Critical phase-transition temperature of a quasi-one-dimensional metal

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We obtain the dependence of the critical temperatures of the dielectric transition (T_p) and of the superconducting transition (T_c) on the parameter w that characterizes the probability of electron tunneling from filament to filament in a quasi-one-dimensional system. It is shown that with decreasing parameter w, owing to the mutual influence of the Peierls and Cooper instabilities, the critical temperature T_c of the superconducting transition increases and reaches a maximum in the region $\omega \approx \Delta$ (where Δ is the binding energy between the particles on one filament). In the region $w < \Delta$ the fluctuation effects suppress the superconducting phase transition, and with further decrease of w the critical temperature either vanishes smoothly as $w \to 0$, or jumpwise at $w = w_c \approx (\epsilon_F T_p^0)^{1/2}$, and at $w < w_c$ a dielectric phase transition takes place. The calculations were performed by the renormalization-group method. The experimental facts concerning phase transitions in quasi-one-dimensional substances are discussed in light of the deduced concepts.

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1. INTRODUCTION

During the last fifteen years, numerous attempts were made to calculate the critical temperatures T_c of various phase transitions in quasi-one-dimensional systems, but without account taken of the transverse coupling between the filaments, so that T_c was calculated approximately within the frameworks of purely one-dimensional models, for which the exact T_c should, according to the Landau theory,[1] be identically equal to zero. The first reliable results for T_c appeared relatively recently, when it was learned to take into account the weak transverse coupling between the filaments, which frequently is of two types: potential—the vertex contains terms with the indices i and j of different filaments, and kinetic, corresponding to the possibility of electron tunneling from filament to filament and characterized by a transverse resonant integral w.

For quasi-one-dimensional systems made up of thick filaments, Efetov and Larkin[2] and Scalapino *et al.*[3] have proposed a self-consistent field method in terms of the transverse coupling, and this method was subsequently extensively used. Other studies[4,5-11] were devoted to quasi-one-dimensional systems based on one-dimensional metallic filaments. These models are more difficult to study. It is known that in the purely one-dimensional case it is necessary to take into account simultaneously two types of instability, Peierls and Cooper,[12,13] and only in the very simplest cases (e.g., for the Tomonaga–Luttinger model[14]) are the one-dimensional parquet equations a sufficiently good approximation. Therefore, say, the results of Gor'kov and Dzyaloshinskiĭ[4] and of Lee *et al*,[8] based on the parquet approximation that yields fair agreement at not too small values of w, cannot be extended to the region of sufficiently small w, where the role of the one-dimensional fluctuations is large. The limits of applicability of the self-consistent-field method with respect to transverse coupling[2,3] are not clear to this day (this, incidentally, will be discussed below), and there are even known cases when this method is certainly not valid.[11]

Thus, calculations at small w should be based on more exact approximation. A suitable method was first described in Ref. 5, where it was proposed to break up the entire temperature (frequency) interval into two regions: one-dimensional (1D), in which only exact one-dimensional solutions or the renormalization-group method in the approximation that follows the parquet approximation must be used,[13] and three-dimensional (3D) where the parquet of ladder approximation can be used for the analysis. In Ref. 5 we also indicated a prescription for matching together the solution on the boundary of the 1D and 3D regions. This method yielded the first results for T_c in the region of small w for systems of one-dimensional metallic filaments[5,6] (in addition, the most accurate values were obtained in Ref. 5 for the critical exponents for the admittances in the 1D region). A similar method was used independently in Ref. 7 for the case of transverse potential coupling. This method was subsequently further developed in Refs. 9 and 10, where the roles of the kinetic[9] and potential[10] couplings were studied. Expressions were obtained for the critical temperatures of the superconducting[9] and dielectric[10] transitions. The most important result is the statement that in the case of a strongly asymmetrical electron spectrum, when the Peierls transition is suppressed (but the corresponding fluctuations in the 1D regions are not small), the temperature T_c of the superconducting transition as a function of w has a maximum in the region

$$w \approx \Delta = \varpi \gamma^{\gamma_0} \exp\{-1/2\gamma\}$$

(see Fig. 2 of Ref. 9), where Δ characterizes the force and the radius of the interaction of the electrons located on one filaments, and specifies the binding energy between the particles for the one-dimensional model.[5,15] Using a similar method, Larkin and Sak[18] have recently reached the same conclusion. Suzumura and Fukuyama[17] also point to the possibility of a maximum of T_c if one moves from the direction of extremely small w.

In the present paper we propose that the electron spectrum is not too asymmetrical, when there exists a

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small-w region in which the Peierls transition is not suppress and is investigated on a par with the superconducting transition, something impossible in the quasi-one-dimensional Tomonaga model.[14] We take into account here the renormalization of the width of the band in the transverse dimension, which occurs in the region of small w. The retardation inherent in the electron–phonon coupling appears here as an effect of softening of one of the end-point frequencies in the two-limit technique of the renormalization groups for the four-fermion interaction. Retardation effects were investigated in detail by Horovitz[18] in the framework of the model with electron–phonon interaction. The $T_c(w)$ and $T_p(w)$ dependences were obtained[19] by the mean-field method, which is valid only when $w \gg \Delta$, and therefore provide not even a qualitative description in the region $w \approx \Delta$ of interest, where all the nontrivial events take place.

2. FORMULATION OF PROBLEM

Depending on the ratio of the parameters w and Δ, we shall distinguish between two cases.

At $w \ll \Delta$ we choose as the initial approximation the solution corresponding to the one-dimensional case, and treat the influence of the transverse motion by perturbation theory, where the small parameter is the ratio w/Δ. The first-approximation result corresponds to allowance for correlated transitions of a pair of particles,[5,20] and in terms of the effective Hamiltonian for the order parameter, it corresponds to the mean-field approximation with respect to transverse kinetic coupling.[2,3]

In the region $w \gg \Delta$ the kinetic energy of the transverse motion must be included in the zeroth Hamiltonian, and the interaction can be treated by perturbation theory.[4,5] If at the same time we confine ourselves to the mean-field approximation in the interaction, then the correction to the interaction will be small in the parameter Δ/w, but not Δ/ε_F, this being due to the enhanced role of the fluctuations in the 1D region compared with the 3D region. We consider below precisely the case of not too small values of the transverse kinetic coupling, $w \gtrsim \Delta$. The system is in this case already close in its properties to a three-dimensiona strongly anisotropic one, but in contrast to the latter the effects to its proximity to one-dimensional are still large. The indicated one-dimensional effects are not restricted merely to an enhancement of the role of the fluctuations near the phase-transition point,[2,4,5] and can appear much earlier in the form of a strong correlation between the fluctuations of the dielectric and superconducting types.[9,11]

There is also another reason why the considered range of w may be of interest. Allowance for the interaction between electrons from different filaments leads as a rule to a dielectric transition. This may be a transiton into an antiferromagnetic state[21] or a transition into the state of a Peierls or anti-Peierls dielectric.[3] Therefore the only method of obtaining a superconducting transition in a quasi-one-dimensional system is to admit of the possibility of electron tunneling from filament to filament. At low values of the transverse kinetic coupling, the dielectric transition will always compete with the superconducting transition. In addition, the value of the critical temperatures turns out to be low because of the destructing action of the fluctuations. One can hope that in the region of not too low values of w the action of such fluctuations turns out to be weakened, and the dielectric transition is suppressed. From the point of view of the superconducting transition, the case considered here is therefore of greatest interest.

It is convenient to carry out the investigation using as an example a simple model[11]—a system of metallic filaments packed to form a planar lattice. Neglecting tunneling, the electron spectrum is flat. Allowance for the transition leads corrugation of the Fermi surface, i.e.,

$$\varepsilon(p)-\varepsilon_F=v(|p_\parallel|-p_0)-w\varphi(p_\perp), \tag{1}$$

where

$$\varphi(p_\perp)=\frac{1}{4}\left\{\cos\frac{ap_x}{\hbar}+\cos\frac{ap_y}{\hbar}\right\}.$$

Although the linearization in the longitudinal momentum, carried out above, is in fact valid only near the Fermi surface, we shall use (1) up to energies of the order of ε_F. At small $w \ll \varepsilon_F$ this can lead to a change in the number under the logarithm sign. We confine ourselves in (1) to the first term of the expansion in w/ε_F, and therefore the equality

$$\varepsilon(p)-\varepsilon_F=-[\varepsilon(p-q_0)-\varepsilon_F], \tag{2}$$

where $q_0=(2p_0; \pi\hbar/a; \pi\hbar/a)$, is satisfied accurate to quantities of order w^2/ε_F. It is just the allowance for these terms which leads to suppression of the dielectric transition.

An important role in the present problem is the interaction of electrons belonging to different parts of the Fermi surface. If we leave out the possible spin dependence, this interaction breaks up into processes with large transfer of the logitudinal momentum of the order of $2p_0$—the constant g_1, and part of processes with small transfer of the longitudinal momentum compared with p_0—the constant[12,13] g_2:

$$\Gamma_{\alpha\beta\gamma\delta}=g_1\delta_{\alpha\gamma}\delta_{\beta\delta}-g_2\delta_{\alpha\delta}\delta_{\beta\gamma}. \tag{3}$$

Although it is difficult to express any judgement concerning the interaction in real systems of the type considered, it is nevertheless useful to examine the relation between the introduced constants for certain interaction mechanisms. In the case when the interaction is due to phonon exhange with large transfer of the longitudinal momentum,

$$g_1\approx-2\lambda^2(q_0)/\Omega(q_0)\approx-2\lambda^2/\omega_D,$$

where λ is the electron–phonon coupling constant and ω_D is an energy is of the order of the Debye frequency. This is a short-range interaction and is cut off at an energy of the order of ω_D, i.e., the energy transferred in the interaction does not exceed ω_D. Phonon exchange with small transfer of the longitudinal momentum can lead either to repulsion or to attraction. For example, in the case of the repulsion, if the phonon spectrum is flat, then $g_2 \approx cg_1$ and $c\approx(\omega_D/\varepsilon_F)^2$, which is much less than the attraction g_1. In the case of attraction we have for an isotropic spectrum $g_2 \approx g_1$. The Little mechanism of interaction via excitation of the electron levels presupposes that $g_1 \approx g_2 < 0$ and that the frequency limit is of

the order of ε_F. In the general case it can be assumed that $g_2 \neq g_1$ and the cutoff parameters for these two interaction constants ω_1 and ω_2 are correspondingly different.

3. CALCULATION OF THE SCATTERING AMPLITUDE AND OF THE GREEN'S FUNCTION

The investigation of the thermodynamic instabilities reduces to a study of the scattering amplitude. The latter can be calculated by summing the most essential diagrams of perturbation theory. In the first-order approximation this is a sequence of diagrams of the pure Peierls and Cooper types, as well as diagrams of the mixed type obtained by combining the former.[12] The diagrams of the last type turn out to be essential only energies that are larger than w. As shown in Ref. 11, at energies lower than w they have an additional smallness due to the decrease of the phase volume in terms of the transverse part of the momenta, in which both instabilities are simultaneously realized, and in the principal logarithmic approximation they can be left out (see incidentally also the sequel and the Appendix).

The indicated diagrams can be summed by Sudakov's method. By separating in each of the diagrams the two-particle cross section, in which the momenta are closer to the Fermi surface than in other sections, we obtain for the amplitudes the graphic equation shown in Fig. 1. The dashed and solid lines represent electrons with $p_{\parallel} > 0$ and $p_{\parallel} < 0$, respectively. We introduce the logarithmic variables corresponding to the separated scattering channels:

$$\eta = \ln \max\left\{\frac{\Omega}{\varepsilon_F};\ \frac{v}{\varepsilon_F}\left|q_{\parallel}\right| - 2p_0\right|;\ \frac{w}{\varepsilon_F}\left|\frac{q_x a}{\pi\hbar}\right| - 1\right|;\ \frac{w}{\varepsilon_F}\left|\frac{q_y a}{\pi\hbar}\right| - 1\right|;\ \frac{T}{\varepsilon_F};\ \frac{w^2}{\varepsilon_F}\right\},$$
$$\xi = \ln \max\left\{\frac{\omega}{\varepsilon_F};\ \frac{v|k_{\parallel}|}{\varepsilon_F};\ \frac{w}{\varepsilon_F}\left|\frac{k_x a}{\pi\hbar}\right|;\ \frac{w}{\varepsilon_F}\left|\frac{k_y a}{\pi\hbar}\right|;\ \frac{T}{\varepsilon_F}\right\}, \tag{4}$$

and define

$$\alpha_1 = \ln\frac{\omega_1}{\varepsilon_F},\quad \alpha_2 = \ln\frac{\omega_2}{\varepsilon_F},\quad \beta = \ln\frac{w}{\varepsilon_F}.$$

Here Ω and q are the transferred frequency and momentum in the Peierls channel, ω and k are the summary frequency and momentum in the Cooper channel, and T is the temperature.

Changing over to dimensionless variables and multiplying them by $N_0/4$, where $N_0 = 2/\pi v\hbar a^2$ is the state density on the Fermi surface, we get for the effective potentials γ_1 and γ_2 the following system of nonlinear integral equations[1)]:

$$\gamma_1(\xi,\eta) = g_1 - \int_{\xi}^{(\alpha_1,\xi)} dt\{\gamma_1(t,\{t,\beta\})\gamma_2(t,\{t,\beta,\eta\}) + \gamma_2(t,\{t,\beta\})\gamma_1(t,\{t,\beta,\eta\})\} - 2\int_{\eta}^{0} dt\gamma_1(\{t,\beta\},t)\gamma_1(\{t,\beta,\xi\},t) + \int_{\eta}^{(\alpha_2,\eta)} dt\{\gamma_1(\{t,\beta\},t)\gamma_2(\{t,\beta,\xi\},t) + \gamma_2(\{t,\beta\},t)\gamma_1(\{t,\beta,\xi\},t)\}; \tag{5}$$

$$\gamma_2(\xi,\eta) = g_2 - \int_{\xi}^{(\alpha_1,\xi)} dt\gamma_1(t,\{t,\beta\})\gamma_1(t,\{t,\beta,\eta\}) - \int_{\xi}^{(\alpha_2,\xi)} dt\gamma_2(t,\{t,\beta\})\gamma_2(t,\{t,\beta,\eta\}) + \int_{\eta}^{(\alpha_2,\eta)} dt\gamma_2(\{t,\beta\},t)\gamma_2(\{t,\beta,\xi\},t). \tag{6}$$

Here $\{t,\beta,\eta\}$ denotes $\max(t,\beta,\eta)$. The appearance of different upper integration limits in (5) and (6) is due to the fact (see Fig. 1) that in one case the integration with respect to the longitudinal momentum (which reduces to integration with respect to energy) is along a closed electron line, i.e., over the entire band, and in the second case the integration region is bounded by the condition that the energy brought in by the wavy line does not exceed ω_1 or ω_2 (the BCS model). It was assumed above that $\omega_1 \leqslant \omega_2$. The system (5) and (6) can be solved exactly. We note for this purpose at $\beta \leqslant (\xi,\eta) \leqslant \alpha_1$ it goes over into the ordinary parquet equations for a one-dimensional system.[12] At $\xi = \eta$ the solution takes the form

$$\gamma_1(\xi) = \frac{\gamma_1(\alpha_1)}{1 - 2\gamma_1(\alpha_1)(\xi - \alpha_1)},\quad \gamma_2(\xi) = g_2 + \frac{1}{2}(\gamma_1(\xi) - \gamma_1(\alpha_1)). \tag{7}$$

The difference between (7) and the corresponding solutions of Bychkov *et al.*[12] is due to the fact that here we have taken into account the renormalization of g_1 on account of the energy region from ω_1 to ε_F, where the solution is represented in the form

$$\gamma_1(\alpha_1 \leqslant \xi \leqslant \alpha_2) = g_1 \exp[-2g_2(\xi - \alpha_2)]\left\{1 - 2g_1\alpha_2 + \frac{g_2}{g_1}(\exp[-2g_2(\xi - \alpha_2)] - 1)\right\}^{-1}; \tag{8}$$

$$\gamma_1(\alpha_2 \leqslant \xi) = \frac{g_1}{1 - 2g_1\xi},\quad \gamma_2(\alpha_1 \leqslant \xi) = g_2. \tag{9}$$

In the region $(\xi,\eta) \leqslant \beta$ Eqs. (5) and (6) can be rewritten in the form

$$\gamma_2(\xi,\eta) = \gamma_2(\beta) + \int_{\eta}^{\beta}\gamma_2^2(\beta,t)dt - \int_{\xi}^{\beta}[\gamma_1^2(t,\beta) + \gamma_2^2(t,\beta)]dt; \tag{10}$$

$$\gamma_1(\xi,\eta) = \gamma_1(\beta) - 2\int_{\eta}^{\beta}[\gamma_1^2(\beta,t) - \gamma_1(\beta,t)\gamma_2(\beta,t)]dt - 2\int_{\xi}^{\beta}\gamma_1(t,\beta)\gamma_2(t,\beta)dt, \tag{11}$$

i.e., the vertex is represented in the form of a sum of two ladders corresponding to independent summation of diagrams of the Peierls and Cooper types, the only difference being that here the irreducible part contains not the bare interaction constants, but $\gamma_1(\beta)$ and $\gamma_2(\beta)$ taken from (7).

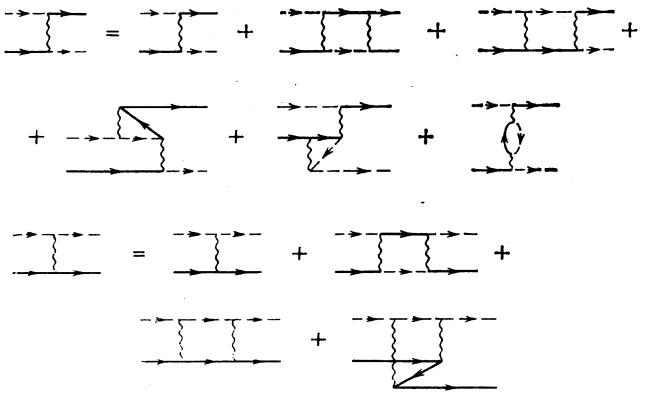

FIG. 1.

The solution (10) and (11) can be written in the form

$$\gamma_1(\xi,\eta)=\frac{1}{2}\frac{\gamma_1(\beta)+\gamma_2(\beta)}{1-[\gamma_1(\beta)+\gamma_2(\beta)](\xi-\beta)}-\frac{1}{2}\frac{\gamma_2(\beta)-\gamma_1(\beta)}{1-[\gamma_2(\beta)-\gamma_1(\beta)](\xi-\beta)}$$
$$-\frac{1}{2}\frac{2\gamma_1(\beta)-\gamma_2(\beta)}{1-[2\gamma_1(\beta)-\gamma_2(\beta)](\eta-\beta)}+\frac{1}{2}\frac{\gamma_2(\beta)}{1+\gamma_2(\beta)(\eta-\beta)}-\gamma_1(\beta); \quad (12)$$

$$\gamma_2(\xi,\eta)=\frac{1}{2}\frac{\gamma_1(\beta)+\gamma_2(\beta)}{1-[\gamma_1(\beta)+\gamma_2(\beta)](\xi-\beta)}+\frac{1}{2}\frac{\gamma_2(\beta)-\gamma_1(\beta)}{1-[\gamma_2(\beta)-\gamma_1(\beta)](\xi-\beta)}$$
$$+\frac{\gamma_2(\beta)}{1+\gamma_2(\beta)(\eta-\beta)}-\gamma_2(\beta). \quad (13)$$

The solution of Eqs. (5) and (6) at another relation between the parameters β and α_1 or α_2 will be given later [see (23)–(27)].

The region of applicability of Eqs. (5) and (6) is restricted by the condition that $\gamma_i(\xi)<1$ be small. At $\gamma_i(\xi)\approx 1$ it is necessary to take into account the corrections of next order. They are due to the first non-parquet diagrams for the vertex function and for the corrections to the mass operator.[13] These corrections have a purely one-dimensional character and turn out to be substantial also only at energies higher than w.[9,11] At energies lower than w they have an additional smallness and can be omitted. Taking into account the character of the obtained corrections, the Green's function must be represented in the form

$$G=\frac{d}{G_0^{-1}+W\varphi(p_\perp)}, \quad G_0=\frac{1}{\omega-v(|p_\parallel|-p_0)}, \quad W=wd. \quad (14)$$

The function d, which describes the renormalization of the residue at the pole of the Green's function is connected with the mass operator Σ in the same manner as in the one-dimensional case: $d=(1-G_0^{-1}\Sigma)^{-1}$. The parameter W in (14) plays the role of the effective width of the band in the transverse direction, and can differ from w. At energies higher than W, the corrections considered coincide with those for a one-dimensional system.[5,9] They can be summed by using the renormalization group method proposed previously for the one-dimensional case.[13] As a result we have

$$d(\xi)=1, \quad \alpha_2\leqslant\xi\leqslant 0; \quad d(\xi)=\exp[g_2^2(\xi-\alpha_2)], \quad \alpha_1\leqslant\xi\leqslant\alpha_2; \quad (15)$$

$$d(\xi)=\exp[g_2^2(\alpha_1-\alpha_2)]\exp\left\{-\int_\xi^{\alpha_1}dt(g_1'^2+g_2'^2-g_1'g_2')\right\}, \quad \xi\leqslant\alpha_1; \quad (16)$$

$$\gamma_i(\xi)=d^{-2}(\xi)g_i'(\xi), \quad (17)$$

where $g_i'(\xi)$ are invariant charges. At $\xi\geqslant\alpha_1$ the charge $g_2'=g_4$ and $g_1'(\xi)$ coincides with $\gamma_1(\xi)$ from (8) and (9), if g_2 is neglected compared with unity.

In the region $\xi\leqslant\alpha_1$, the quantity $g_i'(\xi)$ satisfies the equations

$$\frac{dg_1'}{d\xi}=2g_1'^2(1+g_1'), \quad \frac{dg_2'}{d\xi}=g_1'^2(1+g_1') \quad (18)$$

with initial conditions $g_2'(\alpha_1)=g_2$ and $g_1'(\alpha_1)=\gamma_1(\alpha_1)$.

Expressions (15)–(18) are valid in the energy region $\omega\geqslant W(\xi)[\xi=\ln(\omega/\varepsilon_F)]$ or $\omega\geqslant T_1$, where T_1 is determined from the equation

$$T_1=wd(\delta), \quad \delta=\ln(T_1/\varepsilon_F). \quad (19)$$

The parameter T_1 sets the boundary value of the energy or temperature, below which the behavior of the system changes from one-dimensional to three-dimensional. Generally speaking this transition (the corresponding equations describing such a transition were obtained in Ref. 11) occurs mainly in the Tomanaga-Luttinger model. Moreover, near the critical point on the Fermi surface there are almost flat sections, and therefore the corresponding one-dimensional corrections remain also in the most critical point. However, at low coupling constants or at large values of w the indicated sections constitute a small fraction of the entire Fermi surface. In the reciprocal lattice, they accommodate only a fraction on the order of $(T_{cr}/w)^2$ and they can be omitted relative to this parameter outside the vicinity of the essential singularity of the phase-transition point. In the Appendix we analyze their influence on the phase diagram. Allowance for parquet diagrams of the mixed type leads to the appearance of a certain region in the plane of the bare constants g_1 and g_2, in which the position of the singularities in the Cooper and in the Peierls channels merge, i.e., the purely one-dimensional parquet solution is obtained for the amplitude.[12] However, when account is taken of the succeeding one-dimensional corrections to the vertex function and the Green's function, the solution in the region of small interaction constants or large values of w is again the same as obtained neglecting the pure one-dimensional corrections in the region of energies lower than w.

Thus, at $(\xi,\eta)\leqslant\delta$, taking (12) and (13) into account we get for the amplitude

$$d^2(\delta)\gamma_1(\xi,\eta)=\frac{1}{2}\frac{g_1'(\delta)+g_2'(\delta)}{1-[g_1'(\delta)+g_2'(\delta)][\xi-\delta]}$$
$$-\frac{1}{2}\frac{g_2'(\delta)-g_1'(\delta)}{1-[g_2'(\delta)-g_1'(\delta)][\xi-\delta]}+\frac{1}{2}\frac{2g_1'(\delta)-g_2'(\delta)}{1-[2g_1'(\delta)-g_2'(\delta)][\eta-\delta]}$$
$$+\frac{1}{2}\frac{g_2'(\delta)}{1+g_2'(\delta)[\eta-\delta]}-g_1'(\delta); \quad (20)$$

$$d^2(\delta)\gamma_2(\xi,\eta)=\frac{1}{2}\frac{g_1'(\delta)+g_2'(\delta)}{1-[g_1'(\delta)+g_2'(\delta)][\xi-\delta]}$$
$$+\frac{1}{2}\frac{g_2'(\delta)-g_1'(\delta)}{1-[g_2'(\delta)-g_1'(\delta)][\xi-\delta]}+\frac{g_2'(\delta)}{1+g_2'(\delta)(\eta-\delta)}-g_2'(\delta). \quad (21)$$

Equations (20) and (21) pertain to the region of parameter values specified by the condition $T_1(w)<\omega_1$ or $w<w_1$, where w_1 satisfies the equation

$$T_1(w_1)=\omega_1. \quad (22)$$

The expression for $\gamma(\xi,\eta)$ at $w\geqslant w_1$ can be obtained by solving (5) and (6) and using (15)–(17). As a result we get at $w_1\leqslant w\leqslant\omega_2$ and $(\xi,\eta)\geqslant\alpha_1$

$$\gamma_1(\xi,\eta)=\gamma_1(\delta)\{1-(2\gamma_1(\delta)-g_2)(\eta-\delta)\}^{-1}\{1+g_2(\eta-\delta)\}^{-1}, \quad (23a)$$

$$\gamma_2(\xi,\eta)=\frac{g_2}{2}\exp[-2g_2^2(\delta-\alpha_2)]\left\{\frac{1+g_2(\xi-\delta)}{1-g_2(\xi-\delta)}+\frac{1-g_2(\eta-\delta)}{1+g_2(\eta-\delta)}\right\}, \quad (23b)$$

where $\gamma_1(\delta)$ is obtained from (17); if $(\xi,\eta)\leqslant\alpha_1$, then

$$\gamma_1(\xi,\eta)=\gamma_1(\delta)\{1-(2\gamma_1(\delta)-g_2)(\eta-\delta)\}^{-1}\{1+g_2(\eta-\delta)\}^{-1}-\gamma_1(\delta)$$
$$+\gamma_1(\delta)\{1-(\gamma_2(\delta,\alpha_1)+\gamma_1(\delta))(\xi-\alpha_1)\}^{-1}\{1-(\gamma_2(\delta,\alpha_1)$$
$$-\gamma_1(\delta))(\xi-\alpha_1)\}^{-1}, \quad (24a)$$

$$\gamma_2(\xi,\eta)\exp[2g_2^2(\delta-\alpha_2)]=g_2\{1+g_2(\eta-\delta)\}^{-1}-g_2$$
$$+\gamma_2(\delta,\alpha_1)\{1-(\gamma_2(\delta,\alpha_1)$$
$$+\gamma_1(\delta))(\xi-\alpha_1)\}^{-1}\{1-(\gamma_2(\delta,\alpha_1)$$
$$-\gamma_1(\delta))(\xi-\alpha_1)\}^{-1}, \quad (24b)$$

where $\gamma_1(\delta,\alpha_1)$ is taken from (23b).

At $\omega_2\leqslant w\leqslant\varepsilon_F$ and $\alpha_2\leqslant(\xi,\eta)<\beta$ it follows from (5) and (6) that

$$\gamma_2(\xi,\eta)=g_2, \quad \gamma_1(\xi,\eta)=g_1/(1-2g_1\eta); \quad (25)$$

in the region $\alpha_1\leqslant(\xi,\eta)\leqslant\alpha_2$

$$\gamma_1(\xi,\eta)=g_1\{(1-2g_1\alpha_2)(1+g_2(\eta-\alpha_2))-2g_1\}^{-1}, \quad (26a)$$

$$\gamma_2(\xi,\eta)=\frac{1}{2}\left\{\frac{1+g_2(\xi-\alpha_2)}{1-g_2(\xi-\alpha_2)}+\frac{1-g_2(\eta-\alpha_2)}{1+g_2(\eta-\alpha_2)}\right\}, \quad (26b)$$

and in the region $(\xi,\eta)\leqslant\alpha_1$

$$\gamma_1(\xi,\eta)=g_1\{(1-2g_1\alpha_2)(1+g_2(\eta-\alpha_2))-2g_1\}^{-1}-\gamma_1(\beta)+\gamma_1(\beta)\{1-(\gamma_2(\alpha_1,\alpha_2)+\gamma_1(\beta))(\xi-\alpha_1)\}^{-1}\{1-(\gamma_2(\alpha_1,\alpha_2)-\gamma_1(\beta))(\xi-\alpha_1)\}^{-1}, \quad (27a)$$

$$\gamma_2(\xi,\eta)=g_2(1+g_2(\eta-\alpha_2))^{-1}-g_2+\gamma_2(\alpha_1\alpha_2)\{1-(\gamma_2(\alpha_1,\alpha_2)+\gamma_1(\beta))(\xi-\alpha_1)\}^{-1}\{1-(\gamma_2(\alpha_1,\alpha_2)-\gamma_1(\beta))(\xi-\alpha_1)\}^{-1}. \quad (27b)$$

We have considered above a model with four-fermion interaction, in which ω_1 and ω_2 are defined formally as the interaction cutoff parameters. If ω_1 and ω_2 are regarded as the limiting values of the energies of the excitations whose virtual exchange in fact realizes the interaction of the electrons, then it is necessary to take into account also the renormalization of ω_1 and ω_2.[4,12] In the case of phonons $\omega_1=\Omega(q^0)$ and its renormalized value must be found from the equation

$$\omega_1^2=\omega_D^2(1-g_1P(\{\alpha_1,\delta\})), \quad (28)$$

where ω_D is the value of the renormalized frequency of the phonon spectrum, equal to $\omega_D=\Omega_0(q^0)$, and $P(\eta)$ is the polarization operator, equal to

$$P(\eta)=\frac{-1}{\sigma}\{\exp[-2g_2(\eta-\alpha_2)]-1\}+2\alpha_2, \quad \alpha_1\leqslant\eta\leqslant\alpha_2; \quad (29)$$

$$P(\eta)=2\eta, \quad \alpha_2\leqslant\eta\leqslant 0. \quad (30)$$

The renormalization $\omega_D\to\omega_1$ is due to the softening of the phonon spectrum. Since it is effective only at $q_\parallel\approx 2p_0$ and the constant g_2 describes the interaction with small transfer of the longitudinal momentum, the frequency ω_2 must be regarded as fixed and for the isotropic phonon spectrum $\omega_2\approx\omega_D$.

4. CRITICAL TEMPERATURE

The results obtained in the preceding section are best plotted on the (T,w) plane (see Fig. 2). Different regions of the values of T and w correspond to different equations for the determination of the scattering amplitude. In the region $\omega_2\leqslant T\leqslant\varepsilon_F$ are summed diagrams of the Peierls type, which are responsible for the renormalization of the phonon spectrum at $q_\parallel\approx 2p_0$. At $T\leqslant\omega_2$ the interaction specified by the constant g_3, is turned on and this interaction must be taken into account in the calculation of the polarization operator and of the electron-phonon vertex. In the region $T\leqslant\omega_1$ and $w\leqslant w_1$ it is necessary to take into account all the diagrams of the parquet type, and at $T\ll\omega_1$ also the diagrams of the next order. On going into the region $T\leqslant T_1(w)$ (three-dimensional region) the aggregate of the essential diagrams is limited to diagrams of pure Peierls and Cooper type, the former being important only at $T>T_1(w)w/\varepsilon_F$ because of the suppression effects that occur at lower temperatures.

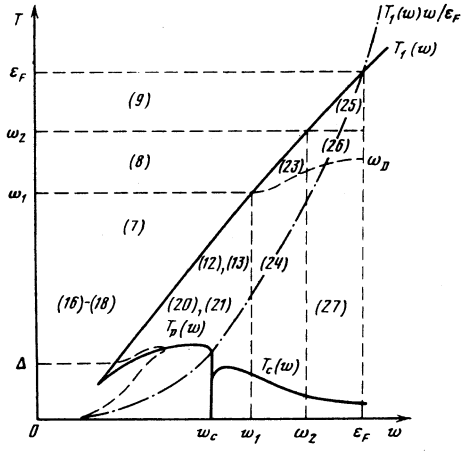

FIG. 2.

The appearance of the singularities in $\gamma_{1,2}(\xi,\eta)$ points to the possibility of a phase transition in the quasi-one-dimensional system, and their position determines the critical temperature. In the region $w\ll w_1$, according to (20) and (21), the temperatures of the superconducting and dielectric transitions are

$$T_c=T_1\exp\left\{\frac{1}{3/2g_1'+a}\right\}, \quad T_\rho=T_1\exp\left\{\frac{1}{3/2g_1'-a}\right\}, \quad (31)$$

where $g_1'=g_1'(\delta)$, $a=-\frac{1}{2}g_1'+g_2'=-\frac{1}{2}\gamma_1(\alpha_1)+g_2$, while $g_1'(\delta)$ and $\gamma_1(\alpha_1)$ are obtained from (18) and (8), respectively. Inverting the dependences of g_1' on $T_1[\delta=\ln(T_1/\varepsilon_F)]$ and of T_1 on w, we have according to (18) and (19)

$$T_1=\Delta\left(\frac{1+g_1'}{|g_1'|}\right)^{1/2}\exp\left(-\frac{1}{2g_1'}\right)$$

$$\Delta=\omega_1\left(\frac{\gamma_1(\alpha_1)}{1+\gamma_1(\alpha_1)}\right)^{1/2}\exp\left(\frac{1}{2\gamma_1(\alpha_1)}\right); \quad (32)$$

$$w=w_1\left(\frac{T_1}{\omega_1}\right)^{1-\sigma^2}\left(\frac{1+\gamma_1(\alpha_1)}{1+g_1'}\right)^{3/2}, \quad w_1=\omega_1\left(\frac{\omega_1}{\omega_2}\right)^{g_2^2}. \quad (33)$$

Equations (31)–(33), in which g_1' was regarded as a parameter that varies in the range $-1\leqslant g_1'\leqslant\gamma_1(\alpha_1)$, determine implicitly the dependences of T_c and T_ρ on w in the region $w\leqslant w_1$.

The singularities of (20) and (21), whose positions are given by the equations

$$1-(g_2'(\delta)-g_1'(\delta))(\xi_c-\delta)=0,$$
$$1+g_2'(\delta)(\eta_c-\delta)=0, \quad (34)$$

correspond to a transition to a superconducting state with triplet pairing (TP) and to an antiferromagnetic transition with formation of an Overhauser spin density wave (SDW).[5] Separating at the given g_1 and g_2 the phase transition with the largest value of the critical temperature, we can obtain the phase diagram of the states shown in Fig. 3 [here $\gamma_1(\alpha_1)$ is connected with g_1 in accord with (8)]. It must be borne in mind here that the dielectric transition (CDW, SDW) is suppressed if it turns out that $\omega/\varepsilon_F\geqslant T_\rho/T_1$. We shall focus our attention below on the regions $\gamma_1(\alpha_1)<0$ or $g_1<0$.

As indicated in our earlier paper,[9] the critical temperature as a function of w can have a maximum. At a

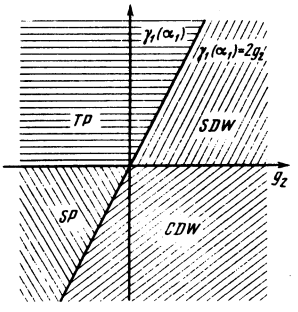

FIG. 3.

$<\gamma_1(\alpha_1)<1$ it occurs at $w \gtrsim T_1 \gtrsim \Delta$, where the parameter Δ [see (32)] coincides with the binding energy in the one-dimensional case.[5,15] On the other hand it follows from (18) that $g_1[\ln(\Delta/\varepsilon_F)] \approx 0.72$, and consequently the condition $w \gtrsim \Delta$ determines the region of applicability of the renormalization-group method ($g_1' < 1$). The value of the parameter g_1' from (31)–(33) at the point of the maximum is determined from the equation[9]

$$g_{1m}'^3 + {}^1/_4 g_{1m}'^2 \mp a g_{1m}' - {}^1/_3 a^2 = 0$$

(the minus and plus signs are taken for T_c and T_ρ, respectively) and turns out to be smaller than $g_1'[\ln(\Delta/\varepsilon_F)]$. If $w \ll \Delta$, then the region $w \lesssim T \ll \Delta$ does not lend itself to a quantitative analysis, for it is impossible at all the separate here the essential diagrams. Nonetheless, Eq. (31) gives a qualitatively correct result also at $w \ll \Delta$:

$$T_{cr} \approx w(w/\Delta)^{\lambda/(1-\lambda)}. \tag{35}$$

Here λ is the exponent of the power-law singularity of the scattering amplitude, calculated at $w = 0(\gamma_1 \propto \omega^{-2\lambda})$. It depends substantially on the presence and on the value of the fixed point of the invariant charge, and in the present approximation its value is $\lambda = \frac{3}{4} + a^2$.

Generally speaking, at $w \ll \Delta$ we can expect the correct result to be obtained by the mean-field method in w (Refs. 2, 20), where λ from (35) is already connected with the exponent of the power-law singularity of the correlator that characterizes the corresponding phase transition ($\chi \propto \omega^{-2(1-\lambda)}$. It must be noted, however, that at $w \ll \Delta$ the region of the essential singularity near the phase transition point,[4,9] $\Delta T_{cr} = T_{cr}(T_{cr}/T_1)^4$, turns out to be $\Delta T_{cr} \approx T_{cr}$. In Fig. 2 this corresponds to the fact that if scaling is observed in the one-dimensional region, then on going to the three-dimensional region we immediately land in the region of developed three-dimensional fluctuations. If we use for an estimate of λ in (35) the results of the next approximation in the renormalization-group method,[22] then $\lambda \approx \frac{1}{2}$, which is already close to the value of λ used in the mean-field method in w.[9,20]

The parameters w_1 or ω_1 in (33) are obtained from the solution of Eqs. (22) and (28). The latter exists in the region of the parameters that satisfy the inequality

$$\Omega_0 > \omega_2 \left(\frac{1+g_2}{-g_1+(1-2g_1\alpha_2)g_2}\right)^{1/2} \left[\frac{g_1-(1-2g_1\alpha_2)g_2}{g_1(1+g_2)}\right]^{-1/2g_2}. \tag{36}$$

At $g_2 \ll g_1$ it can be rewritten in the form

$$T_\rho^0 < (|g_1|)^{1/2}\omega_D, \quad T_\rho^0 = \varepsilon_F \exp\{1/2g_1\} \tag{37}$$

(T_ρ^0 is the temperature of the Peierls transition for a one-dimensional system, calculated in the mean-field approximation). We then have for w_1 and $\omega_1 = \omega_1(w_1)$ in accord with (22) and (28)

$$w_1^2 = \omega_1^2 = \omega_D^2 2g_1 \ln(T_\rho^0/\omega_D). \tag{38}$$

Assume that (36) does not hold or that $T_\rho^0 \gtrsim (|g_1|)^{1/2}\omega_D$ if $g_2 \ll g_1$. It follows from (26), (27), (23), (24), and (8) that

$$T_\rho(w \gtrsim \omega_2) = \omega_2 \exp\left\{\left[\frac{2g_1}{1-2g_1\alpha_2} - g_2\right]^{-1}\right\}, \tag{39}$$

$$T_\rho(\bar{T} \leqslant T_1(w) \leqslant \omega_2) = T_1(w)\exp\left\{\left[\frac{2g_1\exp\{-2g_2(\delta-\alpha_2)\}}{1-2g_1\alpha_2+g_1(\exp\{-2g_2(\delta-\alpha_2)\}-1)/g_2} - g_2\right]^{-1}\right\}, \tag{40}$$

$$T_\rho(T_1(w) \leqslant \bar{T}) = \bar{T} = \omega_2[1-g_2(1-2g_1\alpha_2)/g_1]^{-1/2g_2}, \tag{41}$$

where $T_1(w) = w(w/\omega_2)g_2^2$. It was assumed above that $T_\rho^0 \gtrsim \omega_2$. At $T_\rho^0 \gtrsim \omega_2$, according to (9) and (25), $T_\rho = T_\rho^0$. If $g_2 \ll g_1$, then expressions (39)–(41) can be rewritten in the form

$$T_\rho = T_\rho^0(T_\rho^0/\omega_2)^\nu, \quad \nu = g_2 \ln(T_\rho^0/\omega_2), \tag{42}$$

i.e., the interaction given by g_2 can either enhance or suppress the dielectric transition, depending on the sign of g_2. It should be noted that expressions (39)–(42) describe a transition in an ion system, since the corresponding pole singularity appears then in the expression for γ_1 at $T_\rho > \omega_1$.

At $w = w_c$, where w_c satisfies the equation

$$T_1(w)w_c/\varepsilon_F = T_\rho(w_c)$$

or

$$w_c = (\varepsilon_F T_\rho)^{1/2}, \tag{43}$$

if $g_2 \ll g_1$, the dielectric transition is suppressed, and at $w > w_1$ a superconducting phase transition is possible with a critical temperature [see (26) and (27)]

$$T_c = \omega_2 \exp\{1/g_2\}, \tag{44}$$

if in this case $T_c > \omega_1(w)$, or

$$T_c = \omega_1(w)\exp\left\{\left[\frac{g_2}{1-g_2\ln(\omega_1/\omega_2)} + \frac{g_1}{1-2g_1\beta}\right]^{-1}\right\}, \tag{45}$$

if $T_c \leqslant \omega_1(w)$. Here $w \gtrsim \omega_2$ and $\omega_1^2(w) = \omega_D^2(1-2g_1\beta)$.

At $w \leqslant \omega_2$ it follows from (23) and (24) that

$$T_c = T_1(w)e^{1/g_2}, \tag{46}$$

if $T_1 \gtrsim \omega_1(w)$, where $\omega_1(w)$ equals according to (28)

$$\omega_1^2(w) = \omega_D^2(1-2g_1\alpha_2+g_2(\exp[-2g_2(\delta-\alpha_2)]-1)/g_1). \tag{47}$$

At $T_c \leqslant \omega_1(w)$ we have

$$T_c = \omega_1(w)\exp\left\{\left[g_1\left(\frac{\omega_D}{\omega_1}\right)^2\exp\{-2g_2(\delta-\alpha_2)\} + \frac{g_2}{1-g_2(\alpha_1-\delta)}\right]^{-1}\right\}. \tag{48}$$

In the region of parameters satisfying (36), the critical temperature of the dielectric transition at $w \leqslant w_1, w_1 \leqslant w \leqslant \omega_2$, and $w > \omega_2$ is obtained respectively from (31), (39), and (40). The temperature of the superconducting transition at $w > w_1$ is described as before by Eqs. (44)–(48), and at $w \leqslant w_1$ it is obtained from (31). Figures 4 and 5 show typical examples of plots of $T_c(w)$ and $T_\rho(w)$.

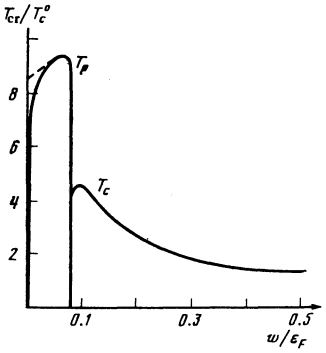

FIG. 4. Dependence of the critical temperatures of the superconducting (T_c) and dielectric (T_ρ) transitions on w at the parameter values $\varepsilon_F/\varepsilon_D = 15$, $T_\rho^0/\varepsilon_F = 0.1$, and $T_c^0/\omega_D = 0.1$. The dashed line shows the behavior of T_ρ in the region of small w when account is taken of the interaction of the electrons from different filaments.

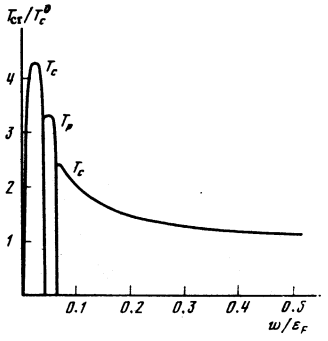

FIG. 5. Dependence of T_c and T_p on w at the parameter values $\omega_D/\omega_F = 0.03$, $T_c^0/\omega_D = 0.03567$, and $T_p^0/\omega_D = 0.2246$.

Here

$$T_c^0 = T_c(\varepsilon_F) = \omega_D \exp\{(g_1+\bar{g}_2)^{-1}\}, \tag{49}$$

where

$$\bar{g}_2 = g_2(1-g_2\ln(\omega_D/\omega_2))^{-1}.$$

We recall that in the three-dimensional BCS model use is made of a short-range interaction,[23] which corresponds to $g_1 = g_2 = g$ and $\omega_1 = \omega_2 = \omega_D$.[12] If it is recognized that the dimensionless coupling constant g is equal here to $VN(0)/4$, then expression (49) for T_c coincides with that obtained in the BCS model.[23]

5. CONCLUSION

One of the most important results of the present paper is the statement that T_c in a quasi-one-dimensional system can be noticeably higher than in a three-dimensional strongly anisotropic metal (see, e.g., Figs. 4 and 5). Although all the calculations were made for a model with four-fermion interaction, we describe the results in terms of electron–phonon interaction, since a correspondence exists between the diagrams for the two types of Hamiltonian. The increase of T_c in the quasi-one-dimensional case is connected both with the softening of the phonon mode and with the inverse effect—the influence of the fluctuations of the superconducting type on the anomalies of the phonon spectrum. Both effects manifest themselves particularly strongly in those energy regions where the influence of the corrugation of the Fermi surface can be neglected, i.e., the increase of T_c has a purely one-dimensional character. Let us illustrate this statement in somewhat greater detail.

In the region $T \gtrsim w$, with decreasing temperature, a softening of the phonon spectrum takes place with a longitudinal momentum of the order of $2p_0$ and with an arbitrary transverse momentum. The width of the longitudinal-momentum strip in which the softening takes place is of the order of T/v. The one-dimensional distinguishing feature is that it is precisely this part of the phonon spectrum with the longitudinal momentum of the order of $2p_0$, with a width of the order of ω_D/v, and with an arbitrary transverse momentum which determines the effective attraction between the electrons ($g_1 \approx -2\lambda^2/\omega_D$). This leads in final analysis to an increase of the effective interaction with decreasing temperature. At $T \ll w$ the softening takes place in a narrow range of transverse-momentum values near $q_{x,y} = \pm\pi\hbar/a$ (it is precisely the narrowness of this interval which we take to mean the smallness of the phase volume). As a result, at $T \ll w$, such a softening manifests itself integrally weakly in the effective interaction, and we omit it. The critical temperature obtainable thereby is only undervalued.

Since the longitudinal-momentum region of the effective softening of the phonon spectrum becomes narrower ($\approx T/v$) at $T < \omega_D$, it seems at first glance that the influence of the softening on the integral coupling constant at $T < \omega_D$ become weaker. This is not so, however, for at $T < \omega_D$ the electron-electron interaction due to phonon exchange is turned on, and an inverse effect sets in—the influence of fluctuations of superconducting type on the softening of the phonon spectrum. It can be seen, with the parquet approximation in the one-dimensional case as an example,[12] that we have here not only a narrowing of the $q_\parallel$ interval in which the softening takes place, but also an increase of the softening itself; therefore the effective interaction continues to increase just as when $T > \omega_D$. This leads to a growth of T_c with decreasing w, like $T_c \approx T_p^0(T_p^0/w)^{1/2}$ in the region $w \gg T_p^0$, and $g_1 = 2g_2$, as was first pointed out by Gor'kov and Dzyaloshinskiĭ.[4] As $w \to T_p^0$, however, the rate of growth of T_c slows down and can no longer be described within the framework of the parquet approximation. This pertains all the more to the region on the left of the maximum, where $T_c \to 0$ as $w \to 0$. In this region the effective coupling constant g tends to saturation, and reaches values of the order of unity, a fact described not by the parquet equations, but by Eqs. (18). From formula (31) it is seen that the decrease of $T_c(w)$ is due to the preexponential factor T_1 [see (32) and (33)], which tends to zero as $w \to 0$. According to Efetov and Larkin,[2] this reflects the increase of the destructive role of the fluctuations, on account of the ever increasing "one-dimensionalization" of the problem.

Much experimental material has by now been accumulated for different classes of quasi-one-dimensional systems.[24] The greater part of the organic compounds of a quasi-one-dimensional type at low temperatures turn out to be dielectrics. This situation is due to their high anisotropy, inasmuch as in the case of a weak transverse coupling the phase transformations are due to interaction of electrons on different filaments, and this contributes to a dielectric transition.[20,21]

At the present time we know of two organic compounds, HMTSeF-TCNQ (Ref. 25) and $(TSeT)_2Cl$ (Ref. 26) which undergo a metal–semimetal transition at low temperatures. The finite conductivity of these compounds, down to infralow temperatures, is an indication that the tunneling of the electrons between the filaments is not too small here. Otherwise the conductivity would tend to zero because of one-dimensional localization of the electrons in the random field of the impurities, or else because of the dielectric transition.

The tunneling transition can be increased by using pressure. The corresponding experiments show (see, e.g., Ref. 27) that at high pressure there exists a metallic phase that is stable at low temperatures. The fact that the system does not become superconducting at these temperatures is due either to the smallness of the effective interaction constant, or means that repulsion

prevails in the interaction. This might lead in principle to a dielectric transition, but the latter is suppressed because of the tunneling of the electrons from filament to filament.

A different situation is realized in the compound $(TTT)_2I_{3+\delta}$.[28] At low temperature its conductivity has a peak of amplitude that increases with increase of the parameter δ that characterizes the deviation from stoichiometric composition or the degree of noncommensurability of the two sublattices for TTT and I. The position of the peak shifts in this case towards lower temperatures, and the peak itself becomes sharper. Abrahams *et al.*[29] believe that the three-dimensional dielectric transition is suppressed by the random phase difference of the wave functions of the holes belonging to different TTT stacks. This is due to the lack of correlation in the positions of the chains made up of the complexes $(I_3)^-$. As a result, the temperature region in which strong one-dimensional fluctuations of the super-conducting type occur becomes broader, as is indeed observed in experiment. Allowance for the possibility of tunneling from filament to filament also leads to this tendency. Another example of a system in which a correlation is observed between the dielectric and the superconduction transitions is provided by the compounds of polychalcogenides of transition metals of the type MX_2 and MX_3, where M → Nb, Ta and X → Se, Te, S. The best investigated among them are niobium triselenide $NbSe_3$ and the $2H$ modification of $NbSe_2$ ($2H$-$NbSe_2$), which has a layered structure.[24] $NbSe_2$ was previously classified as a quasi-one-dimensional substance, but the latest data favor a layered structure.[30] The structural transition in $NbSe_2$ with formation of a charge-density wave does not lead to radical changes of the electron spectrum. Therefore $2H$-$NbSe_2$ remains a metal and goes over at lower temperatures into a superconducting state. Under pressure, a decrease of the critical transition for the critical transition (T_ρ) is observed in $NbSe_2$ (Ref. 31) and in $NbSe_3$ (Ref. 32), and an increase for the superconducting transition. The decrease of T_ρ can be attributed to the increased rigidity of the crystal lattice upon compression. In this case, however, a decrease of T_c should also occur. In $NbSe_3$ (Ref. 32), T_c increases with pressure even more rapidly than the change of T_ρ. It is therefore difficult to attribute this behavior to only a change of the crystal-lattice properties.

In the case of the quasi-one-dimensional spectrum considered above, the system becomes a dielectric or a semimetal after the appearance of the superstructure. The superconducting transition is hindered in this case. This not so for a layered system. The spectrum becomes dielectric here only on congruent sections of the Fermi surface. The dynamic connection between the two singularities in the layered system becomes enhanced with increasing curvature radius of the indicated sections of the Fermi surface. In analogy with the results obtained above (see the Appendix) one can expect this connection in a layered system also to increase as the critical temperatures are approached. This leads to an increase in the value of the lower critical temperature. For a quantitative comparison, a detailed calculation is necessary.

The experimental data on the pressure dependence of T_c in polysulfurnitride $(SN)_x$ (Ref. 33) can be regarded as pertaining to the region of w near the maximum of T_c [it is possible to describe with the aid of w the connection between the individual fibers in the $(SN)_x$] crystal. In this case the increase of T_c with pressure (due to the increase of w) can give way, after going through a maximum, to an abrupt decrease followed by assumption of the three-dimensional value of T_c, or else the superconducting transition is completely suppressed by the dielectric one. The equations derived above allow for such a possibility.

The quasi-one-dimensional state of $Hg_{3-\delta}AsF_6$ state has interesting superconducting properties. No experiments were made, however on the influence of pressure on T_c.

APPENDIX

Proceeding in analogy with our earlier study[11] and using the same approximation, we obtain for the scattering amplitude the following system of equations at energies lower than w:

$$a_1' = {}^3/_4(b_1^2 + b_2^2 - b_3^2 - b_4^2) + \frac{4}{3}a_1^2, \quad a_2' = {}^1/_4(b_1^2 - b_2^2 + 3b_3^2 - 3b_4^2),$$
$$b_1' = b_1(c_1 + a_1 + a_2), \quad b_2' = b_2(c_2 + a_1 - a_2),$$
$$b_3' = b_3\left(c_3 - \frac{1}{3}a_1 + a_2\right), \quad b_4' = b_4\left(c_4 - \frac{1}{3}a_1 - a_2\right), \tag{A.1}$$
$$c_i' = c_i^2 + b_i^2, \quad i = 1, 2, 3, 4,$$

where

$$a_1 = {}^3/_2\mu\gamma_1(\xi, \xi), \quad a_2 = {}^1/_2\mu(2\gamma_2(\xi, \xi) - \gamma_1(\xi, \xi)),$$
$$b_{1,3} = (\mu(1-\mu))^{1/4}(\gamma_2(\xi, \beta) \pm \gamma_1(\xi, \beta)),$$
$$b_2 = (\mu(1-\mu))^{1/4}(2\gamma_1(\beta, \xi) - \gamma_2(\beta, \xi)), \quad b_4 = -(\mu(1-\mu))^{1/4}\gamma_2(\beta, \xi),$$
$$c_{1,3} = (1-\mu)(\gamma_2(\xi, \beta, \beta) \pm \gamma_1(\xi, \beta, \beta)),$$
$$c_2 = (1-\mu)(2\gamma_1(\beta, \xi, \beta) - \gamma_2(\beta, \xi, \beta)), \quad c_4 = -(1-\mu)\gamma_2(\beta, \xi, \beta).$$

Knowing a_i, b_i, and c_i we can obtain the total scattering amplitude $\gamma_i(\xi, \eta, \zeta)$ (Ref. 11). The dimensionless amplitudes introduced above can be interpreted in the following manner: $a_{1,2}$ determine the effective interaction constant of the electron pertaining to the vicinities of the points $\pm\frac{1}{2}q_0$. The corresponding sections of the Fermi surface can be regarded as parallel, and the one-dimensional situation obtains for them. In the reciprocal lattice, a fraction μ of the order of $\mu \approx e^{2(\xi_c - \beta)}$, where ξ_c is the position of the singularity of the amplitude and must be found from the solution of (A.1). The amplitudes $c_{1,3}$ and $c_{2,4}$ correspond to the effective interaction constants in the Cooper and Peierls channels for electrons far from the vicinity of the points indicated above. The amplitudes $b_{1,3}$ and $b_{2,4}$ characterize the interaction between separated groups of electrons in the Cooper and Peierls channels, respectively.

If $\gamma_i(\beta, \beta) < 1$, then $\mu \ll 1$ and consequently $a_i(\beta) \ll 1$ and $b_i(\beta) \ll 1$. Therefore we can leave out in (A.1), outside the phase transition point, the corresponding terms. As a result we arrive at the system of equations considered in the main text of the article.

Near the critical point it becomes important to take into account a_i and b_i. From (A.1) it is easy to see that

in the region of the interaction constants $a_1(\beta) < a_2(\beta) < -a_1(\beta)$ the one-dimensional parquet result[12] is restored in this case, i.e., the dielectric and superconducting transitions have the same transition temperature. Near the critical point we have for the amplitude the following asymptotic behavior:

$$a_1 \propto (\xi - \xi_c)^{-1},\ b_{1,3} \propto (\xi - \xi_c)^{-1/2},\ c_{1,3} \propto (\xi - \xi_c)^{-1/2},$$

and the correlators that characterize the tendency to dielectric and superconducting pairing behave in the following manner:

$$C(\xi) \propto (\xi - \xi_c)^{-1/2},\ P(\xi) \propto (\xi - \xi_c)^{-1/2}.$$

However, near the phase-transition point the next terms of the expansion may also turn out to be substantial. Thus, for example, allowance for the next one-dimensional corrections due to the presence of almost plane sections on Fermi surface, reconstructs the result of the first approximation for the amplitude. Indeed, if we calculate the corrections of next order to the vertex function and to the mass operator, using the same approximations as before in the calculation of the mixed-type diagrams,[11] namely, using the substitution

$$a^2 \int \frac{d^2 p_\perp}{(2\pi\hbar)^2} F(p_\perp) = \mu F\left(\frac{1}{2} q_\perp^0\right) + (1-\mu) F(0),$$

then the following terms are added to the right-hand side of (A.1): in the equation for a_1 the term $\frac{8}{9} a_1^3$ and terms of the type $b_i(a_2^2 + \frac{1}{3} a_1^2$ for b_i. The equations for a_2 and c_i remain unchanged in this case. In this case in the region of small μ, at $a_2 \neq 0$, the position of the singularities in c_1 and c_3 differ, and near the critical point they take as before the pole form:

$$c_1 \approx b_1 \propto (\xi - \xi_c)^{-1}, \quad b_3 \approx c_3 \propto (\xi - \eta_c)^{-1}.$$

The position of the pole singularity is sufficiently well satisfied here by expressions (31)–(33).

[1)]The logarithmic variable ζ corresponding to the third scattering channel was chosen such that $\zeta \leqslant \min|\xi, \eta|$ and in this case γ_1 and γ_2 are independent of ζ (Ref. 11).

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