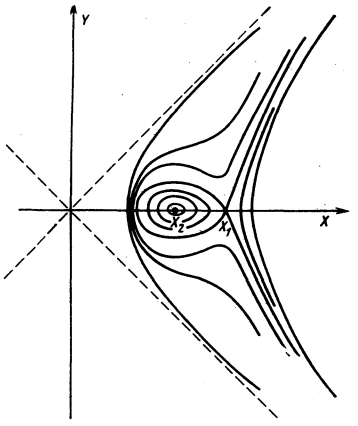

FIG. 7. Phase plane of the system (9): $X_{1,2} = \pm (H^2 - 1)^{1/2} + (H^2 - 1 + C)^{1/2}$.

quencies, generally speaking, of a large number of modes with close k, can be described by introducing the amplitudes of opposing waves that vary slowly along the network ($T > 0$):

$$\begin{aligned} &\partial b_+/\partial x + b_+ + iHb_-^* + i\Delta b_+ + ib_+(|b_+|^2 + 2|b_-|^2) = 0, \\ &-\partial b_-/\partial x + b_- + iHb_+^* + i\Delta b_- + ib_-(|b_-|^2 + 2|b_+|^2) = 0. \end{aligned} \tag{8}$$

In terms of the new variables $X = |b_+|^2 + |b_-|^2$, $Y = |b_-|^2 - |b_+|^2$, $Z = b_+ b_- + b_+^* b_-^*$ we have

$$\begin{aligned} &X' = 2Y,\ Y' = 2X + 2HS,\ Z' = YS; \\ &S^2 = X^2 - Y^2 - Z^2. \end{aligned} \tag{9}$$

By simple transformation we obtain one of the integrals of the system (9) $X^2 - Y^2 + 4HZ = C$. The phase plane is shown in Fig. 7.

Motions close to the separatrix correspond to the experimentally observed single-frequency multimode regimes ("blackout" solitons). On the other hand, motions close to the "center" correspond to regimes with few modes, one of which is shown in Fig. 5a.

The authors thank A. V. Gaponov, V. S. L'vov, and A. L. Fabrikant for helpful discussions.

[1)]Such a line simulates an infinite one-dimensional medium with a continuous frequency spectrum.

[2)]The corresponding measurements were made with a frequency-selective voltmeter.

[3)]By "weakness" of the noise is meant satisfaction of the relation $I_\omega/I_c \lesssim 5 \cdot 10^{-3}$, where I_c is the total excitation intensity in the parametric turbulence regime, and I_ω is the intensity of the noise in the band $0 < \omega < \omega_0$.

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Translated by J. G. Adashko

Metallic screening in a Peierls-Frölich dielectric with pinning to impurities at finite temperatures

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(Submitted 15 May 1978)
Zh. Eksp. Teor. Fiz. **76**, 1000–1009 (March 1979)

The temperature dependence of the metallic properties in a one-dimensional system with a charge-density wave (CDW) in the presence of sparse impurities is investigated. The statistical correlation function of the CDW phase is found, making it possible to determine the effective number of free carriers participating in the Frölich conduction at finite temperatures. The correlation function of the order parameter of the system is also considered.

PACS numbers: 77.90. + k, 66.90. + r

1. INTRODUCTION

1. A significant number of quasi-one-dimensional compounds display anomalous electrical and optical properties. The exceptionally high values of the static and microwave permittivity ε,[1] amounting to $\varepsilon \sim 10^2$–10^4, and also the presence of the peak in the conductivity in the far infrared region,[2] confirmed recently in Ref. 3 by precise measurements, are attracting great attention. These phenomena are observed in a wide range of temperatures, from room temperatures to liquid-helium temperatures. Most of these substances (see Refs. 1 and 4) undergo a transition to the dielectric state, with activation energy

 0038-5646/79/030504-06$02.40

$\approx 10^2$ K, which makes it difficult to explain these effects as a consequence of localization of electron states.[5] The possibility of explaining the anomalous properties as a manifestation of Fröhlich conduction in a Peierls-dielectric state is being widely investigated.[6]

The Fröhlich effect should exist in a system with charge-density waves (CDW) weakly coupled to the basic lattice structure. It is explained by the existence of a low-frequency optically active mode corresponding to the CDW phase.[7] The finite values of the frequency and damping of the Fröhlich mode and, correspondingly, the low-frequency, long-wavelength limit of the permittivity $\varepsilon(k, \omega)$ are determined by the effects of violation of the translational invariance, i.e., by the interaction of the CDW with the periodic structure and random structure of the basic lattice.[7] The interaction of CDWs of opposite sign,[7] which is characteristic of systems with incomplete charge transfer between chains of different types,[4] is equivalent to the former interaction. The latter interaction may be assumed to be weak, in view of the large distances between neighboring chains. The former interaction, which is due to umklapp processes, can be effective when there is commensurability of sufficiently low order between the periods of the CDW (the superstructure) and the basic structure,[8] which is encountered only in particular cases. For incommensurate CDWs this interaction can be realized only with participation of thermal phonons,[9] and this makes it ineffective at temperatures substantially below the Debye temperature.

In such conditions the interaction of CDWs with defects of the basic lattice acquires great importance. In many quasi-one-dimensional compounds the presence of defects is due to the actual structure of the compounds, i.e., to the disordered arrangement of one of the components.[4,5,10] (Here, unlike in Ref. 5, we have in mind those cases in which the defects do not violate the short-range order of the Peierls state.) For example, in KCP (Ref. 10) a disordered arrangement of Br ions with stoichiometric concentration 0.3 instead of 1/3 is equivalent to the pressence of Coulomb impurities with concentration $c = 0.1$.

The influence of impurities on the Fröhlich effect and on the structural properties of quasi-one-dimensional systems has been considered in many theoretical papers.[11-17] The region of low temperatures $T \ll T_c$ (T_c is the temperature of three-dimensional ordering), when the CDWs on neighboring chains are well correlated, has been studied with sufficient thoroughness in Refs. 11, 13, and 15. The case of an isolated chain, which was considered in Refs. 14 and 15, corresponds physically to the region $T \gg T_c$ (typically, $T_c \approx 50$ K); however, the problems arising in this case are not fully solved even for the classical problem with $T = 0$. The low-frequency asymptotic form of $\varepsilon(k, \omega)$ at large concentrations of weak scatterers remains largely unelucidated. The use of a classical description of the CDW phase also requires sufficiently high temperatures $T \gg T_0$,[18] where T_0 is of the order of the frequency of the amplitude oscillations of the CDW. (Typically, $T_0 \approx T_c$.) It is important to note that the problem of the motion of the CDW phase in the presence of interaction with the basic structure is essentially nonlinear, and this, strictly speaking, distinguishes it from the problem of an electronic or other excitation in an external periodic or random potential; therefore, taking thermal or quantum effects into account can change substantially the results obtained.

2. In the present paper we investigate the effect of finite temperatures on the properties of an isolated filament with a CDW in the presence of sparse impurities. Those cases which have been investigated already for the classical model at zero temperature are considered. Although the chief interest is in the dynamical response functions (principally $\varepsilon(k, \omega)$), we have not yet succeeded in investigating these. Instead we shall study the frequency-integrated characteristic $\varepsilon(k)$, calculated in the framework of statistical mechanics. In this case the Debye screening parameter can be obtained, and thus the effective fraction of the CDW charge that is freed from the pinning at the impurities as a result of thermal activation is determined. We obtain a common temperature-dependent factor, reducing the quantities $\varepsilon(k)$, $\mathrm{Re}\varepsilon(k, \omega)$, and the conductivity $\sigma(k, \omega)$. The temperature dependence of the latter two quantities is determined also by the as-yet unknown behavior of the damping of the phase mode of the CDW. The original results of the paper pertain to the pinning to impurities; however, we first give some results on pinning to the periodic basic structure, which are necessary for comparison. The combined effect of the pinnings of different types has been investigated in Ref. 19 (see also the literature cited there).

2. GENERAL DESCRIPTION OF PINNING OF CDWs

1. At temperatures that are low compared with the gap Δ in the electron spectrum ($T \ll \Delta$) the charge density $\rho(x)$ and free energy $\mathcal{F}$ of the CDW are described by the phase $\chi(x)$:

$$\rho(x) = \rho_0 \cos[2p_F x + \chi(x)], \tag{1a}$$

$$\mathcal{F}\{\chi\} = \int dx \left[\frac{v_F}{4\pi} \left(\frac{d\chi}{dx} \right)^2 - \varepsilon_p n(x) \cos\left(m\chi + \int \varkappa(x')\,dx' \right) - \frac{e}{\pi} E\chi \right], \tag{1b}$$

where v_F is the Fermi velocity, e is the electron charge, and E is the intensity of the external electric field. Here and henceforth, all coefficients correspond to the Fröhlich model. The expression (1) describes different forms of pinning, depending on the form of the functions $n(x)$ and $\varkappa(x)$.

a) For the effects of interaction with the basic periodic structure,[8]

$$\varkappa = 2p_F m - Q = \text{const}, \quad \varepsilon_p n(x) = \frac{\pi m^2}{32} \frac{T_p^2}{v_F} = \text{const},$$

where m is the commensurability index, Q is a reciprocal-lattice vector, and T_p is the pinning temperature.

b) For a model of two oppositely charged filaments with CDWs,

$$\varkappa = 0, \quad m = 1, \quad \varepsilon_p n(x) = \text{const} \propto T_c^2 / v_F,$$
$$\chi = \chi_1 - \chi_2, \quad v_F = (v_{F,1}^2 + v_{F,2}^2)/2(v_{F,1} + v_{F,2}),$$

where the subscripts 1 and 2 refer to the two filaments. As a supplement to (1), the energy $\mathcal{F}$ also contains bilinear terms depending on $\tilde{\chi}=\chi_1+\chi_2$. However, in problems of statistical mechanics we can average explicitly over $\tilde{\chi}$, as a result of which we obtain the expression given above for the effective coefficient v_F.

c) For interaction with impurities positioned at the points x_k,

$$m=1,\quad n(x)=\sum_k \delta(x-x_k),$$
$$\varkappa(x)=2p_F+V_f(x)/2v_F, \qquad (2)$$
$$\varepsilon_p=\frac{\Delta}{\lambda v_F}\int V_b(x')\,dx',$$

where $V_f(x)$ and $V_b(x)$ are the potentials for forward and backward scattering of electrons by an impurity, λ is the electron-phonon interaction parameter, and $\Delta\sim\varepsilon_F\exp\{-\lambda^{-1}\}$, where ε_F is the Fermi energy.[14] The phase

$$\varphi(x)=\chi(x)+\frac{1}{2v_F}\int^x V_f(x')\,dx'$$

is a physical quantity.

2. For the cases a) and b) the correlation function $\langle\rho(x)\rho(x')\rangle$ has been investigated as a function of $\varkappa$ and T in Ref. 8. The permittivity ($S_\perp$ is the area per filament)

$$\varepsilon(k)=\frac{4}{\pi}\frac{e^2}{S_\perp}\int\langle\chi(0)\chi(x)\rangle e^{-ikx}dx=4T\int\sigma(k,\omega)\frac{d\omega}{\omega} \qquad (3)$$

can be investigated analogously without any new details. Here we give only the result.

1) For $T\gg T_p$ or for $|\varkappa|\gg T_p/v_F$ and any T the pinning is unimportant and

$$\varepsilon(k)=8Te^2/v_Fk^2S_\perp,$$

as for a free electron gas.

2) For $|\varkappa|\ll T_p/v_F$ and $T\ll T_p$ we have

$$\varepsilon(k)=8\pi e_s^2(n_{s+}+n_{s-})/k^2S_\perp,$$

where $e_s=2e/m$ and n_s are the charge and density of the solitons—the external solutions for the functional (1):

$$n_{s\pm}=\frac{\pi}{4}m^2\frac{T_p}{v_F}\exp\left\{-\frac{T_p\mp v_F\varkappa/m}{T}\right\}.$$

Here the signs ± correspond to solitons with phases changed by $\pm 2\pi/m$. The region 2) has been investigated most fully in Ref. 20.

We see that in the widest (with allowance for the lower bounds on the temperature discussed in the Introduction) region 1), umklapp processes a) and the interaction of opposite charges b) do not exert any substantial influence on the magnitude of the free charge of the CDW. Moreover, the situation 2) goes over into the situation 1) if the condition that the quantum fluctuations be small,

$$m^2u/v_F<1,$$

where u is the phase velocity of the CDW, is not fulfilled.

From the available estimates,[3,6]

$$(u/v_F)^2=M/M^*\sim10^{-2},$$

where M is the band mass and M^* is the effective mass of the CDW,[7] so that certainly for $m\geq4$, and possibly also for $m=3$, the pinning is broken (solitons do not exist) and free charge density is restored. In these cases the interaction of the CDW with the superstructure is taken into account by perturbation theory, and the damping $\gamma(k,\omega)$ appearing in $\varepsilon(k,\omega)$ can be found. This was done in Ref. 8 for the region $T\ll T_0$, and extended in Ref. 9 for the region $T\gg T_0$.

3. INTERACTION WITH IMPURITIES

1. We shall consider the case c)—interaction of the CDW with defects of the basic structure. The principal contribution to the interaction arises from scattering of the electrons constituting the CDW. The defect potential contains a smooth part $V_f(x)$ that determines the forward scattering of the electrons, and a part $V_b(x)$ that determines the backward scattering. In view of the large number of atoms in the planar molecules forming the majority of quasi-one-dimensional systems,[4] or because of the disposition of the charged defects to the side of the conducting chains as in KCP,[4] we may suppose that $V_f(x)$ constitutes the main part of the random potential. This part is long-range and can be described by a random Gaussian field. For the reason indicated above, substantial potentials $V_b(x)$ are relatively few and should be regarded, in the general case, as isolated impurities with a short-range potential. The potentials $V_f(x)$ affect the structure factor of the system (they give rise to broadening of the Brillouin peaks and to the appearance of a central peak[17]) and the three-dimensional ordering of the chains.[17,21] In these problems the potentials $V_b(x)$ can be disregarded in practice. However, the potentials $V_f(x)$ do not affect $\varepsilon(k,\omega)$, and in this problem it is sufficient to take only $V_b(x)$ into account. As a result we arrive at a phenomenological description on the basis of the expression (1), with the parameters (2), for the free energy. The function $\varkappa(x)$ in (2) can be obtained from the microscopic theory of Zavadovskiĭ[16] or directly, for the expression (1), by the method of Ref. 18. The parameter ε_p in (2) was calculated in Refs. 14 and 15.

2. In the problem under consideration there are two energy scales: ε_p and $T^*=cv_F/\pi$, where c is the concentration of impurities. At $T=T^*$ the thermal correlation length $\xi_T=v_F/\pi T$ for the phase is comparable with the average spacing between impurities. As was shown in Ref. 15, the relative magnitude of ε_p and T^* determines the character of the ground state at low temperatures. The ground state and low-frequency dynamics for $T^*\ll\varepsilon_p$ were investigated in Refs. 14 and 15. The region of high concentrations, when $T^*\gg\varepsilon_p$, was investigated in Refs. 13 and 15. However, only the results pertaining to a system of correlated chains can be regarded as reliable.

The free-energy functional (1) of the system has a nonquadratic dependence on the phase $\chi(x)$ only at the

positions x_n of the impurities. Therefore, the distribution function

$$W\{\chi\}=Z^{-1}\exp[-\mathcal{F}\{\chi\}] \tag{4}$$

(Z is the partition function) can be averaged over $\chi(x)$ for $x\neq x_n$ by fixing $\chi(x_n)=\varphi_n-\alpha_n$, where

$$\alpha_n=\frac{1}{2v_F}\int_{x_n}^{x_n} V_1(x)\,dx.$$

The set $\{x_n\}$ should also include the points x_α ($\alpha=1,\ldots,r$), if we are calculating the r-point distribution function

$$G\{x_\alpha;\ \varphi_\alpha\}=Z^{-1}\langle G\{x_n;\ \varphi_n\}\rangle_{n\neq\alpha}\rangle_{av}, \tag{5}$$

where $\langle\ \rangle_{av}$ denotes configurational averaging, the angular brackets denote integration over the $\{\varphi_n\}$ for $n\neq\alpha$, and

$$G\{x_n;\ \varphi_n\}=\int D\chi(x)\,W\{\chi\}\prod_n\delta(\chi(x_n)-\varphi_n+\alpha_n). \tag{6}$$

Calculating the Gaussian integral (6) we obtain, as we should expect, a product of the distribution functions of the phases φ_n at the individual impurities

$$W_1\{\varphi_n\}=\exp\left\{\frac{\varepsilon_p}{T}\cos(\varphi_n+\alpha_n+Qx_n)\right\} \tag{7}$$

with the "transition matrices" between neighboring points x_n and x_{n+1}:

$$G\{x_n,x_{n+1};\varphi_n,\varphi_{n+1}\}=\left[\frac{\xi_T}{4\pi|x_{n+1}-x_n|}\right]^{1/2}\exp\left[-\frac{\xi_T}{4}\frac{(\varphi_{n+1}-\varphi_n)^2}{|x_{n+1}-x_n|}\right]. \tag{8}$$

Namely,

$$G\{x_n;\varphi_n\}=\prod_n G_1\{x_{n+1},x_n;\varphi_{n+1},\varphi_n\}\,W_1\{\varphi_{n+1}\}\propto$$
$$\propto\exp[-\beta\mathcal{F}\{\varphi_n\}], \tag{9}$$

where the effective free energy $\mathcal{F}\{\varphi_n\}$ is equal to ($\beta=1/T$)

$$\beta\mathcal{F}\{\varphi_n\}=\sum_n\left[\frac{\xi_T}{4}\frac{(\varphi_{n+1}-\varphi_n)^2}{|x_{n+1}-x_n|}-\frac{\varepsilon_p}{T}\cos(\varphi_n+\alpha_n+Qx_n)\right]. \tag{9a}$$

3. We shall consider the region $T\gg T^*$ for an arbitrary value of ε_p. We shall expand

$$W_1\{\varphi_n\}=\sum_{l_n}I_{l_n}\left(\frac{\varepsilon_p}{T}\right)\exp\{il_n(\varphi_n+\alpha_n+Qx_n)\}, \tag{10}$$

where $I_l(t)$ are modified Bessel functions. On substitution of (10) into (9) and integration over φ_n in accordance with (5) with the functions (8), the l-th harmonic is carried over from the point x_n to the point x_{n+1} with conservation of the phase α_n+Qx_n but with the reducing factor

$$A^{(l)}_{n,n+1}=\exp\{-l^2|x_{n+1}-x_n|/\xi_T\}.$$

From the condition $T\gg T^*$, $A^{(1)}_{n,n+1}\ll 1$ for $|x_n-x_{n+1}|\propto c^{-1}$ and $l\neq 0$. Therefore, in calculating $Z(T)$ or $\varepsilon(k)$ it is sufficient to retain only the harmonics with $l=0$. The dependence on the arrangement of the impurities disappears, and we immediately obtain

$$Z(T)=Z_0(T)[I_0(\varepsilon_p/T)]^N, \tag{11}$$

where $Z_0(T)$ is the partition function of the chain without impurities and $N=cL$ is the total number of impurities in a chain of length L.

The impurity part C_{im} of the specific heat is equal to ($T\gg T^*$)

$$C_{im}=\begin{cases}{}^1/_2c & \text{for } \varepsilon_p\gg T\\ c\varepsilon_p^2/T^2 & \text{for } \varepsilon_p\ll T\end{cases}.$$

In the calculation of $\varepsilon(k)$ from formula (3) in the long-wavelength limit $|k|\xi_T\ll 1$ the factors $I_0(\varepsilon_p/T)$ in the integral with the propagation function (9) cancel with the normalization factor (11), and $\varepsilon(k)$ is found to be the same as in the pure system. Corrections of the order of

$$c\xi_T=T^*/T\ll 1$$

from cluster effects do not change the functional form of $\varepsilon(k)$.

We conclude that for $T\gg T^*$ the effective free charge of the CDW coincides with its total charge.

4. The calculation of the structure correlation function

$$K_{if}=K(x_i-x_f)=\langle\exp\{i[\varphi(x_i)-\varphi(x_f)]\}\rangle$$
$$=\int G(x_i,x_f;\varphi_i,\varphi_f)\exp\{i[\varphi(x_i)-\varphi(x_f)]\}d\varphi_i d\varphi_f \tag{12}$$

at distances $|x_i-x_f|\gg\xi_T$ gives rise to characteristic difficulties, inasmuch as, in the determination of the correction to the correlation length ξ, we are expanding formally in the large parameter $c|x_i-x_f|$. We shall give this calculation briefly, since it can have methodological interest.

Let m impurities be situated on the segment (x_i,x_f), at the points x_n ($n=1,\ldots,m$). Having substituted (9) into (12) we shall integrate successively over $\varphi_i,\varphi_1,\ldots,\varphi_m,\varphi_f$. First the harmonic with index $l_i=1$ is propagated. If at the impurities x_n we take into account the terms in (10) with $l_n=0$ the index does not change, and the result does not depend on the location of the impurities. If, at some impurity n_1, we take into account first the term with $l_{n_1}=-1$ in the expansion (10), then further on the index will be equal to $l=0$, i.e., the damping ceases. Somewhere further on, at impurity n_2, the index $l=1$ should be restored, in order that we can carry out the integration over φ_f. Transitions between the harmonics with $l=0,\pm 1$ can occur at different impurities an arbitrary number (k) of times. For each distribution $\mathfrak{N}_k$ of indices we obtain a common factor

$$\exp[-\xi_T^{-1}L_1\{\mathfrak{N}_k\}]\mu^k,\qquad \mu=I_1(\varepsilon_p/T)/I_0(\varepsilon_p/T).$$

(We normalize all expressions to $[I_0(\varepsilon_p/T)]^m$.) The length $L_1\{\mathfrak{N}_k\}$ is the traversed path with index $l=1$. In addition we have the product

$$U\{\mathfrak{N}_k\}=\prod_n\exp[il_n(\alpha_n+Qx_n)],$$

the factors of which appear whenever the index changes. In order that the result of the configurational averaging not be equal to zero, they must be cancelled by analogous factors extracted from the normalization factor $Z^{-1}(T,\{x_n\})$. The calculation of $Z(T,\{x_n\})$ can be analyzed analogously as the propagation of a harmonic with initial index $l_i=0$. For the part of Z^{-1} that we

need we always obtain the coordinate dependence

$$\propto \exp\{-L_0\{\mathfrak{M}_k\}/\xi_T\}\mu^k,$$

where $L_0\{\mathfrak{M}_k\}$ is the total length previously covered with index $l=0$. Since

$$L_0+L_1=|x_f-x_i|,$$

each expression does not depend on precisely which impurities the changes of indices occurred at, and it remains only to calculate the combinatoric coefficients. We have

$$K_{if}=A/B,$$
$$A=\sum_{k=1}^{m}\mu^k\sum_{\mathfrak{M}_k}\exp[-\xi_T^{-1}L_1\{\mathfrak{M}_k\}]U\{\mathfrak{M}_k\}, \tag{13}$$
$$B=1+\sum_{k=1}^{m}\mu^k\sum_{\mathfrak{M}_k}\exp[-\xi_T^{-1}L_0\{\mathfrak{M}_k\}]U\{\mathfrak{M}_k\}.$$

We expand the denominator in (13) in powers p of the second term and separate out those products

$$\prod_{\nu=1}^{p}U\{\mathfrak{M}_{k_\nu}\},\quad \sum_{\nu=1}^{p}k_\nu=k,$$

which cancel $U\{\mathfrak{M}_k\}$ in the numerator of formula (13). This partition is equivalent to the problem of placing k objects, chosen from a total number m of objects, in p boxes; in each box ν there can be any number k_ν of objects ($p\geqslant k_\nu>0$). The total number P_k^p of such partitions is known from elementary algebra:

$$P_k^p=\sum_{q=0}^{p}(-1)^q(p-q)^kC_p^q. \tag{14}$$

We obtain

$$K_{if}=K_{if}^{(0)}\sum_{k=0}^{m}C_m^k\mu^{2k}\sum_{p=1}^{k}(-1)^pP_k^p,$$

where $K_{if}^{(0)}=\exp\{-|x_i-x_f|/\xi_T\}$. We substitute (14) and change the order of the summation over p and q:

$$K_{if}/K_{if}^{(0)}=\sum_{k=0}^{m}C_m^k\mu^{2k}\sum_{q=1}^{k}\sum_{p=q}^{k}(-1)^{p+q}(p-q)^kC_p^q$$
$$=\sum_{k=0}^{m}C_m^k\mu^{2k}(-1)^k=(1-\mu^2)^m. \tag{15}$$

Next we must sum over m with a Poisson distribution with mean value $m=c|x_f-x_i|$. Finally, we have

$$K_{if}=\exp\{-|x_i-x_f|(\xi_T^{-1}+c\mu^2)\}. \tag{16}$$

The potentials $V_f(x)$ will give an extra factor to (16):

$$S_{if}=\exp\{i[\alpha(x_i)-\alpha(x_f)]\}=\exp\{-|x_i-x_f|/4l_f\},$$

where l_f is the electron forward-scattering length.[16,21] Thus, the total correlation length ξ is given by

$$\frac{1}{\xi}=\pi\frac{T}{v_p}+c\left[I_1\left(\frac{\varepsilon_p}{T}\right)\Big/I_0\left(\frac{\varepsilon_p}{T}\right)\right]^2+\frac{1}{4l_f}. \tag{16a}$$

5. We now consider the low-temperature region $T\ll T^*$. Here we are forced to confine ourselves to the case of "strong pinning" $T^*\ll\varepsilon_p$, where the structure of the ground state is known.[14,15] It can also be established from the expression (9a) for $\mathcal{F}\{\varphi_n\}$. As approximate equilibrium values of φ_n we can choose

$$\varphi_n^{(0)}=-(\alpha_n+Qx_n)+2\pi l_n,$$

where the integers l_n should ensure that $|\varphi_n^{(0)}-\varphi_{n+1}^{(0)}|$ is a minimum. Substantial deviations $|\varphi_n-\varphi_n^{(0)}|\lesssim\pi$ require energies $\sim\varepsilon_p$, whereas a difference $|\varphi_{n+1}^{(0)}-\varphi_n^{(0)}|\lesssim 2\pi$ over a distance $|x_{n+1}-x_n|\approx c^{-1}$ requires a lower energy $\sim T^*$. Since $T\ll T^*\ll\varepsilon_p$ we can assume that $\varphi_n\approx\varphi_n^{(0)}$ with accuracy

$$|\varphi_n-\varphi_n^{(0)}|\propto T^*/\varepsilon_p,\quad (T/\varepsilon_p)^{1/2}.$$

(The former estimate is associated with making the ground state more precise, and the latter with thermal fluctuations.) With the same accuracy, thermal excitations of the system will consist in a change of the values of all the phases φ_n to one side of a certain point x_{n_0} by $2\pi\delta l$:

$$l_n\to l_n+\delta l\quad\text{for}\quad n>n_0.$$

The segment $(x_{n_0}, x_{n_0\pm1})$ of length s is analogous to the soliton in the problem of pinning to a periodic structure (see Sec. 2, case b). As will be seen from the following, for the important values of s the energy of such an excitation is always large compared with the temperature:

$$\frac{\delta\mathcal{F}}{T}=\pi^2(\delta l)^2\frac{T^*}{T}\frac{1}{cs}\gg 1.$$

Therefore, we can confine ourselves entirely to excitations with $\delta l=\pm1$ and assume them to be isolated.

The thermal variance of the advance of the phase over large distances will be equal to

$$\langle(\delta\varphi_n-\delta\varphi_m)^2\rangle=\sum_{k=m}^{n-1}(2\pi)^2\exp\left\{-\frac{T^*}{T}\frac{\pi^2}{c|x_{k+1}-x_k|}\right\}. \tag{17}$$

Because of the small concentration c^* of excitations, each term in (17) must be averaged independently over the distribution of lengths

$$s_k=|x_{k+1}-x_k|.$$

We obtain

$$\langle[\delta\varphi_n-\delta\varphi_m]^2\rangle_{av}=(2\pi c)^2|x_n-x_m|\int ds\exp\left\{-\frac{T^*}{T}\frac{\pi^2}{cs}-cs\right\}. \tag{18}$$

The integral in (18) converges in the neighborhood of the optimal distance

$$s_0=\pi c^{-1}(T^*/T)^{1/2},\quad s_0\gg c^{-1}.$$

Finally,

$$\langle[\delta\varphi_n-\delta\varphi_m]^2\rangle_{av}=4\pi^3\left(\frac{T^*}{T}\right)^{1/4}\exp\left\{-2\pi\left(\frac{T^*}{T}\right)^{1/2}\right\}c|x_n-x_m|.$$

By virtue of (3) we have

$$\varepsilon(k)=32\pi^2e^2\left(\frac{T^*}{T}\right)^{1/4}\exp\left\{-2\pi\left(\frac{T^*}{T}\right)^{1/2}\right\}\frac{c}{S_\perp k^2}. \tag{19}$$

The anomalous activational dependence in (19) is analogous to the Mott law, inasmuch as here too the change of state occurs principally for an optimal spacing s_0 between impurities.

4. CONCLUSION

In the present paper we have systematized the theoretical investigations on pinning of CDWs. The relationship between the different pinning mechanisms are indicated, for application to the typical parameters of the substances investigated experimentally. An investigation of pinning to impurities at finite temperatures is carried out. The new results are reflected, in their final form, by the formulas (16), (16a), and (19).

It is important to note that, in all cases, when the temperature is raised the static pinning of the CDW weakens, and T^* vanishes completely when $T > T_p$. This dependence agrees with the low-temperature increase, observed in Refs. 22 and 23, of the microwave permittivity $\varepsilon(\omega)$, which, according to preliminary data, becomes a sharp decrease in the region $T \approx 30$–40 K. As a result, apparently, of the increasing role of the damping $\gamma(\omega)$ of the Fröhlich mode, the breaking of the pinning does not lead to high negative values of $\varepsilon(0)$. The pinning effects that permit a dynamical investigation show[8,9] that their contribution γ_p to $\gamma(0)$ is a maximum at $T \sim T_p$, and for $T > T_p$ decreases like

$$\gamma_p \sim \omega_p (T_p/T)^3$$

where $\omega_p \sim (u/v_F)T_p$ is the pinning frequency and u is the phase velocity of the CDW. In view of what has been said above we conclude that the principal role should be played by the damping γ_{ph} of the Fröhlich mode on account of its interaction with the thermal phonons,[9] which increases with temperature. It is necessary that $\gamma_{ph} > \omega_p$ be fulfilled as soon as $T \gtrsim T_p$. This condition can be fulfilled[9] in view of the specially low Debye temperatures $\Theta \gtrsim T_p$ in the substances investigated.

In conclusion the author expresses his deep gratitude to I. E. Dzyaloshinskiĭ for numerous discussions and for critical remarks on reading the manuscript, and also to A. I. Larkin and D. E. Khmel'nitskiĭ for a critical discussion of the paper.

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Translated by P. J. Shepherd