Influence of vacuum polarization by a magnetic field on the propagation of electromagnetic waves in a plasma

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The influence of polarization of the electron–positron vacuum by a magnetic field B on the absorption coefficients and refractive indices, and also on the polarization of normal waves in a magnetoactive plasma is considered. If the plasma density is $N \lesssim N_m = 4.5 \times 10^{28} (B/B_0)^4$ cm^{-3}, where $B_0 = 4.4 \times 10^{13}$ G, this influence significantly changes the polarization, spectrum, and directional diagram of the plasma emission. In particular, near the so-called critical frequencies $\omega_{c1,2} < \omega_B = eB/mc$ at which the influences of the plasma and the vacuum on the linear polarizations are compensated, the vacuum effects change linear polarization of the radiation into circular polarization, while for frequencies $\omega >> \omega_B$ they change circular into linear polarization. The emission spectrum of an optically thick plasma in the region of the critical frequencies has spectral features in the form of absorption or emission lines. Near the critical frequencies, the directional diagram of the emission of an optically thick plasma contracts strongly to the plane perpendicular to the magnetic field (giving rise to a fan diagram), and side lobes appear. These effects must be manifested in the emission of cosmic objects with a strong magnetic field. In particular, the spectral features in the region of the critical frequencies could imitate the "cyclotron" lines observed in x-ray pulsars.

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1. In a strong magnetic field, the electron-positron vacuum causes photons to propagate as in an anisotropic medium with birefringent properties.[1-5] For photons with energy $\hbar\omega \ll mc^2$ in a magnetic field $B \ll B_0 = m^2c^3/e\hbar = 4.4\times10^{13}$G, the vacuum polarization is described by permittivity ε and magnetic permeability μ given by[3]

$$\varepsilon_{ik}^{(v)} = \delta_{ik}(1-2a) + 7a\frac{B_iB_k}{B^2}, \quad a = \frac{\alpha}{45\pi}\left(\frac{B}{B_0}\right)^2,$$
$$\mu_{ik}^{-1(v)} = \delta_{ik}(1-2a) - 4a\frac{B_iB_k}{B^2}, \quad \alpha = \frac{e^2}{\hbar c}. \qquad (1)$$

The departure of ε and μ from unity is determined by a, which is very small ($\sim10^{-8} - 10^{-6}$) even for the very strong fields $B \sim 10^{12} - 10^{13}$G thought to be present near the surface of neutron stars. Nevertheless, vacuum polarization may have a strong influence on the radiation of cosmic objects with strong magnetic fields.

This is due, first, to the large magnitude of the magneto-optical effects (of the Faraday and Cotton-Mouton type) resulting from the phase shift $\chi \sim al/\lambda$ between the normal waves in a magnetized vacuum (for example, for x rays of wavelength $\lambda \sim 10$Å leaving the surface of a neutron star with field $B \sim 3\times10^{12}$ G, the shift is $\chi \sim 1$ over a path length of $l \sim 0.1$ cm). This fact was noted by Novick *et al.*,[6] who concluded erroneously[7] on this basis that the x-ray emission of neutron stars will be completely depolarized.

Second, as is shown in Refs. 7 and 8, the vacuum polarization by the magnetic field has a strong influence on the generation and propagation of radiation in the magnetoactive plasma of a number of cosmic objects. This is because the parameter $v = \omega_p^2/\omega^2$ ($\omega_p = (4\pi Ne^2/m)^{1/2}$ is the electron plasma frequency), which determines the deviation of the plasma permittivity from unity, can be $\lesssim a$ for real values of the electron density N and the frequency ω. For example, the ratio

$$\frac{v}{a} = \frac{N}{1.5\cdot10^{28}[\text{cm}^{-3}]}\left(\frac{B}{B_0}\right)^2\left(\frac{mc^2}{\hbar\omega}\right)^2$$

for $B = 4\times10^{12}$ G and $\hbar\omega = 50$ keV is $\lesssim 1$ if $N \lesssim 10^{24}$ cm^{-3}, which is the case for the emitting region of x-ray pulsars in accordance with current ideas. At $B = 4\times10^8$ G (such fields are present in the surface layers of some white dwarfs) and $\hbar\omega = 2.5$ eV, the ratio $v/a \lesssim 1$ for $N \lesssim 2.5\times10^7$ cm^{-3}, i.e., the vacuum polarization may also influence the propagation of optical radiation in the coronas of magnetic white dwarfs.

Thus, for an interpretation of the x-ray, ultraviolet, and optical observations of astrophysical objects with strong magnetic fields it is important to study the influence of vacuum polarization on the generation, absorption, and propagation of electromagnetic radiation in a magnetoactive plasma. To solve this problem, it is necessary to investigate the complex refractive indices of the system consisting of a plasma and the vacuum in a magnetic field. Approximate formulas for the real parts of the refractive indices were obtained (but not investigated) by Adler[3] for the special case when the contribution of the plasma to the polarizability is small compared with the contribution of the vacuum. Expressions for the complex refractive indices were obtained by Gnedin *et al.*,[7,8] who also drew attention to some qualitative effects of vacuum polarization on the polarization, refraction, and absorption of photons in a magnetoactive plasma. Some effects of this kind were also noted by Mészáros and Ventura[1)] in Ref. 9 for the case when Thomson scattering is the main process of interaction of the photons and the plasma electrons.

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In the present paper, we obtain (in Sec. 2) and investigate in detail (Secs. 3 and 4) expressions for the refractive indices and absorption coefficients, and also for the polarization of normal waves in a system consisting of a plasma and vacuum in a magnetic field. In Sec. 5, we consider the influence of vacuum polarization on the emission of optically thin and optically thick plasmas. The main conclusions of the paper are given in Sec. 6.

2. A plane wave in an anisotropic medium is conveniently regarded as a superposition of normal waves that propagate independently if photon scattering processes are not important. The complex refractive indices $n_j = \varkappa_j - ik_j$ (j is the number of the wave) of these waves are determined by the conditions of the existence of a nontrivial solution of the system of equations for the components of the electric field $\mathbf{E}$ of the wave:

$$\varepsilon_{im}E_m + n^2 e_{iqs} l_q \mu_{sk}^{-1} e_{kpm} l_p E_m = 0, \tag{2}$$

where $\mathbf{l}$ is the direction of propagation of the wave, and e_{iqs} is the unit antisymmetric tensor. Taking the direction l as the z axis, we obtain the relation $E_z = -(\varepsilon_{zx}E_x + \varepsilon_{zy}E_y)/\varepsilon_{zz}$, and using this we can transform (2) into a system of two equations for E_x and E_y. Equating to zero the determinant of this system, we obtain the dispersion relation for n_j:

$$(\tilde{\varepsilon}_{xx} - n_j^2\mu_{yy}^{-1})(\tilde{\varepsilon}_{yy} - n_j^2\mu_{xx}^{-1}) - (\tilde{\varepsilon}_{xy} + n_j^2\mu_{yx}^{-1})(\tilde{\varepsilon}_{yx} + n_j^2\mu_{xy}^{-1}) = 0;$$
$$\tilde{\varepsilon}_{ik} = \varepsilon_{ik} - \varepsilon_{iz}\varepsilon_{zk}/\varepsilon_{zz}, \quad i, k = x, y. \tag{3}$$

The real and imaginary parts of the roots of Eq. (3) determine the refractive indices $\varkappa_j$ and the absorption coefficients $\mu_j = (2\omega/c)k_j$ of normal waves. The polarization of the normal waves is determined by the ratio

$$\alpha_j \equiv -\left(\frac{E_x + iE_y}{E_x - iE_y}\right)_j = -\frac{(\tilde{\varepsilon}_{xy} - i\tilde{\varepsilon}_{xx}) + n_j^2(\mu_{yx}^{-1} + i\mu_{yy}^{-1})}{(\tilde{\varepsilon}_{xy} + i\tilde{\varepsilon}_{xx}) + n_j^2(\mu_{yx}^{-1} - i\mu_{yy}^{-1})} \equiv |\alpha_j| e^{i\delta_j}. \tag{4}$$

The modulus and phase δ_j of this ratio determine the ellipicity K_j (the minor to major axis of the polarization ellipse) and the position angle χ_j between the major axis of the polarization ellipse and the y axis:

$$K_j = (|\alpha_j| - 1)/(|\alpha_j| + 1), \quad \chi_j = \delta_j/2. \tag{5}$$

The sign of K_j determines the direction of rotation of the vector $\mathbf{E}$ ($K_j < 0$ and $K_j > 0$ correspond to right- and left-handed polarization, respectively). For a medium with unit magnetic permeability ($\mu_{sk} = \delta_{sk}$), Eqs. (3) and (4) take the well-known form (see, for example, Ref. 10).

To find n_j and α_j for a magnetoactive plasma with allowance for vacuum polarization, we must determine ε_{ik} and μ_{ik}^{-1}. For a plasma without allowance for vacuum polarization, we can assume $\mu_{ik}^{-1(p)} = \delta_{ik}$, and for $\varepsilon_{ik}^{(p)}$ use the well-known expressions.[10] To take into account not only bremsstrahlung absorption of photons but also Thomson scattering processes, which are important in a low-density high-temperature plasma, we must add the radiative width $\nu_r = 2\omega^2 e^2/3mc^3$ to the effective frequencies of electron-ion collisions. In addition, we must remember that in a strong magnetic field the effective collision frequency depends on the polarization of the radiation, and this can be taken into account by introducing two frequencies $\nu_\parallel$ and $\nu_\perp$ for the polarizations along and at right angles to the magnetic field.[11] Then in a coordinate system with z axis along l and vector $\mathbf{B}$ in the zy plane,

$$\varepsilon_{xx}^{(p)} = 1 - \frac{v(1-i\gamma_\perp)}{(1-i\gamma_\perp)^2 - u},$$
$$\varepsilon_{yy}^{(p)} = 1 - v\frac{(1-i\gamma_\perp)[1-i(\gamma_\parallel\cos^2\vartheta + \gamma_\perp\sin^2\vartheta)] - u\sin^2\vartheta}{(1-i\gamma_\parallel)[(1-i\gamma_\perp)^2 - u]},$$
$$\varepsilon_{xy}^{(p)} = -\varepsilon_{yx}^{(p)} = \frac{-ivu^{1/2}\cos\vartheta}{(1-i\gamma_\perp)^2 - u}, \tag{6}$$
$$\varepsilon_{xz}^{(p)} = -\varepsilon_{zx}^{(p)} = \frac{ivu^{1/2}\sin\vartheta}{(1-i\gamma_\perp)^2 - u},$$
$$\varepsilon_{yz}^{(p)} = \varepsilon_{zy}^{(p)} = \frac{v[u + i(\gamma_\perp - \gamma_\parallel)(1-i\gamma_\perp)]\cos\vartheta\sin\vartheta}{(1-i\gamma_\parallel)[(1-i\gamma_\perp)^2 - u]},$$
$$\varepsilon_{zz}^{(p)} = 1 - v\frac{(1-i\gamma_\perp)[1-i(\gamma_\parallel\cos^2\vartheta + \gamma_\perp\sin^2\vartheta)] - u\cos^2\vartheta}{(1-i\gamma_\parallel)[(1-i\gamma_\perp)^2 - u]},$$

where $v = \omega_p^2/\omega^2$, $u = \omega_B^2/\omega^2$, $\omega_B = eB/mc$ is the electron cyclotron frequency, $\gamma_{\parallel,\perp} = (\nu_{\parallel,\perp} + \nu_r)/\omega$, and ϑ is the angle between the vectors l and $\mathbf{B}$.

Strictly speaking, Eqs. (6) hold only for sufficiently high frequencies $\omega \gg \nu_{\parallel,\perp} + \nu_r$ outside the region of cyclotron resonance $|\omega - \omega_B| \gg \nu_{\parallel,\perp} + \nu_r$, where the Hermitian part of $\varepsilon^{(p)}$ can be assumed to be independent of $\nu_{\parallel,\perp} + \nu_r$, and the anti-Hermitian part to have a linear dependence on it. The dependences of $\nu_{\parallel,\perp}$ on ω, B, and the temperature T in this case are well known.[11,12] Note that in the region $|\omega - \omega_B| \lesssim \nu_D$, where $\nu_D = \omega_B \times (kT/mc^2)^{1/2}\cos\vartheta$ is the Doppler width of the cyclotron line (in the relevant case of x-ray pulsars $\nu_D \gg \nu_{\parallel,\perp} + \nu_r$), Eqs. (6) are invalid because allowance has not been made for spatial dispersion (thermal motion of the electrons along the magnetic field).

For vacuum without plasma, we obtain from (1)

$$\varepsilon_{xx}^{(v)} = \mu_{xx}^{-1(v)} = 1 - 2a, \quad \varepsilon_{yy}^{(v)} = 1 - 2a + 7a\sin^2\vartheta, \quad \varepsilon_{zz}^{(v)} = 1 - 2a + 7a\cos^2\vartheta,$$
$$\mu_{yy}^{-1(v)} = 1 - 2a - 4a\sin^2\vartheta, \quad \mu_{zz}^{-1(v)} = 1 - 2a - 4a\cos^2\vartheta,$$
$$\varepsilon_{ik}^{(v)} = \mu_{ik}^{-1(v)} = 0 \quad (\text{for } i \neq k). \tag{7}$$

The anti-Hermitian part of the tensors (7) is zero, since photon absorption through pair creation when $\hbar\omega \ll mc^2$ is still impossible even in a strong magnetic field.

We shall be interested in a plasma of such low density that $|\varepsilon_{ik}^{(p)} - \delta_{ik}| \ll 1$. A similar condition is satisfied for $\varepsilon_{ik}^{(v)}$ and $\mu_{ik}^{-1(v)}$. Then the polarizability of the system consisting of the plasma and vacuum in a magnetic field is the sum of the polarizability of the plasma without allowance for the vacuum polarization and the polarizability of the vacuum in the absence of the plasma, i.e.,

$$\varepsilon_{ik} = \varepsilon_{ik}^{(p)} + \varepsilon_{ik}^{(v)} - \delta_{ik}, \quad \mu_{ik}^{-1} = \mu_{ik}^{-1(v)}. \tag{8}$$

Under the same conditions $\tilde{\varepsilon}_{ik} \approx \varepsilon_{ik}$ and the roots of the dispersion relation (3) and the ratio (4) have the form

$$n_j = \varkappa_j - ik_j = n_I \pm (n_L^2 + n_C^2)^{1/2}, \tag{9}$$
$$\alpha_j = [-n_C \mp (n_L^2 + n_C^2)^{1/2}]/n_L; \quad j = 1, 2, \tag{10}$$

where

$$n_I = \varkappa_I - ik_I = 1 + (\varepsilon_{xx} + \varepsilon_{yy} - \mu_{xx}^{-1} - \mu_{yy}^{-1})/4,$$
$$n_L = \varkappa_L - ik_L = (\varepsilon_{xx} - \varepsilon_{yy} + \mu_{xx}^{-1} - \mu_{yy}^{-1})/4, \quad n_C = \varkappa_C - ik_C = i\varepsilon_{xy}/2 \tag{11}$$

(in writing down (11), we have remembered that $\mu_{xy}^{-1} = \mu_{yx}^{-1} = 0$, $\varepsilon_{xy} + \varepsilon_{yx} = 0$).

For $\gamma_{\parallel,\perp} \ll 1$ and $|u - 1| \gg \gamma_{\parallel,\perp}, \nu_D/\omega$, we obtain from

(6)–(8) and (11)

$$\varkappa_I = 1 - \frac{v(2-u\sin^2\vartheta)}{4(1-u)} + \frac{11}{4}a\sin^2\vartheta,$$

$$k_I = \frac{v[\gamma_\perp(1+u)(1+\cos^2\vartheta)+\gamma_\parallel(1-u)^2\sin^2\vartheta]}{4(1-u)^2},$$

$$\varkappa_L = -\left(\frac{vu}{1-u}+3a\right)\frac{\sin^2\vartheta}{4}, \tag{12}$$

$$k_L = \frac{v[\gamma_\perp(1+u)-\gamma_\parallel(1-u)^2]\sin^2\vartheta}{4(1-u)^2},$$

$$\varkappa_C = \frac{vu^{1/2}\cos\vartheta}{2(1-u)}, \quad k_C = -\frac{vu^{1/2}\gamma_\perp\cos\vartheta}{(1-u)^2}.$$

Equations (9) and (10) can be simplified for special cases. The regions of applicability of the simplified expressions are determined by the relationships between the real parameters q and p, which determine the polarization of the normal waves:

$$q+ip\equiv -n_L/n_C \equiv -(\varkappa_L - ik_L)/(\varkappa_C - ik_C). \tag{13}$$

If quantities proportional to $\gamma^2_{\parallel,\perp}$ are ignored, then

$$q \approx -\frac{\varkappa_L}{\varkappa_C} = q_0\left(1-\frac{N_m}{N}\frac{u-1}{u^2}\right), \quad p = \frac{q_0}{u}\left[\gamma_\parallel(u-1)+\gamma_\perp\left(1-\frac{2N_m}{N}\right)\right]$$

$$N_m = \frac{1}{60\pi^2}\left(\frac{mc}{\hbar}\right)^3\left(\frac{\hbar\omega_B}{mc^2}\right)^4 \approx 4.5\cdot 10^{28}\left(\frac{B}{B_0}\right)^4 \quad [\mathrm{cm}^{-3}],$$

$$q_0 = u^{1/2}\sin^2\vartheta/2\cos\vartheta. \tag{14}$$

In the region $p^2 \ll q^2$, we obtain from (10) the results[2)]

$$\alpha_j = \frac{1\pm(1+q^2)^{1/2}}{q}\left[1\mp\frac{ip}{q(1+q^2)^{1/2}}\right], \quad K_j = \pm\frac{1}{|q|+(1+q^2)^{1/2}},$$

$$\chi_j = \mp\frac{p}{2q(1+q^2)^{1/2}} - \frac{\pi}{2}\frac{1\mp\mathrm{sign}\, q}{2}\mathrm{sign}\, p. \tag{15}$$

Equation (9) can be simplified similarly in the larger region $p^2 \ll 1+q^2$:

$$\varkappa_j = \varkappa_I \pm \frac{\varkappa_C - q\varkappa_L}{(1+q^2)^{1/2}} \approx \varkappa_L \pm (\varkappa_C^2+\varkappa_L^2)^{1/2},$$

$$k_j = k_I \pm \frac{k_C - qk_L}{(1+q^2)^{1/2}} = k_I \pm \frac{\varkappa_C k_C + \varkappa_L k_L}{(\varkappa_C^2+\varkappa_L^2)^{1/2}}. \tag{16}$$

It is clear from (14) that if vacuum polarization is ignored ($N_m/N \to 0$, "vacuumless" plasma) the condition $\gamma^2_{\parallel,\perp} \ll 1$ is sufficient for the applicability of (15) and (16). For $q=q_0$ we obtain from (15) and (16) the usual[10,13] expressions for a low-density magnetoactive plasma. In the limit $N \to 0$ (magnetized vacuum without plasma) $\gamma_{\parallel,\perp} \to 0$, $p \to 0$, $|q| \to \infty$, and Eqs. (15) and (16) give the result[3]

$$n_1 = 1+(7a\sin^2\vartheta)/2, \quad n_2 = 1+2a\sin^2\vartheta, \quad \chi_1 = 0,$$

$$\chi_2 = \pi/2, \quad K_{1,2} = 0. \tag{17}$$

In this case, the normal waves are polarized linearly for all $\vartheta \neq 0$. Note that normal waves in a magnetized electron-positron plasma have the same polarization, which seems natural, since the vacuum polarization is due to the creation of virtual electron-positron pairs.

As follows from (14), the conditions of applicability of (15) and (16) can be violated[3)] in the region $u > 1$, $N_m/N > 4$ near the (critical) values of the density N_c, the frequencies $\omega_{c1,2}$, and the magnetic field B_c at which the parameter q vanishes:

$$N_c = \frac{N_m(u-1)}{u^2}, \quad \omega^2_{c1,2} = \frac{\omega_B^2}{2}\left[1\pm\left(1-\frac{4N}{N_m}\right)^{1/2}\right] \equiv \omega_B^2 u^{-1}_{c1,2}(N,B),$$

$$B_c^2 = \left(\frac{mc\omega}{e}\right)^2\left(1+\frac{Nu^2}{N_m}\right) \equiv \left(\frac{mc\omega}{e}\right)^2 u_c(N,\omega). \tag{18}$$

The condition $p^2 \ll q^2$ is violated if the difference of any of the parameters N, ω, and B from the corresponding critical value is less than some quantity proportional to γ (the actual expression for this quantity is readily obtained from (14)). The condition $p^2 \ll 1+q^2$ is also satisfied in the region of violation of $p^2 \ll q^2$ provided $\beta \equiv |\pi/2 - \vartheta| \gg \beta_n = |\pi/2 - \vartheta_n|$, where ϑ_n is determined from the condition $p(q=0) = \pm 1$ or

$$\frac{\cos\vartheta_n}{\sin\vartheta_n} = \pm\frac{1}{2u^{1/2}}\left[\gamma_\parallel(u-1)+\gamma_\perp\left(1-\frac{2N_m}{Nu}\right)\right]_{q=0}. \tag{19}$$

In cases of practical interest $\gamma_{\parallel,\perp} \sim 10^{-5}-10^{-3}$, and the angle ϑ_n is close to $\pi/2$ (β_n is near zero), except for a critical point lying near $u=1$ for large N_m/N. For example, if $B = 4.4\times10^{12}$ G ($\hbar\omega_B = 51$ keV, $N_m = 4.5\times10^{24}$ cm^{-3}) and $N = 4.5\times10^{21}$ cm^{-3} for the critical frequencies $\omega_{c1} = (1-5\times10^{-4})\omega_B$ and $\omega_{c2} = 0.032\omega_B$ (at these frequencies, $\gamma_{\parallel,\perp} \approx \nu_r/\omega = \gamma_r$, $\gamma_r(\omega_{c1}) = 0.49\times10^{-3}$, and $\gamma_r(\omega_{c2}) = 1.6\times10^{-3}$) the angles β_n are, respectively, 23.5° and 0.014°.

The meaning of the angle $\vartheta_n = \pi/2 \pm \beta_n$ is clear from the analysis of the mutual orthogonality of the polarizations of the normal waves. The orthogonality is described[14] by the parameter ξ $(0 \le \xi \le 1)$:

$$\xi^2 = [1-(1-\eta^2)^{1/2}]/[1+(1-\eta^2)^{1/2}], \quad \eta = 2p/(1+q^2+p^2). \tag{20}$$

For $p^2 \ll q^2$ (as in a vacuumless plasma and in a pure vacuum) and for $p^2 \gg q^2$, the parameter ξ satisfies $\xi \ll 1$, i.e., the normal waves are virtually orthogonal (the axes of the polarization ellipses are mutually perpendicular, and the ellipses themselves are similar). For $q=0$, $p=\pm1$ (or $\varkappa_L = \pm k_c$, $\varkappa_c = \mp k_L$) we have $\xi = 1$, i.e., the normal waves are maximally nonorthogonal $K_1 = K_2 = 0$, $\chi_1 = \chi_2 = \pm\pi/4$. At the same time, we also have $\varkappa_1 = \varkappa_2$, $k_1 = k_2$, i.e., we have the case of essentially multiple roots of the dispersion relation.[10] Thus, ϑ_n is the angle at which maximal nonorthogonality (coincidence) of the normal waves is attained if any of the parameters N, ω, or B is equal to its critical value.

The interval of angles β in which the nonorthogonality is appreciable ($\xi < 0.5$) is maximal at the critical points, and is determined there by the inequality $0.5\beta_n < \beta < 2\beta_n$. The intervals of nonorthogonality with respect to the parameters N, ω, and B, which are maximal for $\vartheta = \vartheta_n$ at the corresponding critical points, are proportional to $\gamma_{\parallel,\perp}$ and are determined by the condition $|q(\vartheta=\vartheta_n)| \le 1$. In the limit $\gamma_{\parallel,\perp} \to 0$, we have $\vartheta_n \to \pi/2$, and the interval of nonorthonality with respect to any of the parameters collapses to a point. Note that in the region of nonorthogonality the usually employed transfer equations for the normal waves[14] are inapplicable, and the passage of the normal waves through such a region in an inhomogeneous plasma is accompanied by numerous effects associated with the breakdown of the approximation of geometrical optics.[10]

Near the critical points, where Eqs. (15) are invalid, the polarization of the normal waves is conveniently investigated by means of other approximate expressions:

$$\alpha_j = [1\pm(1-p^2)^{1/2}]/ip,$$

$$K_j = \begin{cases}\pm(1-p^2)^{1/2}/(1+|p|) \\ 0\end{cases} \quad \chi_j = \begin{cases}-{}^1/_4\pi\,\mathrm{sign}\,p, & p^2\le 1 \\ \pm{}^1/_2\tan^{-1}(p^2-1)^{-1/2} - {}^1/_4\pi(\mathrm{sign}\,p\pm1), & p^2\ge 1\end{cases} \tag{21}$$

which hold for $p^2 \gg q^2$, $q^2 \ll 1$.

For $p^2 - 1 \gg q^2$, which corresponds to the region $|q(\vartheta = \vartheta_n)| \lesssim 1$, $\beta \ll \beta_n$, in which $p \approx k_L/\varkappa_c$, the refractive indices and absorption coefficients are given by

$$\varkappa_j = \varkappa_I \pm \frac{p\varkappa_L + k_C}{(p^2-1)^{1/2}} \approx \varkappa_I \pm \frac{\varkappa_L k_L + \varkappa_C k_C}{(k_L^2 - \varkappa_C^2)^{1/2}},$$
$$k_j = k_I \pm \frac{pk_L - \varkappa_C}{(p^2-1)^{1/2}} \approx k_I \pm (k_L^2 - \varkappa_C^2)^{1/2}. \qquad (22)$$

Finally, at the critical point $(q=0)$

$$\varkappa_j = \begin{cases} \varkappa_I \pm (\varkappa_C^2 - k_L^2)^{1/2} \\ \varkappa_I \pm k_C (k_L^2 - \varkappa_C^2)^{1/2}/\varkappa_C \end{cases}, \quad k_j = \begin{cases} k_I \pm k_C (\varkappa_C^2 - k_L^2)^{1/2}/\varkappa_C, & (\varkappa_C^2 > k_L^2,\ \beta > \beta_n) \\ k_I \pm (k_L^2 - \varkappa_C^2)^{1/2}, & (\varkappa_C^2 < k_L^2,\ \beta < \beta_n) \end{cases} \qquad (23)$$

In the region of applicability of (22), as of (15) and (16), the normal waves are orthogonal, whereas (21) and (23) describe the behavior of K_j, χ_j, and $\varkappa_j$, k_j in the region of nonorthogonality as well.

3. In accordance with (15), the ellipticity of the normal waves K_j is everywhere, except for narrow regions of widths proportional to γ around the critical points, determined by the parameter q [see (18); Fig. 1]: $K_j \approx \pm 1$ for $|q| \ll 1$ (circular polarization and quasilongitudinal propagation) and $K_j \approx 0$ and $|q| \gg 1$ (linear polarization and quasitransverse propagation). If the electron density satisfies $N \gg N_m |u-1| u^{-2}$ (for given ratio N_m/N, this corresponds to the range of frequencies in which the corresponding continuous curve in Fig. 1 approaches fairly close to the dashed line), then $q \approx q_0$ and the polarization of thenormal waves is determined by the plasma: for $\omega \ll \omega_B$, the normal waves are polarized linearly in a wide range of angles; for $\omega \gg \omega_B$, they are circularly polarized. At the same time, for one wave (in a vacuumless plasma it is usually called the ordinary wave and denoted by the index 2) the angle χ_j is near zero, and $K_j > 0$; for the other (extraordinary; $j=1$) $\chi_j \approx \pm\pi/2$, $K_j < 0$.

For $N \lesssim N_m |u-1| u^{-2}$, the vacuum polarization becomes important, and the behavior of K_j and χ_j changes

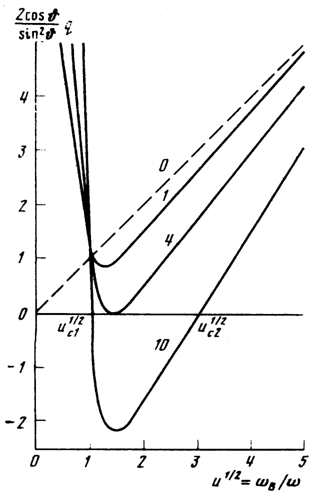

FIG. 1. Influence of vacuum polarization on the dependence $q(\omega_B/\omega)$. The numbers next to the curves give the values of N_m/N. Here and in the remaining figures, the dashed lines correspond to a "vacuumless" plasma.

qualitatively. As N, ω, or B approach the critical values ($u \to u_{c1,2}$ in Fig. 1) $|q|$ decreases, i.e., the polarization of the normal waves becomes nearly circular even for $\omega \ll \omega_B$ (in contrast to a vacuumless plasma and pure vacuum). This occurs[7] because of the compensation of the opposite influences of the plasma and the vacuum near the critical points for $u > 1$ on the linear polarization (in (12), $\varkappa_L \to 0$). At the critical values of N, ω, and B and in the immediate proximity of them, the behavior of K_j and χ_j can, depending on the relationship between β and β_n, correspond to two possibilities (see (21)). Both possibilities are shown in Fig. 2 by the dependences of K_j and χ_j on ω/ω_B at $\vartheta = 85°$. The chosen values $N_m/N = 100$ and $\gamma_\parallel = \gamma_\perp = \gamma_r = 10^{-3}\ \omega/\omega_B$ correspond to $\hbar\omega_B = 105$ keV $(B = 9\times10^{12}$ G), $\omega_{c1} = 0.9949\omega_B$, $\omega_{c2} = 0.1005\omega_B$, $N = 8.1\times10^{23}$ cm^{-3}, $\vartheta_{n1} = 84.3°$, $\vartheta_{n2} = 89.7°$.

The behavior of K_j and χ_j near the points $\omega_{c1,2}$ is different, since $\vartheta > \vartheta_{n1}$ but $\vartheta < \vartheta_{n2}$. The width of the region of variation of the angles χ_j near a critical point is, in accordance with (19) and (21), proportional to $\gamma_{\parallel,\perp}$, and the maximal value of $|K_j|$ for $\beta > \beta_n$ in a wide range of angles is near unity (see footnote[2)]). The changeover due to the influence of the vacuum from quasitransverse to quasilongitudinal propagation near the critical points occurs in a wide range of angles $\beta > \beta_n$, as canbe seen from curves a and c in Fig. 3. For $\beta < \beta_n$, in contrast, the propagation is more nearly quasitransverse the closer ω is to $\omega_{c1,2}$.

Note that on the transition through a critical point there is either a change in each normal wave of χ_j by $\pm\pi/2$ or a change in the sign of K_j. Therefore, the separation of waves into an ordinary an extraordinary wave cannot have the same meaning as in Fig. 2, we ascribe a definite number j to each branch (i.e., continuous curve with continuous derivative) of the dependences of K_j and χ_j on ω, N, or B, the number of the branch in the angular dependences can change when ϑ passes through ϑ_n (see curve a in Fig. 3). The same is true of $\varkappa_j$ and k_j (see Sec. 4, Figs. 4 and 5).

For $N \ll N_m |u-1| u^{-2}$ (corresponding to the regions of strong departure of the continuous curve from the dashed line in Fig. 1), the polarization of the normal waves is determined by the vacuum. This leads to a strong reduction in the region of quasilongitudinal propagation far from the critical points (when $\omega \gg \omega_B$ as well). This effect is illustrated by curves b in Fig. 3. At angles ϑ not too near 0 and π and frequencies not equal to the critical frequencies, the ellipticity K_j is strongly suppressed. Note that a strong decrease in the region of quasilongitudinal propagation for $\omega > \omega_B$ can also occur at very small ratios N_m/N. For example, for $B = 4.4\times10^{11}$ G ($\hbar\omega = 5.1$ keV, $N_m = 4.5\times10^{20}$ cm^{-3}), $\hbar\omega = 5.1$ keV $(u = 0.001)$, $N = 4.5\times10^{21}$ cm^{-3} $(N_m/N = 0.1)$, the parameter satisfies $|q| < 1$ (polarization nearly circular) when $\vartheta < 8°$ instead of $\vartheta < 87°$ in a vacuumless plasma.

4. As follows from (12), vacuum polarization affects the coefficients $\varkappa_I$ and $\varkappa_L$. A change in $\varkappa_I$ leads to changes in $\varkappa_1$ and $\varkappa_2$ by the same amount $(11/4)a \sin^2\vartheta$. The dependence of $\varkappa_L$ on a means that, first, the vacuum

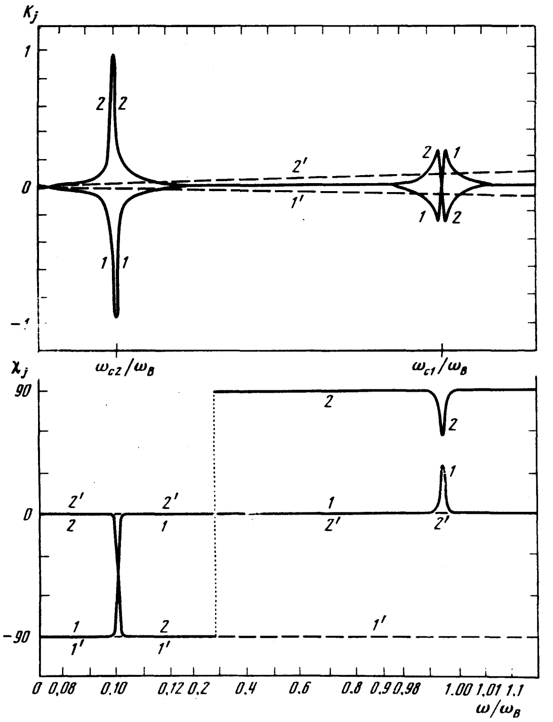

FIG. 2. Dependences of the ellipticities K_j and the angles χ_j, which determine the directions of the axes of the polarization ellipses of the normal waves, on the frequency for $N_m/N=100$, $\vartheta=85°$, $\gamma_{\parallel,\perp}=\gamma_r=10^{-3}\omega/\omega_B$. Here and in Figs. 3–5, the numbers next to the curves (for vacuumless plasma with primes) give the number j of the wave (in the limit of a vacuumless plasma, wave 1 goes over into 1′, the extraordinary wave, and 2 into 2′, the ordinary wave). In the intervals $0.08<\omega/\omega_B<0.12$ and $0.98<\omega/\omega_B<1.01$ near the critical frequencies $\omega_{c1,2}$ the scale is enlarged by a factor 10.

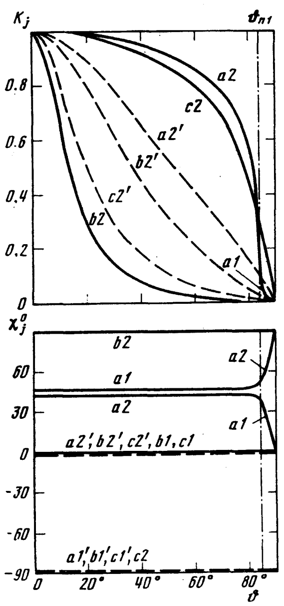

FIG. 3. Angular dependences of K_j and χ_j for $N_m/N=100$ and $\gamma_{\perp,\parallel}=\gamma_r=10^{-3}\omega/\omega_B$ and different values of the frequency. The curves aj, bj, and cj correspond to the frequencies $\omega=0.995\omega_B\approx\omega_{c1}$, $\omega=0.3\omega_B$, $\omega=0.102\omega_B\approx\omega_{c2}$.

polarization influences $\varkappa_1$ and $\varkappa_2$ differently, i.e., changes the birefringent properties of the plasma, and, second, as follows from the foregoing section, it changes the polarization of the normal waves for $\vartheta\neq 0$, $\pi/2$, π. Since the absorption of radiation in an anisotropic medium depends on its polarization, the magnetized vacuum influences k_1 and k_2, although for $\hbar\omega\ll mc^2$ it does not have the direct capacity of absorbing photons.

Figures 4 and 5 show the characteristic changes in the coefficients k_j and $\varkappa_j$ due to the influence of the vacuum polarization. We note first of all the occurrence of additional points of intersection (or close approach) of the k_j and $\varkappa_j$ dispersion curves at $\omega=\omega_{*1,2}$ (Fig. 4). At $\omega=\omega_{*3}$, the curves k_j also intersect in a vacuumless plasma. The additional intersections at ϑ values near $\pi/2$ are brought about by the transition from one vacuumless branch to the other (for $\varkappa_j$, both branches are displaced by the amount $11/4a\sin^2\vartheta$). The values of the parameters at which dispersion curves (denoted by an asterisk) intersect are determined by the condition (see (12)) $\mathrm{Im}(n_L{}^2+n_c{}^2)=-2(k_L\varkappa_L+\varkappa_c k_c)=0$ or

$$[u^2-N^{-1}N_m(u-1)]\times[\gamma_\parallel(1-u^2)-\gamma_\perp(1+u)]\sin^4\vartheta-8u^2\gamma_\perp\cos^2\vartheta=0. \quad (24)$$

If at the same time $\beta>\beta_n$ (as occurs at the point $\omega=\omega_{*2}$ for Fig. 4), then only the k_j curves intersect; if $\beta<\beta_n$ (in Fig. 4, this corresponds to the point $\omega=\omega_{*1}$), only the $\varkappa_j$ curves intersect. The close approaches and intersections of the $\varkappa_j$ and k_j curves are due to the

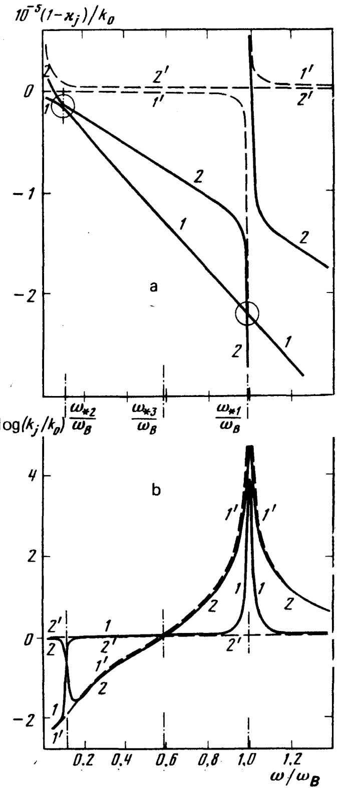

FIG. 4. Frequency dependence of the refractive indices (a) and absorption coefficients (b) of normal waves for the same parameter values as in Fig. 2; $k_0=k_{1,2}(B=0)=(c/2\omega)N\delta_T$. The circles surround the regions of enhanced linear transformation of waves in an inhomogeneous plasma.

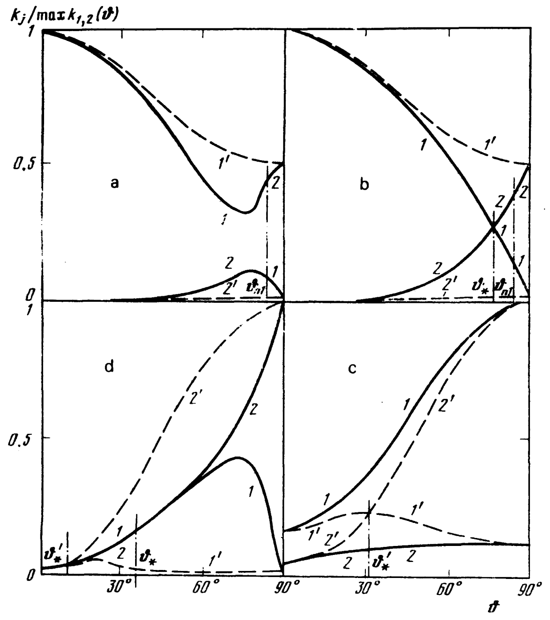

FIG. 5. Angular dependences of the absorption coefficients for $N_m/N=100$ and $\gamma_{\perp,\parallel}=\gamma_r=10^{-3}\omega/\omega_B$ and different frequencies: $\omega/\omega_B=0.995$ (a), 0.994 (b), 0.3 (c), 0.1 (d).

changeover from quasitransverse to quasilongitudinal (with decreasing $|q|$) propagation near the critical points. This can be seen in Eq. (16) if one notes that $\omega_{*1,2}\approx\omega_{c1,2}$ at ϑ values not too far from $\pi/2$. The change in the polarization of the normal waves has a particularly strong influence on the absorption coefficient of the "ordinary" wave[4)] at the critical point near $u=1$: the dependence $k_1(\omega)$, in contrast to $k_{2'}(\omega)$ in a vacuumless plasma, takes on a well defined resonance nature. For $N_m/N\gg 1$, the position of the peak corresponds to the frequency $\omega\approx\omega_{c1}\approx\omega_B(1-N/2N_m)$, and its relative height is $\sim 2(N_m/N)^2$. We emphasize that this is not a cyclotron resonance in the usual meaning of the word, and it has the same origin as the less pronounced peak at $\omega=\omega_{*2}$. We shall see below that the presence of peaks and dips in the absorption coefficients of the normal waves leads to characteristic "vacuum" features in the spectral, angular, and polarization characteristics of the plasma radiation.

As ϑ decreases, the frequency dependences in Fig. 4 change as follows. The peak of the ordinary wave at $\omega=\omega_{*1}$ increases, approaching the dip of the extraordinary wave (this dip cannot be well distinguished in Fig. 4b because of the logarithmic scale). At $\vartheta\leqslant\vartheta_n$, the peak and the dip merge, and a branch intersection of the same type as at the point ω_{*2} occurs. With decreasing ϑ, the points ω_{*1} and ω_{*3} move toward each other, merge, and disappear at $\vartheta=\vartheta_b$, which can be estimated from (24): $\sin^2\vartheta_b/\cos\vartheta_b\approx 4(N/N_m)^{1/2}$ for $N_m/N\gg 1$ ($\vartheta_b\approx 20°$ for $N_m/N=100$). At the same time, the point ω_{*2} moves into the region of low frequencies, tending at $\vartheta\gtrsim\vartheta_b$ to the value $\omega_{*3}(\vartheta)$ for the vacuumless plasma. For sufficiently large β, strong vacuum polarization ($N_m/N\gg 1$) reduces the range of frequencies in which the propagation is quasilongitudinal (see Sec. 3). As a result, the dispersion curves approach each other (see (16)), but, in contrast to the case of ϑ values near $\pi/2$, this occurs in a larger range of frequencies far from the critical point.

If radiation propagates in an inhomogeneous plasma, the values of $\varkappa_1$ and $\varkappa_2$ (and k_1 and k_2) approach each other on the passage through the regions of critical values of N or B. This may be accompanied by violation of the condition of applicability of geometrical optics[15]:

$$\left(\frac{1}{2(1+q^2)}\frac{dq}{dl}\right)^2\ll\left|\frac{\pi^2}{\lambda^2}(\varkappa_1-\varkappa_2)^2+i\frac{\pi}{\lambda}\frac{d}{dl}(\varkappa_1-\varkappa_2)\right|. \tag{25}$$

Violation of (25) leads to linear transformation of the normal waves and changes the polarization of the radiation leaving the plasma. Effects of this kind are important in producing the polarization of the radiation of x-ray pulsars.[7] There are also important changes under the influence of vacuum polarization in the dependences $k_j(\vartheta)$ (see Fig. 5), which are responsible for the angular distribution of the radiation leaving the plasma. In a vacuumless plasma, the curves $k_j(\vartheta)$ intersect at $\vartheta'_*=\pi/2$ if $\omega=\omega_{*3}(\pi/2)$ (at $\gamma_\parallel=\gamma_\perp$, the frequency $\omega_*(\pi/2)=\omega_B/\sqrt{3}$). For $\omega>\omega_{*3}(\pi/2)$, there is no intersection, and as the frequency decreases from $\omega_{*3}(\pi/2)$ the angle $|\pi/2-\vartheta'_*|$ increases monotonically, tending to $\pi/2$. Vacuum polarization (for $N_m/N>4.5$) leads to the appearance of a point of intersection ϑ_* at $\omega_*(\pi/2)<\omega<_{c1}$ (see Figs. 5a and 5b), and to its disappearance at $\omega_{c2}<\omega<\omega_*(\pi/2)$ (Fig. 5c) and a shift in the direction of $\pi/2$ at $\omega\lesssim\omega_{c2}$ (Fig. 5d). Note the approach to each other of the curves $k_j(\vartheta)$ in Fig. 5d (in the scale of the figure, they merge for $\vartheta\lesssim\vartheta_*$), which occurs in a wide range of angles. This effect can lead to the formation of a narrow directional diagram of a new type in the emission of x-ray pulsars.[7]

5. The characteristics of the normal waves considered above determine the generation, propagation, and emergence of the radiation from the plasma. The solution of this problem is complicated by the fact that the inhomogeneity of the concentration that occurs in every real medium entails inhomogeneity of the anisotropy in the presence of vacuum polarization (in contrast to a pure low-density plasma, the parameter q depends on the density). As a result, for each concrete model one must verify the fulfillment of the conditions of geometrical optics, and for inhomogeneous anisotropy these are more complicated than for homogeneous anisotropy. To obtain quantitative results, one must make numerical calculations even for simple models. However, the qualitative features due to vacuum polarization can be understood by studying a medium in which the magnetic field and the density are homogeneous, and $\beta\gg\beta_n$ at the critical frequencies.

For *an optically thin plasma* $\int\mu_j dl\ll 1$ (the integral is taken along the line of sight through the emitting region), and the intensity of the emergent radiation is

$$I_\omega\propto\int\varepsilon_\omega\,dl=\int(\mu_1+\mu_2)S_\omega\,dl,$$

where ε_ω is the emissivity of unit volume, $S_\omega(l)$ is the function that characterizes the distribution of the radiation sources in the medium (in the case of local thermodynamic equilibrium, $S_\omega(l)=B_\omega(T(l))$, B_ω is Planck's function, and $T(l)$ is the local temperature). The Stokes parameters, which determine the polarization of the radiation, are proportional to $\int(\mu_1-\mu_2)S_\omega dl$. Since the vacuum polarization does not influence the value of $\mu_1+\mu_2=(4\omega/c)k_l$, the intensity of the radiation of an optically thin plasma is not changed either. But $\mu_1-\mu_2$, and therefore the spectral and angular dependences of the Stokes parameters can change appreciably. For example, at the frequencies $\omega*_{1,2}$ (see Sec. 4) $k_1=k_2$, i.e., both the circular and the linear polarization of the emergent radiation are zero. Since vacuum polarization strongly reduces the value of $|K_j|$ for $u<1$, $N<N_m u^{-2}$, the polarization of the emergent radiation may be predominantly linear even for $u\ll 1$, in complete contrast to an ordinary plasma. In contrast, near the critical points the linear polarization is reduced and the circular increased, even for $u\gg 1$.

In the radiation of an *optically thick plasma*, one can expect the effects of the vacuum polarization to be more strongly manifested. In this case, the intensity I_j of a normal wave emerging from the plasma is determined by the value of the source function (the temperature $T(l)$ in the case of local thermodynamic equilibrium) at a distance $\sim u_j^{-1}$ from the emitting boundary: $I_j\propto S_\omega(u_j^{-1})$. Because of the nonlinear dependence of I_j on u_j, vacuum polarization changes not only the polarization characteristics but also the spectral and angular dependences of the radiation intensity in the case of the emission of an optically thick plasma with inhomogeneous source function $S_\omega(l)$. In particular, if $S_\omega(l)$ increases as one goes into the emitting medium, the wave with smaller absorption coefficient makes a larger contribution. Then as a result, for example, the peaks in the frequency dependence of this coefficient resulting from the influence of the vacuum near the critical points (the points of intersection) are manifested in the form of dips or "vacuum absorption lines" in the spectrum of the emergent radiation.

As an illustration, Figs. 6 and 7 show the spectral and angular characteristics of the radiation emerging from a semi-infinite medium in local thermodynamic equilibrium in which free-free transitions (bremsstrahlung processes) are predominant,[5)] the density and the magnetic field are homogeneous, and the source function increases linearly with the depth:

$$B_\omega(\tau)=B_\omega(0)(1+\beta_\omega\tau),\quad \beta_\omega=\frac{1}{B_\omega(0)}\left[\frac{\partial B_\omega}{\partial T}\frac{dT}{d\tau}\right]_{\tau=0}$$
$$T=T_0\left(1+\frac{3}{2}\tau\right)^{1/4},\quad \tau=\int_0^l\bar{\mu}\,dl, \tag{26}$$

where $\bar{\mu}$ is the absorption coefficient averaged over the frequency in accordance with Rosseland's procedure. Similar calculations for a vacuumless plasma were described earlier in our paper Ref. 13. Besides the occurrence of the "vacuum absorption lines" near the points $\omega*_{1,2}$, we should also note the change in the directional diagram and the complicated nature of the polarization curves near the critical frequencies. Note that the vacuum lines are narrower, the closer is ϑ to $\pi/2$. For $\vartheta=0$, $\pi/2$, π the vacuum polarization is not manifested in the emission of a sufficiently homogeneous plasma. As can be seen in Fig. 6a, the position and the profile of the vacuum absorption line near ω_B depends weakly on the parameter N_m/N, which renders it hardly sensitive to inhomogeneity of the density and thus favors the appearance of this line in the spectra of real objects with strong magnetic field. This circumstance may, for example, be important in elucidating the nature of the spectral feature at $\hbar\omega\sim 60$ keV observed in the x-ray pulsar Her $X-1$ (see Ref. 16).

CONCLUSIONS

We list the main qualitative effects due to vacuum polarization by a magnetic field for propagation of ra-

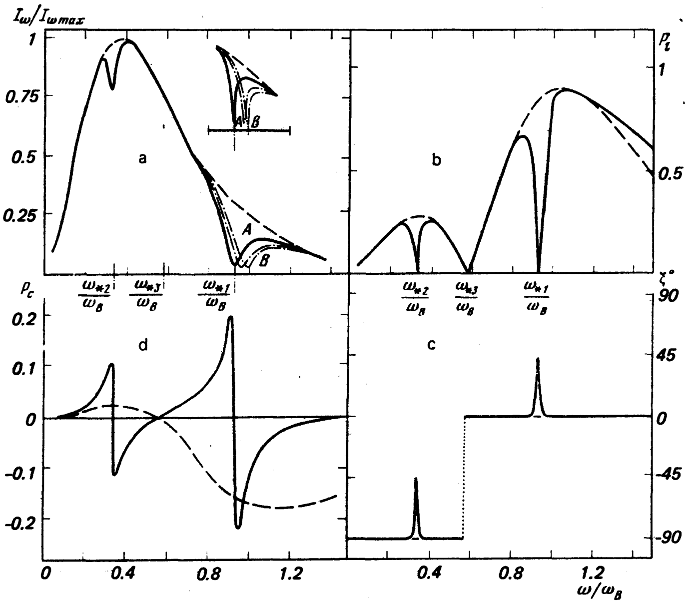

FIG. 6. Spectra of the intensity (a), the degree (b) and polarization angle (c) of linear polarization, and the degree of circular polarization (d) of radiation emerging from an optically thick plasma for $N_m/N=10$ and $\hbar\omega_B/kT=10$ (T is the surface temperature). The magnetic field is parallel to the boundary of the emitting plasma. The angle θ between the normal to the boundary and the line of sight is 5°. The chain curves in Fig. a show the vacuum features near ω_{c1} for $N_m/N=30$ (A) and 4500 (B). In the upper part of Fig. a, a fragment of the spectrum at $\theta=1°$ is shown.

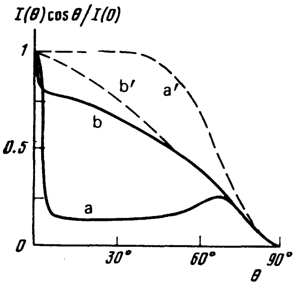

FIG. 7. Angular distribution of the radiation intensity of an optically thick plasma for $N_m/N=10$, $\hbar\omega_B/kT=10$ and frequencies $\omega=0.935\omega_B\approx\omega_{c1}$ and $\omega=0.335\omega_B\approx\omega_{c2}$ (curves a and b).

diation in a magnetoactive plasma.

1. There are critical values of the parameters N, ω, and B (see (18)) at which the influences of the vacuum and the plasma on the linear polarization of the normal waves are compensated. As a result, near these values, the normal waves are polarized in a wide range of angles $\beta>\beta_n$ (see (19)) circularly even for $\omega\ll\omega_B$, in contrast to an "ordinary" magnetoactive plasma.

2. On the transition of any of the parameters N, ω, or B through a critical value for $\beta>\beta_n$, the polarization ellipses of the normal waves are rotated through 90°.

3. At a sufficiently low plasma density, the vacuum polarization changes the circular polarization of normal waves for $\omega\gg\omega_B$ into linear polarization.

4. Near the critical values of the parameters N, ω, and B, the dispersion curves for the refractive indices approach one another, and for $\beta\leqslant\beta_n$ they intersect, and this may lead to linear transformation of the waves even in a weakly inhomogeneous plasma.

5. Near the critical values of N, ω, and B and the angles ϑ_n (see (22)) there is a region of strong nonorthogonality of the normal waves.

6. There are additional points of intersection (or close approach) of the dispersion curves for the absorption coefficients of the normal waves, and this leads to features in the spectral, angular, and polarization characteristics of the plasma radiation.

7. Near the cyclotron frequency for $\vartheta\neq0$, $\pi/2$, π the normal wave with smaller absorption coefficient (in an "ordinary" plasma it is called the ordinary wave) depends "resonantly" on the frequency, but the maximum of the "resonance" is not at the cyclotron frequency, as for the other wave, but at $\omega\approx\omega_{c1}\approx\omega_B(1-N/2N_m)$.

8. The frequency and angular dependences of the polarization characteristics of the radiation of both optically thick and optically thin plasmas vary in accordance with the results of §§1–3, the polarization vanishing at the points of intersection of the dispersion curves $k_j(\omega)$.

9. In the spectral intensity of the radiation emerging from an optically thick thermal plasma there are features in the region of the additional points of intersection (close approach) of the curves $k_j(\omega)$, these taking the form of absorption or emission lines depending on the sign of the temperature gradient.

10. It is possible that the unidentified lines observed in the spectra of some cosmic x-ray sources—and assumed to be cyclotron lines—are in fact "vacuum" lines of radiation formed in optically thick plasma.

11. The close approach of the curves $k_j(\vartheta)$ that takes place as a result of vacuum polarization in a wide range of frequencies leads to shrinking and nonmonotonic behavior of the directional diagram of the radiation of an optically thick plasma.

12. According to modern ideas, the magnetic field and plasma density in the emitting region of x-ray pulsars are such that $N_m/N\gg1$, so that it is necessary to take into account vacuum polarization when interpreting observations and constructing models of these objects.

Discovery of these features in the radiation of astrophysical objects with strong magnetic field would provide a possibility for studying quantum-electrodynamic effects whose observation under laboratory conditions is difficult. Investigation of the features would make it possible to obtain important new information about the emitting objects.

In conclusion, we note that vacuum polarization must also influence other processes of interaction of radiation with matter, and also lead to new processes (for example, the transition radiation in a vacuum with inhomogeneous field considered by Ginzburg and Tsytovich[17]).

We are grateful to Yu. N. Gnedin for numerous discussions.

[1)] They criticize the expression (2) of Ref. 8 for the refractive indices on the ground that it does not coincide with the corresponding expression (5) in Ref. 9, and that its imaginary part does not give the correct result in the limit of a "pure" plasma. In fact, the expression (5) in Ref. 9, which is identical with the expression (16) of the present paper, is invalid near the critical points (see (18)) for $\beta\lesssim\beta_n$ (see (19)). In the region in which it is applicable (for a "pure" plasma when $\nu_{\parallel,\perp}+\nu_r\ll\omega$), it can be readily obtained from the more general expression (2) of Ref. 8 if in that expression the obvious misprint in the sign in front of $i\gamma$ in the denominator of the first fraction in the curly brackets is corrected.

[2)] For $|q|\ll1$, the expressions for K_j and X_j are valid if one ignores quantities $\sim p$ and $\sim p^2/q^2$, respectively.

[3)] This important circumstance was not noted in Ref. 9 (see footnote [1)]).

[4)] In view of the vacuum polarization, the names "ordinary" and "extraordinary" are rather arbitrary (see Sec. 3). Near the cyclotron frequency, it is natural to say that the wave with the smaller absorption coefficient is the "ordinary" wave.

[5)] Although Thomson scattering is more important under real conditions, this does not affect the qualitative manifestations of vacuum polarization.

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The problem of comparing the observation results with the theoretical data to check on relativistic effects in the solar system

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To develop a noncontradictory relativistic theory of the motion of celestial bodies, the dynamic effects must be determined (from the equations of motion of the bodies) and the measured quantities calculated (using the light-propagation equations) in a single coordinate frame. The choice of the frame itself is arbitrary. By way of illustration we consider complanar circular motions of the earth and of an inner planet in the sun's graviational field; this motion is described by an arbitrary parametrized metric in arbitrary quasi-Galilean coordinates. In radar observations, the measured quantitites are the signal-propagation time intervals, and in angle measurements these are the angles between the directions to the planet and to a remote immobile source (quasar) or between the directions to the planet and to the sun. These quantities can be calculated as functions of only the measured initial values of any form, irrespective of the employed coordinate frame. However, the relativistic corrections to the values of the physically measured quantities depend on the makeup of the initial measurements. Direct measurements of the angles uncover new possibilities of checking relativistic theories of gravitation.

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1. INTRODUCTION

The analysis of relativistic effects in the motion of celestial bodies, of light rays, and of radio waves in the solar system calls for the use of the metric of the gravitational field in the post-Newtonian approximation. Small perturbations of the metric are solutions of the field equation of a concrete gravitational theory and can be obtained by successive approximations accurate to four arbitrary functions, which determine the choice of some coordinate frame (coordinate conditions). It is obvious that this choice is quite arbitrary and in problems of relativistic celestial mechanics it can be restricted only by the requirement that the coordinate system be quasi-Galilean. The relativistic equations of the motion of celestial bodies, are expressed in the form of the equations of ordinary dynamics and their solutions therefore take different forms in different coordinate frames. As a result, in the general case the relativistic corrections to the Newtonian motion of celestial bodies (such as, e.g., the corrections to rectangular coordinates, semi-axes, etc.) likewise become coordinate-dependent and can therefore not be directly compared with the observation results. A qualitative way out of this situation becomes clear if one recalls that the dynamic theory is compared with the observations with the aid

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