field intensity of the Langmuir oscillations.

Thus, in those regions where an intense Langmuir field is present, magnetic fields can be generated. In the corona of a laser plasma, this can occur not only at densities close to crictical, but also in the vicinity of a density close to one-quarter the critical value.

We note in conclusion that the onset of strong magnetic fields in a laser plasma can greatly influence the transport coefficients. If the magnetic field has a regular character and its spatial structure is characterized by a wavelength λ (which coincides according to (6) with the wavelength of the Langmuir oscillations), this influence will be appreciable if the Larmor radius of the electron is small compared with λ. This condition can be rewritten in the form

$$\delta B \geqslant (T_e^{1/2}/\lambda)\,[\mathrm{MG}], \tag{19}$$

where T_e is in kiloelectron volts and λ is in microns. For a plasma with an electron temperature ~1 keV and a perturbation wavelength amounting to some tenths of λ_0, the condition (19) is realized for a magnetic field of several megagauss. We note that the transport phenomena can be influenced not only by the regular but also by stochastic turbulent magnetic fields.[14]

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Theory of electron dragging by a spin wave in a layered medium

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A surface spin wave produces in a semiconductor layer static extraneous currents of conduction electrons. If the layer is galvanically open-circuited, then two types of extrinsic currents flow: the dragging current and the thermoelectric current. These currents are calculated by time-averaging the phenomenological expression for the instantaneous current density. It is shown that the principal role is played by the dragging current. The dragging leads to formation of current eddies and to the onset of a magnetic moment of the sample. The problem of the distribution of the electric field over the layer, with the eddies taken into account, is solved. The longitudinal and transverse potential differences are calculated and the induced magnetic moment is estimated.

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The dragging of electrons by a spin wave (SW) was first observed apparently in thin metallic nickel films.[1] This effect, which turned out to be quite sizable, was subsequently observed in the magnetic simiconductor $HgCr_2Se_4$ (Ref. 2) and ultimately in a structure consisting of flat layers of yttrium iron garnet (YIG) and n-InSb in contact with one another.[3] The dragging effect was later investigated experimentally in detail both in magnetic semiconductors[4-6] and in layered structures[7,8] (see also the review[9]). Despite the large number of experiments, the theory of the effect is seemingly only in its initial stage. All that were discussed mainly were the possible nonlinearity mechanisms that contribute to the time-averaged electron current in the SW field.[5,10,11]

The present paper is devoted to the theory of the dragging effect in layered semiconductor + ferrite structures. Starting from a phenomenological formula derived and discussed in Ref. 12 for the electron-current density, we obtain by time-averaging two types of extrinsic currents in a galvanically open-circuited sample. The first type is the dragging current $I^{(d)}$ that satisfies the Weinreich relation.[13]

$$I^{(d)} \sim \mu_d \alpha W / v_{\mathrm{ph}}, \tag{1}$$

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where μ_d is the electron drift mobility, α is the coefficient of wave absorption by the electrons, $v_{\rm ph}$ is the phase velocity, and W is the energy flux density in the wave. Relation (1), as is well known, corresponds to the idea of momentum transfer to electrons when the wave is absorbed, which in fact determines the dragging force acting on the electrons. The extraneous current of the second kind is the result of the decrease of the wave intensity as it becomes absorbed, and the associated inhomogeneous heating of the electrons by the wave. This is the thermoelectric current.

The SW field peneterates into the semiconductor to a depth of the order of the wavelength or of the skin layer. The extrinsic currents can therefore differ from zero only at this depth. Under the influence of the extrinsic currents, the electric charge and field become redistributed over the entire thickness of the semiconductor (if its thickness exceeds the penetration depth). Then, even if the sample is open-circuited, circular currents (eddies) continue to flow in it. If the eddies are disregarded, the following logitudinal potential differences should be observed in the direction of the SW propagation:

$$V_{\parallel}\sim\int\limits_0^L\frac{I^{(d)}(x)\,dx}{|e|N\mu_d},\tag{2}$$

where L is the sample length, N is the electron concentration, and $e<0$ is their charge. In fact, however, as will be shown below, the eddies cause $V_{\parallel}$ to be several orders of magnitude smaller than given by (2). The eddy currents short-circuit, as it were, the semiconductor surface layer adjacent to the ferromagnet, and this semiconducting layer carries extrinsic currents that consequently decrease $V_{\parallel}$. At the same time, the same eddy currents increase the transverse potential difference $V_{\perp}$. Finally, the eddy currents induce a magnetic moment in the sample.

The general solution obtained in this paper for the problem of the field distribution under the influence of extrinsic currents concentrated at the surface pertains equally well to the dragging by ultrasound and by other field types. In this sense it can be regarded as a generalization of earlier papers,[14-16] where certain particular problems were solved and the essential role of the eddy currents was demonstrated. We note also that, as shown in Ref. 17, when electrons are dragged in an unbounded medium (and not in a layer as in our case), current eddies, $V_{\perp}$, and a magnetic moment can sometimes also be produced.

1. EXTRANEOUS CURRENTS

The frequencies ω and the wave numbers q of the SW will be assumed to be small enough and we shall use for the current density the phenomenological formula[12]

$$j_i=|e|N\mu_d\left\{E_i+\frac{\mu_H}{c}[\mathbf{H}\times\mathbf{E}]_i+\frac{\mu_F^2}{c^2}H_i\mathbf{E}\times\mathbf{H}\right\}$$
$$-eD_{ik}\frac{\partial N}{\partial x_k}-eN\alpha_{ijk}^{(E)}\frac{\partial E_j}{\partial x_k}-eN\alpha_{ijk}^{(H)}\frac{\partial H_j}{\partial x_k}$$
$$-e\beta_{ij}E_j\frac{\partial N}{\partial t}-eN\gamma_{ij}^{(E)}\frac{\partial E_j}{\partial t}-eN\gamma_{ij}^{(H)}\frac{\partial H_j}{\partial t},\tag{3}$$

in which the electric and magnetic fields are generally speaking sums of static ($\mathbf{E}_0,\mathbf{H}_0$) and oscillating ($\delta\mathbf{E},\delta\mathbf{H}$) components, the kinetic coefficients in (3) $\mu_d, D_{ik}, \alpha_{ijk}^{(E)}$ and others depend on the fields $\mathbf{E}$ and $\mathbf{H}$, and c is the speed of light.

We are interested in the time-averaged increment $I_i\equiv\langle\delta j_i\rangle$, due to the oscillations δN, $\delta\mathbf{E}$, and $\delta\mathbf{H}$ in the SW. In the principal order in the SW intensity, I_i is quadratic in these oscillations ($\sim W$). When calculating I_i, we recognize that for the surface spin wave (SSW) of Damon and Eshbach[18] the relations $\delta N=0$, $\mathbf{H}_0\cdot\delta\mathbf{H}=0$, and $\mathbf{H}_0\parallel\delta\mathbf{E}$ are satisfied if $\mathbf{E}_0\cdot\mathbf{H}_0=0$. We recognize also that in the considered case of an open-circuited sample the field $\mathbf{E}_0$ is produced by weak extraneous currents I_i and is therefore small ($\sim W$). We choose the coordinate frame shown in Fig. 1, and make $\mathbf{H}_0\parallel z$ and $\mathbf{q}\parallel x$. Then the contribution of the local terms in (3) (i.e., those containing no derivatives with respect to x and t) is

$$\begin{aligned}I_x^{(d)}&=\frac{4\pi|e|N\mu_d}{c^2}\left\{-\mu_H w_x+\mu_F\frac{\mu_F H_0}{c}w_y\right\},\\ I_y^{(d)}&=\frac{4\pi|e|N\mu_d}{c^2}\left\{-\mu_H w_y-\mu_F\frac{\mu_F H_0}{c}w_x\right\},\end{aligned}\tag{4}$$

where we have introduced the local energy flux density $\mathbf{w}=c\delta\mathbf{E}\times\delta\mathbf{H}/4$.

Expressions (4) describe the extrinsic dragging current. It is seen that the energy flux and the current are proportional to each other, as they should in the sense of the Weinreich relations. Because of the Hall effect, the energy flux say along the x axis (w_x) produces a dragging current not only along the x axis but also along y. In order of magnitude we get from (4)

$$I_x^{(d)}\sim\mu_d\frac{q}{q^2l^2}\frac{w_x}{v_\Phi},\tag{5}$$

where $l=c/(4\pi|e|N\mu_H\omega)^{1/2}$ is the depth of the skin layer. The quantity q/q^2l^2 in (5) has the meaning of the local absorption coefficient.

To reduce the estimate (5) to the form (1), we must express $I_x^{(d)}$ in terms of the integral coefficient α of the absorption of the wave energy and in terms of the average energy flux density

$$W\sim q\int\limits_{-\infty}^{\infty}w_x dy.\tag{6}$$

If $ql>1$, then the SSW penetrates to equal depths ($\sim q^{-1}$) into the ferrite and the semiconductor. The order-of-magnitude difference between W and w could then be only the result of the existence of transverse energy fluxes at different values of y, and of a strong cancellation of

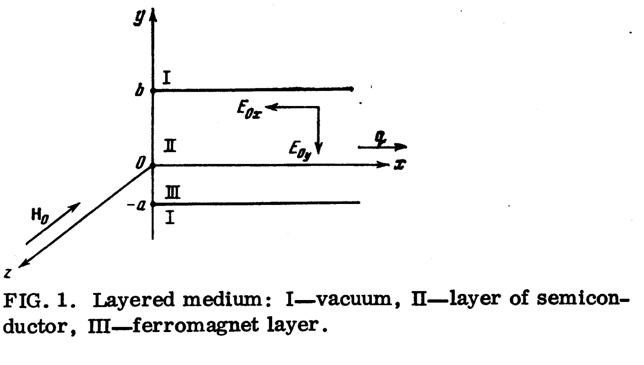

FIG. 1. Layered medium: I—vacuum, II—layer of semiconductor, III—ferromagnet layer.

these fluxes when the integration is carried out in (6). To estimate the degree of this cancellation, we can start from the assumption that it is precisely this cancellation that leads to a slowing down of the flow of the SSW energy, by a factor v_{gr}/v_{ph}, where $v_{gr} \equiv d\omega/dq$ is the group velocity. Hence $w \sim W v_{ph}/v_{gr}$. Substituting this value in (4) and recognizing further that

$$\alpha \sim \frac{q}{(ql)^2} \frac{v_{ph}}{v_{gr}}$$

at $ql > 1$ (Ref. 12), we obtain exactly the Weinreich relation (1).

The nonlocal terms in (3) also make a finite contribution $I_i^{(nl)}$ to the time-averaged current. Using the expressions for the vibrational parts of the kinetic coefficients, derived in Ref. 12, we get

$$I_x^{(nl)} = |e| N \varkappa \left[\left\langle \delta E_x \frac{\partial \delta E_x}{\partial x} \right\rangle - \theta \left\langle \delta E_z \frac{\partial \delta E_z}{\partial y} \right\rangle \right]$$

$$I_y^{(nl)} = |e| N \varkappa \left[\theta \left\langle \delta E_z \frac{\partial \delta E_z}{\partial x} \right\rangle + \left\langle \delta E_x \frac{\partial \delta E_x}{\partial y} \right\rangle \right], \qquad (7)$$

where $\varkappa \sim \mu_d^2 \tau_e$ and $\theta \sim \mu_H |H_0|/c$. According to (7) $I_i^{(nl)}$ is proportional to the components of the gradient of the increment of the effective temperature $\langle \delta T \rangle \sim |e| \mu_d \tau_e \langle \delta E_z^2 \rangle$, i.e., it is the thermoelectric current in the magnetic field $\mathbf{H}_0$.

We must now substitute in (4) and (7) the components of the SSW field in the semiconductor layer. At $e^{-2qb} \ll 1$ these components depend on x, y, and t like $\exp[\lambda y + i(qx - \omega t)]$, where $\lambda = q[1 - \omega^2 \varepsilon_s / q^2 c^2 - i/q^2 l^2]^{1/2}$, and ε_s and ε_f are the permittivities of the semiconductor and of the ferrite.[12] This yields for the current

$$I_x = I_{0x} \exp(-2q'' x - 2\lambda' y), \qquad (8)$$

with $I_{0x} = I_{0x}^{(d)} + I_{0x}^{(nl)}$, $q = q' + iq''$, $\lambda = \lambda' + i\lambda''$, and analogous expressions obtained by the interchange $x \to y$.

We next obtain

$$I_{0x}^{(d)} = -\frac{|e| N \mu_d}{2} \frac{\omega}{q'c} \frac{\mu_H}{c} |A|^2,$$

$$I_{0y}^{(d)} = -\frac{|e| N \mu_d}{2} \frac{\omega}{q'c} \left[\frac{\mu_H}{c} \left(\frac{1}{2q'^2 l^2} - \frac{q''}{q'} \right) + \frac{\mu_F}{c} \frac{\mu_F H_0}{c} \right] |A|^2, \qquad (9)$$

$$I_{0x}^{(nl)} = -\frac{|e| N \varkappa}{2} (q'' - \theta\lambda') \left(\frac{\omega}{q'c} \right)^2 |A|^2,$$

$$I_{0y}^{(nl)} = -\frac{|e| N \varkappa}{2} (\theta q'' + \lambda') \left(\frac{\omega}{q'c} \right)^2 |A|^2.$$

The constant $|A|^2$ can be connected with the energy flux W by substituting in (6) the standard expressions for the SSW fields in all the contacting media (see, e.g., Ref. 12). It turns out then that this connection can be expressed with sufficient accuracy in the form $|A|^2 \approx 8\pi W / v_{gr}$. It is clear therefore that at fixed W and other conditions equal, the increase of the thickness a of the ferrite layer, which is accompanied by a sharp decrease[12] of v_{gr}, should lead to an increase of all the extraneous currents (9) and accordingly to an increase of $V_\parallel$ and $V_\perp$.

It is convenient to compare the current components in (9) in the case of greatest interest, when the conditions

$$\frac{q''}{q'} \ll 1, \quad (q'l)^{-2} \ll 1, \quad \frac{\mu_H |H_0|}{c} \left[\frac{q''}{q'} + \frac{1}{2q'^2 l^2} \right] \ll 1,$$
$$\frac{(\omega_H + \omega_m/2)^2 (\varepsilon_s + \varepsilon_f)}{q^2 c^2} \ll 1, \quad \alpha_{\text{exch}} q^2 \ll 1, \qquad (10)$$

are satisfied, where $\omega_H = |gH_0|$, $\omega_m = 4\pi |gM_0|$, $g < 0$ is the gyromagnetic ratio for the electron, M_0 is the saturation magnetization ($|H_0|$ exceeds the saturation field), and α_{exch} is the inhomogeneous-exchange constant. The longitudinal dragging current $I_{0x}^{(d)}$ is estimated from (5) to (1). The ratio is

$$\frac{I_{0y}^{(d)}}{I_{0x}^{(d)}} \sim \frac{\mu_F |H_0|}{c} + \left| \frac{q''}{q'} - \frac{1}{(q'l)^2} \right|.$$

We see that the finite current $I_{0y}^{(d)}$ is obtained either because $I_{0x}^{(d)}$ is deflected by the Hall effect, or because a y component of the momentum is transferred when the wave is absorbed. This component is proportional to

$$\lambda'' = q' \left[\frac{q''}{q'} - \frac{1}{(q'l)^2} \right].$$

If $\mu_F |H_0|/c \ll 1$, then by virtue of (10) the current $I_{0y}^{(d)} \ll I_{0x}^{(d)}$. The situation for the components $I_i^{(nl)}$ is reversed, namely, at $\mu_H |H_0|/c \ll 1$ the ratio is

$$\frac{I_{0y}^{(nl)}}{I_{0x}^{(nl)}} \sim \left| \frac{q''}{q'} + \frac{\mu_H |H_0|}{c} \right|^{-1} \gg 1.$$

The reason is that the components $I_i^{(nl)}$ depend on the degree of inhomogeneity of the electron heating. The longitudinal component $I_{0x}^{(nl)}$ is proportional to the longitudinal heating gradient that results from the damping of the SSW along the x axis, and is therefore proportional to q''. On the other hand, the transverse component $I_{0y}^{(nl)}$ is due to the finite depth of penetration of the SSW into the semiconductor, and is therefore $\sim q' \gg q''$. On the other hand if $\mu_H |H_0|/c \gtrsim 1$, then $I_{0x}^{(nl)}$ becomes of the order of $I_{0y}^{(nl)}$ because of the rotation of $I_{0y}^{(nl)}$ through the Hall angle. The total longitudinal current I_{0x} practically always coincides with the dragging current. In fact, from (9) we have

$$\frac{I_{0x}^{(nl)}}{I_{0x}^{(d)}} \sim \omega \tau_e \frac{\mu_d}{\mu_H} \left| \frac{q''}{q'} + \theta \right| \ll 1$$

under real conditions. In a transverse current, on the other hand, the dragging and the thermoelectric power can give comparable contributions, because it follows from (9) that

$$\frac{I_{0y}^{(nl)}}{I_{0y}^{d}} \sim \omega \tau_e \frac{\mu_d}{\mu_H} \left[\left| \frac{q''}{q'} - \frac{1}{2(q'l)^2} \right| + \frac{\mu_H |H_0|}{c} \right]^{-1},$$

which can quite readily be close to unit at $\mu_H |H_0|/c \ll 1$.

2. POTENTIAL DIFFERENCES AND MAGNETIC MOMENTS INDUCED BY THE WAVE

Under the influence of the extrinsic currents I, a certain distribution $\mathbf{E}_0(x, y) = -\nabla\varphi(x, y)$ of the static field is established in the semiconductor layer. This distribution can be obtained from the time-averaged Maxwell equation

$$\text{rot}\langle \mathbf{H} \rangle = \frac{4\pi}{c} (\mathbf{j}_0 + \mathbf{I}), \qquad (11)$$

in which $\mathbf{I}$ is taken from (8) and (9), and j_0 is given by expression (3) in which we substitute the static fields $\mathbf{E}_0$ and $\mathbf{H}_0$ and discard the nonlocal terms. The latter can be discarded because, first, we confine ourselves to currents linear in W, and $E_0 \sim W$ and $\alpha_{ijk} \sim E_0$.[12] Second, the derivatives with respect to t vanish after averaging.

Third, we are not considering changes of $\mathbf{E}_0$, $\mathbf{H}_0$, and N_0 over distances of the order of the Debye radius r_D, which is assumed to be much less than q^{-1}. In this approach the field $\mathbf{E}_0$ inside the semiconductor layer can be found with the aid of the equation

$$\operatorname{div}(\mathbf{j}_0+\mathbf{I})=0, \tag{12}$$

which follows from (11). This field is entirely independent of the charges and currents that are extraneous with respect to the layer. The influence of these charges and currents is screened over distances $\sim r_D$. On the boundary of the layer there should be satisfied the condition

$$\mathbf{n}(\mathbf{j}_0+\mathbf{I})=0, \tag{13}$$

which expresses the fact that the sample is open-circuited ($\mathbf{n}$ is the normal to the boundary surface).

Replacing the sought function $\varphi(x, y)$ by

$$u(x, y)=\varphi(x, y)+\varphi_0 \exp(-2q''x-2\lambda'y),$$

where $\varphi_0=2|e|N\mu_d(q''I_{0x}+\lambda'I_{0y})/(q''^2+\lambda'^2)$, we reduce the problem to the standard potential-theory form

$$\Delta u=0 \tag{14}$$

inside the semiconductor layer and

$$B(s)\frac{\partial u}{\partial x}+C(s)\frac{\partial u}{\partial y}=D(s) \tag{15}$$

on the layer boundary, with $B(s)$, $C(s)$, and $D(s)$ dependent on the coordinates along the layer boundary s and determined from (13). The problem is solved by conformally mapping the layer on a unit circle.[19] The final form of the solution is significantly simplified if it is recognized that the SSW usually attenuates strongly on the layer length L, so that

$$2q''L\gg 1. \tag{16}$$

This enables us to replace the real layer by a semi-infinte one.

Leaving out the details of these calculations, we present the final expression for the longitudinal potential difference

$$V_{\parallel}=[\varphi(L)-\varphi(0)]|_{y=0}=-\int_0^L dx E_{0x}(x,0), \tag{17}$$

which can be measured directly in the experiment. We get

$$\begin{aligned}\frac{|e|N\mu_d V_{\parallel}}{bI_{0x}}=&\frac{1}{2q'b}\left[\frac{1}{2q'^2l^2}+\frac{\mu_F}{\mu_H}\frac{\mu_F H_0}{c}+\frac{\mu_H'}{\mu_H}\frac{\omega\tau_e}{[1+\mu_H^2H_0^2/c^2]}\right]\\ &+\frac{1}{\pi}\left(1+\frac{\mu_F^2H_0^2}{c^2}\right)\left(1+\frac{\mu_H^2H_0^2}{c^2}\right)^{-1}\left\{\int_{\pi/2}^{3\pi/2} d\varphi Q(\varphi)\exp\left[-\frac{2q'b}{\pi}\beta(\varphi)\right]\right.\\ &-\frac{q''}{q'}\left[\exp(-2q'b)\int_0^{\pi/2} d\varphi Q(\varphi)\left[\operatorname{ctg}\frac{\varphi}{2}+\left(\operatorname{ctg}^2\frac{\varphi}{2}-1\right)^{1/2}\right]^{-\varkappa}\right.\\ &+\int_{3\pi/2}^{2\pi} d\varphi\left[-\operatorname{ctg}\frac{\varphi}{2}+\left(\operatorname{ctg}^2\frac{\varphi}{2}-1\right)^{1/2}\right]^{-\varkappa}\\ &\left.\left.\times\left(\frac{\mu_H H_0}{c}[2\cos\varphi(1-\cos\varphi)]^{1/2}+Q(\varphi)\right)\right]\right\},\end{aligned} \tag{18}$$

where $\mu_H'\sim\mu_H$, b is the semiconductor thickness (see the figure), $\varkappa=2q''b/\pi$,

$$Q(\varphi)=\frac{1+\cos\varphi}{\pi\sin\varphi(1+\cos\varphi-\sin\varphi)}\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^{1/4} \times\ln\left|\frac{1+\cos\varphi+\sin\varphi}{2\sin\varphi}\right|, \tag{19}$$

$$\beta(\varphi)=\begin{cases}\pi-\operatorname{arctg}(\operatorname{tg}^2(\varphi/2)-1)^{1/2}, & \pi/2\leqslant\varphi\leqslant\pi\\ \operatorname{arctg}(\operatorname{tg}^2(\varphi/2)-1)^{1/2}, & \pi\leqslant\varphi\leqslant 3\pi/2\end{cases} \tag{20}$$

$$0\leqslant\operatorname{arctg}(\operatorname{tg}^2(\varphi/2)-1)^{1/2}\leqslant\pi/2.$$

The contribution of the thermoelectric current to (18) is given by the term proportional to μ_H', which is always small if $\omega\tau_e\ll 1$, as was already assumed. Consequently, only the dragging effect is essential in the formation of $V_{\parallel}$. At $2q'b>1$ we get from (18) the order-of-magnitude estimate

$$\frac{eN\mu_d V_{\parallel}}{bI_{0x}}\sim\frac{1}{2q'b}\left(1+\frac{\mu_H|H_0|}{c}+\ln 2q'b+\frac{1}{2q''b}\right). \tag{21}$$

As expected on the basis of (16), $V_{\parallel}$ is independent of the length L. At $2q''b\gg 1$, the quantity $V_{\parallel}$ changes only slightly with changing b, and at $2q''b\ln 2q'b\ll 1$ we get $V_{\parallel}\sim b^{-1}$, i.e., $V_{\parallel}$ increases with decreasing thickness. If the eddy currents were disregarded and if $V_{\parallel}$ were estimated from (2), we would get $|e|N\mu_d V_{\parallel}/bI_{0x}\sim(2q''b)^{-1}$. Comparing this with (21), we see that the eddy currents actually decrease the longitudinal effect by a factor

$$\frac{q''}{q'}\left(1+\frac{\mu_H|H_0|}{c}+\ln 2q'b+\frac{1}{2q''b}\right)$$

i.e., possibly by several orders of magnitude.

This result can be understood qualitatively as a consequence of diversion of part of the current into the volume of the layer, and this can only decrease $V_{\parallel}$. If P is the absorbed electromagnetic power consumed in the excitation of the SSW, then the flux is $W\sim qP/L_z$, where L_z is the width of the layer in the direction of the z axis. Then, substituting I_{0x} from (9) in (21) and having $|A|^2\sim 8\pi W/v_{gr}$, we obtain a formula that is convenient for estimates

$$V_{\parallel}\sim 2\pi\left(\frac{v_{ph}}{v_{gr}}\right)\frac{\mu_H P}{c^2 L_z}\left(1+\frac{\mu_H|H_0|}{c}+\ln 2q'b+\frac{1}{2q''b}\right). \tag{22}$$

We recall that (22) is valid at $2q''L>1$ and $2q'b>1$. For a thin semiconductor layer, when $2q'b<1$, we must replace the round bracket in (22) by the factor $\sim q'/q''$, meaning that the eddy currents are neglected. Choosing $P\sim 1$ MW, $\mu_H\sim 5\times 10^3$ cm^3/V-sec (n-InSb film at 300 K), $L\sim 0.3$ cm, $2q'b<1$, and $v_{ph}q'/v_{gr}q''\sim\omega'/\omega''=Q\sim 10^3$ (Q is the factor FMR quality, we get $V_{\parallel}\sim 1$ mV, which is of the order of the experimental values.[8]

Experiment yields also the transverse potential difference

$$V_{\perp}=[\varphi(b)-\varphi(0)]|_{x=0}=-\int_0^b dy E_{0y}(0,y). \tag{23}$$

The calculated value of this quantity is

$$\begin{aligned}\frac{|e|N\mu_d V_{\perp}}{bI_{0x}}=&\frac{1-e^{-2q'b}}{2q'b}\left[\frac{1}{2q'^2l^2}+\frac{\mu_F}{\mu_H}\frac{\mu_F H_0}{c}\right.\\ &\left.+\frac{\mu_H'}{\mu_H}\frac{\omega\tau_e}{1+\mu_H^2H_0^2/c^2}\right]+\frac{1}{\pi}\left(1+\frac{\mu_F^2H_0^2}{c^2}\right)\left(1+\frac{\mu_H^2H_0^2}{c^2}\right)^{-1}\\ &\times\left\{-\int_{\pi/2}^{3\pi/2} d\varphi\exp\left[-\frac{2q'b}{\pi}\beta(\varphi)\right]\left[\frac{\mu_H H_0}{c}[-2\cos\varphi(1-\cos\varphi)]^{-1/2}+R(\varphi)\right]\right.\\ &+\frac{q''}{q'}\left[\exp(-2q'b)\int_0^{\pi/2} d\varphi R(\varphi)\left[\operatorname{ctg}\frac{\varphi}{2}+\left(\operatorname{ctg}^2\frac{\varphi}{2}-1\right)^{1/2}\right]^{-\varkappa}\right.\\ &\left.\left.+\int_{3\pi/2}^{2\pi} d\varphi R(\varphi)\left[-\operatorname{ctg}\frac{\varphi}{2}+\left(\operatorname{ctg}^2\frac{\varphi}{2}-1\right)^{1/2}\right]^{-\varkappa}\right]\right\},\end{aligned} \tag{24}$$

where

$$R(\varphi)=\frac{1}{\pi(1-\sin\varphi-\cos\varphi)}\left(\frac{1-\sin\varphi}{1+\sin\varphi}\right)^{1/4}\ln\frac{|\cos\varphi|}{1-\sin\varphi}. \tag{25}$$

At $2q'b>1$ we get from (25) the order-of-magnitude estimate

$$\frac{|e|N\mu_d V_\perp}{bI_{0x}}\sim\frac{1}{2q'b}\left(1+\frac{\mu_H|H_0|}{c}+\ln 2q'b+2q''b\right), \tag{26}$$

and it is of interest to compare it with (21). There is no dependence on L in (26) just as in (21), so that (16) is satisfied. Since, as a rule

$$1+\mu_H|H_0|/c+\ln 2q'b\sim 1,$$

we get at $2q''b\gg 1$ that $V_\perp\sim b$, and is practically independent of b at $2q''b\ll 1 V_\perp$. The ratio is

$$V_\perp/V_\parallel\sim 2q''b. \tag{27}$$

This means that the transverse effect can be either larger or smaller than the longitudinal one. However, if the semiconductor layer is thin enough, when $2q''b\ll 1$, we have $V_\perp\ll V_\parallel$, as was in fact observed in experiment. The estimate (26) for $V_\perp$ is due almost entirely to the eddy currents. W ere it not for them we would obtain for V a value

$$\left\{\frac{\mu_H|H_0|}{c}+\left|\frac{q''}{q'}-\frac{1}{2q'^2l^2}\right|+\omega\tau_e\right\}$$
$$\times\left\{\left(1+\frac{\mu_H^2H_0^2}{c^2}\right)\left(1+\frac{\mu_H|H_0|}{c}+\ln 2q'b+2q''b\right)\right\}^{-1}$$

times smaller than from (26). We see therefore that at $\mu_H|H_0|/c\ll 1$ the eddy currents can increase $V_\perp$ by several orders. This is simple to explain: in the absence of the eddy currents $V_\perp$ exists only because of the transverse extraneous current I_{0y}, which is much less than I_{0x} if $\mu_H|H_0|/c\ll 1$.

The eddy currents induce a magnetic moment that can be easily estimated by starting from (11). Integrating (11) we get

$$4\pi M\sim\langle H_z\rangle-H_0\sim\frac{4\pi}{c}\left[\int_0^b dy\,(j_{0x}+I_x)+\int_0^L dx\,(j_{0y}+I_y)\right]. \tag{28}$$

When (16) is satisfied and $2q'b>1$, the result of the integration does not depend on the lengths b and L, and takes the form

$$4\pi M\sim\frac{2\pi}{c}\frac{I_{0x}}{q}\left(1+\frac{\mu_F^2H_0^2}{c^2}\right). \tag{29}$$

Substituting the numerical data from our example and assuming furthermore $N\sim 3\times 10^{16}$ cm^{-3}, we get $4\pi M\sim 3\times 10^{-2}$ G. This moment is small but is quite easy to measure in experiment.

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