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Electromagnetic and scalar fields around an infinite filament and around other bare singularities of the Kasner type

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It is shown that the intensity of the electric field of an infinite charged filament increases to infinity with increasing distance from it, as a result of self-gravitation, and forms an inhomogeneous singularity. An object of this kind with $H_\varphi \neq 0$ and $H_z \neq 0$ cannot exist in general relativity theory. The scalar field of a filament is considered and it is shown that in contrast to the electromagnetic field such a field is capable of upsetting the oscillatory approach to bare singularities and replacing it by a power-law approach for which a formula is given. The effect of such fields on other bare singularities of the Kasner type is also investigated.

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1. INTRODUCTION

This paper is devoted to a study of the influence of electromagnetic and scalar fields on bare singularities of the Kasner type and primarily on the simplest among them, described by a Kasner spatial metric [Eq. (10) at $c=0$]. This metric describes the gravitational field around an infinitely long and thin filament with mass[1] (see also Ref. 2). A situation is investigated wherein this filament is a source of an electromagnetic or a scalar field. Thus, in the next section we consider the classical electrostatic problem of finding the electric field intensity around an infinite charged filament. Its solution within the framework of general relativity theory differs substantially from the result obtained in the Newtonian approximation. With increasing distance to the source, the field intensity first decreases, but later the gravitational interaction of the electric field with the filament and with itself causes the field to increase and to tend to infinity at a finite distance from the source. This distance is the limit for the given model, and a position at a greater distance from the filament is impossible. This phenomenon cannot be avoided without resorting to a source with a negative and infinite "nonrenormalized" linear mass density, a situation having hardly any physical meaning.

In Sec. 3 we consider the effect exerted on the spatial Kasner matrix by an electric or magnetic field that depends on one variable. It turns out that no object that might be described as an infinitely long and thin charged filament with finite positive linear mass density, surrounded by a magnetic field with nonzero components H_z and H_φ, can exist within the framework of general relativity.

In Sec. 5 we consider the scalar field around a linear source. For a zero-mass field, a metric is obtained at arbitrary distance from the filament, similar to that obtained by V.A. Belinskiĭ and I.M. Khalatnikov for singularities attainable on spacelike hypersurfaces in the presence of a scalar zero-mass field.[3] For a scalar field with mass, asymptotic expressions can be obtained for the metric near the singularity, where it coincides with the solution for the zero-mass field, and far from the axis, where the field attenuates exponentially and does not influence the metric. It is proved that a phenomenon analogous to the increase of the electric field intensity on account of self-gravitation does not exist for a scalar field. It is shown in the same section that in the presence of a scalar field near a bare singularity the metric cannot have an oscillatory character similar to the oscillatory regime of V.A. Belinskiĭ, E.M. Lifshitz, and I.M. Khalatnikov (BLKh). This oscillatory form can be possessed by the metric near bare singularities of general type in the absence of nongrav-

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itational fields. This is seen from the fact that we can generalize the Kasner spatial metric in the manner used by BLKh to generalize the usual temporal Kasner metric. The solution obtained differs x in practice from the BLKh solution only by the substitution $t \rightleftharpoons x$, describes the bare singularity, and contains the necessary number of physical arbitrary functions for the description of a gravitational field in vacuum.[1)] If a scalar field is present, however, the metric has a power-law asymptotic form near bare singularities. The influence of an electromagnetic field on this oscillatory solution is the subject of Sec. 4. It is proved there that the presence of such a field can not upset this solution.

In Sec. 6 is considered the influence of an electromagnetic field and a scalar field on more complicated singularities of the Kasner type with complex or equal exponents (Ref. 4, p. 492 of original, p. 388 of translation). It is shown that the presence of an electromagnetic field upsets a metric with complex exponents. On the other hand a metric with equal exponents, which describes a gravitational wave of zero frequency in vacuum or around a linear source,[5] admits of only electromagnetic waves of zero frequency, i.e., a static field with $|\mathbf{E}| = |\mathbf{H}|$ and $\mathbf{E} \perp \mathbf{H}$. The presence of a scalar field admits of both types of singularity.

2. FIELD OF A CHARGED FILAMENT WITH MASS

We obtain the electromagnetic and gravitational fields around an infinitesimally thin and long charged rod with mass. We seek a metric in the form

$$dS^2 = a^2(\rho)\,dt^2 - d\rho^2 - b^2(\rho)\,d\varphi^2 - c^2(\rho)\,dz^2, \tag{1}$$

where ρ, φ, and z are cylindrical coordinates. Maxwell's equations

$$\frac{\partial F_{ik}}{\partial x^l} + \frac{\partial F_{li}}{\partial x^k} + \frac{\partial F_{kl}}{\partial x^i} = 0, \tag{2}$$

$$F^{ik}{}_{;k} = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi j^i \tag{3}$$

yield in this case

$$F_{01} = \lambda a b^{-1} c^{-1}, \quad F^{01} = -\lambda (abc)^{-1}; \quad \lambda = \text{const}. \tag{4}$$

The remaining components F_{ik} vanish identically. We can now substitute (4) in the formula for the energy-momentum tensor of the electromagnetic field

$$T_{ik} = \frac{1}{4\pi}\left(-F_{il}F_k^{\ l} + \frac{1}{4}F_{lm}F^{lm}g_{ik}\right) \tag{5}$$

and formulate the Einstein equations. Introducing the quantities

$$\alpha = \ln a, \quad \beta = \ln b, \quad \gamma = \ln c,$$

and a new variable ξ defined by $d\xi = (abc)^{-1}d\rho$, and setting the speed of light and the gravitational constant equal to unity, we get

$$\alpha_{\xi\xi} = -\beta_{\xi\xi} = -\gamma_{\xi\xi} = \lambda^2 e^{2\alpha}, \tag{6}$$

$$\alpha_\xi\beta_\xi + \beta_\xi\gamma_\xi + \alpha_\xi\gamma_\xi = -\lambda^2 e^{2\alpha}. \tag{7}$$

From the first equation we get

$$\alpha_\xi^2 = \text{const} + \lambda^2 e^{2\alpha}, \quad \xi = \int \frac{d\alpha}{(\text{const} + \lambda^2 e^{2\alpha})^{1/2}}. \tag{8}$$

Depending on the sign of the constant, we have three different solutions.

1) If const >0, then

$$\begin{gathered}
\alpha = \mu p_1\xi - \ln\left(1 - \frac{\lambda^2}{4\mu^2 p_1^2}e^{2\mu p_1\xi}\right),\\
\beta = \mu p_2\xi + \ln\left(1 - \frac{\lambda^2}{4\mu^2 p_1^2}e^{2\mu p_1\xi}\right),\\
\gamma = \mu p_3\xi + \ln\left(1 - \frac{\lambda^2}{4\mu^2 p_1^2}e^{2\mu p_1\xi}\right),\\
p_1+p_2+p_3 = p_1^2+p_2^2+p_3^2 = 1, \quad \mu = \text{const} \neq 0, \quad p_1 \neq 0.
\end{gathered} \tag{9}$$

Introducing

$$x = \mu^{-1}e^{\mu\xi}, \quad c^2 = \frac{\lambda^2}{4p_1^2}\mu^{2(p_1-1)}$$

and changing the scales of the axes t, φ, and z, we reduce the solution (9) to the form

$$\begin{gathered}
dS^2 = \frac{x^{2p_1}}{(1-c^2x^{2p_1})^2}dt^2 - (1-c^2x^{2p_1})^2(dx^2 + x^{2p_2}d\varphi^2 + x^{2p_3}dz^2),\\
F_{01} = 2p_1 c x^{2p_1-1}(1-c^2x^{2p_1})^{-2}.
\end{gathered} \tag{10}$$

At $c=0$, i.e., in the absence of an electromagnetic field, this solution goes over the spatial Kasner metric that describes the gravitational field around an infinitely narrow and long linear source.[1] The metric has here a singularity at $x=0$, which cannot be removed if there are no zeros among the numbers (p_1, p_2, p_3). The coordinate x is radial, and this singularity corresponds to a source.

On the other hand if $c \neq 0$, then on top of the singularity $x=0$ we have also at $x^{p_1} = c^{-1}$ a power-law singularity with exponents $(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3) = (-1/2, 1/2, 1/2)$. Assuming that $x=0$ corresponds to the axis of the charged source, as was the case in the absence of a charge, we see that the electric field intensity first decreases with increasing distance from the source, and then increases and tends to infinity as $x \to c^{-1/p_1}$. This is the consequence of the gravitational influence of the electric field on itself. Thus, the radial component x, and with it also the distance to the source axis, has an upper bound:

$$x_{max} = c^{-1/p_1}, \quad l_{max} = \frac{2|p_1|c^{-1/p_1}}{2p_1+1}.$$

It is impossible to continue the coordinate x beyond the singularity, despite the conservation of the signature of the metric, since the singularity at $x = x_{max}$ is a true one, as seen from the fact that the invariants of the curvature tensor as well as the electric field intensity diverge at this point. At $p_1 > 0$ and at small x the metric is asymptotically equal to the metric in the absence of charge. At $p_1 < 0$ the situation changes. In the absence of charge, Eq. (10) describes the gravitational field around a filament of unity mass and of length $\mu = p_1/2 < 0$. At $c \neq 0$ the asymptotic form changes as $x \to 0$. As a result, the linear mass density of the charged filament is equal to $\mu = |p_1|/2 > 0$, a physically clearer physical result.

The Gauss theorem written with allowance for the fact that the field depends only on x

$$\int j^0\sqrt{-g}\,dV = -\frac{1}{4\pi}\oint \sqrt{-g}\,F^{01}d\varphi dz \tag{11}$$

yields an expression for the charge $a = p_1 c$ per unit filament length.

The gravitational and electric fields around a filament with negative mass density can be obtained by identifying in the metric (10) the singularity $x = c^{-1/p_1}$ with the source. In this case the distance to the filament can assume arbitrarily large values. However, when we calculate the mass of the filament and the field in a cylinder of unit length and radius x_0 we find that at small x_0 this mass diverges like $-x_0^{-1}$ and tends to negative infinite mass per unit length of source, while at large x_0 it tends to the limit $\mu = -|p_1|/2 < 0$. This case is therefore of no physical interest.

2) We turn now to Eq. (8). If const $= 0$, then at $\lambda = 0$ we have a Galilean metric expressed in a cylindrical coordinate frame, and at $\lambda \neq 0$ we have a new solution

$$\alpha = -\gamma = -\ln[\lambda(c-\xi)], \quad \beta = A\xi + \ln[\lambda(c-\xi)],$$
$$c = \text{const}, \quad A = \text{const}. \tag{12}$$

Introducing $x = \lambda c A^{-1} e^{A\xi}$, we rewrite the metric in the form

$$dS^2 = \left(1 - (Ac)^{-1} \ln \frac{Ax}{\lambda c}\right)^{-2} dt^2 - \left(1 - (Ac)^{-1} \ln \frac{Ax}{\lambda c}\right)^2 (dx^2 + x^2 d\varphi^2 + dz^2),$$
$$F_{01} = \left[\lambda A c^3 x \left(1 - (Ac)^{-1} \ln \frac{Ax}{\lambda c}\right)^2\right]^{-1}, \quad q = (2\lambda A c^3)^{-1}. \tag{13}$$

This metric also has two non-removable singularities, $x = 0$ and $x = x_{\max} = \lambda c A^{-1} e^{Ac}$, and is similar in character to (19). Unlike the latter, it corresponds to the case when the electric field is in vacuum and becomes concentrated by its own gravitational field. The absence of a source on the axis does not prevent the field from reaching a maximum intensity at $x = x_{\max}$. This metric admits also of an interpretation wherein the singularity $x = x_{\max}$ is associated with an axial source having a negative mass density, an interpretation that seems to have no connection with reality. In contrast to such an interpretation of (10), the metric (13) corresponds to a case when the total mass of the singularity and of the field surrounding it is equal to zero.

3) If we assume const $= -k^2 < 0$ in (8), we obtain a third solution:

$$dS^2 = k^2 \lambda^{-2} \cos^{-2}(k\xi)\, dt^2 - \lambda^2 \cos^2(k\xi)\,(e^{2(A+B)\xi} d\xi^2 + e^{2A\xi} d\varphi^2 + e^{2B\xi} dz^2),$$
$$AB = -k^2, \quad F_{01} = k^2 \lambda^{-1} \cos^{-2}(k\xi), \quad q = \lambda/2. \tag{14}$$

This metric has singularities at $\cos(k\xi) = 0$. We must therefore consider a region of ξ bounded by two singularities. This solution describes the gravitational and electric fields around a filament with negative negative mass density. However, the charge density here is larger than in the cases of metrics (10) and (13), in which x changes from $x_{\max}$ to infinity. Therefore the total mass of the filament of the field, contained in a cylinder of radius ξ_0, becomes equal to zero at $\sin k\xi_0 = 0$, and when ξ_0 increases it becomes positive and the aforementioned concentration of the electric field by its own field with production of a singularity takes place. This solution is likewise of no physical interest.

3. ELECTRIC AND MAGNETIC FIELDS THAT DEPEND ON ONE SPATIAL VARIABLE

We consider now the case when besides the electric field there exists also a magnetic field that depends only on one coordinate. For the metric (1) we have from (2) and (3)

$$F_{01} = \lambda a b^{-1} c^{-1}, \quad F_{12} = \lambda_2 b a^{-1} c^{-1}, \quad F_{13} = \lambda_3 c a^{-1} b^{-1}. \tag{15}$$

Using the old notation, we write down the Einstein equations for this case in the form

$$\begin{aligned} \alpha_{\xi\xi} &= \lambda^2 e^{2\alpha} + \lambda_2^2 e^{2\beta} + \lambda_3^2 e^{2\gamma}, \\ \beta_{\xi\xi} &= -\lambda^2 e^{2\alpha} - \lambda_2^2 e^{2\beta} + \lambda_3^2 e^{2\gamma}, \\ \gamma_{\xi\xi} &= -\lambda^2 e^{2\alpha} + \lambda_2^2 e^{2\beta} - \lambda_3^2 e^{2\gamma}, \\ \alpha_\xi \beta_\xi + \beta_\xi \gamma_\xi + \gamma_\xi \alpha_\xi &= -\lambda^2 e^{2\alpha} + \lambda_2^2 e^{2\beta} + \lambda_3^2 e^{2\gamma}. \end{aligned} \tag{16}$$

We consider first the case $\lambda = \lambda_3 = 0$. We then have the metric

$$dS^2 = (1 + c_2^2 x^{2p_2})^2 (x^{2p_1} dt^2 - dx^2 - x^{2p_3} dz^2) - \frac{x^{2p_2}}{(1 + c_2^2 x^{2p_2})^2} d\varphi^2.$$

We use here the same notation as in (10), and $c_2^2 = \lambda_2^2 (4 p_2^2)^{-1} \mu^{2(p_2 - 1)}$. In contrast to the metric (10), there is no singularity at finite x. This solution practically coincides with the metric of one transition (alternation of epochs) under the influence of the perturbation in the vibrational approach to the singularity reached on the timelike hypersurface for the pseudohomogeneous Bianchi models VIII and IX. The difference between this regime and the BLKh oscillatory solution is that the replacement $t \rightleftarrows x$ is made that the signature is changed. For the case of homogeneous models, the BLKh oscillatory regime goes over into a spatial oscillatory regime in pseudohomogeneous models.

Thus, under the influence of the terms with $\lambda_2 \neq 0$ in the right-hand sides of (16), a transition takes place at $\lambda = \lambda_3 = 0$ from one Kasner regime to another with different Kasner exponents. The terms with $\lambda_3 \neq 0$ lead at $\lambda = \lambda_2 = 0$ to an analogous transition. On the other hand the terms containing λ differ in sign from the perturbations in the pseudohomogeneous models. Therefore, while they do lead to a transition to another Kasner regime, the metric has the singularity (10) in the course of this transition. If the signs of the terms with λ^2 in the right-hand sides of (16) were different, then at $\lambda \neq 0$, $\lambda_2 \neq 0$, and $\lambda_3 \neq 0$ the sought metric would differ from the BLKh solution only by the substitution $t \rightleftarrows x$. Consequently, at $\lambda \neq 0, \lambda_2 \neq 0, \lambda_3 \neq 0$ the solution of (16) is a regime oscillatory in the variable x, in which power-law singularities with exponents $(p_1, p_2, p_3) = (-1/2, 1/2, 1/2)$, which are not connected by the relations $p_1 + p_2 + p_3 = p_1^2 + p_2^2 + p_3^2 = 1$, are present in each transition from one Kasner regime to another (change of epochs) under the action of the perturbation $\lambda^2 e^{2\alpha}$. We therefore break up the x axis into segments by an infinite sequence of such singularities, which condenses to the point $x = 0$. We need consider only one such interval of x.

The metric in this interval describes the electromagnetic and gravitational fields around the source with infinite negative mass density, corresponding to the singularity that bounds this interval from the side of small x. On going away from this interval, the summary linear density of the masses of the source and of the electromagnetic field increases, becomes positive, followed by a concentration of the electromagnetic field by the gravitational one, such that the electric field intensity increases to infinity. This corresponds to the singularity that bounds the interval on the side of large x. In the gap between the two singularities the metric experiences one or several transitions with interchange of the Kasner exponents. However, the need for introducing a negative infinite mass density of the source makes this solution unacceptable from the point of view of physical meaning.

4. INFLUENCE OF ELECTROMAGNETIC FIELD ON THE SPATIAL OSCILLATORY REGIME NEAR BARE SINGULARITIES

In the absence of a field, the metric near bare singularities of general type is oscillatory and differs from the BLKh solution in the replacement $t \rightleftarrows x$ and in a suitable change of signature. The question of the influence of the electromagnetic on the metric will be considered first for the case of pseudohomogeneous models, in which the metric takes the form

$$dS^2 = -dx^2 + [a^2(x) e_\alpha^{(0)} e_\beta^{(0)} - b^2(x) e_\alpha^{(2)} e_\beta^{(2)} - c^2(x) e_\alpha^{(3)} e_\beta^{(3)}] dx^\alpha dx^\beta, \quad (17)$$

where $\mathbf{e}^{(a)}$ stands for the three reference vectors of the given space, $(a) = (0, 2, 3)$. We change to the reference vector components of the tensors used by us (Ref. 4, p. 488 of original, p. 386 of translation). From (3) we get

$$v e_{(c)}^{\alpha} \frac{d(F^{(c)1} abc)}{dx} + F^{(c)(b)} abc \frac{\partial (v e_{(c)}^{\alpha} e_{(b)}^{\beta})}{\partial x^\beta} = 0, \quad v = (\mathbf{e}^{(0)} [\mathbf{e}^{(2)} \times \mathbf{e}^{(3)}]). \quad (18)$$

The reference components $F_{(a)(b)}$ depend here only on x. All the symbols in the equations connected with the analysis for the case of pseudohomogeneous spaces are the same as in Ref. 4.

We multiply (18) by $e^{(a)}$. Noting that in view of the antisymmetry of $F^{(c)(b)}$

$$F^{(c)(b)} e_{(b)}^{\beta} \frac{\partial e_{(c)}^{\alpha}}{\partial x^\beta} = \frac{1}{2} F^{(c)(b)} \left[e_{(b)}^{\beta} \frac{\partial e_{(c)}^{\alpha}}{\partial x^\beta} - e_{(c)}^{\beta} \frac{\partial e_{(b)}^{\alpha}}{\partial x^\beta} \right] = \frac{1}{2} F^{(c)(b)} C_{(b)(c)}^{(d)} e_{(d)}^{\alpha}, \quad (19)$$

we get

$$v (F^{(a)1} abc)' + abc F^{(a)(b)} \frac{\partial (v e_{(b)}^{\beta})}{\partial x^\beta} + \frac{1}{2} abcv F^{(c)(b)} C_{(b)(c)}^{(a)} = 0. \quad (20)$$

The prime denotes hereafter differentiation with respect to x. The second term in (20) can be simplified. For example,

$$\frac{\partial (v e_{(0)}^{\beta})}{\partial x^\beta} = \frac{\partial [\mathbf{e}^{(2)} \times \mathbf{e}^{(3)}]_\beta}{\partial x^\beta} = \operatorname{div}[\mathbf{e}^{(2)} \times \mathbf{e}^{(3)}]$$
$$= \mathbf{e}^{(3)} \operatorname{rot} \mathbf{e}^{(2)} - \mathbf{e}^{(2)} \operatorname{rot} \mathbf{e}^{(3)} = v (C^{(2)(3)} - C^{(3)(2)}). \quad (21)$$

Similar expressions are obtained for $\partial(v e_{(2)}^{\beta})/\partial x^\beta$ and $\sigma(v e_{(3)}^{\beta})/\partial x^\beta$.

Equation (2) yields the following relations for the reference components of the electromagnetic-field tensor:

$$F_{(a)(d)} C_{(b)(c)}^{(d)} + F_{(b)(d)} C_{(c)(a)}^{(d)} + F_{(c)(d)} C_{(a)(b)}^{(d)} = 0, \quad (22)$$

$$F_{1(a)} C_{(b)(c)}^{(a)} + F'_{(b)(c)} = 0. \quad (23)$$

In the case of interest to us, when

$$C_{(3)(2)}^{(0)} = -C_{(2)(3)}^{(0)} = \lambda, \quad C_{(0)(3)}^{(2)} = -C_{(3)(0)}^{(2)} = \mu, \quad C_{(2)(0)}^{(3)} = -C_{(0)(2)}^{(3)} = \nu, \quad (24)$$

and the remaining $C_{(b)(c)}^{(a)}$ vanish identically, relation (22) is always valid, and from (20) and (23) we get

$$\left(F_{1(0)} \frac{bc}{a}\right)' = -\lambda \frac{a}{bc} F_{(2)(3)}, \quad \left(F_{1(2)} \frac{ac}{b}\right)' = -\mu \frac{b}{ac} F_{(3)(0)},$$
$$\left(F_{1(3)} \frac{ab}{c}\right)' = -\nu \frac{c}{ab} F_{(0)(2)}, \quad \lambda F_{1(0)} = F'_{(2)(3)}, \quad (25)$$
$$\mu F_{1(2)} = F'_{(3)(0)}, \quad \nu F_{1(3)} = F'_{(0)(2)}.$$

We consider the case of Bianchi type VIII or IX, when $|\lambda| = |\mu| = |v| = 1$. The solution of (25) is

$$F_{1(0)} = A \frac{a}{bc} \sin\left(\int \frac{a}{bc} dx\right), \quad F_{(2)(3)} = -\lambda A \cos\left(\int \frac{a}{bc} dx\right),$$
$$F_{1(2)} = B \frac{b}{ac} \sin\left(\int \frac{b}{ac} dx\right), \quad F_{(3)(0)} = -\mu B \cos\left(\int \frac{b}{ac} dx\right), \quad (26)$$
$$F_{1(3)} = C \frac{c}{ab} \sin\left(\int \frac{c}{ab} dx\right), \quad F_{(0)(2)} = -\nu C \cos\left(\int \frac{c}{ab} dx\right).$$

Calculating from this the reference components of the energy-momentum tensor, we substitute them in the Einstein equations. The off-diagonal components $R_{(a)(b)}$ vanish identically. Equating to zero the off-diagonal components $T_{(a)(b)}$, we get

$$AB(1+\lambda\mu) = AC(1+\lambda\nu) = BC(1-\nu\mu) = AB(\mu-\lambda)$$
$$= AC(\nu-\lambda) = BC(\nu+\mu) = 0. \quad (27)$$

It is easily seen that these equations are satisfied only if two out of the three quantities A, B, and C are equal to zero. This requirement together with relations (26) leads to a qualitative difference between the form of the electro-magnetic field tensor for Bianchi cases I and VII (IX).

We consider now the diagonal components of the Einstein equations. We introduce as before the quantities α, β, and γ and the variable ξ. Then the equations will differ from (16) in that λ, λ_2 and λ_3 are replaced by A, B, and C, respectively, and in that terms are added to the right-hand side to account for the action of the spatial curvature and ensure in the absence of a field an oscillatory approach to the singularity. We see that the terms with B^2 and C^2 cannot upset this regime, since they act in the same direction as the terms due to spatial curvature. We need therefore consider only a transition in which the terms with A^2 become substantial. The Einstein equations for this term take the form

$$\alpha_{\xi\xi} = -\beta_{\xi\xi} = -\gamma_{\xi\xi} = A^2 e^{2\alpha} - {}^1/_2 e^{4\alpha}. \quad (28)$$

Hence

$$\alpha_\xi^2 = \text{const} + A^2 e^{2\alpha} - {}^1/_4 e^{4\alpha}. \quad (29)$$

This equation is readily integrated. The question of interest to us, however, is whatever singularities are present in the metric when the epoch is changed by a perturbation in the form of the right-hand side of (28), It is easy to see that no such singularity can exist. When α increases the right-hand side of (29) becomes negative, contradicting the positiveness of the left-hand side. Therefore the range of variation of α is bounded from above, thus excluding the singularity.

Thus, an electromagnetic field may either not upset the oscillatory regime (Bianchi type VIII, IX) or can produce such a regime (type II, VI, VII). We note the rather unnatural oscillatory behavior of the field in models VIII and IX and even other Bianchi types (26). The field $F_{1(0)}$ is very strong and oscillates rapidly near the maxima of a, but is small and varies slowly near the minima of a/bc. We note also that two out of the three quantities A, B, and C are zero—a requirement peculiar to types VIII and IX.

In the general case, the oscillatory regime is not upset also by the presence of an electromagnetic field. This follows from the fact that the terms resulting from the action of the spatial curvature contain a higher power e^{α} than the terms due to the electromagnetic field. The singularities at which $e^{\alpha} \to \infty$ are therefore excluded, inasmuch the different parts of the Einstein Equations can then have opposite signs.

5. FILAMENT WITH MASS AS A SOURCE OF A SCALAR FIELD

Consider a scalar field with an energy-momentum tensor

$$T_{ik} = \varphi_{,i}^{*}\varphi_{,k} + \varphi_{,k}^{*}\varphi_{,i} - g_{ik}(\varphi_{,l}^{*}\varphi^{,l} - m^2\varphi^{*}\varphi) \tag{30}$$

and with a source in the form of an infinitely long and thin filament. Both the field and the metric, the latter chosen in the form (1), depend only on the coordinate ρ. The Einstein equations take the form

$$R_1^1 = -m^2\varphi^{*}\varphi - 2\varphi_{,\rho}^{*}\varphi_{,\rho}, \tag{31}$$

$$R_\alpha^\beta = -\delta_\alpha^\beta m^2\varphi^{*}\varphi. \tag{32}$$

The Greek indices α and β run through the values (0, 2, 3) and the coordinate $x^1 = \rho$. In the case of a zero-mass field, $m = 0$, the solution of (31) and (32) is

$$\begin{gathered} dS^2 = \rho^{2p_1}dt^2 - d\rho^2 - \rho^{2p_2}d\varphi^2 - \rho^{2p_3}dz^2, \\ p_1+p_2+p_3=1, \quad p_1^2+p_2^2+p_3^2 = 1-2|\varphi_0|^2, \quad \varphi = \varphi_0 \ln\rho. \end{gathered} \tag{33}$$

For the field with mass (33) is the asymptotic expression for the metric and field at small ρ.

We introduce the quantity $\chi = [(\ln\sqrt{-g})']^{-1}$. Equations (31) and (32) yield

$$\chi' = 1 + 3m^2|\varphi|^2\chi^2 > 0. \tag{34}$$

Inasmuch as at small ρ we get $\chi = \rho > 0$ from (33), it follows that $\chi > 0$ at any ρ, thus excluding at finite ρ any singularity with $\chi = 0$. At large the metric tends asymptotically to the Kasner spatial matrix, and $\varphi \sim e^{-m\rho}\rho^{-1/2}$. Since all the exponents (p_1, p_2, p_3) in the metric (33) can be non-negative at small ρ, the presence of a scalar field upsets the oscillatory regime, which is then replaced by the power-law asymptotic form (33) generalized in the sense of E.M. Lifshitz and I.M. Khalatnikov:

$$\begin{gathered} dS^2 = -dx^2 + (x^{2p_1}e_\alpha e_\beta - x^{2p_2}m_\alpha m_\beta - x^{2p_3}n_\alpha n_\beta)dx^\alpha dx^\beta, \\ p_1+p_2+p_3=1, \quad p_1^2+p_2^2+p_3^2 = 1-2|\varphi_0|^2, \quad \varphi = \varphi_0 \ln x, \end{gathered} \tag{35}$$

where $\mathbf{e}, \mathbf{m}, \mathbf{n}, \varphi_0$, and p_1 are functions of $x^\alpha = (t, y, z)$.

6. BARE SINGULARITIES OF KASNER TYPE WITH EQUAL OR COMPLEX EXPONENTS IN THE PRESENCE OF AN ELECTROMAGNETIC OR SCALAR FIELD

Besides the spatial Kasner metric with real exponents [Eq. (10) with $c = 0$] there exist also two exact vacuum solutions that depend on a single spatial variable. This is the Kasner metric with complex and equal exponents (Ref. 4, p. 492 of original, p. 388 of translation). Their physical meaning was explained in Ref. 5. Let us seen the form that these solutions assume in the presence of an electromagnetic and field and a scalar field. We shall first generalize the metric with complex exponents, expressing it in the form

$$dS^2 = -dx^2 + e^{2\alpha(x)}\cos 2\psi(x)(dx_2^2 - dx_1^2) + 2e^{2\alpha}\sin 2\psi dx_1dx_2 - e^{2\gamma(x)}dy^2. \tag{36}$$

For the electromagnetic field we have from (2) and (3)

$$\begin{gathered} F_{x_1x} = e^{-\gamma}(\lambda_1\cos 2\psi + \lambda_2\sin 2\psi), \\ F_{xx_2} = e^{-\gamma}(\lambda_1\sin 2\psi - \lambda_2\cos 2\psi), \\ \lambda_1 = \text{const}, \quad \lambda_2 = \text{const}. \end{gathered} \tag{37}$$

For the metric (36) we have $R_{x_1}^{x_1} = R_{x_2}^{x_2}$, and from (37) we get

$$T_{x_1}^{x_1} = -T_{x_2}^{x_2} = \frac{\lambda_1^2+\lambda_2^2}{8\pi}\cos 2\psi e^{-2(\gamma+\alpha)}. \tag{38}$$

Consequently, a metric of type (36) can exist only in the absence of an electro-magnetic field. In the presence of a massless scalar field, the Kasner metric with complex exponents takes the form

$$\begin{gathered} dS^2 = -dx^2 + x^{2p'}\cos 2\psi(dx_2^2 - dx_1^2) + 2x^{2p'}\sin 2\psi\, dx_1dx_2 - x^{2p_3}dy^2, \\ \psi = p''\ln(x/a), \quad 2p'+p_3=1, \quad 2p'^2 - 2p''^2 + p_3^2 = 1 - |\varphi_0|^2, \\ \varphi = \varphi_0\ln x, \quad a = \text{const}. \end{gathered} \tag{39}$$

For a field with mass, (39) is the asymptotic expression for the metric at small x. At large x the metric takes the form (39) at $\varphi_0 = 0$. Then $\varphi \sim e^{-mx}x^{-1/2}$. The proof that there are no singularities at finite x is the same as for the preceding metric.

A generalization of the spatial Kasner metric with equal roots is

$$dS^2 = -dx^2 - b(x)e^{2\alpha(x)}dx_2^2 - 2e^{2\alpha}dx_1dx_2 - e^{2(\gamma)x}dy^2. \tag{40}$$

It follows from (2) and (3) that

$$F_{xx_1} = \lambda_1 e^{-\gamma}, \quad F_{xx_2} = \lambda_2 e^{-\gamma} + b\lambda_1 e^{-\gamma}. \tag{41}$$

Hence

$$T_{x_1}^{x_1} = -T_{x_2}^{x_2} = \frac{1}{8\pi}b\lambda_1^2 e^{-2(\gamma+\alpha)}. \tag{42}$$

But since $R_{x_1}^{x_1} = R_{x_2}^{x_2}$, we must put $\lambda_1 = 0$ solving the Einstein equations with allowance for this fact, we get two possibilities

$$\begin{gathered} dS^2 = -dx^2 + [\pm\ln(x/a) + {}^9/_4\lambda^2x^{4/3}]x^{4/3}dx_2^2 - 2x^{4/3}dx_1dx_2 - x^{-2/3}dy^2, \\ F_{xx_2} = \lambda x^{1/3} \end{gathered} \tag{43}$$

and

$$\begin{gathered} dS^2 = -dx^2 + [\pm\ln(x/a) + 2\lambda^2\ln^2 x]dx_2^2 - 2dx_1dx_2 - x^2dy^2, \\ F_{xx_2} = \lambda x^{-1}. \end{gathered} \tag{44}$$

For both metrics $F_{ik}F^{ik} = e^{iklm}F_{ik}F_{lm} = 0$. Thus, a metric of type (40) admits only a field in which $|\mathbf{E}| = |\mathbf{H}|$, $\mathbf{E} \perp \mathbf{H}$, which can be regarded as the limiting case of electromagnetic waves, i.e., a wave of zero frequency. We note that in the absence of a field the metric (40) describes a gravitational wave of zero frequency near a linear source with a definite mass density (43) and in vacuum (44), as proved in Ref. 5.

In the presence of a massless scalar field, the metric (40) takes the form

$$dS^2=-dx^2\pm\ln(x/a)x^{2p_1}dx_2^2-2x^{2p_1}dx_1dx_2-x^{2p_3}dy^2, \tag{45}$$

$$p_1=\frac{1\pm(1-3|\varphi_0|^2)^{1/2}}{3},\quad p_3=\frac{1\mp2(1-3|\varphi_0|^2)^{1/2}}{3},\quad \varphi=\varphi_0\ln x. \tag{46}$$

For a massive scalar field, Eqs. (45) and (46) are the asymptotic expressions for the metric at small x. At large x we have (45) with $(p_1, p_3)=(2/3, -1/3)$ or $(0, 1)$ and $\varphi\sim e^{-mx}x^{-1/2}$. That there are no singularities at finite x is proved in the same manner as in the preceding case.

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The baryonic asymmetry of the Universe

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We discuss a possible process of appearance of an excess of baryons and antileptons during the early stage of expansion of a charge-symmetric hot universe in the framework of a unified gauge theory of strong, weak, and electromagnetic interactions. According to the estimates of the present paper, the baryon asymmetry $A = N_B/N_\gamma$ (the ratio of the mean baryon density to the density of quanta of the background radiation, which, up to a numerical factor, equals the ratio of the number of baryons to the initial entropy of the hot universe in the same comoving volume element) has the order of magnitude $A\sim\alpha^3\vartheta^3\delta_\alpha$ ($\alpha = g^2$ is the coupling constant of the gauge field, ϑ is a quantity of the order of the Cabibbo angle, δ_α is the phase of the complex quark mixing). The numerical coefficient in this formula may contain an additional small parameter. The paper presents some arguments relative to the "multifoliated" (many-sheeted) model of the universe previously proposed by the author.

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§1. INTRODUCTION. ESTIMATE OF THE EFFECT

In 1966 the author has proposed a hypothesis for the appearance of an observable baryon asymmetry of the Universe (and of a conjectured lepton asymmetry) during an early stage of the cosmological expansion out of a charge-symmetric initial state. Such a process is possible owing to effects of CP-violation under nonstationary expansion conditions, if one assumes nonconservation of the baryonic and leptonic charges.[1]

In 1978 an analogous idea has been formulated in a paper of Yoshimura.[2] Yoshimura indicates that in unified gauge theories of strong, weak, and electromagnetic interactions (cf. Ref. 3 and subsequent papers quoted in Ref. 2) baryon number is not conserved, due to interactions in which the "leptoquark" intermediate boson participates, and this together with the violation of CP-invariance leads unavoidably to an excess of baryonic charge (baryon number) during the early stages of the expansion of the hot Universe. Yoshimura indicates the possibility of a quantitative calculation of this effect by means of perturbation theory methods. While he was working on the present paper, the author also learned about the paper of Dimopoulos and Susskind[4] devoted to the same problem.

Below we obtain for the baryon asymmetry an estimate which is close to the one given by Dimopoulos and Susskind,[4] but was obtained from a more detailed consideration of the kinetics of mutual transformation of particles and does not make use of the assumption that the mass of the leptoquark boson has the order of magnitude of the Planck mass $M_0=10^{19}$ GeV. Section 5 contains some considerations related to the "multifoliated model of the Universe" ("many-sheeted model of the Universe") proposed earlier by the author.

The other sections contain the reasoning behind the estimate of the baryon and lepton asymmetry. Here we summarize briefly the main points of this reasoning.

Deviations from particle-antiparticle symmetry manifest themselves only on account of the nonstationarity caused by the expansion of the Universe. We denote the

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