Effect of interband transitions on the current states in systems with electron-hole pairing

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We consider various types of low-lying linear and nonlinear collective excitation in excitonic phases (at temperatures $T = 0$ and $T \to T_c$) and undamped inhomogeneous fluxes of electrons e and holes h in systems with spatially separated and nonequilibrium e and h. We analyze the influence on the excitations by interband (or interfilm) transitions on various types, which fix the phase φ of the order parameter in the ground state and make the existence of homogeneous solutions of the type $\varphi(\mathbf{R}) = \mathbf{P}\cdot\mathbf{R}$ impossible in the excited state. The obtained nonlinear equations that describe the distribution of the phase of the order parameter in the excited states take the form of the sine-Gordon equation or of its simple generalizations. We consider collective excitations that manifest themselves in excitonic phases as polarization-density waves in intrinsic antiferroelectrics or as magnetization-density waves. The restrictions imposed on the symmetry of the wave functions of the pairing particles for which a second order phase transition into the excitonic phase is possible are indicated.

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1. INTRODUCTION

It is known that restructurings that can be described as condensates of electron-hole (e-h) pairs are possible in a number of e-h systems. These restructurings can occur: 1) in semi-metals in which the e and h Fermi surfaces have sections of almost the same form, or in semiconductors with narrow band gaps[1-7]; 2) in a semimetal in a quantizing magnetic field[8] or in a quasi-one-dimensional structure (see, e.g., Refs. 9 and 10); an excitonic phase can be produced also in nonequilibrium systems (Bose condensation of a rarefield exciton gas[11,12] or a transformation of a nonequilibrium e-h system into a coherent dielectric liquid[13]); 4) finally, formation of an e-h pair condensate is possible also for spatially separated e and h, and this circumstance is of interest in connection with the recently analyzed possibility of superfluidity of the charges in such systems (e-h superconductivity) (Refs. 14–16).[1)] As shown in Ref. 14, in the case of the system 4), all the processes with transitions between the bands of the pairing e and h make the existence of strictly homogeneous current states impossible. However, as indicated in Ref. 16 (and later in Ref. 19) and as will be analyzed in the present paper in greater detail and on the basis of realistic models, excited states with spontaneously broken translational symmetry exist and correspond to inhomogeneous superfluid current states, and the characteristics of the inhomogeneity are determined by the matrix elements of interband transitions.

The ground state of the system remains in this case translationally invariant and only the constant phase of the order parameter is fixed in it.[5,20,7]

This paper will deal, in particular, with inhomogeneous superfluid current states in systems with pairing of spatially separated e and h, and account will be taken of the influence exerted on them by all the hitherto not considered types of interband transitions. Allowance for the latter causes the equation obtained for the order parameter in the stationary and one-dimensional pieces to be analogous to the equation for the physical pendulum with a rotating oscillating plane. The fluxes in systems with spatially separated e and h, in constrast to systems of type 1) and 2), carry charges, i.e., they give rise (in the case of films) to a macroscopic experimentally observable undamped electric current (the system does not have anomalous diamagnetism[14]).

We discuss also a system of nonequilibrium paired e and h, in which the condensate flux carries excitation energy. It will be shown below that when a semiconductor is irradiated with an electromagnetic wave, the fluxes are also spontaneously inhomogeneous, and the inhomogeneity length is determined by the matrix elements of the interband transitions induced by the field.

We obtain (for $T = 0$ and $T \to T_c$, where T_c is the phase-transition temperature) an equation for the weakly inhomogeneous phase $\phi(\mathbf{R}, t)$ of the order parameter, with account taken of hybridization and of all the interband transitions due to the Coulomb interactions, and for equilibrium excitonic phases. The functions $\varphi(\mathbf{R}, t)$ determine the low-lying collective excitations of the system. We note that, in contrast to Refs. 5 and 21, where density waves of orbital and spin currents were investigated in the *ground* state of an excitonic dielectric, the "inhomogeneous" fluxes considered by us are connected with an inhomogeneous phase $\varphi(\mathbf{R}, t)$ and are *excited* states of the system. These excited states can exist at any order parameter in the ground state (real or imaginary) and, in particular, should occur also in all the systems considered in Refs. 5 and 21.

If a phase transition is possible in the system, then the free-energy term that fixes the phase and is linear and the order parameter should vanish by virtue of the symmetry of the system (see Sec. 2). In this case the phase fixing can be due only to free-energy-functional terms that are quadratic (or of higher order) in the order parameter (Sec. 2)—in contrast to the case consid-

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ered for singlet pairing in Ref. 22 for layered and homogeneous excitonic phases.

In this paper we consider various types of linear and nonlinear collective excitations in systems with singlet and triplet e-h pairing (including systems of low density, Sec. 3). Besides the high-frequency plasma oscillations of the e-h system, which change little (if $\Delta \ll \omega_p$) on going to the excitonic phase, in a system with e-h pairing there occur low-lying collective excitations which can be described in the general case by an inhomogeneous order parameter $\Delta_{\alpha\beta}(\mathbf{R}, t)$:

$$\Delta_{\alpha\beta}(\mathbf{R}, t) = (\delta_{\alpha\beta}\cos\eta + i(\mathbf{n}\sigma_{\alpha\beta})\sin\eta)\Delta_0 e^{i\varphi},$$

where $\sigma_{\alpha\beta}$ is a vector whose components are Pauli matrices and $\mathbf{n}$ is a unit vector. In correspondence with the four independent real functions $(\eta(\mathbf{R}, t), \varphi(\mathbf{R}, t), \mathbf{n}(\mathbf{R}, t))$ it is possible to have four types (see also Ref. 5) of collective excitations in the system: two modes corresponding to "oscillations" of the magnetization vector $\mathbf{n}$, one mode corresponding to the oscillations of the phase φ of the wave function of the e-h pair condensate, and one mode connected with the oscillations of η. The η oscillation corresponds to the appearance, in the excited state, of an inhomogeneous admixture of triplet e-h pairs if the ground state corresponds to a singlet condensate, or to an impurity of singlet pairs for a triplet condensate in the ground state.

From the point of view of collective excitations and of their contribution to the observable physical quantities, three-dimensional equilibrium excitonic phases have no "superproperties," and do not differ in principle from ordinary dielectrics, antiferromagnets, etc., which are described by other models, (see Sec. 6). All this pertains also to layered or quasi-one-dimensional crystals with alternating microscopic layers of e and h, where the electric currents along the layers play the role of "molecular currents" and are not quantities that can be directly determined in experiment. Such systems should likewise exhibit no superproperties of their behavior in external fields.

Collective excitations can manifest themselves as nonlinear polarization waves (Sec. 6) or nonlinear magnetization waves (Sec. 4), domain walls, etc.

2. EQUATION FOR THE PHASE OF THE ORDER PARAMETER IN HIGH-DENSITY SYSTEMS

We consider first an e-h system of type 1) and of a system with spatially separated e and h, which are listed in the Introduction [the qualitatively predicted effects occur also for the system 2)]. The Hamiltonian of such systems can be represented in the form $H = H_0 + H_1 + H_2$, where H_0 is the Hamiltonian of the e-h system without allowance for the interband transitions; H_1 corresponds to tunneling between the films for a system with spatially separated e and h or to hybridized interation in the Kohn-Luttinger representation for the systems 1)

$$H_1 = \sum_{p,\alpha} M(\mathbf{p}) a^+_{1p\alpha} a_{2p\alpha} + \text{H.c.};$$

H_2 corresponds to interband transitions due to screening by the Coulomb interaction

$$H_2 = \sum_{\substack{p,p',k\\ \alpha,\beta}} \Big[\frac{1}{2} V_1(\mathbf{k}) (a^+_{1p\alpha} a^+_{2p'\beta} a_{1p'+k\beta} a_{2p-k\alpha} + a^+_{1p\alpha} a^+_{1p'\beta} a_{2p'+k\beta} a_{2p-k\alpha}) + V_2(\mathbf{k}) a^+_{1p\alpha} a^+_{1p'\beta} a_{1p'+k\beta} a_{2p-k\alpha} + V_3(\mathbf{k}) a^+_{1p\alpha} a^+_{2p'\beta} a_{2p'+k\beta} a_{2p-k\alpha} + \text{H.c.} \Big].$$

Here $a_{1p\alpha}, a_{2p\alpha}$ are the operators of electron annihilation in the first and in the second bands, or on films 1 and 2, while α and β are the spin indices. The hybridization[23] is given by the formula

$$M(\mathbf{p}) = \frac{1}{m}(\mathbf{p}\mathbf{P}_{12}), \quad \mathbf{P}_{12} = -i\int \varphi_{1k_0}(x)\nabla\varphi_{2k_0}(x)\,d\mathbf{x},$$

(here and below $\hbar = 1$), $\varphi_{k0}(\mathbf{x})$ are Bloch functions, $\mathbf{k}_0$ is the quasimomentum of the band extrema, $V_{1,2,3}(\mathbf{k})$ are the matrix elements of the screened Coulomb interaction and correspond to transitions of electrons from the first to the second band. We have, for example, for small frequency transfers:

$$V_2(\mathbf{k}) = e^2 \int \dot{\varphi}_{1k_0}(x_1)\dot{\varphi}_{1k_0}(x_2)\frac{\exp[-|\mathbf{x}_1-\mathbf{x}_2|/r_D]}{|\mathbf{x}_1-\mathbf{x}_2|} \times \varphi_{1k0}(x_2)\varphi_{2k_0}(x_1)\exp[i\mathbf{k}(\mathbf{x}_1-\mathbf{x}_2)]\,d\mathbf{x}_1\,d\mathbf{x}_2. \tag{1}$$

Since $\varphi_{1k0}(x)$ and $\varphi_{2k0}(x)$ are orthogonal, the matrix elements $V_{1,2,3}(\mathbf{k})$ can be small compared with the pairing potential $V_0(\mathbf{k})$ which enters in H_0 and is connected with the direct Coulomb interaction of e and h; the quantities $V_2(\mathbf{k})$ and $V_3(\mathbf{k})$ do not differ greatly, and we shall henceforth assume for simplicity $V_3(\mathbf{k}) = V_2(\mathbf{k})$. For a system of spatially separated e and h, the matrix elements $V_{0,1,2}(\mathbf{k})$ are Fourier transforms of the "screened" (see, e.g., Ref. 14) e and h Coulomb potential averaged over the wave functions of the transverse particle motion in the films. The quantities $M(\mathbf{p}), V_{1,2}(\mathbf{k})$ can be small compared with the pairing potential $V_0(\mathbf{k})$ because of the weak overlap of the wave functions of the transverse motion of the particles in different films. As shown by Keldysh and Kopaev[1], a high-density e-h system is unstable to pairing of e and h in the case when the Fermi surfaces of e and h are close in shape.

We derive an equation for the phase of the order parameter and an expression for the current in the system.

A. Temperature $T = 0$. We introduce the normal and anomalous Green's functions[24]:

$$G^{1,2}_{\alpha,\beta}(\mathbf{p}, \mathbf{P}, t, t') = -i\langle T(a_{1,2p\alpha}(t)a^+_{1,2p+P\beta}(t'))\rangle,$$

$$F^+_{\alpha\beta}(\mathbf{p}, \mathbf{P}, t, t') = -i\langle T(a_{2p\alpha}(t)a^+_{1p+P\beta}(t'))\rangle.$$

In this section we consider singlet pairing of the e and h, so that $F_{\alpha\beta} = \delta_{\alpha\beta}F^+$. We consider weakly inhomogeneous excited states. We assume therefore that the characteristic momenta $\mathbf{P}$ and the frequencies corresponding to the motion of the pair as a whole are much less than the momenta $\mathbf{p}$ and the frequency ω of the relative motion. Taking the Fourier transforms of the frequencies ω of the relative motion with respect to t-t' and of the coordinate $\mathbf{R}$ of the center of gravity with respect to $\mathbf{P}$, we change over to the functions $G_{p\omega}{}^1(\mathbf{R}, t)$ and $F_{p\omega}{}^*(\mathbf{R}, t)$. For the latter at $P/p \ll 1$ we obtain the Gor'kov equations for a system with Hamiltonian H:

$$\Big(\omega + i\frac{\partial}{\partial t} - \varepsilon(\mathbf{p}+\hat{\mathbf{P}})\Big) G_{p\omega}{}^1(\mathbf{R}, t) = \tilde{\Delta}_p(\mathbf{R}, t)F_{p\omega}{}^+(\mathbf{R}, t) + 1,$$
$$\Big(\omega + i\frac{\partial}{\partial t} + \varepsilon(\mathbf{p}+\hat{\mathbf{P}})\Big) F_{p\omega}{}^+(\mathbf{R}, t) = \tilde{\Delta}_p{}^*(\mathbf{R}, t)G_{p\omega}{}^1(\mathbf{R}, t), \tag{2}$$

where $\tilde{\Delta}^*(\mathbf{R},t)$ is the effective order parameter:

$$\tilde{\Delta}_p^*(\mathbf{R},t)=M(\mathbf{p})+i\int V_0(\mathbf{p}-\mathbf{k})F_{k\omega'}^+\,d\mathbf{k}\,d\omega'$$
$$+i\int V_1(\mathbf{p}-\mathbf{k})F_{k\omega'}\,d\mathbf{k}\,d\omega'-2iV_1(0)\int(F_{k\omega'}+F_{k\omega'}^+)\,d\mathbf{k}\,d\omega'$$
$$-2iV_2(0)\int(G_{k\omega'}^1+G_{k\omega'}^2)\,d\mathbf{k}\,d\omega'$$
$$+i\int V_2(\mathbf{p}-\mathbf{k})(G_{k\omega'}^1+G_{k\omega'}^2)\,d\mathbf{k}\,d\omega'. \qquad (3)$$

For the sake of simplicity we chose the dispersion laws in the form $\epsilon_{1,2}(p)=\pm(p^2/2m+E_g/2)$, where E_g is the overlap of the bare bands for the initial semimetal.

B. We consider first the simplest case $M(\mathbf{p})=M$ = const: This should take place for a system of spatially separated e and h. The function $F_{p\omega}^+(\mathbf{R},t)$ depends[2)] on $|\mathbf{p}|$ and in expression (3) for $\tilde{\Delta}_p^*(\mathbf{R},t)$ we can integrate $V_{0,1,2}(\mathbf{p}-\mathbf{k})$ with respect to the angles $d\Omega_k$. The functions $V_{0,1,2}(\mathbf{p}-\mathbf{k})$ vary over momenta $|\mathbf{p}-\mathbf{k}|\approx 1/r_D$, and we therefore replace the angle-integrated potentials

$$\int V_{0,1,2}(\mathbf{p}-\mathbf{k})\,d\Omega_k$$

by the constants $V_{0,1,2}$ at $|\mathbf{p}-\mathbf{k}|<1/r_D$ and by zero at $|\mathbf{p}-\mathbf{k}|>1/r_D$. Thus, we have for $\tilde{\Delta}_p^*(\mathbf{R},t)$

$$\tilde{\Delta}_p^*(\mathbf{R},t)\equiv\Delta^*(\mathbf{R},t)=\tilde{M}+i\tilde{V}_0\int F_{p\omega'}^+\,d\mathbf{k}\,d\omega'-i\tilde{V}_1\int F_{k\omega'}\,d\mathbf{k}\,d\omega',$$
$$\tilde{M}=M+\lambda,\quad \lambda=-i(2V_2(0)-V_2)\int(G_{k\omega'}^1+G_{k\omega'}^2)\,d\mathbf{k}\,d\omega', \qquad (4)$$
$$\tilde{V}_0=V_0-2V_1(0),\ \tilde{V}_1=2V_1(0)-V_1,$$
$$V_{0,1,2}=\frac{1}{\Omega_0}\int V_{0,1,2}(\mathbf{p}-\mathbf{k})\,d\Omega_k|_{p=k=p_F},$$

p_F is the Fermi momentum, and $\Omega_0=2\pi$ for two-dimensional and $\Omega_0=4\pi$ for three-dimensional systems. Eliminating $G_{p\omega}'(\mathbf{R},t)$ from Eqs. (2), we obtain for $F_{p\omega}^+(\mathbf{R},t)$:

$$\{\omega^2-\xi^2-|\Delta|^2+\hat{B}_{p\omega}(\Delta^*)\}F_{p\omega}^+(\mathbf{R},t)=\Delta^*(\mathbf{R},t). \qquad (5)$$

Here $\hat{B}_{p\omega}(\Delta^*)$ includes all the terms that contain the "small operators" $\partial/\partial t$ and $\hat{\mathbf{P}}=-i\partial/\partial\mathbf{R}$:

$$\hat{B}_{p\omega}(\Delta^*)=-\frac{\partial^2}{\partial t^2}-\hat{\Gamma}^2+2\left(\omega i\frac{\partial}{\partial t}-\xi\hat{\Gamma}\right)$$
$$-\frac{1}{\Delta^*}\left[i\left(\frac{\partial\Delta^*}{\partial t}\right)-(\hat{\Gamma}\Delta^*)\right]\left(i\frac{\partial}{\partial t}+\omega+\xi+\hat{\Gamma}\right),$$
$$\varepsilon(\mathbf{p}+\hat{\mathbf{P}})=\varepsilon(p)+\hat{\Gamma}\equiv\xi+\hat{\Gamma}.$$

We substitute in (4) the formal solution for $F_{p\omega}^+$ from (5). Expanding the operator $(\omega^2-\xi^2-|\Delta|^2+\hat{B}_{p\omega}(\Delta^*))^{-1}$ in powers of $\hat{B}_{p\omega}(\Delta^*)/(\omega^2-\xi^2-|\Delta|^2)$ and assuming the change of the modulus $|\Delta|$ to be small, we obtain as a result an equation for the phase of the order parameter $\varphi(\mathbf{R},t)$, $(\Delta=|\Delta|e^{i\varphi}\equiv\Delta_0 e^{i\varphi})$:

$$\nabla^2\varphi-\frac{1}{u^2}\frac{\partial^2\varphi}{\partial t^2}=\frac{1}{L_1^2}\sin\varphi-\frac{1}{L_2^2}\sin(2\varphi), \qquad (6)$$

$$\frac{1}{L_1^2}=\frac{1}{\xi_0^2}\frac{\tilde{M}}{\Delta_0}\left(\frac{8}{N(0)V_0}\right)\qquad \frac{1}{L_2^2}=\frac{1}{L_1^2}\left[\frac{\tilde{V}_1}{\tilde{V}_0}-\frac{\tilde{M}^2}{2\Delta_0^2N(0)V_0}\right]\frac{\Delta_0}{\tilde{M}}, \qquad (7)$$

$u^2=v_F^2/2$, $N(0)=m/2\pi$; the term $\tilde{M}^2/(2\Delta_0^2N(0)V_0$ in $1/L_2^2$ takes into account in the approximation of lowest order in M the weak change of the modulus of the order parameter. The current of the charges in one film[3)] is

$$\mathbf{j}^1=\frac{e}{m}(\nabla_{x'}-\nabla_x)G^1(xt,x't')$$
$$\underset{x'\to x}{t'\to t+0}$$
$$=-\frac{ie}{m}\int(2\mathbf{p}+\hat{\mathbf{P}})\frac{1}{\Delta^*}\left(i\frac{\partial}{\partial t}+\hat{\Gamma}+\xi+\omega'\right)F_{p\omega'}^+(R,t)\,d\mathbf{p}\,d\omega'. \qquad (8)$$

In first order in the small operators $\partial/\partial t$ and $\hat{\mathbf{P}}$ we get from (8)

$$\mathbf{j}^1=\frac{en_0}{2m}\nabla\varphi(R,t),\qquad n_0=\frac{p_F^2}{2\pi}. \qquad (9)$$

The particle density in one film, in the same approximation, is

$$n'=-2iG^1(xt,xt')=n_0-N(0)\frac{\partial\varphi}{\partial t}. \qquad (10)$$
$$t'\to t+0$$

The current $\mathbf{j}^1$ in one film is determined by the solution of equation (6) with boundary condition

$$\frac{en_0}{2m}\nabla\varphi|_s=\mathbf{j}_s,$$

and in the nonstationary problem and initial conditions, for example,

$$\varphi(R,0)=\varphi_0,\qquad N(0)\frac{\partial\varphi(\mathbf{R},0)}{\partial t}=n_0-n^1(\mathbf{R}),$$

where $\mathbf{j}_s$ is the current on the boundary s, and $n_1(\mathbf{R})$ is the distribution of the charges at the instant $t=0$. We neglect here the reaction of the induced magnetic field on the current. It is easily seen from (9) and (10) that Eq. (6) follows from the continuity equation for the total current in the system:

$$e\frac{\partial n^1}{\partial t}+\operatorname{div}\mathbf{j}^1=I_{12},$$

where I_{12} is the tunnel current between the e and h films.

C. For three-dimensional excitonic phases $\int M(\mathbf{p})d\Omega_p=0$, so that the terms of first order in $M(\mathbf{p})$ drop out of the equation for the phase. In the lowest order in $|M(\mathbf{p})/\Delta|\ll 1$ we obtain, an analogy with the derivation given above, an equation of the form (6) for the phase of the order parameter, but with different coefficients:

$$\frac{1}{L_1^2}=\frac{1}{\xi_0^2}\frac{\tilde{V}_2}{\tilde{V}_0}\left(\frac{12}{N(0)V_0}\right),$$
$$\frac{1}{L_2^2}=\frac{1}{\xi_0^2}\left[\frac{\tilde{V}_1}{\tilde{V}_0}+\frac{M^2N(0)V_0}{2\Delta_0^2}-\frac{\tilde{V}_2^2}{2\Delta_0^2N(0)V_0}\right]\left(\frac{12}{N(0)V_0}\right), \qquad (11)$$
$$u^2=\frac{v_F^2}{3},\qquad M^2=\frac{1}{4\pi}\int M^2(\mathbf{p})\,d\Omega_p|_{p=p_F},\qquad N(0)=\frac{mp_F}{2\pi^2},$$

where $\tilde{V}_{0,1,2}$ is defined in (4). The last term in $1/L_2^2$ in the approximation of lowest order in $(\tilde{V}_2/\tilde{V}_0)$ takes into account the weak change of the modulus $|\Delta|$. Concerning the effective boundary conditions for the excitonic phases, see Ref. 25.

If the Bloch functions of the electrons and holes, which enter in the definition of $V_2(\mathbf{k})$ [see (1)] transform in accordance with different representations of the symmetry group of the crystal, then, as follows from (4), $\tilde{V}_2=0$ and consequently $1/L_1^2=0$. In this case $V_1\neq 0$, since V_1 contains the Bloch-function combination $(\varphi_{1k}^*\varphi_{2k}^*\varphi_{1k}\varphi_{2k})$, and consequently $1/L_2^2\neq 0$. Only in the case $V_2=0$ does a phase transition exist in the system (see subsection E). The coefficient $1/L_1^2$ may vanish also for a system with spatially separated e and h, for example if the different films have the same lattice symmetry, but the Bloch functions of the pairing quasiparticles transform in accordance with different representations of the symmetry group.

We write out the conditions for the applicability of

Eq. (6) and expressions (9) and (10) for the densities of particles and of the current. We used in their derivation the slowness of the variation of Δ over the coherence length ξ_0, as well as the condition $|\Delta| \cong$ const. This imposes restrictions on the values of M, V_1, and V_2 and on the boundary conditions. Estimating the terms discarded in the derivation of (6), we arrive at the inequalities

$$|\tilde{V}_{1,2}/\tilde{V}_0| \ll 1, \quad |M/\Delta| \ll 1, \quad |\nabla\varphi|_s| \ll \xi_0^{-1}.$$

These conditions can be easily satisfied for a system with spatially separated e and h, for example, by increasing the distance between the films. In semimetals and semiconductors, the conditions for the smallness of the interband matrix elements, sufficient for the existence of collective excitations connected only with the phase $\varphi(\mathbf{R}, t)$, can be satisfied in rather special cases.

D. For three-dimensional excitonic phases at a temperature T close to critical, $(T_{c0} - T)/T_{c0} \ll 1$, we can derive a time-independent equation for the phase of the order parameter (T_{c0} is the phase-transition temperature in the absence of interband transitions). We start with the Gor'kov equations for the thermodynamic Green's functions[24] $G^1(\mathbf{P}) = -\langle T(a_{1\beta} a^+_{i\rho+P})\rangle$ and $F^+(\mathbf{P}) = \langle T(a_{2\beta} a^+_{+P})\rangle$. These equations coincide with equations (2) if we make the substitutions $\omega + i\partial/\partial t \to i\omega$, $\tilde{\Delta}_p^* \to -\tilde{\Delta}_p^*$, $i\tilde{V}_{0,1} d\omega' \to V_{0,1} T\sum$. The order parameter $\Delta^* = \tilde{\Delta}_p^* - M(\mathbf{p}) \neq 0$ in the entire temperature region if $\lambda \neq 0$ (see (4)). As $T \to T_{c0}$, however, we have $|\Delta| \sim T_{c0}^{1/3}\lambda^{1/3}$ and consequently we can expand the Gor'kov equations in terms of the parameter $|\Delta_0/T_{c0}|$ near T_{c0}, if $|\lambda/T_{c0}| \ll 1$:

$$F_{\rho\omega}^+(R) = G_{-\omega}^0(\Delta + M(\mathbf{p}))^* G_\omega^0 - G_{-\omega}^0(\Delta + M(\mathbf{p}))^* \times G_\omega^0(\Delta + M(\mathbf{p})) G_{-\omega}^0(\Delta + M(\mathbf{p}))^* G_\omega^0, \; G_\omega^0 = (i\omega - \varepsilon(\mathbf{p} + \hat{\mathbf{P}}))^{-1}.$$

Assuming slowness of the variation of $\Delta(\mathbf{R})$ over the length ξ_0, we obtain an equation for $\psi^*(\mathbf{R}) = [7\zeta(3)N/8(\pi T_{c0})^2]^{1/2}\Delta(\mathbf{R})$, where N is the particle density:

$$\frac{1}{4m}(i\nabla)^2\psi^* + \tilde{\alpha}\psi^* + \beta|\psi|^2\psi^* + \gamma\psi^* - \tilde{\lambda} = 0,$$

$$\tilde{\alpha} = \alpha + 2\left(\frac{M^2}{T_{c0}^2}\right)\frac{3T_{c0}^2}{4\varepsilon_F}, \quad \gamma = \frac{\tilde{V}_1}{\tilde{V}_0}\eta^{-1}\ln\left(\frac{\omega_0}{T_{c0}}\right) + \frac{M^2}{T_{c0}^2}\frac{3T_{c0}^2}{4\varepsilon_F}, \tag{12}$$

$$\omega_0 = \frac{v_F}{r_D}, \quad \tilde{\lambda} = \frac{\lambda}{N(0)\tilde{V}_0}[9(\pi T_{c0})^2 N/14\zeta(3)\varepsilon_F^2]^{1/2}.$$

We do not present here the remaining coefficients of (12), since they coincide in the case of three-dimensional excitonic phases with the corresponding coefficients of superconductivity theory.[24] Equation (12) corresponds to a free-energy density:

$$F\{\psi\} = F_0 + \frac{|i\nabla\psi|^2}{4m} + \tilde{\alpha}|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{\gamma}{2}(\psi^2 + \psi^{*2}) - \tilde{\lambda}(\psi + \psi^*). \tag{13}$$

The last two terms lead to fixation of the phase of the order parameter. From (13) assuming $|\psi| \equiv n = \text{const}$, which is valid only at $|\nabla\varphi| \ll n/v_F$, $|\tilde{V}_1/\tilde{V}_0| \ll 1$ and $|M^2/T_{c0}^2| \ll 1$, we obtain an equation for the phase of the order parameter:

$$\nabla^2\varphi = \left(\frac{4m\tilde{\lambda}}{n}\right)\sin\varphi - 4m\gamma\sin(2\varphi). \tag{14}$$

For a zero-gap excitonic impurity dielectric at $T \to T_{c0}$ we can obtain for the phase $\varphi(\mathbf{R}, t)$, with the aid of trivial transformations in the Gor'kov-Éliashberg theory, a time-dependent equation that describes the excited states and corresponds to oscillations of the condensate relative to the particles in excess of the condensate.

E. Influence of interband transitions on the phase transition in a three-dimensional system. It is seen from (13) that the quantity $\tilde{\lambda}$, which is determined by interband transitions of the type $V_2(\mathbf{k})$, plays the role of an external field conjugate to the order parameter ψ (and ψ^*). Similarly, the quantity γ connected with the interband transitions of type $V_1(\mathbf{k})$ and with the second order in the hybridization, plays the role of an external field conjugate to the square of the order parameter ψ^2 (and ψ^{*2}).

We consider now the homogeneous case. By virtue of $\tilde{\lambda} \neq 0$ we know (see Ref. 26) that the phase transition of second order in the temperature vanishes and the order parameter is different from zero in the entire temperature region. The jump of the specific heat is smeared out in this case in the region $|T_{c0} - T| \approx \tilde{\lambda}^{2/3}\beta/\alpha_0$, where $\alpha = \alpha_0(T - T_{c0})$.

At $V_2 = 0$ (i.e., $\tilde{\lambda} = 0$) and $\tilde{V}_1 \neq 0$ (i.e., $\gamma \neq 0$), it can be easily verified by using (13) that the minimum of $F\{\psi\}$ is reached at $\psi = |\psi| e^{i\varphi_0}$, where $\varphi_0 = \pi/2$ at $\gamma > 0$, i.e., the phase of the order parameter is fixed[5,7,20] and the order parameter is in this case pure imaginary. The gain in free energy due to pairing is $\delta F = F - F_0 \sim |\psi|^4$, so that a second order phase transition should take place in the system, just as when no account is taken of the interband transitions.[4)] Only a renormalization of the transition temperature T_c takes place:

$$T_c = T_{c0}\left(1 + \frac{\tilde{V}_1}{\tilde{V}_0}\ln\left(\frac{\omega_0}{T_{c0}}\right) - \frac{M^2}{T_{c0}^2}\frac{7\zeta(3)}{8\pi^2}\right), \quad \gamma > 0.$$

Besides the phase transition with respect to temperature, phase transitions with respect to other parameters (pressure, etc.) can occur in the system. For example, at $T = 0$ a first-order phase transition can occur when the pressure is varied.[7,20]

3. EXCITED STATES IN *e-h* SYSTEMS OF LOW DENSITY

By way of examples of equilibrium systems with low e-h pair density we point out the following: 1) e-h films with low concentrations of e and h (Ref. 14); 2) the excitonic phase produced upon restructuring of a semiconductor with a bare gap width E_g only negligibly smaller than the exciton binding energy Ry*: $(\text{Ry}^* - E_g)/\text{Ry}^* \ll 1$.[2] In such systems, the possibility of transition into the excitonic phase is not connected with the rather stringent requirements that the shapes of the Fermi surfaces (or of sections of these surfaces) of the electrons and holes coincide, a mandatory requirement in the case of dense e-h systems.

We obtain now an equation for the homogeneous order parameter. We introduce the function $\Delta(\mathbf{x}t, \mathbf{x}'t') = iV_0(\mathbf{x} - \mathbf{x}')F(\mathbf{x}t, \mathbf{x}'t')$. For simplicity we assume $V_1 = V_2 = 0 (M \neq 0)$. Their role is analogous to the high-density case considered above. The equation for $\Delta(\mathbf{x}t, \mathbf{x}'t')$ and

$T=0$ in the self-consistent approximation is of the form (to abbreviate the notation we leave out the integration variables):

$$\Delta(xt,x't')=iV_0(x-x')\left\{\int[G_1^0(\Delta+M)G_2^0+G_1^0\Delta G_2^0\Delta^*F]\right\}, \quad (15)$$

where $G^0_{1,2}$ are the exact Green's functions in the absence of pairing and $V_0(x-x')$ is the unscreened Coulomb potential. We discard in (15), assuming that $|M/\Delta|\ll 1$, the terms of second order in M. Equation (15) does not take into account the e-h pair correlation interactions which are essential for low-density systems. But all these terms which are not accounted for in (15) are proportional to Δ^3 and to higher powers of the small (for systems with low e-h pair density) quantity Δ. Therefore the correlation effects will be taken into account by replacing the coefficient of Δ^3, as in Ref. 12, by a phenomenological constant. We do not take into account here the possibility of formation of a condensed e-h phase (see Ref. 13).

We change over, just as in Sec. 2, to a mixed representation (R,t,p,ω) and consider slow variations with respect to the coordinates R and time t of the motion of the center of gravity of the e-h pair. In this case we can expand the terms in the right-hand side of (15) in the gradients of the function Δ. Introducing

$$\Delta_p(R,t)=\int\Delta_{p\omega'}(R,t)\,d\omega'$$

and

$$\psi_p(R,t)=\left[2\varepsilon(p)+\frac{\hat P^2}{4m}-i\frac{\partial}{\partial t}-\frac{M}{\Delta}2\varepsilon(p)+\frac{|\Delta|^2}{2\varepsilon(p)}\right]^{-1}\Delta_p(R,t),$$

we obtain from (15) for $\psi_p(R,t)$:

$$\left[2\varepsilon(p)+\frac{\hat P^2}{4m}-i\frac{\partial}{\partial t}+\frac{|\Delta_p(R,t)|^2}{2\varepsilon(p)}\right]\psi_p(R,t) -M(p)=\int V_0(p-p')\psi_{p'}dp', \quad (16)$$

the dispersion laws of e and h are assumed for simplicity to be quadratic. Since it was assumed that all the quantities change slowly relative to R and t, we can carry out an adiabatic separation of the variables in (16). The equation for the fast subsystem (variable p) is taken in the form

$$\left[2\varepsilon(p)+\frac{|\Delta_p(R,t)|^2}{2\varepsilon(p)}-E(R,t)\right]\varphi(p)=\int V_0(p-p')\varphi_{p'}dp'. \quad (17)$$

Then, introducing $\Phi_n(R,t)$: $\psi_p=\sum_n\varphi_n(p)\Phi_n(R,t)$, we obtain for the slow subsystem

$$\left[\frac{\hat P^2}{4m}-i\frac{\partial}{\partial t}+E_n(R,t)\right]\Phi_n(R,t)-M_n=0, \qquad M_n=\int M(p)\varphi_n(p)\,dp. \quad (18)$$

In the derivation of (18) we have discarded all the nonadiabatic corrections, a procedure permissible if $|M/\Delta|\ll 1$. From (17) we get $E_n(R,t)$ and $\varphi_n(p)$, regarding $|\Delta(R,t)|^2/2\varepsilon(p)$ as a small perturbation:

$$E_n(R,t)=E_{ex}(n)+E_g+\int\varphi_n(p)\frac{|\Delta_p(R,t)|^2}{2\varepsilon(p)}\varphi_n(p)\,dp, \quad (19)$$

where $E_{ex}(n)$ are the levels and $\varphi_n(p)$ are the wave functions of the hydrogenlike e-h pair. Let E_g be such that $\alpha_1<0$ but $\alpha_n>0$ at $n\geq 2$, where $\alpha_n=E_{ex}(n)+E_g$ (i.e., the width of the bare forbidden band is less than the binding energy of the pair, but still exceeds its binding energy in the first excited state). We then obtain from (18) and (19) an equation that describes the change of the density of the Bose condensate of e-h pairs in a low-density system:

$$i\frac{\partial\Phi}{\partial t}=\frac{\hat P^2}{4m}\Phi+\alpha_1\Phi+\beta_1\Phi^2-M_1. \quad (20)$$

As already mentioned, β_1 includes the correlation interactions. Linearizing (20), we obtain the spectrum of linear collective excitations (see also Ref. 20):

$$\omega^2(k)=\left(\frac{k^2}{4m}+\frac{M}{\Phi_0}\right)\left(\alpha_1+3\beta_1\Phi_0^2+\frac{k^2}{4m}\right); \qquad \alpha_1\Phi_0+\beta_1\Phi_0^2-M_1=0. \quad (21)$$

For spatially separated e and h, the particle flux on one film

$$j^1=\frac{e}{m}(\nabla_{x'}-\nabla_x)\,G^1_{\substack{t'\to t+0\\ x'\to x}}(xt,x't')=C\frac{ie}{2m}[\Phi^*\nabla\Phi-(\nabla\Phi^*)\Phi], \quad (22)$$

and the particle density on the film is

$$n^1=-2i\,G^1_{t'\to t+0}(xt,xt')=C|\Phi|^2, \quad (23)$$

where the coefficient C takes into account the correlation effects ($C=1$ in the average-field approximation). The quantities j^1 and n^1 are connected with the continuity equation that follows from (20). Equation (20) differs from the phenomenological Ginzburg-Pitaevskiĭ equation in the presence of a source of e-h pairs connected with the tunneling (or hybridization). Allowance for the other types of the interband transitions leads, just as in a dense system, to the appearance of terms of the type $\gamma(\Phi^*+\Phi)$ in (20).

In the stationary case (20) goes over into an equation of the type (12), from which we get, assuming $|\Phi|$ to be constant, an equation of type (14) for the phase $\varphi(R,t)$ of the wave function $\Phi(R,t)$.

4. INHOMOGENEOUS SOLUTIONS IN TRIPLET PAIRING OF e AND h

The anomalous Green's function ($T\to T_c$) in the case of triplet pairing takes the form $F_{\alpha\beta}{}^t=(n\cdot\sigma_{\alpha\beta})F^t$. In triplet pairing of e and h from bands separated by a vector w equal to half the reciprocal-lattice vector, antiferromagnetic ordering of the spins appears, and the vector n characterizes the magnetization direction.[2]

We consider collective excitations corresponding to oscillations of the vector n and to an inhomogeneous distribution of the phase $\varphi(R,t)$ of the condensate with allowance for all the interband transitions[5]. For oscillations of n in a plane perpendicular to the z axis, we introduce $e^{i\varphi_n}=n_x+in_y$. Using a procedure similar to that described in Sec. 2, we obtain as $T\to T_c$ equations for the phase φ (see also Ref. 27) of the parameter connected with $F_{p\omega}(R)$, and for the phase φ_n of the vector n:

$$\nabla^2\varphi=4\gamma' m\sin(2\varphi),\quad \nabla^2\varphi_n=0,\quad \gamma'=\eta^{-1}\frac{V_1}{V_0}\ln\left(\frac{\omega_0}{T_c}\right). \quad (24)$$

We note that Eq. (24), for φ, in the case of triplet pairing, in contrast to Eq. (14) for singlet pairing, does not contain terms connected with M, $V_1(0)$ and V_2. The point is that in the case of triplet pairing the sing-

le-particle interband (tunnel) transitions between the pairing e and h (Figs. 1a and 1b) are impossible, since the particle spin cannot be flipped by the Coulomb interactions with the spin-orbit interaction neglected. There remains, however, a possibility of two-particle tunneling between e and h belonging to the e-h pairs with different spin projections. It is these processes to which the terms $\sim V_1(\mathbf{k})$ of the Hamiltonian correspond and determine the right-hand side of the (24).

Equation (24) admits of the following: 1) "linear" excitations with $\varphi_n = \mathbf{P}\cdot\mathbf{o}_{xy}$, where ρ_{xy} is a vector in the xy plane (similar modes correspond to oscillations of $\mathbf{n}$ in the planes xz and yz) and with fixed phase $\varphi = \pi n/2 (n = 0, 1, \ldots)$; 2) "nonlinear" excitations with $\Phi \neq \text{const}$. The magnetic-moment density corresponding to the inhomogeneous phase $\varphi(\mathbf{R}, t)$ is given by

$$\mathbf{M}(\mathbf{R}, t) = 2n\rho_x \Delta_0 \sin\varphi(\mathbf{R}, t), \quad \rho_x = \sum_{\substack{p, \\ |\xi_p| < \omega_D}} e^{i\mathbf{p}\mathbf{R}} u_{1p}(R)\, u_{2p+\mathbf{Q}}(R)\, \frac{1}{(\xi^2+\Delta^2)^{1/2}},$$

where $e^{i\mathbf{p}\mathbf{x}} u_p(x) = \varphi_p(x)$ are Bloch functions. The foregoing oscillation modes corresponding to excited states of an antiferromagnet corresponds to large scale inhomogeneity of $\mathbf{M}(\mathbf{R}, t)$.

At $T = 0$, using the method of Sec. 2, we can obtain a time-dependent equation of the sine-Gordon type for $\varphi(\mathbf{R}, t)$ and a wave equation for φ_n. From the equation for φ we can obtain the spectrum of the collective excitations with gap at small $\mathbf{k}$, and from the equation for φ_n we can obtain the zero-gap collective excitations (spin waves). The spin-orbit interaction can lead to fixing of the phase φ_n and to the appearance of a gap in the spin-wave spectrum.

Excited states connected with the phase of η. Fixing of the phase of the parameter η (see the Introduction) is the result of the term $\sim\lambda$ (see Sec. 2A). Consequently at $\lambda \neq 0$ in the ground state of an excitonic dielectric there coexist triplet and singlet e-h pairs. We direct the z axis along the vector $\mathbf{n}$, and then $\Delta_{\alpha\beta} = (\delta_{\alpha\beta}\cos\eta + i\sigma_{\alpha\beta}^{z}\sin\eta)\Delta_0 \exp(i\varphi_0)$. By the method described in Sec. 2, we obtain an equation for η in the excited state (at $T = 0$):

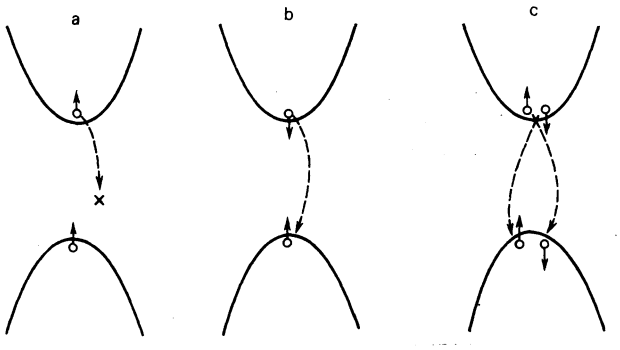

FIG. 1. a) Triplet pairing; "single-particle" interband transition is impossible; b) singlet pairings; single-particle interband transition is possible; c) triplet pairing; "two-particle" interband transition is possible and is connected with the terms $\sim V_1(k)$ in the Hamiltonian $\hat{H}$, where $V_1(k)$ corresponds to transition between e and h belonging to e-h pairs with different spin projections.

$$\nabla^2\eta - \frac{1}{u^2}\frac{\partial^2\eta}{\partial t^2} = \frac{1}{L_1^2}\sin(\varphi_0+\eta) - \frac{1}{L_2^2}\sin 2(\varphi_0+\eta), \quad \varphi_0 = \pi n.$$

The coefficients L_1 and L_2 are defined by (11).

5. QUALITATIVE INVESTIGATION OF THE SOLUTIONS OF THE EQUATIONS FOR THE PHASE

The time-dependent equation for the phase (6) can be set in correspondence with the Hamiltonian

$$H = \int \left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial t'}\right)^2 + \frac{1}{2}(\nabla\varphi)^2 - \cos\varphi + \frac{\lambda}{2}\cos(2\varphi)\right] dx'.$$

We have introduced the dimensionless variables $x' = x/L_1$, $t' = tu/L_1$ and put $(L_1^2/L_2^2) = \lambda$.

In the homogeneous ground state ($\partial\varphi/\partial t = \nabla\varphi = 0$) the minimum of the functional H is reached at $\varphi = \varphi_0 = 0$ if $\lambda < 1/2$, and $\varphi = \varphi_{01,2} = \pm \arccos(1/2\lambda)$, if $\lambda > 1/2$. It is these points which correspond to the fixed order-parameter phase in the ground state. In the stationary one-dimensional case the time-independent equations for the phase of the order parameter, (6) and (14), coincide with the equations of motion of a physical pendulum with rotating oscillating plane and a potential energy $U(\varphi) = \cos\varphi - 1/2\lambda\cos(2\varphi)$, and the variable x' plays in this case the role of the time. The points $\varphi_0, \varphi_{01}, \varphi_{02}$ are the points of the extrema of the potential energy of the pendulum. Qualitatively, the character of the solutions can be easily understood by considering the plot of the potential energy $U(\varphi)$ (Fig. 2). Three different types of solutions of the stationary equation for φ are possible: 1) Small oscillations of the pendulum about the equilibrium positions φ_0 and φ_0^*; It can be shown that solutions of this type are unstable. 2) Rotations of the pendulum about the suspension axis. 3) Solutions of the soliton type, corresponding to an infinitesimally slow rotation of the pendulum from position 1 to position 2 or from position 2 to position 3 (at $\lambda > 1/2$). Multisoliton solutions also exist. Solutions of type 2) can be realized only if the gradient of the phase on the boundary is $|\nabla\varphi|_s| > A$, where $A = L_1^{-1}(2 + 1/2\lambda + 2\lambda)^{1/2}$ at $\lambda > 1/2$ and $A = 2/L_1$ at $\lambda < 1/2$. Each type of solution corresponds to a definite distribution of φ and $\Delta\varphi$ in space. For a system of spatially separated e and h, the solutions of the first type correspond to closed circular flows in regions of dimensional L_1 (and L_2). Solutions of the second type correspond to a weakly modulated (over lengths L_1 and L_2) superfluid flux of particles in the system. Solutions of the third type correspond to

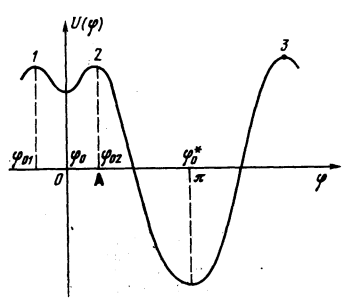

FIG. 2. Dependence of the energy $U(\varphi) = \cos\varphi - \frac{1}{2}\lambda\cos 2\varphi$ on the phase; $A = \arccos\frac{1}{2}\lambda$.

solitary currents in limited regions with dimensions L_1 or L_2. Outside these regions there is no flow. The average flow differs from zero, just as in case 2).

The spectrum of the linear collective excitations of the system does not have an acoustic character, as was the case in the absence of terms that fix the phase of the order parameter (see also Refs. 28 and 20). Indeed, linearizing the nonstationary equation (8) near $\varphi = 0$, we get $\omega(k) = u(k^2 + L_1^{-2} + L_2^{-2})^{1/2}$.

There exist also "moving" solutions $\varphi(x' - vt')$ of (6), obtained from the solutions $\varphi(x')$ of the stationary equation by the substitution $x' \to (x' - vt')/(1 - v^2)^{1/2}$. With increasing matrix elements M of the interband transitions, the energy of the inhomogeneous excited states in the system increases, and at $|M/\Delta| \gtrsim 1$ their energy becomes larger than the energy of the normal phase. Thus, at $|M/\Delta| \gtrsim 1$ the collective excitations of the type considered above cannot be realized.

6. ABSENCE OF SUPERPROPERTIES OF THREE-DIMENSIONAL EXCITONIC PHASES. POLARIZATION WAVES

The inhomogeneous phases φ_n, φ, η correspond to collective excitations. These collective excitations can contribute to the specific heat and to other quantities that characterize the state of the system. Their contribution to the thermodynamic characteristics of the system is negligibly small, since the excitations that correspond to the inhomogeneous phase involve a very large number of particles in a region with dimensions $L \gtrsim \xi_0$. Collective excitations of this type can, in addition, contribute also to the transport properties of the system. Thus, for example, energy transport can be realized with the aid of an excitation of the soliton type, (i.e., moving domain walls). This raises the question whether the excitonic phases have some superproperties, for example superthermal conductivity. It is obvious that a condensate of e-h pairs which is at rest as a whole cannot carry any heat (its entropy is zero). On the other hand, the contribution of the single-particle excitations to the thermal-conductivity coefficient is finite (in contrast to ^{4}He), since the latter always have a finite mean free path connected with the scattering by the lattice defect, impurities, phonons, etc. (see also Ref. 29). Only moving (time-dependent) collective excitations of the considered type, for example soliton solutions of the type $\varphi(x - vt)$ (moving domains), can contribute to the thermal-conductivity coefficient. However, these inhomogeneous collective excitations are stopped and scattered by the lattice defects etc., and consequently likewise do not lead to superthermal conductivity of the system. It is important here that the state corresponding to the soliton or the inhomogeneous "flux" solution, moving with velocity v, is not separated by an energy barrier from the state of the same type moving with a changed velocity. Therefore the change of the velocity v of the entire structure $\varphi(x - vt)$, for example the stopping of a soliton, is a process of no little probability (but a change of the type of structure, for example the production of a soliton (domain), does consume much energy and has therefore low probability). Thus, the dissipation of moving collective excitations of the condensate is substantial.

On the other hand, in a system with spatially separated e and h, an electric current $\mathbf{j} \sim \nabla\varphi$ exists also in the case when the corresponding distribution of the phase $\varphi(x)$ does not move—the soliton was pinned. Moreover, in analogy with type-II superconductors, the ideal conductivity can be realized only if this pinning did take place, for otherwise dissipation will be connected with the motion of the inhomogeneous solution $\varphi(x - vt)$.

Polarization waves. We consider a case when a dipole moment due to electrons and holes can appear in a crystal having the symmetry of a pyroelectric. The dipole moment of the unit cell is equal to

$$\mathbf{d}_i = \int_{v_{cl}} \mathbf{d}(\mathbf{x}t)\,d\mathbf{x},\quad \mathbf{d}(\mathbf{x}, t) = 2\mathbf{d}[G^1(\mathbf{x}t, \mathbf{x}t) + G^2(\mathbf{x}t, \mathbf{x}t) + 2\,\mathrm{Re}\,F(\mathbf{x}t, \mathbf{x}t)],\quad \mathbf{d} = e\mathbf{x},$$

$\mathbf{d}(\mathbf{x}, t)$ is the local density of the dipole moment. The first two terms in the density $\mathbf{d}(\mathbf{x}, t)$ can lead to pyroelectricity even in the absence of e-h pairing. We shall analyze here the contribution of the third term, which is due to e-h pairing. In this case

$$\mathbf{d}_i(\mathbf{R}, t) = -2\mathbf{d}_{0i}\Delta_0 \cos\varphi(\mathbf{R}, t),$$

$$\mathbf{d}_{0i} = \sum_{\substack{p,\\ |\xi_p| < \omega_0}} \int_{v_{cl}} u_{1p}(x)\,\dot{u}_{2p}(x)\,\frac{\mathbf{d}}{(\xi^2 + \Delta^2)^{1/2}}\,\frac{dx}{v_c}. \tag{25}$$

In the ground state of the system, the phase is fixed $\varphi = \varphi_0$ and $\mathbf{d}_i = \mathrm{const} \neq 0$, if $\varphi_0 \neq \pi/2$. If the extrema of the bands coincide, then in the ground state the total dipole moment of the crystal is $\mathbf{D} = \sum_i \mathbf{d}_i \neq 0$ (see Refs. 30 and 21). At $V_2 = 0$ (see Sec. 2) a second-order phase transition is possible in the system, accompanied by a dipole moment proportional to the order parameter, i.e., the system is a ferroelectric. At $V_2 \neq 0$ there is no phase transition, and the system is a pyroelectric. If the extrema of the bands are separated by a vector $|\mathbf{w}| = G/2$, where $\mathbf{G}$ is the reciprocal-lattice vector, then the dipole moment of the unit cell is given by formula (25), where $\mathbf{d}_{0i}$ must be replaced by

$$\mathbf{d}_{wi} = \sum_{\substack{p,\\ |\xi_p| < \omega_0}} \int_{v_{cl}} u_{1p}(x)\,\dot{u}_{2p+w}(x)\,e^{i\mathbf{w}x}\,\frac{\mathbf{d}}{(\xi^2 + \Delta^2)^{1/2}}\,\frac{dx}{v_c}.$$

In the case of a displacement equal to the lattice period, $\mathbf{d}_{wi}$ reverses sign ($\mathbf{d}_{wi} = -\mathbf{d}_{wi+1}$). In this case the total dipole moment is $\mathbf{D} = 0$, and we have an antiferroelectric in the ground state.

We consider now the excitation of the antiferroelectric state. It corresponds to an inhomogeneous distribution of the phase of the order parameter, $\mathbf{d}(\mathbf{R}, t) \sim \cos\varphi(\mathbf{R}, t)$, where $\varphi(\mathbf{R}, t)$ satisfies Eq. (6). Thus, a polarization-density wave (with a period L_1 or L_2) appears in the excited state of the antiferroelectric. It is possible to introduce the tensor Γ_{ik} of the flux of the vector $\mathbf{d}(\mathbf{R}, t)$, which we determine from the continuity equation for $\mathbf{d}(\mathbf{R}, t)$. For Γ_{ik} we obtain (in the zeroth order in the gradients of the function $\varphi(\mathbf{R}, t)$):

$$\Gamma_{ik} = d_{ik}\Delta_0 \sin\varphi,\quad d_{ik} = \frac{1}{m} \sum_{\substack{p,\\ |\xi_p| < \omega_0}} \int_{v_c} [(\nabla_k \varphi_{1p})\dot{\varphi}_{2p+w} - \varphi_{1p}\nabla_k \dot{\varphi}_{2p+w}] \times \frac{d_i}{(\xi^2 + \Delta^2)^{1/2}}\,\frac{dx}{v_c}.$$

In the ground state $\Gamma_{ik}=0$ for a real order parameter. We note that the flux Γ_{ik} is not directly connected with $\nabla\varphi$.

7. FIXING OF THE PHASE AND INHOMOGENEOUS CURRENT STATES IN A NONEQUILIBRIUM SYSTEM

We consider an intrinsic semiconductor $E_g \gg E_{ex}$ in the field of a strong electromagnetic wave of frequency $\Omega(\Omega \gtrsim E_g)$, which realizes the inversion of the quasi-Fermi population in the system. After a unitary transformation corresponding to a transition to specified Fermi quasilevels of the nonequilibrium e and h (see Ref. 31), the Hamiltonian of the system takes the form

$$\hat{H}=\sum_p \varepsilon_1(p)a_{1p}^+a_{1p}+\varepsilon_2(p)a_{2p}^+a_{2p}+(\lambda(\mathbf{p})a_{1p}^+a_{2p}+\text{H.C.})$$
$$+\sum_{p,p',k} V_0(k)a_{1p}^+a_{2p'}^+a_{2p'+k}a_{1p-k}+\hat{H}',$$
$$\varepsilon_1(p)=-\varepsilon_2(p)=\frac{p^2}{2m}-\frac{p_0^2}{2m},\ \frac{p_0^2}{2m}=\frac{\Omega-E_g}{2}.$$

$\hat{H}'$ takes into account the interband transitions in the transformed Hamiltonian (for example, hybridization corresponds to the term $[M(\mathbf{p})e^{i\Omega t}a_{1p}+a_{2p}+\text{H.c.}]$), and $\lambda(\mathbf{p})$ is the matrix element of the interband transitions in the field of a monochromatic wave).[31]

The interband transitions that enter in $\hat{H}'$ can be treated by perturbation theory and yield rapidly oscillating ($\sim e^{i\Omega t}$) corrections to the density and to the particle flux (with amplitude $\sim |M/\Omega| \ll 1$), which vanish for an averaged over time intervals $t \gg 1/\Omega(\Omega \approx E_g)$. We shall therefore leave them out hereafter. Then the Hamiltonian $\hat{H}$ does not depend on the time and reduces to the form considered by us in Sec. 2 above (the Hamiltonian H and $V_1=V_2=0$ and $\lambda(\mathbf{p})=M(\mathbf{p})$). The description of the pairing of the nonequilibrium e and h is in this case similar to the equilibrium case.[7] The term $\lambda(\mathbf{p})$ connected with the electromagnetic field plays the same role as the hybridization interaction in the analysis of the flux states. Calculations similar to those in Sec. 2 yield for the phase φ of the order parameter:

$$\nabla^2\varphi-\frac{1}{u^2}\frac{\partial^2\varphi}{\partial t^2}=\left(\frac{6\lambda^2}{\Delta_0^2}\right)\frac{1}{\xi_0^2}\sin(2\varphi),\ \lambda^2=\frac{1}{4\pi}\int\lambda^2(\mathbf{p})\,d\Omega_p|_{p=p_0}.$$

We note that if

$$\lambda=\int\lambda(\mathbf{p})\,d\Omega_p|_{p=p_0}\neq 0,$$

then the equation for φ will contain the term $\sim(\lambda/\Delta_0)\sin\varphi$. The expressions for the flux and density of the particles coincide with (9) and (10), in which $N(0)$ is in this case the state density on the Fermi quasilevels. In contrast to the equilibrium case, the inhomogeneous exciton flux is accompanied by energy transport: $\mathbf{j}_\varepsilon=E_g n_0\nabla\varphi/4m$. The inhomogeneity length L of the flux and the concentration of the excitons are determined by the intensity of the external field: $L\approx 1/\lambda$; for $\lambda=(10^{-2}-10^{-4})$ eV, $L\approx(1-10^4)\xi_0$ at $\Delta_0=10^{-2}$ eV.

In conclusion, we are deeply grateful to B. A. Volkov, V. L. Ginzburg, Yu. M. Kagan, D. A. Kirzhnits, Yu. V. Kopaev, L. A. Maksimov and V. I. Petviashvili for interest in the work and for useful discussions of a number of questions touched upon here. We are very grateful to V. F. Elesin, I. V. Lerner, and V. I. Yudson for critical remarks.

Note added in proof (25 December 1978). In a paper by E. B. Sonin (Sov. Phys. JETP **47**, 1091, 1978), the equation for the angle φ_n that characterizes the rotation of the magnetization vector (which follows from the phenomenological Landau-Lifshitz equation) is erroneously set in correspondence with a derived microscopic equation for another quantity—the phase φ of the condensate (cf. Sec. 4 of the present paper). The "superfluid" spin fluxes discussed by Sonin are in a more adequate language simply magnetization domains. The solutions of the Landau-Lifshitz equations corresponding to magnetic solitons (domains) were first obtained in the very interesting papers of A. E. Borovik (see A. E. Borovik, JETP Lett. **28**, 1978 and the literature cited therein).

[1)] Pairing is possible here at[15] $1/\lambda_1+1/\lambda_2 \geq \xi_0^{-1}$, where λ_1 and λ_2 are the mean free paths of e and h, and $\xi_0^{-1}=|\Delta|/\hbar v_F$ is the coherence length. This is realizable in the following: 1) In two semiconducting films in which the charged separation is due to the difference between the work functions. 2) In two semimetallic films A and B in which the Fermi surfaces of the electrons of A and of the holes of B are close in form. 3) In inversion layers of e and h type on the internal surfaces of films adjacent to a dielectric liner. On the other hand, in a system of films of n- and p-type semiconductors,[17] in which the number of carriers is smaller than the number of impurities, the opposite inequality holds, so that e-h pairing is impossible. Superfluidity of spatially separated e and h was not considered in Ref. 17. The most realistic for the observation of e-h superconductivity are systems with small and intermediate density, considered in Refs. 14, 18, and 13.

[2)] For simplicity we assume that the pairing potential is $V_0(k)=V_0(k)$.

[3)] For simplicity we disregard the effects of periodicity for e-h films. In addition, in expressions (8) and (10) we have replaced, accurate to terms of second order in the derivatives of the function $\varphi(\vec{R},t)$, the total Green's function on one film $G_{\text{tot}}=-i\langle T((\psi_1(x_1)+\psi_2(x_1))(\psi_1^*(x_1')+\psi_2^*(x_1'))\rangle$ ($\vec{x}_1$ and $\vec{x}_1'$ lie in the first film) by a function G^1 which does not take into account the weak overlap of the wave function of the particles on different films. This pertains also to formulas (21) and (22) below.

[4)] Allowance for the possibility of formation of an e-h liquid[13] leads to a transformation of the phase transition into a first-order transition. According to Gor'kov and Mnatsakanov,[6] anisotropy can also lead to a similar effect.

[5)] The phase fixing connected with transitions between the subsystems of the e-h pairs with different spin projections, and its connection with the possibility of homogeneous superfluid fluxes in these subsystems, were analyzed in Ref. 14 for spatially separated e and h. Spin fluxes in antiferromagnets were considered in Ref. 27.

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Quantum kinetics of phase transitions at low temperatures

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A theory is developed of quantum decay of a metastable state in a class of problems in which quantum fluctuations governing subbarrier evolution of virtual nuclei of the new phase are related to the local motion of single particles. A model of a crystal with two positions (states) of an atom in a unit cell is considered. The amplitude of the tunnel creation of a critical nucleus is found. It is shown that the associated characteristic sum over various "paths" can be found employing the conventional statistical methods. The discrete nature of the energy structure of the levels makes it necessary to allow for the interaction with phonons, which is done within the framework of the kinetic equation for the density matrix. The method can be applied also to analyze the decay of a metastable state via formation of finite clusters, when the macroscopic description is impossible.

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1. INTRODUCTION

Lifshitz and Kagan[1] (see also Iordanskiĭ and Finkel'shteĭn[2]) developed quantum kinetics of phase transitions at temperatures close to $T = 0$. Decay of a metastable state is found to be associated with subbarrier tunneling of a virtual nuclei of the new phase in the configuration space. This virtual growth of nuclei causes an initially homogeneous system to pass through a sequence of locally inhomogeneous states. Lifshitz and

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