Quantum kinetic equation for electrons in optical breakdown of gases

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The photon energy $\hbar\omega$ in optical breakdown of a gas can become comparable with the average electron energy, with the classical energy accumulated in the field, as well as with the energy of the transitions in the atomic or molecular system with which the electron interacts. In this case, a quantum approach must be used [see, e.g., Zel'dovich and Raĭzer, Sov. Phys. JETP **29**, 772 (1965), where a quantum kinetic equation was first derived from physical consideration for a weak field that does not resonate with the atomic system]. The oscillations of the distribution function in the quantum case can greatly influence the rate constants of various processes and the breakdown thresholds (their magnitude and their dependence on the frequency ω), especially if the cross sections of the inelastic collisions of the electrons have a resonant character. A quantum kinetic equation is derived for electrons in a gas in the field of a strong wave in cases when resonance is present or absent between the radiation and the transitions in atomic systems. Expressions are presented for the absorption coefficient and for the real part of the permittivity at the frequency of the incident radiation. The problem of energy accumulation by an electron scattered by an atom (molecule) that is at resonance with the field, and the lowering of the threshold intensities of the optical breakdown of the gas in this case are estimated.

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A classical analysis of electron heating in the field of an electromagnetic wave (of intensity E and frequency ω) by collisions with atomic particles can be carried out if the quantity $\hbar\omega$ is small compared with the average energy $\langle\varepsilon\rangle$ and with the classical energy gain[1,2] $m_e v_T v_E$ (v_T is the characteristic average velocity, $v_E = eE/m_e\omega$ is the vibrational velocity of the electrons in the field of the wave). In the case of gas breakdown by optical radiation, the conditions $\hbar\omega \sim \langle\varepsilon\rangle$ and $N = m_e v_T v_e/\hbar\omega \sim 1$ are realized, and a quantum treatment becomes essential. Zel'dovich and Raĭzer[1] were the first to present, on the basis of physical considerations, a quantum kinetic equation for the zeroth harmonic (in the expansion of the Fourier series in the frequency ω) of the spherically symmetrical part of the electron distribution function. A numerical solution of this equation[3-5] for a number of gases has shown that the distribution function acquires characteristic oscillations with energy spacing $\hbar\omega$. This behavior of the distribution function can exert a strong influence on the rate constants of the excitation and ionization of atoms and molecules (and consequently on the breakdown thresholds I_{br}), leading to a nonmonotonic dependence of I_{br} on the frequency, especially if the cross sections for the inelastic collisions have a resonant character.

In the breakdown of gases by IR radiation it is necessary to take into account multiphoton processes of energy acquisition by the electrons, inasmuch as at threshold intensities the parameter N which characterizes the average number of absorbed photons,[6,7] is larger than unity. Thus, for example, in the breakdown of air at atmospheric pressure by CO_2 laser radiation ($I_{br} \approx 5 \times 10^9$ W/cm^2, Ref. 8) we have $N \approx 4$. A quantum kinetic equation was presented in Ref. 9 in analogy with Ref. 1 for elastic scattering by ions in laser heating of a plasma at $N > 1$. In the problem of optical breakdown at $N > 1$, the probability of the elastic loss of energy by the electrons to the excitation of atomic particles is substantially modified. For a number of problems it is necessary to have more detailed information on the electron distribution function, for example on its first harmonic, from which we can calculate the nonlinear current of a weakly ionized plasma (and accordingly the absorption coefficient and the real part of the permittivity ε_d) at the frequency ω. There is no published quantum kinetic equation for an arbitrary harmonic of the electron distribution function in the field of a strong electromagnetic wave ($N > 1$) in the literature on optical breakdown.[1)]

The most interesting peculiarity of optical breakdown of gases occurs in the case of resonance between the radiation and the atomic (molecular) system. It was noted in Refs. 13–15 that the threshold intensities of the breakdown can in this case be noticeably decreased because of the absorption of energy when the electrons are scattered by the field-induced resonant dipole moment of the atom. The derivation of a quantum kinetic equation for resonant breakdown is complicated by the fact that a two-level system evolves in the fields, and the off-diagonal elements of the atomic density matrix are important. A derivation of the corresponding equation shows that the probabilities of the transitions for the electrons with a change of energy $l\hbar\omega \pm s\hbar\Omega$ (l is the number of absorbed photons, $s = 0 \pm 1$, and Ω is the Rabi frequency of the two-level system) assume different forms for a short radiation pulse ($\tau \ll T_2$, T_2 is the relaxation time of the phase of the resonant system) and for long one ($\tau \gg T_2$).

In Sec. I of the present paper we derive a quantum kinetic equation for electrons in nonresonant breakdown. Expressions are presented for the absorption coefficient and for the real part of ε_d of a weakly ionized plasma in the field of a strong electromagnetic wave. It is shown

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that at $v_E/v_T \ll 1$ the equations for the zeroth harmonic of the distribution function of the electrons coincide in the generalized and kinematic momentum representations, even though the initial equations are different in these two cases: the quantum equation is finite-difference for the generalized momentum and differential for the kinematic momentum (owing to the presence of a field term[16-18] $e\mathbf{E}\partial f/\partial \mathbf{p}$ in the latter case). The transition to the classical limit ($\hbar\omega \ll \langle\varepsilon\rangle, N \gg 1$) is considered. It is shown that the classical equation takes a diffusion form (cf. Ref. 2) and is obtained at $N \gg 1$ if the condition $v_E \ll v_T$ is simultaneously satisfied.

In Sec. II we consider the problem of electron scattering by atoms (molecules) in a resonant strong field. A characteristic feature of the resonant acquisition of energy is the interference between the process of absorption of l photons by the electrons and processes in which the electron emits $l \pm 1$ photons, while one photon is given up (or acquired) by the resonating system.

In Sec. III we derive a quantum equation for the distribution function of the electrons in resonant breakdown and present estimates of the lowering of the threshold intensity for atoms and molecules.

1. QUANTUM KINETIC EQUATION FOR ELECTRONS SCATTERED BY ATOMS IN A NONRESONANT FIELD OF A STRONG ELECTROMAGNETIC WAVE

We consider the derivation of a quantum kinetic equation for electrons in the field of an electromagnetic wave with a photon energy $\hbar\omega$ not equal to the energies of the transitions in the atomic system. We choose for the wave field a gauge in the form

$$\mathbf{A}=\mathbf{A}_0 \cos \omega t \quad (A_0=cE_0/\omega_0), \quad \varphi=0.$$

The equation for the density matrix $\rho_{nm}^{\mathbf{kk}'}$ [where $\hbar\mathbf{k}$ and $\hbar\mathbf{k}'$ are the generalized momenta of the electron; n and m are quantum numbers that characterize the atomic (molecular) subsystem] takes the form

$$i\hbar\dot{\rho}_{nm}^{\mathbf{kk}'}=\left[\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}-\frac{e\hbar}{m_e c}(\mathbf{k}-\mathbf{k}', \mathbf{A}(t))+E_n-E_m\right]\rho_{nm}^{\mathbf{kk}'} + \sum_{\mathbf{k}''p}\left(U_{np}^{\mathbf{k}''}\rho_{pm}^{\mathbf{k}''\mathbf{k}'}-\rho_{np}^{\mathbf{kk}''}U_{pm}^{\mathbf{k}''\mathbf{k}'}\right). \tag{1}$$

Here $\varepsilon_{\mathbf{k}}$ is the electron energy, $U_{np}^{\mathbf{kk}'}$ is the matrix element of the potential of the interaction of the electron with the atoms regarded as an aggregate of randomly distributed structural impurities (cf. Ref. 19). The distribution function of the electrons with respect to the generalized momenta $\rho(k, t)$ is obtained from $\rho_{nm}^{\mathbf{kk}'}$ in the following manner:

$$\rho(k,t)=\left\langle\sum_n \rho_{nn}^{\mathbf{kk}}\right\rangle, \tag{2}$$

where the symbol denotes averaging over the random spatial distribution of the atoms and, as follows from (1) and (2), satisfies the equation

$$i\hbar\dot{\rho}(k,t)=\left\langle\sum_{n,m,\mathbf{k}'}\left(U_{nm}^{\mathbf{kk}'}\rho_{mn}^{\mathbf{k}'\mathbf{k}}-\rho_{nm}^{\mathbf{kk}'}U_{mn}^{\mathbf{k}'\mathbf{k}}\right)\right\rangle. \tag{3}$$

To obtain a closed equation on the basis of (3) we must solve (1) for the off-diagonal elements of the density matrix. In the lowest order in the impurity concentration and in the Born approximation in the electron interaction with the atoms, we have

$$i\hbar\dot{\rho}_{nm}^{\mathbf{kk}'}=\left[\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}-\frac{e\hbar}{m_e c}(\mathbf{k}-\mathbf{k}', \mathbf{A}(t))+E_n-E_m\right]\rho_{nm}^{\mathbf{kk}'} + U_{nm}^{\mathbf{kk}'}(\rho_{mm}^{\mathbf{k}'}-\rho_{nn}^{\mathbf{k}}). \tag{4}$$

Equation (4) can be solved out of the condition $\rho_{nm}^{\mathbf{kk}'}(t_0) = 0, t_0 \to -\infty$:

$$\rho_{nm}^{\mathbf{kk}'}=-\frac{i}{\hbar}U_{nm}^{\mathbf{kk}'}\int_{-\infty}^{t}\exp\left\{-\frac{i}{\hbar}(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}+E_n-E_m)(t-t') + \frac{ie(\mathbf{k}-\mathbf{k}')}{m_e c}\int_{t'}^{t}\mathbf{A}(t'')\,dt''\right\}(\rho_{mm}^{\mathbf{k}'}-\rho_{nn}^{\mathbf{k}})\,dt'. \tag{5}$$

To integrate explicitly in (5), we expand the quantities $\rho_{nn}^{\mathbf{k}}(t)$ in Fourier series in the frequency of the external field, assuming the expansion coefficients to be slow functions of the time (for example, on account of avalanche multiplication of the number of electrons):

$$\rho_{nn}^{\mathbf{k}}(t)=\sum_{l=-\infty}^{\infty}\rho_{nn}^{\mathbf{k}}(l,t)e^{il\omega t}. \tag{6}$$

Substituting (6) in (5) and integrating, we have

$$\rho_{nm}^{\mathbf{kk}'}=U_{nm}^{\mathbf{kk}'}\sum_{lrs=-\infty}^{\infty}(\rho_{nn}^{\mathbf{k}}(l)-\rho_{mm}^{\mathbf{k}'}(l))J_r(N_{\mathbf{kk}'})J_s(N_{\mathbf{kk}'}) \times\frac{\exp[i(l-s+r)\omega t]}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}+E_n-E_m+(l-s)\hbar\omega-i\delta}, \tag{7}$$

where $J_l(x)$ is a Bessel function and $N_{\mathbf{kk}'}=e(\mathbf{k}-\mathbf{k}', \mathbf{E}_0)/m_e\omega^2$. Representing $\rho_{nn}^{\mathbf{k}}(l)$ in the form $\rho_{nn}^{\mathbf{k}}(l)=N_n\rho_l(k)$, where N_n is the population of the state n with energy E_n (the off-diagonal elements $\rho_{nm}^{\mathbf{k}}$ can be neglected in the nonresonant case) we obtain from (3) and (7), after averaging over the random distribution of the scatterers), an equation for the lth harmonic of the electron distribution function

$$i\hbar\frac{\partial\rho_l(k,t)}{\partial t}-l\hbar\omega\rho_l(k,t)=\sum_{\substack{\mathbf{k}'nm\\rs}}W_{nm}^{\mathbf{kk}'}(r,s) \times[N_m\rho_{l+s-r}(k',t)-N_n\rho_{l+s-r}(k,t)]. \tag{8}$$

Here

$$W_{nm}^{\mathbf{kk}'}(r,s)=|V_{nm}^{\mathbf{kk}'}|^2J_r(N_{\mathbf{kk}'})J_s(N_{\mathbf{kk}'})\{[\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}+E_n -E_m+(l-r)\hbar\omega-i\delta]^{-1}+[\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}+E_m-E_n+(l+s)\hbar\omega-i\delta]^{-1}\}, \tag{9}$$

$V_{nm}^{\mathbf{kk}'}$ is the matrix element of the potential of the interaction of the electron with one atom.

Equation (8) is solved with initial condition $\rho_l(-\infty)=0$ ($l \neq 0$). The solution of the homogeneous equation (8), which contains a fast time dependence $\sim e^{il\omega t}$, vanishes, so that it is perfectly legitimate, for this class of solutions, to take the Fourier coefficients $\rho_l(k,t)$, which depend "slowly" on the time, outside the integral sign in (5). The next terms of the expansion of the integral (5), which are obtained for example by integrating by parts, and which contain the derivatives $\partial\rho_l/\partial t$, have an extra smallness $\sim v/\omega$ compared with the derivative in the left-hand side of the (8). In fact, the derivative $\partial\rho_l/\partial t$ in (8) should be discarded in the first approximation (so long as the avalanche development growth rate is small compared with ω, as is practically always

the case) for all but the zeroth harmonic ($l=0$).

From (8) and (9) we can obtain a quantum-kinetic equation for the zeroth harmonic distribution function of the electrons, which is valid in the limit $\omega \gg \nu$ (ν is the electron collision frequency):

$$\frac{\partial \rho_0}{\partial t}=\frac{2\pi}{\hbar} \times \sum_{\substack{k'nm \\ l}} J_l^2(N_{kk'})\,|V_{nm}^{kk'}|^2[N_m\rho_0(k')-N_n\rho_0(k)]\,\delta(\varepsilon_k-\varepsilon_{k'}+E_n-E_m-l\hbar\omega). \tag{10}$$

The right-hand side of (10) characterizes the processes of acquisition (emission) of energy by the electrons in the electromagnetic-wave field in elastic and inelastic collisions with the atoms (cf. Refs. 10–12 for electrons interacting with phonons in a solid). In a weak field ($N_{kk'}\ll 1$) it suffices to retain the terms with $l=\pm 1$, if $m=n$ (elastic scattering), and the terms with $l=0$ if $m\neq n$. After averaging over the orientations of the initial momenta of the electron (transition to the symmetrical part of the distribution function), we obtain from (10) the quantum-kinetic equation of Zel'dovich and Raĭzer.[1]

Using (8), we can express the first harmonic $\rho_1(k,t)$ of the electron distribution function, retaining in the right-hand side only the terms with ρ_0:

$$\rho_1(k,t)=\frac{2\pi}{\hbar\omega}\sum_{\substack{k'nm \\ l}}|V_{nm}^{kk'}|^2(N_n\rho_0(k)-N_m\rho_0(k')) \times\left[i\frac{J_l^2(N_{kk'})}{N_{kk'}}\delta(\varepsilon_k-\varepsilon_{k'}+E_n-E_m-l\hbar\omega) -\frac{1}{\pi}J_l(N_{kk'})\cdot\frac{\partial J_l(N_{kk'})}{\partial N_{kk'}}\mathrm{P}\frac{1}{\varepsilon_k-\varepsilon_{k'}+E_n-E_m-l\hbar\omega}\right]. \tag{11}$$

With the aid of the obtained expression we can calculate the absorption coefficient $\varkappa$ of a strong electromagnetic wave $N_{kk'}>1$ in elastic and inelastic collisions of electrons with atoms:

$$\varkappa=\overline{\mathbf{jE}}/I_0, \tag{12}$$

where the superior bar denotes averaging over the period of the high-frequency field, $I_0=cE_0^2/8\pi$ is the incident intensity, and $\mathbf{j}$ is the current density given by

$$\mathbf{j}=\frac{e}{m_e}\sum_{\mathbf{k}}\left(\hbar\mathbf{k}-\frac{e}{c}\mathbf{A}\right)\rho(k,t). \tag{13}$$

As a result we get an expression for the absorption coefficient (cf. Ref. 6, where the case of elastic scattering by ions is considered):

$$\varkappa=\frac{1}{I_0}\frac{2\pi}{\hbar}\sum_{\substack{kk'nm \\ l}}l\hbar\omega J_l^2(N_{kk'})\,|V_{nm}|^2N_n\rho_0(k)\,\delta(\varepsilon_k-\varepsilon_{k'}+E_n-E_m-l\hbar\omega). \tag{14}$$

In the general case, to find the absorption coefficient it is necessary to solve Eq. (10) for the zeroth harmonic of the distribution function; this can be done numerically in the quantum limit.[3-5] It should be noted that in a number of cases it is necessary to take into account in (14) the elastic and inelastic collisions with the entire aggregate of the excited states.[20]

From (11) we can obtain also an expression for the real part of the nonlinear permittivity at frequency $\omega \gg \nu$ (we can consider analogously the nonlinear properties of an electron gas at other frequencies):

$$\varepsilon_d(\omega)=1-\frac{\omega_p^2}{\omega^2}-\frac{8\pi e}{m_e E_0^2\omega^2}\sum_{\substack{kk'nm \\ l}}(\mathbf{k}-\mathbf{k}',\mathbf{E}_0)N_n\rho_0(k) \times|V_{nm}^{kk'}|^2\frac{\partial J_l^2(N_{kk'})}{\partial N_{kk'}}\mathrm{P}\frac{1}{\varepsilon_k-\varepsilon_{k'}+E_n-E_m-l\hbar\omega}. \tag{15}$$

The correction to the plasma permittivity (the last term) is of strictly quantum origin (it vanishes as $\hbar\omega\to 0$, since $\sum_l J_l^2(x)=1$ and cannot be obtained by the usual substitution $\omega\to\omega+i\nu$, which adds increments quadratic in ν/ω to the real part of the classical permittivity. Expression (15) is an even function of the frequency, as it should. In a weak field it is necessary to retain in the sum over l the terms with $l=0,\pm 1$. The result is an answer of the order of $\omega_p^2\hbar\nu/\omega^2\langle\varepsilon\rangle$ which is finite as $E_0\to 0$.

The quantum kinetic equation (10) obtained for the distribution of the electrons in the generalized momenta is a finite-difference equation. The quantum-kinetic equation given in Refs. 16–18 was written for the distribution function in the kinematic momenta. It is simplest in this case to use directly the gauge $\mathbf{A}=0, \varphi=-e\mathbf{Er}$. The expression for the density matrix $P_{nm}^{pp'}$ then takes the form

$$i\hbar\left(\frac{\partial P_{nm}^{pp'}}{\partial t}+e\mathbf{E}(t)\frac{\partial P_{nm}^{pp'}}{\partial \mathbf{p}}+e\mathbf{E}(t)\frac{\partial P_{nm}^{pp'}}{\partial \mathbf{p}'}\right) =(\varepsilon_p-\varepsilon_{p'}+E_n-E_m)P_{nm}^{pp'}+\sum_{\substack{p''n' \\ p''m'}}(U_{nn'}^{pp''}P_{n'm}^{p''p'}-P_{nn'}^{pp''}U_{n'm}^{p''p'}). \tag{16}$$

The diagonal elements of the matrix P^{qq} satisfy the equation

$$i\hbar\left(\frac{\partial P(p,t)}{\partial t}+e\mathbf{E}(t)\frac{\partial P(p,t)}{\partial \mathbf{p}}\right)=\left\langle\sum_{\substack{p'nm}}(U_{nm}^{pp'}P_{mn}^{p'p}-P_{nm}^{pp'}U_{mn}^{p'p})\right\rangle. \tag{17}$$

Retaining in the sum of the right-hand side of (16), just as in (4), only the diagonal elements $P_{mm}^{p'p'}$ and P_{nn}^{pp}, and solving the resultant equation, we obtain for $P_{nm}^{pp'}$

$$P_{nm}^{pp'}=-\frac{i}{\hbar}U_{nm}^{pp'}\int_{-\infty}^{t}\left[N_mP\left(\mathbf{p}'+e\int_t^{t'}\mathbf{E}d\tau\right)-N_nP\left(\mathbf{p}+\int_t^{t'}\mathbf{E}d\tau\right)\right] \times\exp\left\{i\int_t^{t'}\left[\varepsilon\left(\mathbf{p}+e\int_t^{t''}\mathbf{E}d\tau\right)-\varepsilon\left(\mathbf{p}'+e\int_t^{t''}\mathbf{E}d\tau\right)\right]dt''-\frac{i}{\hbar}(E_n-E_m)(t-t')\right\}dt'. \tag{18}$$

Substituting (18) in (17) and averaging over the locations of the impurities, we can obtain the quantum equation for $P(p,t)$. The closed-form equation for the harmonics $P_l(p,t)$ is already much more complicated, because of the shift of the argument under the collision integral (cf. Ref. 19). The left-hand side of (17) contains a derivative with respect to the momenta, so that the quantum equation for $P(p,t)$ is in the general case a differential equation and not a finite-difference equation as is (8). Taking into account the relation [cf. (5) and (18)][17,18]

$$P_{nm}^{pp'}=\rho_{nm}^{kk'},\quad P(p,t)=\rho(k,t) \tag{19}$$

(where $\mathbf{p}=\hbar\mathbf{k}-e\mathbf{A}(t)/c$), we can see that at $v_E/v_T\approx e\mathbf{E}/\omega p\ll 1$ the fundamental harmonics of the distribution functions $P_0(p)$ and $P_1(p)$ are connected with $\rho_0(k)$ and $\rho_1(k)$ by the following relations:

$$P_1(p)=\rho_1(p)+\frac{eE}{2\omega}\frac{\partial\rho_0}{\partial p},\quad P_0(p)=\rho_0(p)+\frac{eE}{2\omega}\left(\frac{\partial\rho_{-1}}{\partial p}+\frac{\partial\rho_1}{\partial p}\right). \qquad (20)$$

It follows therefore that at $eE/\omega p \ll 1$ the quantum equation for the zeroth harmonic of the distribution function in the kinematic momenta coincides with (10), i.e., is also a finite-difference equation. The imaginary parts of $P_1(p)$ and $\rho_1(p)$, which determine in accordance with (13) the absorption coefficient, coincide similarly. This can be obtained directly by expanding (8) in powers of the parameter v_E/v_T up to second order. The terms containing the derivatives with respect to the momenta in the left-hand side of (17) are then cancelled out by the terms of the expansion of the collision integral (18).

The quantum kinetic Eq. (10) admits of a transition to the classical limit at $\hbar\omega \ll \langle\varepsilon\rangle$. Averaging over the orientations of the initial momentum k and expanding the obtained (at $n=m$) expression in powers of $l\hbar\omega \ll \hbar^2k^2/2m_e$ up to second order, we reduce this expression to the form (cf. Ref. 2, the condition $l \ll \hbar k^2/2m_e\omega$ is satisfied up to $l \sim N \sim eEk/m_e\omega^2$, if $v_E/v_T \ll 1$)

$$\frac{\partial\rho_0}{\partial t}=\frac{1}{6}\left(\frac{eE_0}{m_e\omega}\right)^2\sum_n N_n\frac{\partial}{\partial v}\left(v^3\sigma_{tr}^n\frac{\partial\rho_0}{\partial v}\right)+\mathrm{St}_{in}(\rho_0), \qquad (21)$$

where

$$\sigma_{tr}^n=\int\frac{d\sigma^n}{d\Omega}(1-\cos\vartheta)\,d\Omega$$

is the transport scattering cross section on the level n, and $\mathrm{St}_{in}(\rho_0)$ is the inelastic-collision integral. We have used here the summation formula[21]

$$\sum_l l^2J_l^2(x)=x^2/4.$$

At $v_E/v_T=eE/\omega p \gtrsim 1$ the expansion in $\hbar\omega$ is possible only for terms with $l \lesssim \hbar k^2/2m_e\omega$. On the other hand, an appreciable contribution to the sum over l is made by the terms with $\hbar k^2/2m_e\omega \lesssim l \lesssim eEk/m_e\omega^2$. The equation for ρ_0 does not reduce to a differential equation of second order. In Ref. 22, the classical Eq. (21) was solved in the limit $v_E \gg v_T$, a procedure that the foregoing analysis shows to be invalid.

2. SCATTERING OF ELECTRONS BY ATOMS IN THE FIELD OF A RESONANT ELECTROMAGNETIC WAVE

For convenience in the interpretation of the quantum kinetic equation for the zeroth harmonic of the distribution function in the case of resonance of a wave with a pair of atomic (molecular) levels $E_2-E_1-\hbar\omega=\hbar\Delta \ll \hbar\omega$ (see Fig. 1), we consider the problem of scattering of electrons by atoms for this case. Various aspects of this problem were discussed in Refs. 13, 14, 23, and 24. The wave functions of electrons in the field of a wave characterized by a potential $\mathbf{A}=\mathbf{A}_0\cos\omega t$, are of the form[6,7]

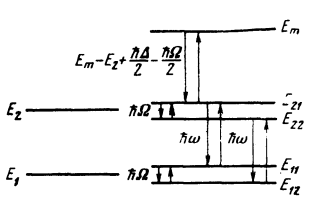

FIG. 1. Energy level scheme of an atom in a resonant electromagnetic field.

$$\varphi_p(\mathbf{r},t)=\exp\left\{\frac{i}{\hbar}\mathbf{pr}-\frac{i}{\hbar}\int_0^t\frac{(\hbar\mathbf{k}-ec^{-1}\mathbf{A}(t'))^2}{2m_e}dt'\right\}. \qquad (22)$$

The wave functions ψ_j (characterized by the indices $j=1$ and 2, which depend on the initial state at which the atom was at $t=0$, (cf. Ref. 13) of a resonating pair of levels are given by

$$\psi_j(\mathbf{R},t)=\sum_{\alpha,\gamma=1}^{2}F_{j\gamma}^{\alpha}\varphi_\alpha\exp\left\{-\frac{i}{\hbar}E_{\alpha\gamma}t\right\}. \qquad (23)$$

Here φ_α are the unperturbed atomic wave functions ($\alpha=1,2$), $E_{\alpha\gamma}$ are the system quasienergy levels in the resonant field (see Fig. 1):

$$E_{\alpha\gamma}=\tfrac{1}{2}[E_1+E_2+(2\alpha-3)\hbar\omega-(2\gamma-3)\hbar\Omega],$$

where $\Omega=(\Delta^2+|K|^2)^{1/2}$ is the Rabi frequency, $\hbar\Delta$ is the detuning from resonance, $K=-i\mathbf{E}_0\mathbf{d}_{12}/\hbar$; $\mathbf{d}_{12}$ is the matrix element of the dipole moment. The coefficients $F_{j\gamma}^{\alpha}$ take the following form:

$$F_{11}^{\ 1}=F_{22}^{\ 2}=\frac{\Omega-\Delta}{2\Omega},\quad F_{12}^{\ 1}=F_{21}^{\ 2}=\frac{\Omega+\Delta}{2\Omega},$$

$$F_{22}^{\ 1}=-F_{21}^{\ 1}=(F_{12}^{\ 2})^*=(F_{11}^{\ 2})^*=\frac{K}{2\Omega}.$$

For the remaining atomic states we choose the initial unperturbed wave functions (the fields E_0 are assumed small compared with the atomic fields):

$$\psi_m(\mathbf{R},t)=\psi_m(\mathbf{R})\exp\{-iE_mt/\hbar\}\quad (m\neq j).$$

For the expansion coefficients $C_{n_0m}^{p_0p}$ (p_0, n_0 are the initial quantum numbers for the electron and atom) of the total wave function in the introduced states (22) and (23) we obtain in the usual manner expressions in first-order perturbation theory. For example, for $n_0=j_0=1, n=j$ we have as $t\to\infty$

$$C_{kj}^{k_0k}=-2\pi i\sum_{l\alpha\beta\gamma\delta}(F_{j\gamma}^{\alpha})^*F_{j_0\delta}^{\beta}V_{\alpha\beta}^{k_0k}J_{l+\alpha-\beta}(N_{k_0k})\delta(\varepsilon_k-\varepsilon_{k_0}-l\hbar\omega+\hbar\Omega(\delta-\gamma)). \qquad (24)$$

The summation in (24) is over l; $\alpha,\beta,\gamma,\delta=1,2$. It follows from (24) that the same change $\varepsilon_k-\varepsilon_{k0}$ of the electron energy amounting to $l\hbar\omega$ or $l\hbar\omega\pm\hbar\Omega$, corresponds to interference of three processes: 1) l phonons are absorbed by the electron at $l>0$ (or are emitted at $l<0$), and the atomic system remains in the state with the $\alpha=\beta$; 2) $l+1$ photons are absorbed by the electron, but the electron gives up one photon to the atomic system; 3) $l-1$ photons are absorbed by electrons, but one photon is taken by it from the atom ($\alpha-\beta=-1$). All these processes interfere at a given j. Transitions to different states j do not interfere.

The probabilities $w_j^{k_0k}(l,s)$ of transitions in which the energy of the electron $\varepsilon_k-\varepsilon_{k0}$, scattered by the two-level system in the initial state j changes by an amount $l\hbar\omega+s\hbar\Omega(s=0,\pm1)$ are of the form

$$w_j^{k_0k}(l,0)=\frac{2\pi}{\hbar}\Big\{J_l^2(N_{k_0k})|V_{11}^{k_0k}+V_{22}^{k_0k}|^2\frac{|K|^2}{4\Omega^2}$$
$$+\Big|J_l(N_{k_0k})V_{jj}^{k_0k}\frac{\Delta}{\Omega}-(2j-3)\Big[J_{l-1}(N_{k_0k})V_{12}^{k_0k}\frac{K^*}{2\Omega}$$
$$+J_{l+1}(N_{k_0k})V_{21}^{k_0k}\frac{K}{2\Omega}\Big]\Big|^2\Big\}\delta(\varepsilon_k-\varepsilon_{k_0}-l\hbar\omega),\quad j=1,2; \qquad (25)$$

$$w_j^{k_0k}(l,s)=\frac{2\pi}{\hbar}\frac{s\Omega+(2j-3)\Delta}{2s\Omega}\Big|J_l(N_{k_0k})(V_{11}^{k_0k}-V_{22}^{k_0k})\frac{K^*}{2\Omega}$$
$$-J_{l-1}(N_{k_0k})V_{12}^{k_0k}\frac{K^*}{K}\frac{s\Omega+\Delta}{2\Omega}+J_{l+1}(N_{k_0k})V_{21}^{k_0k}\frac{s\Omega-\Delta}{2\Omega}\Big|^2\delta(\varepsilon_k-\varepsilon_{k_0}-l\hbar\omega-s\hbar\Omega),$$
$$s=\pm1. \qquad (26)$$

The problem of scattering of an unchanged particle by a two-level system in a resonant field was considered in Ref. 13. The corresponding probabilities follow from (25) and (26), if we let $N_{kk0} \to 0$, or equivalently, replace $J_l(N_{kk0})$ by δ_{l0}. In a weak field ($N_{kk0} \ll 1$, $|K| \ll \Omega$) only the terms with $l = 1$ are of importance; for the stimulated bremsstrahlung scattering we obtain in this case

$$w_1^{kk_0}(1,0) = \frac{2\pi}{\hbar}\left|\frac{N_{kk_0}}{2}V_{11}^{kk_0} + V_{12}^{kk_0}\frac{K^*}{2\Omega}\right|^2 \delta(\varepsilon_k - \varepsilon_{k_0} - \hbar\omega).$$

The first term characterizes here the bremsstrahlung in elastic scattering of an electron, where the second term characterizes absorption of a photon through the system. The amplitudes of these interfering processes correspond to the diagrams of Fig. 2 (cf. Ref. 25). Near resonance, the contribution of the second process may be decisive.[26,27] The remaining processes (26) go over in a weak field either into elastic scattering $w_1^{kk0}(0,0)$, or into elastic scattering: 1) excitation $(1 \to 2) - w_1^{kk0} \times (-1,-1)$, 2) the inverse process $(2 \to 1) - w_2^{kk0}(1,1)$.

Interest attaches also to the probabilities of transitions from states j, intermixed by the field, into states m which are not perturbed by the wave:

$$w_{mj}^{kk_0}(l,s) = \frac{2\pi}{\hbar}\left|J_l(N_{kk_0})V_{mj}^{kk_0}\frac{s\Omega + (2j-3)\Delta}{2\Omega} - J_{l+2j-3}(N_{kk_0})V_{mi}^{kk_0}\frac{K^{2-j}(K^*)^{j-1}}{2\Omega}\right|^2 \delta(\varepsilon_k - \varepsilon_{k_0} - E_j + E_m - l\hbar\omega + (2j-3)\frac{\hbar\Delta}{2} - s\frac{\hbar\Omega}{2}), \quad i=1,2;\ i\neq j;\ s=\pm 1. \tag{27}$$

Here we have interference of transitions with excitation $1 \to m$ and with absorption by the electron of l photons, and excitation $2 \to m$ with absorption of $l-1$ photons (one photon is given up to the electron by the atomic system). In a weak field ($N_{kk0} \ll 1$) at $l = 1$, $s = -1$ these processes correspond to the diagrams of Fig. 3. The inelastic scattering $1 \to m$ in a weak field corresponds to the probability $w_{m_1}^{kk0}(0,-1)$, while the transition $2 \to m$ corresponds to $w_{m_2}^{kk0}(1,1)$. The probabilities of the inverse processes are given by the same relations (27).

3. QUANTUM-KINETIC EQUATION FOR ELECTRONS SCATTERED BY ATOMS IN A RESONANT FIELD OF AN ELECTROMAGNETIC WAVE

To describe resonant breakdown of gases by radiation that is at resonance with some transition in an atomic (molecular) system, it is necessary to have a kinetic equation for the zeroth harmonic of the electron distribution function. To derive this equation we write down an equation for the density matrix $\rho_{nm}^{kk'}$ is an external field that acts both on the electron and on the atomic subsystem (details of the derivation are given in the

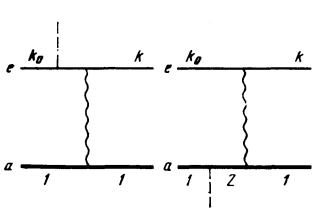

FIG. 2.

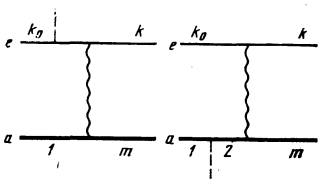

FIG. 3.

Appendix)

$$i\hbar\frac{\partial\rho_{nm}^{kk'}}{\partial t} = \left[\varepsilon_k - \varepsilon_{k'} - \frac{e\hbar}{m_e c}(\mathbf{k}-\mathbf{k}',\mathbf{A}) + E_n - E_m\right]\rho_{nm}^{kk'} - \mathbf{E}\sum_{n'}(\mathbf{d}_{nn'}\rho_{n'm}^{kk'} - \rho_{nn'}^{kk'}\mathbf{d}_{n'm}) + \sum_{n'k''}(U_{nn'}^{kk''}\rho_{n'm}^{k''k'} - \rho_{nn'}^{kk''}U_{n'm}^{k''k'}). \tag{28}$$

Here $\mathbf{d}_{nn'}$ is the matrix element of the dipole moment of the atom. We do not consider here the broadening of the atomic lines in the resonant field (see Refs. 23 and 28). Therefore, the theory developed here is valid at $\Omega > \Gamma$, where Γ is the corresponding line width. The quantities $\rho_{nm}^{kk}(t)$ are given by

$$\rho_{nm}^{kk}(t) = \rho_{nm}^{0}(t)\rho(k,t),$$

where $\rho_{nm}^0(t)$ is the atomic density matrix. In the general case the system for $\rho_{nm}^0(t)$ must be derived (see, e.g., Ref. 28) and solved in a self-consistent manner with the equation for the electron distribution function. In the breakdown problem, so long as the number of the electrons is small and they do not influence the atomic subsystem, at radiation-action times $\tau < T_2$ (T_2 is the phase-relaxation time and depends generally speaking on the intensity[23,28]) the matrix is obtained from the homogeneous solution of the system (8.2) with $q_{nm} = 0$ (coherent breakdown). For this case we obtain

$$\rho_{11}^0 = N_1 + \frac{|K|^2}{2\Omega^2}(N_1 - N_2)(\cos\Omega t - 1),$$
$$\rho_{12}^0 = (\rho_{21}^0)^* = e^{i\omega t}(N_1 - N_2)\left[\frac{K\Delta}{2\Omega^2}(1-\cos\Omega t) - \frac{iK}{2\Omega}\sin\Omega t\right], \tag{29}$$
$$\rho_{11}^0 + \rho_{22}^0 = N_1 + N_2;$$

N_1 and N_2 are the initial populations (we have put here for simplicity $t_0 = 0$).

For the time of action of the radiation $\tau > T_2$ one can choose ρ_{nm}^0 to be the steady-state values in a saturating field: $\rho_{11} \approx \rho_{22} = (N_1 + N_2)/2 \gg \rho_{12}, \rho_{21}$. In a diatomic molecule the entire vibrational band is usually saturated, and not only the rotational transition at resonance in the IR band.

In the coherent case, using for $\rho_{nm}^0(t)$ $(n,m = 1,2)$ their values (29), we obtain for the zeroth harmonic of the double Fourier series in the frequencies of the external field and in the Rabi frequencies, the quantum kinetic equation ($\omega, \Omega \gg v$):

$$\partial\rho_{00}/\partial t = \mathrm{St}_1(\rho_{00}) + \mathrm{St}_2(\rho_{00}) + \mathrm{St}_3(\rho_{00}). \tag{30}$$

The first term in the right-hand side of (30) characterizes transitions between resonating states 1 and 2; the second characterizes transitions between states 1 or 2 and the aggregate of unperturbed states m; the third characterizes transitions between the states m and m' which are not perturbed by the field. For the

corresponding collision integrals we have

$$\begin{aligned}
\mathrm{St}_1(\rho_{00}) &= \sum N_j[w_j^{kk'}(l,s)\rho_{00}(k') - w_j^{k'k}(l,s)\rho_{00}(k)],\\
\mathrm{St}_2(\rho_{00}) &= \sum [(N_j w_{mj}^{kk'}(l,s) + N_m w_{mj}^{k'k}(l,s))\rho_{00}(k')\\
&\quad - (N_j w_{mj}^{k'k}(l,s) + N_m w_{mj}^{kk'}(l,s))\rho_{00}(k)],\\
\mathrm{St}_3(\rho_{00}) &= \sum [N_{m'} w_{mm'}^{kk'}(l)\rho_{00}(k') - N_m w_{m'm}^{k'k}(l)\rho_{00}(k)].
\end{aligned} \tag{31}$$

The summation is over $l, s, k', j = 1, 2$; $m, m' \neq 1.2$. In (30) the probability $w_j^{kk'}(l,s)$ of transitions between resonating states is given by expressions (25) and (26), while the probabilities $w_{mj}^{kk'}(l,s)$ are given by (27); the transition probabilities $w_{mm'}^{kk'}(l)$ are equal to

$$w_{mm'}^{kk'}(l) = \frac{2\pi}{\hbar} J_l^2(N_{kk'}) |V_{mm'}^{kk'}|^2 \delta(\varepsilon_k - \varepsilon_{k'} + E_m - E_{m'} - l\hbar\omega),$$

as given in Eq. (10).

The condition $\Omega \gg \nu$, used in the derivation of (30) is valid for sufficiently low pressures ($10^3 p_{\text{torr}} \lesssim E[V/\text{cm}]$). At $\nu \geq \Omega$, those terms of (8.6) which oscillate with the Rabi frequency can be taken outside the integral sign. We do not present here the corresponding from of the kinetic equation. We consider in greater detail another important limiting case of a long radiation pulse ($\tau \gg T_2$) in a saturating field. The kinetic equation then takes the form (30), (31), with modified expressions for the transition probabilities:

$$\begin{aligned}
w_j^{kk'}(l,0) &= \frac{2\pi}{\hbar}\left[J_l^2(N_{kk'})\left|\frac{V_{11}^{kk'} + V_{22}^{kk'}}{2}\right|^2 + \left|J_l(N_{kk'})\right.\right.\\
&\times (V_{11}^{kk'} - V_{22}^{kk'})\frac{\Delta}{\Omega} + J_{l-1}(N_{kk'})V_{12}^{kk'}\frac{K^*}{2\Omega} + J_{l+1}(N_{kk'})V_{21}^{kk'}\frac{K}{2\Omega}\left.\left|^2\right.\right]\\
&\times \delta(\varepsilon_k - \varepsilon_{k'} - l\hbar\omega),
\end{aligned} \tag{32}$$

$$\begin{aligned}
w_j^{kk'}(l,s) &= \frac{\pi}{\hbar}\left| J_l(N_{kk'})(V_{11}^{kk'} - V_{22}^{kk'})\frac{K^*}{2\Omega}\right.\\
&\quad - J_{l-1}(N_{kk'})V_{12}^{kk'}\frac{K^*}{K}\frac{s\Omega+\Delta}{2\Omega}\\
&\left.+ J_{l+1}(N_{kk'})V_{21}^{kk'}\frac{s\Omega-\Delta}{2\Omega}\right|^2 \delta(\varepsilon_k - \varepsilon_{k'} - l\hbar\omega - s\hbar\Omega).
\end{aligned} \tag{33}$$

We obtain similarly for $w_{mj}^{kk'}(l,s)$:

$$\begin{aligned}
w_{mj}^{kk'}(l,s) &= \frac{2\pi}{\hbar}\frac{s\Omega-\Delta}{4s\Omega}\left|J_l(N_{kk'})V_{m1}^{kk'} - J_{l-1}(N_{kk'})V_{m2}^{kk'}\frac{s\Omega+\Delta}{K}\right|^2\\
&\times \delta\left(\varepsilon_k - \varepsilon_{k'} + E_m - E_1 - l\hbar\omega - \frac{\hbar\Delta}{2} - s\frac{\hbar\Omega}{2}\right),\quad j=1,2;\ s=\pm 1.
\end{aligned} \tag{34}$$

The probabilities of the transitions $m \to m'$ retain their previous form. In the noncoherent case for a saturating field we have $N_1 \approx N_2$. At resonance with a vibrational-rotational transition in the molecule, the fraction of the resonating particles Γ_{j_0} is small:

$$\Gamma_{j_0} = (2j_0+1)\frac{B}{T}\exp\left[-\frac{Bj_0(j_0+1)}{T}\right]$$

(B is the rotational constant and T is the gas temperature). For $j \neq j_0$ the transition probabilities have a nonresonant character. As noted in Sec. II, near resonance in a weak field the contribution of the absorption via the atomic system becomes larger than the direct absorption of the photon by the electron. In this case we obtain from (32)

$$w_1^{\text{ch}}(1,0) \approx \frac{2\pi}{\hbar}\left[\frac{N_{kk'}^2}{4}\frac{|V_{11}^{kk'}|^2 + |V_{22}^{kk'}|^2}{2} + |V_{12}^{kk'}|^2\frac{|K|^2}{4\Omega^2}\right]\delta(\varepsilon_k - \varepsilon_{k'} - \hbar\omega)$$

($N_{kk'} \ll 1$, the interference of the two processes vanishes after averaging over the angles). The second term corresponds to resonant absorption through the atomic system. Its ratio to the probability of direct absorption (the first term) is of the order of

$$\xi \sim \frac{\sigma_{in}}{\sigma_{el}}\left(\frac{\omega}{\Delta}\right)^2 \frac{\hbar\omega}{\langle\varepsilon\rangle} f_{12},$$

where σ_{in} and σ_{el} are the cross sections for inelastic and elastic scattering of the electron by the atom, $f_{12} \approx (m_e\omega/e^2\hbar)|d_{12}|^2$ is the oscillator strength of the corresponding atomic transition. At $\sigma_{in}/\sigma_{el} \sim 0.1$ $\omega/\Delta \sim 10^3 \div 10^5$, $\langle\varepsilon\rangle \sim I$ (I is the ionization potential) and $\hbar\omega/I \sim 0.1 f_{12} \sim 1$ we have $\xi \sim 10^4 - 10^8$. In scattering by an atom, ξ characterizes the efficiency of resonant acquisition of energy and correspondingly the decrease of the threshold intensity in resonant breakdown. When scattering by a molecule, the quantity ξ must be multiplied by m_e/M and by the fraction Γ_{j_0} of the resonating particles. In addition, in the infrared band the corresponding frequencies are smaller by one order of magnitude than in the estimate given above. As a result of all this, a decrease by a factor 2–10 of the threshold intensity in resonant breakdown of a molecular gas is possible within the framework of this model. It must be noted that in a saturating field (at $N_{kk'} \ll 1$) an important role in the acquisition of the energy can be played, by photon absorption in inelastic collisions with excited particles. This process corresponds to the diagram of Fig. 4 and has a probability $w_2^{k'k}(2,1)$ [see (33)]:

$$w_2^{k'k}(2,1) \approx \frac{2\pi}{\hbar}\frac{N_{kk'}^2}{4}|V_{12}^{k'k}|^2\delta(\varepsilon_{k'} - \varepsilon_k - 2\hbar\omega - \hbar\Omega).$$

In a molecular gas, on account of vibrational $v-v$ exchange, an appreciable number of excited particles can be produced by resonant radiation. The electron acquires thereby additional possibilities of accumulating energy on account of impacts of the second kind.

In the case of resonant breakdown in the IR band we have $\hbar\omega \ll \langle\varepsilon\rangle$, and it is possible to expand the kinetic Eq. (30) in powers of $\hbar\omega$ up to second order (just as in Sec. I, we assume that $eE/\omega p \ll 1$). The terms that describe electron heating in a resonant field take the diffusion form [cf. (21)]

$$\frac{1}{v^2}\frac{\partial}{\partial v}D\frac{\partial\rho_{00}}{\partial v},$$

where the diffusion coefficient is

$$D = \sum_{l=1}^{\infty}\sum_{j=1}^{2} N_j\left(\frac{l\hbar\omega}{m_e}\right)^2 v\sigma_j^l. \tag{35}$$

Here σ_j^l is the cross section of the corresponding process, whose probabilities are given by expression (25) for the coherent regime and by expression (32) for the noncoherent one. In the latter case, the inelastic pro-

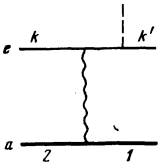

FIG. 4.

cesses characterized by the probabilities $w_j^{kk'}(l,s)$, $s=\pm 1$ [cf. (33)] also reduce to a diffusion form.

As already noted, in resonant breakdown of an atomic gas one should expect a decrease, by many (4–8) orders of magnitude, of the threshold breakdown intensity. An indication that this effect is real is provided by experiments on the breakdown of Rb and Cs vapor by a ruby laser.[29] In Ref. 15, an appreciable contribution to energy absorption via the atomic system was suggested in this connection, although the radiation was not fully resonant ($\Delta \gg |K|, \Delta \sim 0.2\omega$ for Cs and $\Delta \sim 0.1\omega$ for Rb). Estimates made for the molecular case (in the IR band) at atmospheric pressure in the approximation (35) yield, in analogy with Refs. 2 and 30, a decrease of the threshold intensity, by a factor of two, for diatomic molecules (such as CO, HF, etc.). In polyatomic molecules one can expect a substantial decrease of the threshold intensity.

The phenomenon of resonant optical breakdown can be used to produce an optical shutter, a plasma mirror, and optical discharge at lower laser intensities than in the nonresonant case. Extensive possibilities are uncovered by the use of dye lasers and tunable molecular high-pressure lasers.

In conclusion, the authors are deeply grateful to A. M. Dykhne for interest in the work and for helpful remarks.

APPENDIX

We consider first transitions between resonating levels: $n,m=1,2$. In the lowest order in the atom concentration and in the Born approximation in the interaction between the electrons and the atoms it suffices to retain in the last sum of the right-hand side of (28), for the off-diagonal elements $\rho_{nm}^{kk'}$ $(k\neq k')$, only the terms with $k''=k'$ in the first addena and with $k''=k$ in the second. Representing the solution for $\rho_{nm}^{kk'}$ in the form

$$\rho_{nm}^{kk'}=\rho_{nm}\exp[-i\hbar^{-1}(\varepsilon_k-\varepsilon_{k'})t-i\omega(n-m)t+iN_{kk'}\sin\omega t], \tag{A.1}$$

we obtain for ρ_{nm} the system of equations

$$\begin{aligned} &i\dot\rho_{11}+\frac{K}{2}\rho_{21}-\frac{K^*}{2}\rho_{12}=q_{11}, \quad i\dot\rho_{12}+\Delta\rho_{12}-\frac{K}{2}(\rho_{11}-\rho_{22})=q_{12},\\ &i\dot\rho_{21}-\Delta\rho_{21}+\frac{K^*}{2}(\rho_{11}-\rho_{22})=q_{21}, \quad i\dot\rho_{22}-\frac{K}{2}\rho_{21}+\frac{K^*}{2}\rho_{12}=q_{22}, \end{aligned} \tag{A.2}$$

where

$$\begin{aligned} q_{mn}=\frac{1}{\hbar}\sum_{n'}(U_{nn'}^{kk'}\rho_{n'm}^{k'k}-\rho_{nn'}^{kk'}U_{n'm}^{k'k})\exp\Big[\frac{i}{\hbar}(\varepsilon_k-\varepsilon_{k'})t\\ +i\omega(n-m)t-iN_{kk'}\sin\omega t\Big]. \end{aligned} \tag{A.3}$$

Introducing the fraction $\eta=N_1+N_2$ of the atomic (molecular) particles that resonate with the field of the wave, we get from (A.2)

$$\rho_{11}+\rho_{22}=\eta-i\int_{t_0}^{t}(q_{11}+q_{22})\,dt\equiv\eta-iQ(t). \tag{A.4}$$

It is seen that it suffices to solve the system of the first three equations of (A.2) for $\rho_{11},\rho_{12},\rho_{21}$, from which we eliminate ρ_{22} with the aid of (A.4). We next obtain ρ_{22} from (A.4). To take into account those transitions from states 1 or 2 which are mixed by the resonant field, to the unperturbed states m, we seek the solution of the system (28) in the form $(j=1,2)$

$$\rho_{jm}^{kk'}=\rho_{jm}\exp\Big[-\frac{i}{\hbar}(\varepsilon_k-\varepsilon_{k'})t-i\omega\Big(j-\frac{3}{2}\Big)t+i\frac{E_m}{\hbar}t+iN_{kk'}\sin\omega t\Big]. \tag{A.5}$$

As a result we obtain a system of inhomogeneous equations with constant coefficients for ρ_{jm}

$$\begin{aligned} &i\dot\rho_{1m}+\rho_{1m}\Big(-\frac{\omega}{2}-\frac{E_1}{\hbar}\Big)+\frac{K}{2}\rho_{2m}=q_{1m},\\ &i\dot\rho_{2m}+\rho_{2m}\Big(\frac{\omega}{2}-\frac{E_2}{\hbar}\Big)+\frac{K^*}{2}\rho_{1m}=q_{2m}, \end{aligned} \tag{A.6}$$

where

$$\begin{aligned} q_{jm}=\frac{1}{\hbar}(U_{jm}^{kk'}\rho_{mm}^{k'k}-\rho_{jj}^{k}U_{jm}^{kk'}-\rho_{jj}^{k}U_{jm}^{kk'})\\ \times\exp\Big\{\frac{i}{\hbar}\Big[\varepsilon_k-\varepsilon_{k'}+(2j-3)\frac{\hbar\omega}{2}-E_m\Big]t-iN_{kk'}\sin\omega t\Big\}. \end{aligned} \tag{A.7}$$

The equation for the electron distribution function is obtained from (3), in the right-hand side of which it is necessary to substitute (A.1), (A.5), and (5), with account taken of the solutions of the systems (A.2) and (A.6).

[1)] It must be noted that in solid-state physics the quantum kinetic equation for electrons that interact in a magnetic field with phonons was derived in a number of papers.[10-12]

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Longitudinal waves and two-stream instability in a relativistic plasma

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We investigate the problem of longitudinal waves in a relativistic plasma, with phase velocity lower than the velocity of light. We show that such waves exist at arbitrary particle momentum distribution function that falls off rapidly enough at large momenta. Earlier papers dealing with these questions are analyzed in connection with the problem of pulsar radiation, and the causes of the unclear and confused views advanced in some of these papers are discussed. Relativistic-plasma instabilities due to Cerenkov buildup of longitudinal waves by beams of high-energy and low-energy particles are investigated. The quasilinear relaxation of a beam of high-energy particles in a relativistic plasma is considered and the main regularities of this relaxation are explained.

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1. INTRODUCTION

Laboratory experiments have stimulated the development of a theory of collective processes induced by a beam of relativistic electrons in a nonrelativistic plasma.[1] This branch of plasma theory is used also in the interpretation of astrophysical phenomena (see, e.g., Chap. 3 of Ref. 2). At the same time, interest attaches in a number of astrophysical problems to the interaction of a beam of relativistic particles with a relativistic plasma. An example is the problem of radioemission from pulsars (see Ref. 1 and the literature cited therein).

Since two-stream instability is due to Cerenkov interaction of the particles with the waves, $\omega = kv$ (ω, k are the frequency and wave number, and v is the particle velocity), the question of two-stream instability in a relativistic plasma is frequently associated with the question of longitudinal (potential) waves of such a plasma with a phase velocity lower than that of light, $v_{ph} \equiv \omega/k < c$. Longitudinal waves in a relativistic plasma were considered initially by Silin[4] and by him with Rukhadze[5] for the case of an isotropic Boltzmann distribution of the particles in energy. Only waves with $\omega/k > c$ were observed in the cited studies. Silin and Rukhadze have therefore made a general statement that waves with $\omega/k < c$ cannot exist in a relativistic plasma, and consequently Cerenkov interaction of longitudinal waves of particles is impossible. The problem of longitudinal waves of particles is impossible. The problem of longitudinal waves in a relativistic plasma was later investigated by Tsytovich.[6] He has considered a plasma having the same momentum distribution function as Silin and Rukhadze,[4,5] but, on the contrary, found waves with $\omega/k < c$ and calculated their damping decrement due to the Cerenkov interaction with particles. This result contradicts the aforementioned conclusion of Silin and Rukhadze, but Tsytovich made no note of this contradiction and did not explain its causes.

Interest in longitudinal waves in the relativistic plasma has increased after the discovery of pulsars. The question of longitudinal waves with $\omega/k < c$ was considered anew by Tsytovich and Kaplan[7] (without reference to the earlier paper[6]!), but no longer for an isotropic Boltzmann distribution function, as in Refs. 4–6, but for one-dimensional power-law distributions. Tsytovich and Kaplan[6,7] have stated that longitudinal waves with $\omega/k < c$ exist and that Cerenkov generation

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