are absent, so that the invariant charges here will coincide with the physical charges.[20] An important feature of the problem is the nondiagonality of the propagator. As a result, even in the one-loop approximation for the vertex part three different integrals arise:

$$I_1 = K_{\alpha\alpha\alpha\alpha}, \quad I_2 = K_{\alpha\beta\alpha\beta}, \quad I_3 = K_{\alpha\alpha\beta\beta};$$
$$K_{\alpha\beta\gamma\delta} = \frac{1}{2} \int G_{\alpha\beta}(q) G_{\gamma\delta}(q) \frac{dq}{(2\pi)^3}. \tag{4}$$

In the limit $\varkappa \ll q_D$ these integrals depend only on f and $\varkappa$, and $\partial I_i / \partial \varkappa^2 = -I_i/2\varkappa^2$. Differentiating the diagrammatic expansion for the vertex $\Gamma_{\alpha\beta\gamma\delta}(q, q', q'')$ with respect to $\varkappa_0^2$ using the Ward identity

$$\partial G_{\alpha\beta}^{(0)} / \partial \varkappa_0^2 = -G_{\alpha\tau}^{(0)} G_{\tau\beta}^{(0)} \tag{5}$$

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and expressing the derivative with respect to $\varkappa_0^2$ in terms of the derivative with respect to $\varkappa^2$, we can obtain the following equations for the physical charges $\gamma_1 = \frac{1}{3}\Gamma_{\alpha\alpha\alpha\alpha}(0,0,0)$ and $\Gamma_2 = \Gamma_{\alpha\alpha\beta\beta}(0,0,0)$:

$$\frac{d\gamma_1}{d\varkappa^2} = \frac{1}{2\varkappa^2}[9I_1\gamma_1^2 + 12I_2\gamma_1\gamma_2 + 2(I_1+I_2)\gamma_2^2],$$

$$\frac{d\gamma_2}{d\varkappa^2} = \frac{1}{2\varkappa^2}[9I_2\gamma_1^2 + 6(I_1+I_2)\gamma_1\gamma_2 + (I_1+7I_2+4I_3)\gamma_2^2]. \tag{6}$$

For what follows it is convenient to go over to the dimensionless charges $g_i = \gamma_i/\varkappa$ and the dimensionless integrals $J_k = \varkappa I_k$, and also to introduce the new independent variable $t = -\ln\varkappa$ in place of $\varkappa$. Then Eqs. (6) take the form

$$dg_1/dt = g_1 - 9J_1g_1^2 - 12J_2g_1g_2 - 2(J_1+J_2)g_2^2,$$

$$dg_2/dt = g_2 - 9J_2g_1^2 - 6(J_1+J_2)g_1g_2 - (J_1+7J_2+4J_3)g_2^2. \tag{7}$$

These are the GML equations, whose solutions describe the thermodynamics of our system in the critical region.

3. THE PHASE TRAJECTORIES, SEPARATRICES, AND DIAGRAM OF STATES

Even a cursory glance at the formulas (3) and (4) is sufficient to make it clear that the integrals I_i (J_i) cannot be calculated analytically. However, before turning to a computer for help, it is useful to investigate the system of GML equations in general form, without specifying the values of the parameters occurring in it. As usual, we shall be interested in the separatrices and singular (fixed) points of the system (7), whose type, number, and location determine the character of the critical behavior of the crystal. In the case of a system of the type under consideration, the search for the singular points can be simplified radically by noting that all its singular points should lie on rectilinear phase trajectories emerging from the coordinate origin. The proof of this statement is elementary, and we only point out that the presence of this property is due to the lowest (second) order of the right-hand sides of Eqs. (7) and the special form of the linear terms. Thus, in order to establish an upper bound on the number of singular points, and also to obtain some idea of their disposition in the phase plane, it is sufficient to find all the straight phase trajectories of the system (7) that pass through the coordinate origin. In their turn, these phase trajectories also possess a useful property: the ratio g_1/g_2 does not change as we move along them. Therefore, if we find the "equation of motion" of the quantity $z = g_1/g_2$ in "time" t, the positions of the required rectilinear phase trajectories will be determined simply by the roots of its right-hand side.

The derivation of the "equation of motion" for z presents no difficulty. Combining Eqs. (7) in the appropriate way, we shall have

$$\frac{dz}{dt} = g_2[9J_2z^3 + (6J_1-3J_1)z^2 + (J_1-5J_2+4J_3)z - 2J_1 - 2J_2]. \tag{8}$$

By direct inspection one can convince oneself that the "Ising" straight line $g_2 = 0$ is not a phase trajectory of the system (7).[3] Consequently, the polynomial in the square brackets should vanish on all the rectilinear phase trajectories. This polynomial is a homogeneous

function of the integrals J_i, and its properties depend only on their ratios. Therefore, it is convenient to introduce the new parameters

$$\xi = \frac{1}{9}\left(\frac{J_1}{J_2} - 8\right), \quad \zeta = \frac{4}{9}\left(\frac{J_1-J_3}{J_2} - 2\right), \tag{9}$$

after which the equation for the stationary points of (8) takes the form

$$z^3 - (2+3\xi)z^2 + (3+5\xi-\zeta)z - 2(1+\xi) = 0. \tag{10}$$

The parameters ξ and ζ, which depend on f, have been set up in such a way that $\xi = \zeta = 0$ when $f = 0$, i.e., in the absence of crystal anisotropy in the propagator. In this "isotropic" limit Eq. (10) has a single real root, as is easily seen by rewriting its left-hand side in factored form:

$$(z-1)(z^2-z+2) = 0. \tag{11}$$

The phase trajectory corresponding to the root $z = 1$ coincides with the "Heisenberg" straight line $g_1 = g_2$, and on it there is a singular point of the "saddle" type. The phase diagram of the system of GML equations for this case is given in Ref. 17.

In the general case, however, the elucidation of the structure of the phase plane of the system (7) is a considerably more complicated problem than for $f = 0$, and its solution is left to the Appendix. There it is shown, in particular, that for all values of f within the physically interesting interval $(-1, 20)$ the system of GML equations has one rectilinear phase trajectory of the form $g_1 = z_0 g_2$, where z_0 is the real root of Eq. (10). This straight line passes through the first quadrant, and the nontrivial singular point lying on it is a saddle point. The pattern of the phase trajectories for the case under consideration is depicted in Fig. 1. An extremely important feature is that, unlike in the "isotropic" limit ($f = 0$), the separatrix $g_1 = z_0 g_2$ here does not coincide with the "Heisenberg" straight line. The angle of inclination ψ of the separatrix is determined by the magnitude of the anisotropy parameter f; the corresponding curve, together with a graph of the dependence $z_0(f)$, is given in Fig. 2. It can be seen that the angle ψ can differ substantially from 45° even for comparatively small values of f.

Thus, all the phase trajectories of the system of GML equations that originate in the region of stability of the Hamiltonian (1) ($g_1 > 0$, $2g_2 > -g_1$) go outside this
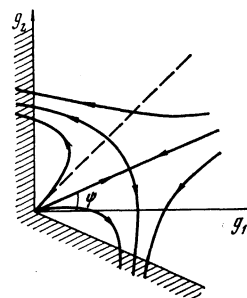

FIG. 1. Pattern of the phase trajectories of the system of GML equations for finite anisotropy of the propagator. The "Heisenberg" straight line $g_1 = g_2$ is shown by the dashed line, and the region of instability of the Hamiltonian (1) is shaded.
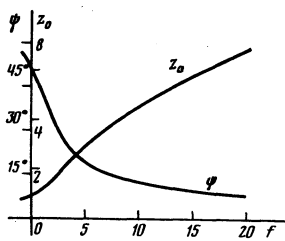
FIG. 2. Graphs of the dependences of the magnitude of the real root of Eq. (10) and the angle of inclination of the separatrix $g_1 = z_0 g_2$ on the anisotropy parameter f.

region as $t \to \infty$. From this, as is well known, it follows that the phase transition in a cubic ferromagnet (ferroelectric) with an anisotropic correlation function should be a first-order transition. The same result was obtained earlier for the case $f = 0$.[17] Thus, there is a definite analogy between the critical behaviors of the system with the Hamiltonian (1) and the model with $f = 0$. But there are also serious differences between them. The point is that, when the crystal anisotropy of the correlator of the fluctuations is taken into account, phase transitions to an ordered phase that is not energetically the most favorable from the standpoint of the Landau theory turn out to be allowed in the system. It is easy to see this by considering Fig. 1. Since the angle of inclination of the separatrix $g_1 = z_0 g_2$ for $f \neq 0$ differs from 45°, the system of GML equations has phase trajectories intersecting the "Heisenberg" straight line (two of them are depicted in the Figure). This means that a situation is possible in which a cubic crystal having anisotropy of the bare coupling constants of the "rhombohedral" type ($\gamma_1^{(b)} > \gamma_2^{(b)}$) will undergo a phase transformation to a tetragonal phase, and vice versa.

The mechanism of this curious phenomenon can be elucidated as follows. In the critical region the fluctuations of the order parameter give rise to strong renormalizations of the anharmonicity constants of the original Hamintonian. The fluctuation corrections to the constants $\gamma_1^{(0)}$ and $\gamma_2^{(b)}$ are expressed in terms of the correlation function $G_{\alpha\beta}(q)$ and depend, accordingly, on the anisotropy parameter f. Through these corrections, the anisotropy of the spectrum of the critical fluctuations enters into the equations controlling the evolution of the effective coupling constants γ_1 and γ_2, and this leads to violation of the symmetry of these equations and their solutions. But since the form of the free energy of the crystal as a function of the $\langle \varphi_\alpha \rangle$, and, in particular, the positions of its minima, are determined by the relative sizes of γ_1 and γ_2, the structure of the ordered phase is also found to depend on the anisotropy of the fluctuation spectrum.

We shall try to elucidate further the appearance of the diagram of states of a cubic ferromagnet in the region in which phase transitions to the anomalous (not describable by the Landau theory) ordered phase are possible. We note that such phase transitions are due exclusively to the interaction of the critical fluctuations. At the same time, it is clear that far from the critical region, where the fluctuations are small, the phase predicted by the Landau theory is thermodynamically

stable. Therefore, on the diagram of states there should be a boundary separating the regions of existence of these two ordered phases. When this boundary is crossed a first-order phase transition will occur in the crystal, and the boundary itself, obviously, should lie in a region of rather strong critical fluctuations. Thus, the region of stability of the anomalous phase does not exceed the critical region in size. Specifically, the extent of the region of the anomalous phase is determined by the parameters f, $\gamma_1^{(b)}$, and $\gamma_2^{(b)}$ of the effective Hamiltonian. As can be seen from Eqs. (7) and Fig. 1, for values of f, $\gamma_1^{(b)}$, and $\gamma_2^{(b)}$ close to those which they take on the line of coexistence of all three phases, the region of the anomalous phase should be very narrow. For fixed $\gamma_1^{(b)}$ and $\gamma_2^{(b)}$ this region will grow with increase of $|f|$, and for constant f will decrease with increase of the anisotropy of the anharmonicity (i.e., the difference $|\gamma_1^{(b)} - \gamma_2^{(b)}|$). As a result, the diagram of states of a cubic ferromagnet in the coordinates $\varkappa^2$, f, $(\gamma_2^{(b)} - \gamma_1^{(b)})$ will have the form depicted in Fig. 3.

The most characteristic feature of this phase diagram is the twisting of the surface separating the two ordered phases in the critical region. We recall that this surface is planar in the framework of the Landau theory. Such a deformation of the phase-separation surface specified in the coordinates $\varkappa^2$, f, $(\gamma_2^{(0)} - \gamma_1^{(0)})$ should also lead, obviously, to the appearance of certain anomalies in the structure of real phase diagrams of crystals, in which experimentally controllable quantities play the role of the independent variables. To see this, we suppose, e.g., that the correlation length $\varkappa^{-1}$ depends primarily on the temperature T, the anisotropy $(\gamma_2^{(b)} - \gamma_1^{(0)})$ of the anharmonicity depends on the pressure P, and the parameter f is insensitive to variation of these quantities and is nonzero. Then, in the $P - T$ diagram of the cubic ferromagnet, the line separating the two ordered phases and the boundary of the region of the paramagnetic phase will form a characteristic "beak" pointing in one direction or the other depending on the sign of f. An idea of the form of the diagram of states of the crystal in this case is given by sections of the three-dimensional phase diagram of Fig. 3 with $f = $ const.

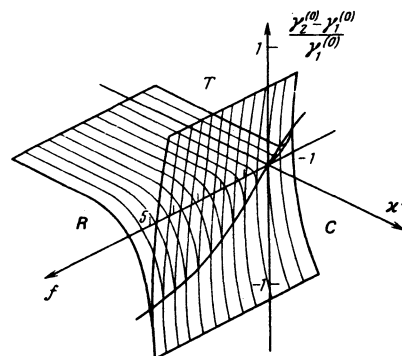The appearance of a "beak" in the diagram of states can have extremely interesting consequences. Indeed,

FIG. 3. Diagram of states of a cubic ferroelectric (ferromagnet). All three surfaces are surfaces of first-order phase transitions. The letters C, T, and R denote the regions of existence of the cubic, tetragonal, and rhombic phases.

if the anisotropy of our crystal is such that sign $f = -\mathrm{sign}(\gamma_2^{(6)} - \gamma_1^{(6)})$, and the difference $\gamma_2^{(6)} - \gamma_1^{(6)}$ is not too large, on lowering of the temperature this crystal will successively undergo two first-order phase transitions, close in temperature. Inasmuch as the Landau theory predicts here one continuous phase transition, there are grounds for speaking of a splitting of the phase transition into two under the influence of critical fluctuations with an anisotropic spectrum. The splitting effect, as can be seen from Fig. 3, is more strongly pronounced the larger the crystal anisotropy of the correlation function, and disappears completely in the "isotropic" limit $f = 0$.

4. CONCLUSION

We shall discuss possible experiments in which it might be possible to observe the effects predicted above. Evidently, the fluctuational change of the anisotropy of the effective anharmonic potential is the most easily susceptible of direct experimental detection. It can be detected without difficulty by measuring the components of the nonlinear magnetic (dielectric) susceptibility tensor as functions of the temperature in the para-phase. The observation of the change of sign of the difference $\gamma_1 - \gamma_2$ in the critical region, which would be evidence of the possibility of a phase transition to an anomalous (not describable by the Landau theory) phase, could be of special interest here. Another way of seeking such a transition is to study experimentally the diagram of states of the crystal in the region of strongly developed critical fluctuations with the aim of observing the "beak." In this case, of course, we must direct our attention primarily to substances which have phase transitions close in temperature and in which the Ginzburg-Levanyuk parameter, which determines the size of the critical region, is not too small.

The question of the experimental observation of the phenomena described is intimately connected with the problem of the effect of strictional interactions on the critical thermodynamics of the crystal. The point is that the Aharony-Fisher Hamiltonian that we have taken as the starting point does not take into account the interaction of the field of the fluctuations of the order parameter with noncritical degrees of freedom, e.g., with acoustic phonons. At the same time, it is well known that such interaction can radically change the critical behavior of the system.[22] It is very difficult at present to say what will be the result of taking this into account in the given case, since even for considerably simpler models (uniaxial models,[23] models with an isotropic propagator,[24] and models without dipolar forces[25]) the analysis of the situation is extremely complicated and does not always give unambiguous results.[25] Therefore, we confine ourselves here to remarking that our treatment is applicable only to crystals with weak striction, i.e., to those systems in which first-order phase transitions occur up to the point at which strictional interactions become important. Obviously, these crystals should be borne in mind when the corresponding experiments are set up.

APPENDIX

In this Appendix we shall elucidate the structure of the phase diagram of the GML equations (7) for nonzero anisotropy of the propagator. As pointed out in the main text, to solve this problem it is necessary to find the real roots of Eq. (10) for nonzero values of the parameters ξ and ζ. In principle, the values of ξ and ζ are uniquely determined by the value of the anisotropy constant f. We, however, shall assume first that ξ and ζ are independent variables, with the aim of investigating the question of the roots of Eq. (10) in general form. We are encouraged to carry out such an investigation by the fact that the GML equations lying at the basis of the theory are themselves approximate. Therefore, besides information on the properties of the system (7) it is also very desirable to have data on the extent to which these properties are stable against any particular perturbation of the initial equations.

Thus, we shall consider the discriminant of the cubic equation (10), which is given by the following expression:

$$D(\xi, \zeta) = -\tfrac{1}{108}[9\xi^4 - 2\xi^2(45\zeta + 19) + \xi^2(9\zeta^2 + 18\zeta + 37) - 4\xi(12\zeta^2 - 17\zeta - 5) + 4(\zeta^3 - 8\zeta^2 + 3\zeta - 7)]. \tag{A.1}$$

We recall that Eq. (10) has three real roots when $D < 0$ and one when $D > 0$. Correspondingly, the equality

$$D(\xi, \zeta) = 0 \tag{A.2}$$

defines in the (ξ, ζ) plane a boundary separating two regions, each of which is characterized by its own number of real roots of Eq. (10). It is possible to establish the shape of this boundary by solving Eq. (A.2) for ξ or ζ. To solve Eq. (A.2) directly, however, is too cumbersome. It is natural, therefore, to attempt to seek a more or less convenient parametrization for the function $\xi(\zeta)$ satisfying this equation. It is indeed possible to find such a parametrization. It has the form

$$\xi = 2\frac{x^3 - x^2 + 1}{3x^2 - 2}, \quad \zeta = \frac{(x-2)(x-1)^2(3x+2)}{3x^2 - 2}. \tag{A.3}$$

The two branches of the curve specified by these formulas are shown in Fig. 4; the region where Eq. (10) has three real roots is shaded.

We can now concentrize the values of the parameters ξ and ζ. They depend on f through the integrals J_i appearing in the GML equations. These integrals were cal-
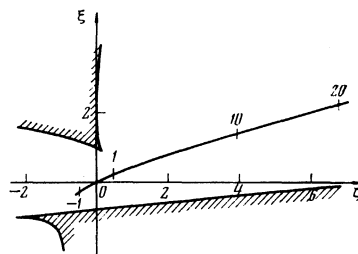

FIG. 4. For the investigation of Eq. (10).

culated on a computer for 27 values of f lying in the interval $(-1, 20)$,[4] and the parameters ξ and ζ were then found from the formulas (9). The results of these calculations are shown in Fig. 4 in the form of a curve which plots the function $\xi(\zeta)$ established from its computer parametrization $\xi(f)$, $\zeta(f)$. The numbers alongside the curve are equal to the values of f at the corresponding points.

As can be seen from Fig. 4, the curve $\xi(\zeta)$ lies entirely in the region in which Eq. (10) has one real root. This means that for any reasonable values of the anisotropy constant f the system of GML equations can possess not more than one nontrivial singular point. Since the signs of all the coefficients in the square brackets in Eqs. (7) coincide, the system does indeed have this singular point. We note that the curve $\xi(\zeta)$ in Fig. 4 passes approximately through the middle of the region in which Eq. (10) possesses one real root. Therefore, the properties of Eqs. (7) discussed here should be stable against not-too-strong variations of the GML functions. This fact gives us grounds to hope that the exact GML equations of our problem also possess analogous properties.

An extremely important point is that the rectilinear phase trajectory on which the nontrivial singular point lies is not, for $f \neq 0$, the "Heisenberg" straight line. Its angle of inclination ψ is determined by the magnitude of the root z_0 of Eq. (10): $\psi = \operatorname{arccot} z_0$. The curves of the dependences of z_0 and ψ on f in the interval $(-1, 20)$, obtained by computer solution of Eq. (10), are given in Fig. 2. As we see, the root z_0 is positive for all values of f of interest to us. Knowing this it is not difficult to deduce that the nontrivial singular point of the system (7) should lie in the first quadrant of the phase plane. But this information is already sufficient to establish the type of the fixed point, with the aid of Eq. (8). In fact, the cubic polynomial in the right-hand side of (8), having a positive coefficient of z^3 and one positive real root, should have a positive derivative at the point z_0. Since $g_2 > 0$ in the first quadrant, $dz/dt > 0$ for $z > z_0$ and $dz/dt < 0$ for $z < z_0$. This means that as we move along the phase trajectories of the system of GML equations we move ever further from the straight line $g_1 = z_0 g_2$. But in moving along this straight line itself we inevitably arrive at the nontrivial singular point, since the point $g_1 = g_2 = 0$ is unstable. It is clear from this that the nontrivial singular point is a saddle point. The entire pattern of the phase trajectories of the system of equations (7) is established without difficulty from these data. This pattern is depicted in Fig. 1.

[1]We also include in this class of systems the two-component model usually used to describe tetragonal crystals.

[2]Thos who have not been convinced by this argument can verify its validity in a purely formal way by considering the Dyson equation for $G_{\alpha\beta}(\mathbf{q})$.

[3]This is also understandable physically: because of the presence of the "nondiagonal" dipolar forces the components of the fluctuating vector field φ turn out to be coupled to each other even when there is no bare anharmonic interaction between them ($\gamma_2^{(0)} = 0$). Therefore, the phase trajectories in such cases can intersect the "Ising" straight line without hindrance.[17,21]

[4]This interval certainly includes all values of the parameter f that are of physical interest.

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