Theory of bremsstrahlung of relativistic electrons and positrons in crystals

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The radiation of soft photons from relativistic electrons and positrons moving in a crystal near one of the crystal planes is considered. General formulas are obtained for the spectral distribution and intensity of the radiation; these formulas are valid both when there is channelling and when there is no channelling, and hold for arbitrary interaction potentials between the particles and the crystal. The difference between the radiation from electrons and that from positrons is examined. The dependence of the intensity of the radiation on the angular spread of the beam of particles is determined.

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1. INTRODUCTION

The well known theory of the bremsstrahlung of relativistic particles moving in a crystal, which was developed in papers by Ferretti,[1] Ter-Mikaelyan,[2,3] and Überall,[4] does not hold when channelling of the particles sets in.[5] The radiation under channelling conditions has been studied in several papers.[5-9] These treatments, however, either involved a very special assumption about the potential of the interaction between the particle and the lattice, or else were restricted to estimates of the spectral intensity of the radiation. Besides this, there has so far been no consideration of the radiation from such particles in the transition region between channeling and nonchannelling conditions. Meanwhile, the influence of the lattice is as important in this transitional range of angles as in the channelling region.

In this paper a theory is developed for the bremsstrahlung of relativistic electrons and positrons moving in a crystal near one of the crystallographic planes. As is well known,[10,11] in this case the motion of a particle in a crystal can be described in the framework of classical mechanics. We shall be concerned with the radiation of soft photons, and therefore we can use classical electrodynamics to describe the emission process. The relative simplicity of this description enables us to use the same approach in treating the radiation from particles both when there is channelling and when there is none, without assuming any sort of model for the interaction potential between the particle and the crystal.

The motion of a particle near one of the crystal planes can be described by a potential averaged over this plane, so that the problem becomes one-dimensional. Owing to this, closed expressions can be obtained for the spectrum and the total intensity of the radiation with an arbitrary potential (Sec. II). The general formulas so obtained cover both cases of channelling and of no channelling, and also the intermediate region.

In Sec. III we consider the degree to which the intensity and spectral distribution of the radiation depend on the form of the interaction potential between the particle and the crystal. In particular, it turns out that an oscillator potential gives an intensity of the radiation higher than that found with more realistic potentials. Meanwhile an oscillator potential is used in a number of papers dealing with bremsstrahlung in a crystal when there is channelling. We note also that an accidental property of the oscillator potential, namely that the frequency of the vibrations is independent of the amplitude, leads to the occurrence of a sharp maximum in the spectral distribution of the radiation, which does not appear with more plausible potentials.

Next, an analysis is given of the influence of the sign of the particle's charge on its bremsstrahlung. It is shown that there is an important difference between the radiations from electrons and positrons in the case of channelling (Sec. III). This is in qualitative agreement with the available experimental data.[12]

Finally, in Sec. IV we take the angular spread of the particles in the incident beam into account and show that this spread smooths out the difference between the radiations from electrons and from positrons.

2. A GENERAL EXPRESSION FOR THE INTENSITY OF THE RADIATION

As is well known, the energy of the radiation from a relativistic particle in the frequency range $(\omega, \omega+d\omega)$ is given in classical electrodynamics by the formula[13]

$$\frac{dE}{d\omega}=\frac{e^2}{4\pi^2}\int[\mathbf{k}\mathbf{I}]^2 do, \tag{2.1}$$

where do is an element of solid angle in the direction of the wave vector $\mathbf{k}$ and

$$\mathbf{I}=\int_{-\infty}^{\infty} dt \exp\{i[\omega t-\mathbf{k}\mathbf{r}(t)]\}\frac{d}{dt}\frac{\mathbf{v}(t)}{\omega-\mathbf{k}\mathbf{v}(t)}. \tag{2.2}$$

Here $\mathbf{v}(t)=d\mathbf{r}(t)/dt$ and $\mathbf{r}(t)$ is the trajectory of the particle in the external field

$$\frac{d}{dt}\frac{m\mathbf{v}(t)}{[1-v^2(t)]^{1/2}}=-\nabla U(\mathbf{r}), \tag{2.3}$$

where $U(\mathbf{r})$ is the potential energy of the interaction of the particle with the external field. (We use a system of units in which the speed of light is $c=1$.)

When the energy of the incident particle is high, the deflection from the direction of the original motion is small, and therefore in first approximation we can represent the velocity $\mathbf{v}(t)$ of the particle in the form

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$$\mathbf{v}(t)=\mathbf{v}(1-{}^1/_2v_\perp^2)+\mathbf{v}_\perp(t), \qquad (2.4)$$

where $\mathbf{v}$ is the velocity of the incident particle and $\mathbf{v}_\perp(t)$ is the component of $\mathbf{v}(t)$ which is perpendicular to $\mathbf{v}$ and $v_\perp \ll v$. We shall assume that the stronger inequality $v_\perp/v \ll m/\varepsilon$ holds; this means that the scattering angle $\vartheta \sim v_\perp/v$ is small in comparison with the characteristic angle $\vartheta_k \sim m/\varepsilon$ at which a relativistic particle emits radiation (m and ε are the mass and energy of the incident particle). Then in Eqs. (2.1) and (2.2) we can make an expansion in terms of the quantities $kv_\perp/(\omega - kv)$ and $\mathbf{k}\cdot\mathbf{r}_\perp(t)$. Keeping the main term of the expansion and integrating over do, we find

$$\frac{dE}{d\omega}=\frac{e^2}{2\pi}\int_\delta^\infty dq\frac{\omega \mathbf{W}^2(q)}{q^2}\left[1-2\frac{\delta}{q}\left(1-\frac{\delta}{q}\right)\right], \qquad (2.5)$$

where

$$\delta=\omega m^2/2\varepsilon^2,\quad \mathbf{W}(q)=\int_{-\infty}^{\infty} dt\dot{\mathbf{v}}_\perp(t)\exp(iqt).$$

To the same accuracy the total energy emitted by the relativistic particle is given by the formula

$$\Delta E=\frac{2e^2}{3}\left(\frac{\varepsilon}{m}\right)^4\int_{-\infty}^{\infty} dt\dot{\mathbf{v}}_\perp^2. \qquad (2.6)$$

If the particle deviates periodically from the original direction of motion, $\mathbf{v}_\perp(t)=\mathbf{v}_\perp(t+T)$, then according to Eq. (2.5)

$$\frac{dE}{d\omega}=N\frac{e^2}{T}\sum_{g>\delta}\frac{\omega \mathbf{W}_T^2(g)}{g^2}\left[1-2\frac{\delta}{g}\left(1-\frac{\delta}{g}\right)\right], \qquad (2.7)$$

where T is the period of the oscillation, N is the number of cycles ($N \gg 1$), and

$$g=2\pi n/T\quad (n=1,2,\ldots),\quad \mathbf{W}_T(g)=\int_0^T dt\dot{\mathbf{v}}_\perp\exp(igt).$$

The total energy of the emitted radiation is given by

$$\Delta E=N\frac{2e^2}{3}\left(\frac{\varepsilon}{m}\right)^4\int_0^T dt\dot{\mathbf{v}}_\perp^2. \qquad (2.8)$$

When the particle moves in a crystal the potential energy $V(\mathbf{r})$ of its interaction with the lattice can be written in the form

$$V(\mathbf{r})=\sum_n U(\mathbf{r}-\mathbf{r}_n), \qquad (2.9)$$

where $U(\mathbf{r}-\mathbf{r})$ is the potential energy of the particle's interaction with the n-th atom, which is at the point $\mathbf{r}_n$. We shall consider the case of a particle which enters the crystal at a small angle θ with the crystallographic plane (x,y) (see Fig. 1), and shall also regard the angle ψ between v and the z axis as small. If $\psi \ll R/a$ and $\theta \ll \psi(R/a)$, where R is the screening radius of the atom and a is the lattice constant ($R \ll a$), the particle will interact with a large number of atoms located in the crystallographic plane (x,y). In this case, as is well known,[10,11] one can describe the motion of the particle in the crystal by using the potential energy (2.9) averaged over the plane (x,y),

$$U_c(x)=\frac{1}{N_yN_za^2}\int dydzV(\mathbf{r}), \qquad (2.10)$$

where N_i is the number of sites along the axis i ($i=x,y,z$). The conditions that have been stated are, however, insufficient for the use of the one-dimensional

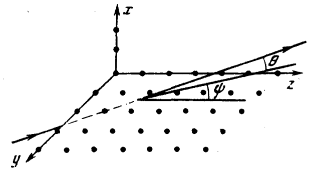

FIG. 1.

potential energy (2.10) to describe the emission of radiation. The point is that the characteristic length along the momentum of the incident particle over which the radiation from an ultrarelativistic particle is formed is of the order of magnitude of δ^{-1}. Therefore for $\delta^{-1} \lesssim a$ the radiation is affected by the location of individual atoms in the plane (x,y), and for $a < \delta^{-1} \lesssim a/\psi$ it is affected by the location of particular chains of atoms in that plane. The one-dimensional potential energy (2.10) can be used to describe the emission of radiation by particles only if the condition $\delta^{-1} \gg a/\psi$ is satisfied.

According to Eqs. (2.3) and (2.4) the trajectory $x(t)$ of the particle in the field $U_c(x)$ can be determined from the equation

$$\ddot{x}=-\frac{1}{\varepsilon}\frac{\partial}{\partial x}U_c(x). \qquad (2.11)$$

A first integral of this equation is

$$v_\perp(t)\equiv\dot{x}(t)=\pm\{2[\varepsilon_\perp(x_0)-U_c(x)]/\varepsilon\}^{1/2}, \qquad (2.12)$$

where $\varepsilon_\perp(x_0)=\varepsilon_\perp+U_c(x_0)$, $\varepsilon_\perp=\varepsilon\theta^2/2$, and x_0 is the impact parameter (the value of the coordinate x at which the particle enters the crystal). From this we find

$$t=\int_{x_0}^{x}\frac{dx}{\dot{x}}. \qquad (2.13)$$

Accordingly, the spectrum $dE(x_0)/d\omega$ and the total emitted energy $\Delta E(x_0)$ for an individual particle with the impact parameter x_0 is given by Eqs. (2.7), (2.8) and (2.11)–(2.13). Integrating with respect to $ds = N_y$ and x_0 and using the fact that the potential is periodic along the x axis, we find the spectrum $dI/d\omega$ and the total intensity I of the radiation in the case of a beam of particles of unit density:

$$\frac{dI}{d\omega}=N_xN_ya\int_0^a\frac{dE(x_0)}{d\omega}dx_0, \qquad (2.14)$$

$$I=N_xN_ya\int_0^a\Delta E(x_0)dx_0. \qquad (2.15)$$

The quantities $W(g)$, $B=\int_0^T dt\dot{v}_\perp^2$, and T that appear in the formulas for $dE/d\omega$ and ΔE can be expressed directly in terms of the potential (2.10):

$$W(g)=\int\frac{\ddot{x}}{\dot{x}}e^{igt}dx,\quad B=\int\frac{\ddot{x}^2}{\dot{x}}dx,\quad T=\int\frac{dx}{\dot{x}}, \qquad (2.16)$$

where $\ddot{x}$, $\dot{x}$, and $t(x)$ are given by Eqs. (2.11)–(2.13). In the expressions (2.16) the integration is taken along the path traversed by the particle during the time T. The limits of integration depend on the value of the impact parameter x_0, the sign of the particle's charge, and the ratio of the quantities $\varepsilon_\perp$ and $|U_c|_{max}$, where $|U_c|_{max}$ is the maximum value of the potential of the crystal plane.

3. THE RADIATION WITH AND WITHOUT CHANNELLING

Let us consider the case of unchannelled particles, $\varepsilon_\perp > |U_c|_{\max}$. The transverse energy $\varepsilon_\perp = \varepsilon\theta^2/2$ is sufficient to carry it over the potential barrier (see Fig. 2), so that the limits of the integrations in the expressions (2.16) will be x_0 and (x_0+a). These formulas can be used both for electrons and for positrons, the only difference being that for electrons $U_c = -|U_c|$, and for positrons $U_c = |U_c|$.

If $\varepsilon_\perp \gg |U_c|_{\max}$, an expansion in terms of the parameter $U_c/\varepsilon_\perp$ can be used in Eq. (2.16). Setting $N = N_z a/T$ and including terms proportional to $U_c/\varepsilon_\perp$, we find

$$\frac{dI}{d\omega} = N_c\frac{2e^2}{3}\frac{a\delta}{\theta}\frac{1}{\theta}\sum_{\tilde g\geqslant\delta/\theta}\left[1-2\frac{\delta}{\tilde g\theta}\left(1-\frac{\delta}{\tilde g\theta}\right)\right]|U_c(\tilde g)|^2 \times\left\{1+\frac{2}{\varepsilon a\theta^2}\frac{1}{\tilde g U_c(\tilde g)}\sum_{\tilde g'}\frac{(\tilde g-\tilde g')^2}{\tilde g'}U_c(\tilde g')U_c(\tilde g-\tilde g')+\ldots\right\}. \tag{3.1}$$

$$I=N_c\frac{2e^2}{3m^2}\left(\frac{\varepsilon}{m}\right)^2 a\int_0^a dx\left(\frac{\partial U_c}{\partial x}\right)^2\left(1+\frac{U_c}{\varepsilon\theta^2}+\ldots\right), \tag{3.2}$$

where $N_c = N_x N_y N_z$ is the number of atoms in the crystal, $g = 2\pi n/a$ ($n = 0, \pm1, \pm2, \ldots$) is the reciprocal lattice vector, and $U_c(g)$ is the Fourier component of the potential,

$$U_c(\tilde g)=\int_0^a dx\,U_c(x)\exp(i\tilde g x). \tag{3.3}$$

The first term in Eq. (3.1) agrees with the result of Überall's paper,[4] which he derived in the first Born approximation, and which naturally does not depend on the sign of the charge of the particle. The correction, which does depend on the sign of the charge, agrees with the result of Ref. 14, which was derived in quantum electrodynamics with the second Born approximation in the interaction of the particle with the field of the crystal.

In the case $\varepsilon_\perp < |U_c|_{\max}$ the transverse energy is too small for the particle to pass over the potential barrier, so that channelling of the particle is possible.[10,11] The condition for a particle with given x_0 to be channelled is $\varepsilon_\perp(x_0) < (U_c)_{\max}$ for positrons and $\varepsilon_\perp(x_0) < 0$ for electrons. The inequality $\varepsilon_\perp(x_0) > U(x)$ defines the range of x over which the motion of a channelled particle occurs for given x_0 and $\varepsilon_\perp$.

All particles can be divided into two groups, channelled and unchannelled. Depending on which of these groups includes an incident particle, there will be different limits of integration in the formulas (2.16) for the quantities $W(g)$, B, and T. For unchannelled particles the limits are as before, x_0 and (x_0+a). For channelled electrons the region of integration consists of two intervals, $(-x_0, x_0)$ and $(x_0, -x_0)$, and for channelled positrons

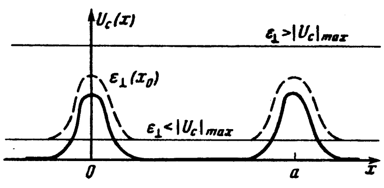

FIG. 2.

it consists of the intervals $(x_0, a-x_0)$ and $(a-x_0, x_0)$. For $\varepsilon_\perp = 0$ all particles are channelled.

Let us now consider the case in which the energy of interaction between the particle and the particle crystal plane is of the form

$$U(x)=V_0\varphi(|x|/R), \tag{3.4}$$

where V_0 is a constant and $\varphi(x/R)$ is a dimensionless function which falls off rapidly for $|x| \gg R$. If, for example, the potential of an individual atom is a screened Coulomb potential

$$U(r)=(Ze|e|/r)\exp(-r/R),$$

then according to Eq. (2.10)

$$U(x)=V_0\exp(-|x|/R),\quad V_0=\frac{2\pi Ze|e|}{a}\frac{R}{a}. \tag{3.5}$$

Substituting Eq. (3.4) in Eq. (3.2) and separating out all dimensional quantities, we get for the total intensity of the radiation when there is no plane channelling of particles the formula

$$I_\pm=N_c\frac{4e^2(aV_0)^2}{3m^2}\frac{\varepsilon^2}{m^2R}\int_0^\infty ds(\varphi')^2\left(1\pm\frac{|V_0|}{\varepsilon\theta^2}\varphi+\ldots\right), \tag{3.6}$$

where $\varphi' = d\varphi(s)/ds$, and the index "$-$" refers to electrons, "$+$" to positrons.

In the case $\varepsilon_\perp = 0$, when all particles are channelled, the total energy of the radiation from positrons and from electrons is given, according to Eqs. (2.15) and (2.17), by

$$I_\pm=N_c\frac{4e^2(aV_0)^2}{3m^2}\frac{\varepsilon^2}{m^2R}\begin{cases}\dfrac{R}{a}A_+\\ A_-\end{cases}, \tag{3.7}$$

where

$$A_+\approx 2\int_0^\infty ds(\varphi(s))^{1/2}\int_s^\infty\frac{[\varphi'(u)]^2}{[\varphi(s)-\varphi(u)]^{1/2}}du, \tag{3.8}$$

$$A_-\approx\int_0^\infty ds\left(\int_0^s\frac{[\varphi'(u)]^2}{[\varphi(u)-\varphi(s)]^{1/2}}du\right)\left(\int_0^s\frac{d\sigma}{[\varphi(\sigma)-\varphi(s)]^{1/2}}\right)^{-1} \tag{3.9}$$

are numerical coefficients of the order of unity.

In the derivation of Eqs. (3.6) and (3.7) we have used the fact that the function $\varphi(|x|/R)$ falls off rapidly.

It is seen from the equations that the total radiated intensity is different for electrons and positrons. For $\varepsilon_\perp > |U_c|_{\max}$ the radiation from positrons is greater than that from electrons, and the difference increases as the angle θ decreases. For $\varepsilon_\perp = 0$, i.e., for $\theta = 0$, the radiation from positrons is smaller than that from electrons by a factor a/R (see Fig. 3). Furthermore, according to Eqs. (3.6) and (3.7), the total radiation from channelled positrons is of order of magnitude a/R times smaller than for unchannelled particles, whereas the radiation of electrons is of the same order of magnitude whether or not there is channelling. The reason for this is that in the field (3.4) the oscillation periods of channelled positrons and electrons differ by a factor a/R.

In this connection we shall examine the radiation of positrons in a crystal in more detail. If, as is often done,[6,7] we take as the interaction potential the oscillator potential

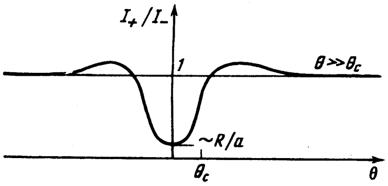

FIG. 3.

$$U_c(x)=4V_0(x-a/2)^2/a^2,$$

then in the absence of channelling the spectrum and the total intensity of the radiation are found from Eqs. (3.1) and (3.2) to be

$$\frac{dI}{d\omega}=N_c\frac{16e^2(aV_0)^2}{\pi^3m^2}\frac{\delta}{g_\theta}\frac{1}{\theta}\sum_n\frac{1}{n^4}\left[1-2\frac{\delta}{ng_\theta}\left(1-\frac{\delta}{ng_\theta}\right)\right], \qquad (3.10)$$

$$I=N_c\frac{32e^2(aV_0)^2}{9m^2}\frac{\varepsilon^2}{m^2a}, \qquad (3.11)$$

where $n\geqslant\delta/g_\theta$, $g_\theta=2\pi\theta/a$. Under conditions of channelling, for $\varepsilon_\perp=0$ we get from Eqs. (2.14) and (2.16)

$$dI/d\omega=N_c\frac{4e^2(aV_0)^2}{3m^2}\frac{\delta}{g}\frac{1}{\theta_c}\left[1-2\frac{\delta}{g}\left(1-\frac{\delta}{g}\right)\right],\quad \delta\leqslant g, \qquad (3.12)$$

$$dI/d\omega=0,\quad \delta>g,$$
$$I=N_c\frac{16e^2(aV_0)^2}{9m^2}\frac{\varepsilon^2}{m^2a}, \qquad (3.13)$$

which agrees with the results of Ref. 6. Here $g=2\pi/T$, $T=\pi a/\theta_c$, where θ_c is the critical angle for channelling, $\theta_c=(2V_0/\varepsilon)^{1/2}$.

Accordingly, for motion of positrons in an oscillator potential the total intensities of the radiation with and without channelling differ by a factor two. The spectral distributions in the two cases are decidedly different. With channelling the spectrum has a single sharp maximum at $g=\delta$, while in the absence of channelling there are many sharp maxima at frequencies which satisfy the conditions $\delta/g_\theta=1,2,\ldots$. The first maximum is order of magnitude smaller than the maximum intensity with channelling by a factor $2\theta/\theta_c$ [see Eq. (3.12)].

Let us now proceed to the study of the spectral distribution of the radiation in the case of the more general potential (3.4). It follows from Eq. (3.1) that in the absence of channelling the spectra of the radiations from electrons and positrons contain a maximum in the range of frequencies $\omega\sim\varepsilon^2\theta/m^2R$. In order of magnitude we have for the maximum intensity

$$\frac{dI}{d\omega}\sim\frac{dI_{\text{am}}}{d\omega}\frac{R^2}{a^2\theta}, \qquad (3.14)$$

where $dI_{\text{am}}/d\omega$ is the intensity of the radiation from an amorphous medium in this same frequency range, $dI_{\text{am}}/d\omega=N_ce^2(Ze^2/m)^2$. In addition, there are sharp maxima and minima in the spectrum (3.1) at $a\delta/2\pi\theta=1,2,\ldots$. With decreasing θ, the intensity of the radiation at the maximum must increase, according to Eq. (3.14). This increase, however, occurs up to the point at which the particles fall into the channellized condition, i.e., at which $\theta>\theta_c$.

We note that with decreasing θ there can also occur a violation of the inequality $\vartheta\ll m/\varepsilon$, which is the condition for the applicability of the general formula (2.7). We shall not deal with this case here; it is considered in detail in earlier papers,[15,16] and we remark only that the inequality in question can be violated only at energies $\varepsilon>10$ GeV.

According to Eqs. (2.14) and (2.16), for channelled particles with $\theta=0$ the maximum radiation for both electrons and positrons lies in the frequency range $\omega\sim\varepsilon^2/m^2T$. The intensities at the maximum are different for electrons and for positrons; for electrons,

$$\frac{dI_-}{d\omega}\sim\frac{dI_{\text{am}}}{d\omega}\frac{R^2}{a^2\theta_c}, \qquad (3.15)$$

whereas for positrons,

$$\frac{dI_+}{d\omega}\sim\frac{dI_{\text{am}}}{d\omega}\frac{R^3}{a^3\theta_c}. \qquad (3.16)$$

Next, we note that in the spectrum (2.7) of the radiation from a single channelled particle there are sharp maxima and minima at $\delta T/2\pi=1,2,\ldots$ However, in the field (3.4) the vibration periods of particles with different impact parameters x_0 are different, so that the positions of the maxima are different for different particles. The result is that the radiation from channelled particles has a spectrum which is a smooth function of the frequency (in contrast with the case discussed earlier of the radiation from unchannelled particles).

We call attention to the fact that the realistic potential (3.4) leads to a picture of the distribution of the radiation which is qualitatively different from that given by the oscillator potential. In the case of the oscillator potential there is a single sharp maximum (inclusion of a small correction to the oscillator potential to allow for anharmonicity[7] leaves this result practically unchanged), whereas in the case of a more realistic potential no sharp maximum appears.

4. INFLUENCE OF THE ANGULAR SPREAD OF THE PARTICLES IN A BEAM ON THE RADIATION

As is well known, actual sources of particles are characterized by a certain spread in the angle θ, and therefore in $\varepsilon_\perp$. When the beam goes through a crystal this spread varies with the depth l (in particular, owing to dechannelling), so that it is natural to describe a beam with a distribution $f(\varepsilon_\perp, l)$. We shall analyze the question of the influence of this spread of the particles in the beam on the radiation.

For ultrarelativistic particles the characteristic distance X over which there is a major change in the distribution of the particles in $\varepsilon_\perp$ is large in comparison with the distance the particle moves during the period of a vibration and with the distance $\delta^{-1}\sim a/\theta_c$ which is characteristic for the radiation when there is channelling (for $\varepsilon=1$ BeV, for example, the dechannellizing length is[11] $X_d>1$ cm, and $a/\theta_c\sim10^{-4}$ cm). In other words, the emission from each layer $(l, l+\Delta l)$, with $a/\theta_c<\Delta l<X$, occurs independently, so that for the spectral distribution and the total intensity we can use the formulas

$$\frac{dI}{d\omega}=N_xN_ya\int_0^a dx_0\int_0^\infty d\varepsilon_\perp\int_0^L dl f(\varepsilon_\perp, l)\frac{d^2E(x_0,\varepsilon_\perp,l)}{d\omega dl}, \qquad (4.1)$$

$$I=N_xN_ya\int_0^a dx_0\int_0^\infty d\varepsilon_\perp\int_0^L dl f(\varepsilon_\perp, l)\frac{d[\Delta E(x_0,\varepsilon_\perp,l)]}{dl}, \qquad (4.2)$$

which are direct generalizations of Eqs. (2.14) and (2.15). Here $d^2E(x_0, \varepsilon_\perp, l)/d\omega dl$ is the energy emitted by the particle in the frequency interval $(\omega, \omega+d\omega)$ per unit length traversed in the crystal, $d\Delta E(x_0, \varepsilon_\perp, l)/dl$ is the total energy emitted per unit path length in the crystal, and L is the thickness of the crystal.

If $L \ll X$, the distribtuion function of the particles can be regarded as independent of l and completely determined by the external source. We shall examine this case in more detail. We confine ourselves to the consideration of the emission from channelled particles, which, as was shown in Sec. III, shows most distinctly the dependence of the radiated intensity on the sign of the charge.

For a strictly parallel beam of particles entering the crystal along a crystal axis [for this case the distribution function is $f(\varepsilon_\perp) = \delta(\varepsilon_\perp)$, where $\delta(\varepsilon_\perp)$ is the delta function], Eq. (3.7) shows that the main contribution to the total intensity comes from particles with impact parameters $(na - R) \lesssim x_0 \lesssim (na + R)$, where n is the number of the plane. Most of the particles in a beam with $\varepsilon_\perp = 0$ [those with impact parameters $(na + R) \lesssim x_0 \lesssim [(n+1)a - R]$ play practically no part in the emission.

Let us now determine the contribution to the radiation from particles that enter the crystal between lattice planes if the distribution function of these particles differs appreciably from zero in the region $\varepsilon_\perp \lesssim |U_c|_{max}$. We shall confine ourselves to the simplest model:

$$f(\varepsilon_\perp) = \begin{cases} 1/|U_c|_{max}, & \varepsilon_\perp \leqslant |U_c|_{max} \\ 0, & \varepsilon_\perp > |U_c|_{max} \end{cases}. \tag{4.3}$$

Since in this case characteristic values of $\varepsilon_\perp$ are of the order of magnitude of $|U_c|_{max}$, particles that enter the crystal in the range of impact parameters $(na - R) \lesssim x_0 \lesssim (na + R)$ will have enough energy to pass over the potential barrier (see Fig. 2). According to Eqs. (2.15) and (2.16) the radiation from each of these particles (in contrast with the case just now considered, that of a strictly parallel beam with $\varepsilon_\perp = 0$ will be comparable with that from particles that enter the crystal in the range of impact parameters $(na + R) \lesssim x_0 \lesssim [(n+1)a - R]$. Since there are many more of these latter particles, the contribution of particles with $(na - R) \lesssim x_0 \lesssim (na + R)$ to the radiation can be neglected. Then to find the total intensity of the radiation we can set the quantity $U_c(x_0)$ equal to zero and carry out the integration over x_0 in Eq. (4.2). The result is

$$I = N_x N_y a^2 \int d\varepsilon_\perp f(\varepsilon_\perp) \Delta E(\varepsilon_\perp), \tag{4.4}$$

where the function $\Delta E(\varepsilon_\perp)$ is given as before by Eq. (2.8), and we have for positrons

$$B = \frac{1}{\varepsilon}\sqrt{\frac{2}{\varepsilon}} \int_{x_1}^{a-x_1} dx \frac{(\partial U_c/\partial x)^2}{(\varepsilon_\perp - U_c(x))^{1/2}}, \quad T = \sqrt{2\varepsilon} \int_{x_1}^{a-x_1} \frac{dx}{(\varepsilon_\perp - U_c(x))^{1/2}}, \tag{4.5}$$

and for electrons

$$B = \frac{1}{\varepsilon\sqrt{2\varepsilon}} \int_0^a dx \frac{(\partial U_c/\partial x)^2}{[\varepsilon_\perp + |U_c(x)|]^{1/2}}, \quad T = \sqrt{\frac{\varepsilon}{2}} \int_0^a \frac{dx}{[\varepsilon_\perp + |U_c(x)|]^{1/2}} \tag{4.6}$$

with x_1 determined from the condition $\varepsilon_\perp = U_c(x_1)$.

Using Eqs. (3.4) and (4.3)–(4.6), we now finally find

$$I_\pm = N_c \frac{4e^2(aV_0)^2}{3m^2} \frac{\varepsilon^2}{m^2 R} \tilde{A}_\pm, \tag{4.7}$$

where

$$\tilde{A}_+ \approx \int_0^\infty ds \varphi'(s)\sqrt{\varphi(s)} \int_s^\infty du \frac{[\varphi'(u)]^2}{\sqrt{\varphi(s) - \varphi(u)}},$$

$$\tilde{A}_- \approx \int_0^\infty ds (\varphi')^2 \left(\sqrt{1+\varphi} + \varphi \ln \frac{\sqrt{\varphi}}{1+\sqrt{1+\varphi}} \right)$$

are numerical coefficients of the order of unity.

Comparing Eqs. (3.7) and (4.9), we see that in the case of channelled positrons the angular spread of the particles in the beam has a major effect on the radiation. The radiation from a strictly parallel beam of positrons has an intensity smaller by a factor a/R than that emitted by a beam with the distribution function (4.3).

In the case of channelled electrons the intensities of the radiation emitted from parallel and from nonparallel beams are of the same order of magnitude.

It follows from Eqs. (3.7) and (4.7) that as the spread of the particles in the beam increases the difference between the intensities radiated by channellized electrons and positrons becomes smaller.

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