Investigation of the subharmonic structure on the current–voltage characteristic of a superconducting point contact

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We investigated the subharmonic structure of the current–voltage ($I-V$) characteristics of superconducting Nb–Nb point contacts in an external microwave field of frequency 60–70 GHz. Distinct current "substeps" satisfying the generalized Josephson relation $2eV_0=(n/m)\hbar\omega_k$ (e is the electron charge, V_0 is the bias voltage, $\hbar$ is Planck's constant, ω_k is the frequency of the external microwave radiation, and n and m are integers) were observed at a temperature 4.2 K on the $I-V$ characteristics of point contacts with large critical currents I_c. The dependence of the amplitude of the substeps on the external radiationpower is investigated in detail. A theoretical model is proposed for the mechanism whereby the subharmonic structure is produced, and it is shown that allowance for the action of the contact radiation proper and of the harmonics of the external microwave field describes well the complicated character of the dependence of its amplitude on the microwave power. The model explains the "gap subharmonics" $2\Delta/m$ and the "gap harmonics" $n\Delta$ observed on the $I-V$ characteristics of tunnel junctions and point contacts in a zero external field.

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1. INTRODUCTION

It is known that in some cases the current-voltage ($I-V$) characteristic of a narrow thin-film superconducting bridge exposed to external microwave radiation of frequency ω_k acquire an additional subharmonic structure that satisfies the generalized Josephson relation[1,2]

$$2eV_0=\frac{n}{m}\hbar\omega_k. \quad (1)$$

Here n and m are integers, V_0 is the dc bias voltage, e is the electron charge, and $\hbar$ is Planck's constant. This substructure is usually attributed to the relatively large capacitance of the thin-film bridges[2] and to the nonsinusoidal relation between the current and the phase.[3] The absence of a subharmonic structure on the $I-V$ characteristics of superconducting point contacts (PC), which have a very small capacitance ($10^{-1}-10^{-2}$ pF) seems to attest to the validity of the first assumption. However, first of all, it is shown in a number of theoretical papers, e.g., Ref. 4, that a nonsinusoidal current-phase relation is realized only at the temperatures far from the critical temperature T_c of the superconductor, whereas the substructure on the characteristics of the superconducting bridges is usually observed at temperatures close to T_c. Second, a substructure satisfying the relation (1) (with $m=2$ and $n=1,2,\ldots$) was observed[2] also in the case of thin Nb–Nb contacts with small capacitance. This, neither the capacitance nor the deviation from the sinusoidal current-phase relation can apparently explain fully the appearance of the additional structure of the $I-V$ characteristics of superconducting point contacts. In our opinion, this additional subharmonic structure is due to the interaction of the Josephson radiation proper with the alternating current that flows in the bridge or in the point contact. It appears that this substructure is of the same nature as the well known gap subharmonics observed on the $I-V$ characteristics of point contacts and film tunnel junctions.

We present here the results of an experimental investigation of the subharmonic structure of the $I-V$ characteristics of Nb-Nb point contacts in an external microwave field, and propose a theoretical model to explain the observed phenomenon.

2. EXPERIMENTAL RESULTS

We investigated Nb–Nb point contacts placed in a 1.8×3.6 mm waveguide or in a constriction of a 3-cm waveguide measuring 23×2 mm. Liquid helium was made to flow over the parts of the waveguide with the point contact, and all the measurements were made at a temperature 4.2° K. The points and anvils of the point contacts were made of niobium wire of 1 mm diameter with a superconducting-transition temperature 9.2 K. The contact point was sharpened in the usual manner to 1–2 μm.

In the experiments, the point contact was connected to the current source, adjusted to the necessary value of the critical current I_c, and exposed to external radiation of frequency 60–70 GHz. In the course of the experiment we plotted the $I-V$ characteristics of the contact and the dependence of its differential resistance $R_d=dV/dI$ on the bias voltage V_0 both without any external radiation and at various external-radiation power levels. The modulation signal amplitude used when recording the $dV/dI(V_0)$ plots was 0.1–0.2 μV.

Figure 1 shows a typical $I-V$ characteristic of an Nb-Nb point contact exposed to radiation of frequency 63 GHz. One can clearly see an additional structure between the fourth and eighth current steps, corresponding to the principal harmonics of the external-radiation frequency; this additional structure satisfies the relation (1) with $m=2$

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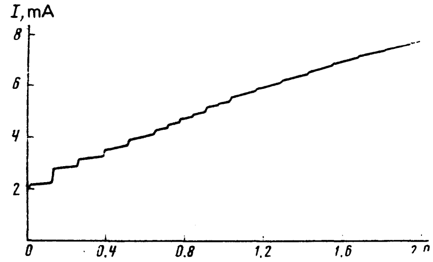

FIG. 1. I–V characteristic (V is in mV) of an Nb–Nb point contact in a microwave field of frequency 63 GHz; $T = 4.2$ K.

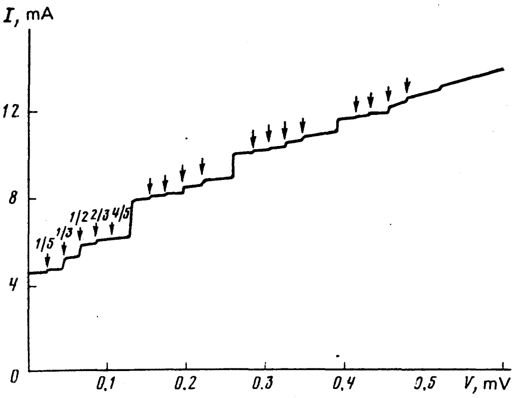

FIG. 3. I–V characteristic of Nb–Nb point contact in an external microwave field of frequency 63 GHz at $T = 4.2$ K. The arrows mark the subharmonic structures with numbers $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{4}{5}$.

$$2eV_0 = \frac{n}{2}\hbar\omega_k. \tag{2}$$

The indicated section of the characteristic reveals not only multiplication of the frequency ω_k of the incident radiation, but also its "division" by two. This manifests itself most clearly in Fig. 2, which shows the section of the I–V characteristic from the eighth to the twentieth current steps, and its derivative $-dV/dI(V_0)$, for an Nb–Nb point contact in an external field of frequency ~63 GHz. Here, just as in the preceding case, there is a distinct substructure in addition to the usual steps corresponding to the harmonics $n\omega_k$.[1)] The entire structure is well described by relation (2).

Attention is called to the oscillatory character of the dependence of the amplitude of the subharmonic structure on the number n. With increasing amplitude of the fundamental current steps, the size of the substeps decreases, and vice versa. The apparent reason is that part of the power of the harmonics $n\omega_k$ is consumed in the formation of the subharmonics and $n\omega_k/2$.

Within the framework of the resistive model of point contacts (see, e.g., Ref. 1), the amplitude of the principal current steps in its I-V characteristic on the radiation power is similar to a Bessel function and is furthermore proportional to the critical current I_c of the contact. Subharmonics of higher order with $m > 2$ were therefore expected to appear on the characteristics of the contacts with the higher critical current. Figs. 3 and 4 show the I–V characteristics and the plot of $-dV/dI(V_0)$ for two different Nb–Nb point contacts, having relatively large values of I_c. As proposed, a different subharmonic structure is resolved on the characteristics of these contacts at $m > 2$.

When the external radiation power was changed, the amplitudes of the current steps corresponding to the principal harmonics $n\omega_k$ varied almost like the Bessel functions, in accord with the resistive model, whereas the dependence of the substeps on the microwave power was more complicated. The points in Fig. 5 show this dependence for the first three substeps with numbers $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$.

3. THEORETICAL MODEL

To explain the possible mechanism of formation of a subharmonic structure on the I–V characteristics of the point contacts, we make use in the present paper of the effect of the action of the Josephson radiation proper on the contact itself. We start from the resistive model of a superconducting point contact. In this case the current through the contact (disregarding the capaci-

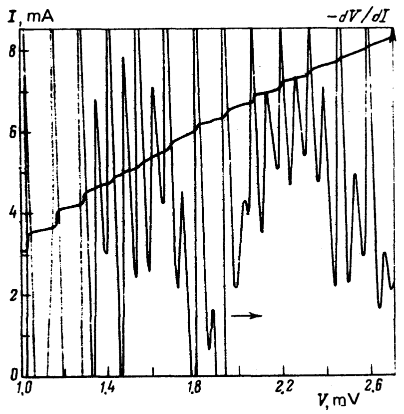

FIG. 2. Section of I–V characteristic from the 8-th to the 20-th current step, and its derivative $-dV/dI\,(V_0)$, for an Nb–Nb contact in a microwave field of frequency ~ 63 GHz; $T = 4.2$ K.

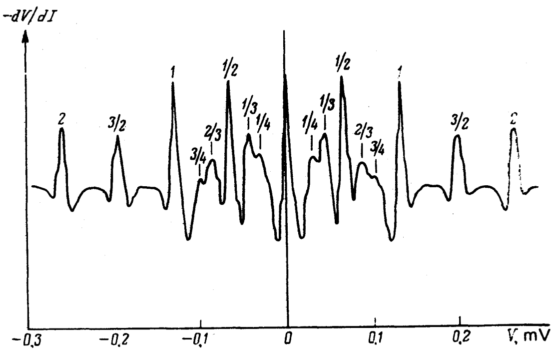

FIG. 4. Plot of $-dV/dI$ against V_0 for an Nb–Nb point contact in a microwave field of frequency ~ 63 GHz at $T = 4.2°$ K. The numbers mark the harmonic and subharmonic structures in the initial region of the I–V characteristic.

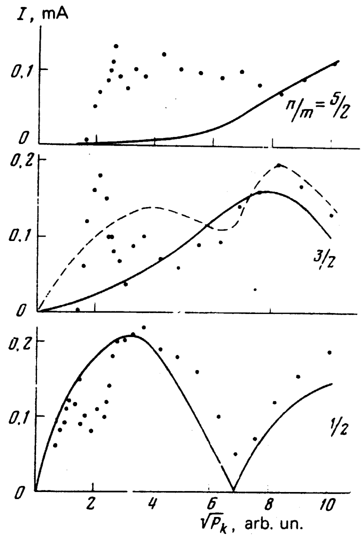

FIG. 5. Dependence of the substeps with numbers $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ on the power of the external radiation of frequency 63 GHz. The points are the experimental results. The solid curves correspond to the moduli of the Bessel functions of first, third, and fifth order. The dashed curve reflects the qualitative theoretical dependence with account taken of the self-induced alternating voltage on the contact.

tance) takes the form

$$I=I_c \sin\varphi+\frac{V}{R_N},\quad \varphi=\frac{2e}{\hbar}\int^t V(t)\,dt. \tag{3}$$

According to the assumed model, the voltage V on the contact consists of a dc bias voltage V_0, an alternating voltage $A\cos\omega_k t$ induced by the external microwave radiation, and an alternating voltage $B\cos\omega_0 t$ due to the proper radiation of the contact at the frequency $\omega_0=2eV_0/\hbar$, i.e.,

$$V=V_0+A\cos\omega_k t+B\cos\omega_0 t. \tag{4}$$

Substituting (4) in (3), we obtain an expression for the superconducting current in the contact

$$I_s=I_c\sin\left(\varphi_0+\omega_0 t+\frac{2eB}{\hbar\omega_0}\sin\omega_0 t+\frac{2eA}{\hbar\omega_k}\sin\omega_k t\right)$$

$$=I_c\sin(\varphi_c+\omega_0 t+\gamma_0\sin\omega_0 t+\gamma_k\sin\omega_k t), \tag{5}$$

where

$$\gamma_0=\frac{2eB}{\hbar\omega_0},\quad \gamma_k=\frac{2eA}{\hbar\omega_k}. \tag{6}$$

The amplitude A of the induced voltage is determined by the external microwave radiation power P_k and is proportional to $P_k^{1/2}$. The amplitude B, meaning also γ_0, depends on the critical current I_c of the contact, on the geometry of the point contact, and on the electrodynamic system in which the contact is connected.

The quantity B can be represented as a product $B=I_c\rho$, where ρ does not depend on I_c and has the dimension of resistance. If the point contact is located in the wave guide as shown in Fig. 6, then the value of ρ can be determined in the following manner. Starting from Maxwell's equations, we obtain the field $\mathbf{E}(x, y, z, t)$ excited in the waveguide by the current density

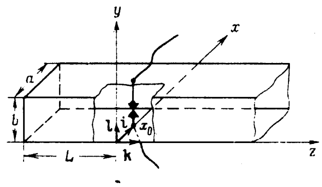

FIG. 6. Placement of the point contact in the waveguide.

$$\mathbf{j}=I_c\sin\omega_0 t\,\delta(z)\,\delta(x-x_0)\,\mathbf{l},$$

which describes the real situation when a current $I_c\sin\omega_0 t$ flows in the contact. This field acts on the point contact and induces across it a voltage

$$V=\int_0^b E_y dy=B\sin\omega_0 t=I_c\rho\sin\omega_0 t,$$

which can be written in the form

$$V=I_c\frac{8\pi}{c}\left(-\frac{\omega_0}{ch_1}\right)\sin^2\frac{\pi}{a}x_0\sin h_1 L\left(\frac{l}{b}\right)\left(\frac{b}{a}\right)\sin(\omega_0 t+\alpha). \tag{7}$$

Here c is the speed of light, $h_1=(\omega_0^2/c^2-\pi^2/a^2)^{1/2}$, L is the distance from the end of the waveguide to the point contact, and l is the region of radiation of the contact.

For the case when $a\approx 23$ mm, $b\approx 2$ mm, $L\approx 3$ cm and at the customarily assumed limiting frequency of a niobium contact $\omega_c=2eI_cR_N/\hbar\approx 10^{12}$ Hz, the value of γ_0 should be close to unity, i.e.,

$$\gamma_0=2eB/\hbar\omega_0\sim 1,$$

which is, as will be shown below, the necessary condition for observing a large number of subharmonics.

To investigate this structure of the $I-V$ characteristic of a point contact under the foregoing assumptions, we expand the expression (5) for the current in a series of Bessel functions:

$$I_s=I_c\sum_{m,n=-\infty}^{\infty}J_m(\gamma_0)J_{-n}(\gamma_k)\sin[\varphi_0+(m+1)\omega_0 t-n\omega_k t], \tag{8}$$

where $J_m(\gamma_0)$ is a Bessel function of order m. From this expression we see that at $\omega_0=n\omega_k/(m+1)$ beats with zero frequency are produced in the contact and the current acquires a dc component. As a result, current steps appear on the $I-V$ characteristic of the point contact, at voltages satisfying the equation

$$\frac{2eV_0}{\hbar}=\omega_0=\frac{n}{m+1}\omega_k.$$

The case with $m=0$ corresponds to the ordinary harmonics of the external radiation, and the case with $m\neq 0$ corresponds to subharmonics.

It follows also from (8) that the sizes of the substeps on the I-V characteristic are proportional to the products of the Bessel functions

$$J_m(\gamma_0)J_{-n}(\gamma_k). \tag{9}$$

If γ_0 is small ($\gamma_0\ll 1$), then the amplitudes of the substeps (they are proportional to γ_0^m) are also small and should decrease strongly with increasing m. To observe subharmonics of high order it is therefore necessary to have γ_0 close to unity; this is realized at large values of I_c and ρ. This situation obtains in tunnel junctions with bridges of type-II superconductors (see, e.g., Refs. 6 and 7) or in the case of Nb–Nb point contacts operating in a microbridge regime with large I_c, as was the case in Ref. 5 and as is the case in the present study.

4. DISCUSSION OF RESULTS

1. The proposed model explains the appearance of a subharmonic structure on the $I-V$ characteristics of point contacts, but it is of interest to compare the experimental results with the theory.

It follows from (8) that the magnitude of the $n/(m+1)-st$ substep is proportional to the product of two Bessel functions $J_m(\gamma_0)J_{-n}(\gamma_k)$ and when the power of the external microwave radiation changes it should vary like $J_{-n}(\gamma_k)$, since $\gamma_k \sim P_k^{1/2}$ only. The solid lines in Fig. 5 shows the moduli of the Bessel functions for $n=1,3$, and 5. It is seen that in the case of the substep numbered $\frac{1}{2}$ the agreement between theory and experiment is quite good, but this cannot be said of the substeps numbered $\frac{3}{2}$ and $\frac{5}{2}$.

To explain this discrepancy we must recognize that the external radiation produces in the point contact alternating current not only of the frequency of the external radiation ω_k, but also of frequencies $2\omega_k$, $3\omega_k$, etc. [see Eq. (8)]. The radiation due to these currents also acts on the point contacts, and induces on it an alternating voltage. We shall discuss only the $2\omega_k$ harmonic. In this case we must write in place of (5) and (8)

$$I_s=I_c\sin(\varphi_0+\omega_0t+\gamma_0\sin\omega_0t+\gamma_k\sin\omega_kt+\gamma_{k2}\sin2\omega_kt)$$

$$=I_c\sum_{m,n,l=-\infty}^{\infty}J_m(\gamma_0)J_{-n}(\gamma_k)J_{-l}(\gamma_{k2})\sin[\varphi_0+(m+1)\omega_0t-(n+2l)\omega_kt], \quad (8')$$

where $\gamma_{k2}\sin2\omega_kt$ is the voltage induced on the contact by the intrinsic radiation of frequency $2\omega_k$. The condition for the appearance of steps on the $I-V$ characteristic of the contact now takes the form

$$\omega_0=\frac{n+2l}{m+1}\omega_k.$$

The amplitude, for example, of the substeps with $n+2l=N$ will be proportional to the sum

$$\sum_{\substack{n,l\\ n+2l=N}}J_m(\gamma_0)J_{-n}(\gamma_k)J_{-l}(\gamma_{k2}). \quad (9')$$

Thus, (9′) has in comparison with (9) additional terms, allowance for which permits a better description of the experimental dependence of the amplitude of the substeps on the power of the external microwave radiation. The sizes of the substeps with number $\frac{3}{2}$ now become proportional not to $J_3(\gamma_k)$, as assumed above, but to a sum of products of Bessel functions

$$J_1(\gamma_0)J_{-3}(\gamma_k)J_0(\gamma_{k2})+J_0(\gamma_0)J_{-1}(\gamma_k)J_{-1}(\gamma_{k2})+\ldots. \quad (10)$$

Since γ_0 does not depend on the microwave power, and the parameter γ_{k2} is small (because the effective interaction of the contact with its own radiation due to the induced current is "secondary"), expression (10) can be rewritten in the form

$$CJ_3(\gamma_k)+DJ_1(\gamma_k)+\ldots, \quad (10')$$

where C and D are coefficients that depend little on the microwave power. Thus, in first-order approximation, the amplitude of the substep numbered $\frac{3}{2}$ should change, with changing external radiation, like a sum of the two Bessel functions $J_3(\gamma_k)$ and $J_1(\gamma_k)$. This situation is illustrated in Fig. 5 by the dashed line. It is seen that allowance for the influence of $J_1(\gamma_k)$ alters substantially the character of the dependence of the amplitude of the substep on the microwave power. Allowance for the next terms in (10) will obviously improve gradually the agreement between theory and experiment.

2. The model considered above explains the "gap subharmonics" $2\Delta/m$ very frequently observed on the $I-V$ characteristics of superconducting tunnel junctions and of point contacts in the absence of external microwave radiation, as well as the previously observed[6,7] "gap harmonics" $n\Delta$ (2Δ is the energy gap of the superconductor that forms the point contact or the tunnel junction). In studies of tunnel junctions[6,7] based on lead, tin, and Nb_3Sn, both ordinary gap subharmonics at $V_0<2\Delta/e$ were observed[6] up to $m=18$, as well as gap harmonics with $n=1-10$ at $V_0>2\Delta/e$.[6,7] A Δ/m subharmonic structure was also observed[7] at voltages larger than $2\Delta/e$.

The subharmonic and harmonic structures on the $I-V$ characteristics of film tunnel junctions in a zero external field are observed mainly if the junctions have superconducting bridges or "microshorts" through the insulator layer.[2] In this case at $I>I_c$ and at nonzero voltage V_0 on the tunnel barrier, a constant single-particle current flows through the bridges as a result of breaking of the superconducting electron pairs. So long as the films of the tunnel junction remain superconducting, processes of breaking and formation of superconducting pairs take place near the bridge boundaries, as a result of which an energy 2Δ is released and an electromagnetic radiation of frequency $\omega_\Delta=2\Delta/\hbar$ is produced. This radiation is concentrated between the superconducting films that make up the tunnel junction, and interacts with the Josephson alternating current in the bridge. Thus, to describe this situation it is necessary to replace in Eq. (8) for the current the external-radiation frequency ω_k by the frequency ω_Δ. It follows then from (8) that a self-induced structure can be observed on the $I-V$ characteristics of tunnel junctions with bridges or of point contacts at a zero external field, at voltages satisfying the expression

$$\frac{2eV_0}{\hbar}=\frac{n}{m+1}\frac{2\Delta}{\hbar}. \quad (11)$$

The relation (11) describes simultaneously the harmonics and subharmonics of the gap.

It is clear from (11) that the period of the harmonics of the gap ($m=1$) should equal Δ, and not 2Δ—a fact that would be difficult to explain by the previously assumed theoretical models. Equation (8) with $\omega_k=\omega_\Delta$ explains readily also the substructure observed in Ref. 7 at $V_0=n\Delta/4e$ ($m\neq1$).

It should be noted in conclusion that in experiments with tunnel junctions made up of different superconductors, at $V_0<(\Delta_1+\Delta_2)/e$, one observes most frequently "even" gap subharmonics $(\Delta_1+\Delta_2)/2m$ ($2\Delta_1$ and $2\Delta_2$ are the energy gaps of the superconductors making up the tunnel junction). In fact, according to (11), the gap subharmonics should in this case appear at a voltage

$$V_0=\frac{\Delta_1+\Delta_2}{2e}\frac{n}{m+1}.$$

The even gap subharmonics correspond to the case $n=1$, and the odd ones to $n=2$. However, as noted above, owing to the "secondary" character, observation of subharmonics with $n=1$ is more probable than that of $n=2$.

[1)] The very fact that substructures of higher orders are observed at voltages exceeding the limits V_c for Josephson structures of niobium ($V_c=I_cR_N\approx\pi\Delta(0)/2e\approx 2$ mV and R_N is the resistance of the normal state) is evidence of the extremely high nonlinearity of the point contacts used in the present study.

[2)] This is usually indicated by the form of the I–V characteristics of such tunnel junctions.

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