is determined by the imaginary part of the magnetic susceptibility, cannot be described by a single phenomenological constant α_T (see Ref. 6, § 31, and Ref. 18). Second, the perturbations of the magnetization field that are due to the nonlinear wave are not small and are not determined solely by the linear susceptibility. Our approach, of course, takes account of both these facts and may prove useful for analysis of the nature of the dissipation of nonlinear waves in any nonlinear media with dispersion.

The authors are deeply grateful to V. G. Bar'yakhtar and to A. A. Slutskin for a number of valuable suggestions and discussions. One of the authors (B.I.) sincerely thanks I. E. Dzyaloshinskiĭ, A. M. Kosevich, and V. M. Tsukernik for very fruitful discussions.

[1)] In contrast, for example, to the problem of the damping of a dislocation in a metal[4] or ferromagnet,[5] in which the elastic strain field due to the dislocation may be regarded simply as an external force acting on the electron or magnon subsystem.

[2)] Such "minimal" inclusion of dipole energy is necessary, since without allowance for magnetic dipole energy a DW in a uniaxial FM cannot move at all (see Ref. 12).

[3)] This formula takes no account of the attenuation γ of spin waves. Analysis shows that it is important only at very small DW velocities ($V < x_0\gamma \lesssim$ (1–10) cm/sec).

[4)] The value of (in our notation) B/B_0 at room temperature was measured in Ref. 16. The value obtained was $B/B_0 \approx 6$. Allowance for dipole scattering increases B/B_0, but this is unimportant for analysis of the results of Ref. 17, since in that work ferrite films with $\beta/4\pi \approx 30$ were used.

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Translated by W. F. Brown, Jr.

Nonlinear theory of the electron temperature superlattice in semiconductors

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(Submitted 5 December 1978)
Zh. Eksp. Teor. Fiz. **76**, 1713–1726 (May 1979)

A nonlinear theory is developed for the electron temperature superlattice (Bénard phenomenon) in semiconductors with hot electrons. The stability conditions of the superlattice and the amplitude of the spatial oscillations of the electron temperature are determined as functions of the voltage applied to the sample. The asymptotic distribution of the electron temperature T, which is established upon superheating, $(T - T_0)T_0^{-1}$ (T_0 is the lattice temperature), is also obtained when the superheating is suffficiently large but not so great that scattering of the energy by optical phonons is appreciable. The interchange of the energy and momentum scattering mechanisms which occurs at a sufficiently high electron temperature is also taken into account. The asymptotic distribution is found to be one-dimensional and stable, at any rate, on a small scale.

PACS numbers: 72.20.Ht

1. INTRODUCTION AND SETUP OF THE PROBLEM

The electron analog of the hydrodynamic problem of Bénard—the appearance of a spatially inhomogeneous distribution of the electron temperature in a nonuniformly heated electron gas—has been investigated in

 0038-5646/79/050871-08$02.40

previous researches.[1,2] It has been proposed that the heating is due to electromagnetic radiation absorbed in the intraband transitions (the situation in the case of interband absorption has been studied elsewhere[3]). It was shown that free convection arises in the electron gas under certain conditions, and that the one-dimensional static distribution of the electron temperature and of the electric field intensity in the sample become unstable. It becomes three dimensional, while in the xy plane perpendicular to the flux of the heating light (see the drawing) this distribution turns out to be spatially periodic (super-lattice). The period of the superlattice is controlled by the parameters of the material, and also by the intensity of the heating light and (or) the voltage V applied to the sample (according to the field effect scheme).

The results of Refs. 1–3 were obtained, however, under conditions of weak heating and, in addition, in an approximation that is linear in the quantity

$$\xi=\xi_s(z)+\delta\xi(x, y, z)=(T-T_0)/T_0. \quad (1)$$

Here T and T_0 denote respectively the electron and lattice temperatures; $\xi_s(z)$ is the relative heating under static conditions, while the component $\delta\xi$ arises in the presence of convection. Later,[4,5] the condition of the smallness of ξ was removed to a known extent, and it was shown that it is not always justified. However, an explicit solution of the problem, which is valid over the entire range of values of ξ_s, has not been obtained and, what is especially important, the theory has remained linear in $\delta\xi$. This did not permit calculation of the amplitude of the spatial oscillations of the electron temperature and of the intensity of the electric field in the superlattice, but it is necessary to know them for a complete description of the effects which must be expected here.[6] The investigation of the second-order (as a minimum!) approximation in $\delta\xi$ is also necessary for the solution of the question as to the stability of the superlattice. In the present work, a corresponding nonlinear theory for a nondegenerate gas is proposed; as a preliminary step, the static solution is found in explicit form. This solution is valid without any limitations on the value of the relative superheating.

We shall use the same assumptions on the parameters of the system as previously,[1,2,4,5] assuming in particular that the inequalitites

$$r_0\ll\lambda_0^{-1}\ll\gamma^{-1}\ll l \quad (2)$$

are satisfied. Here r_0 is the screening length, γ is the absorption coefficient of the heating light, assumed, for simplicity, to be independent of the electron temperature, $\lambda_0^{-1}=[(2/3)\varkappa_0\tau_0]^{1/2}$ is the cooling length. By $\varkappa_0$ and τ_0 are denoted the electronic thermal diffusivity and the energy relaxation time, respectively; the index 0 here and in what follows means that the corresponding quantity is taken at $T=T_0$. The inequalities (2) make it possible to use the condition of quasineutrality (outside of narrow regions of space charge at the boundaries of the sample), and also to consider the absorption of light as a volume effect. The sample in this case can be regarded as semi-infinite in the direction of the z axis; the plane $z=0$ coincides with the irradiated surface.

We shall consider a nondegenerate gas of positive carriers (yet calling them electrons). As before,[2] we introduce the following units of measurement: length—λ_0^{-1}, time—τ_0, drift velocity (u)—$\lambda_0^{-1}\tau_0^{-1}$, electron k thermal diffusivity—$\varkappa_0$, differential thermal emf (α) – $(3/2)e$, where e is the absolute value of the electron charge, temperature—T_0, potential (φ) and electric field intensity—$3T_0/2e$ and $3T_0\lambda_0/2e$, respectively, mobility (μ)—$2e/3T_0\tau_0\lambda_0^2$, light energy flux (I)—$(3/2)n_0T_0u_0$. Here n_0 is the constant concentration of the carriers in the unilluminated material outside the space charge regions. The carrier concentration n will be measured in units of n_0; under conditions of quasineutrality, it is equal to unity. We note that, by virtue of (2), the dimensionless absorption coefficient turns out to be small in comparison with unity. This inequality will play the fundamental role in what follows.

It is convenient to specify the dependence of the kinetic coefficients on the electron temperature in the following frequently used form (we preserve the same notation for the dimensionless and dimensional quantities, since the latter are not encountered further in the fundamental part of the paper):

$$\mu=\mu_0(1+\xi)^r, \quad \varkappa=(1+\xi)^{r+1}, \quad \tau=(1+\xi)^p, \quad (3)$$

and in this system of units $\mu_0=(9/2)(2r+5)$. The values of r and p for the various scattering mechanisms are given in Ref. 1 and in a number of books (see, for example, Ref. 7).

The equations of continuity and energy transfer, and also the expression for the drift velocity outside the space charge regions have the form

$$\frac{\partial\xi}{\partial t}-\operatorname{div}(\varkappa\nabla\xi)+\frac{5+2r}{3}\operatorname{div}[\mathbf{u}(1+\xi)]+\mathbf{u}\nabla\varphi+\xi\tau^{-1}(\xi)=\gamma I_m e^{-\gamma z}, \quad (4)$$

$$\operatorname{div}\mathbf{u}=0, \quad (5)$$

$$\mathbf{u}=-\mu(\nabla\varphi+\alpha\nabla\xi). \quad (6)$$

Here I_m is the light energy flux (the light intensity) on the irradiated boundary of the sample (at $x=+0$—see the drawing).

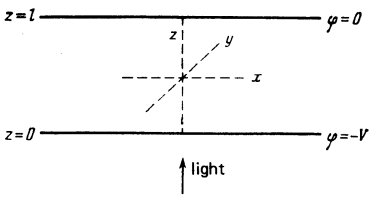
For the thermal emf, we have

$$\alpha=\frac{5+2r}{3}+\frac{2}{3}\frac{F}{1+\xi}, \quad (7)$$

where $F(T)$ is the absolute value of the difference between the Fermi level and the edge of the corresponding band, expressed in units of T_0. It is convenient to set (as before, outside of the space charge region)

$$\nabla\varphi+\alpha\nabla\xi=\nabla\Phi, \quad (8)$$

where Φ is a new unknown function. It is easy to esta-



blish the fact that Eq. (8) is always solvable; in particular, under the conditions (7) and $n=n_0$,

$$\Phi=\varphi+{}^1/_3(5+2r)\xi+{}^2/_3F_0\xi +(1+\xi)\ln(1+\xi)-\xi+\text{const}, \tag{8'}$$

where $F_0=F(T_0)$. Then

$$\mathbf{u}=-\mu\nabla\Phi, \tag{6'}$$

and Eq. (5) takes the form

$$\nabla^2\Phi+\frac{d\ln\mu}{d\xi}\nabla\Phi\cdot\nabla\xi=0. \tag{5'}$$

The boundary conditions at $z=0$ and in the absence of bending of the bands ($V=0$) have the form

$$\frac{\partial\Phi}{\partial z}=0, \tag{9a}$$

$$\varkappa\frac{\partial\xi}{\partial z}=\nu[\xi]\xi. \tag{9b}$$

Here $\nu[\xi]$ is the phenomenological coefficient introduced perviously[4] describing the energy losses of the carriers on the boundary (here it is dimensionless—expressed in units of u_0). It is clear beforehand that intensive exchange of energy with the surrounding medium prevents the development of effects connected with the heating of the electron gas. Therefore, there is sense in considering only the conditions under which $\nu\ll 1$, to which we shall limit ourselves in what follows (actually, the indicated inequality is by far not rigorous, since the unit of velocity u_0 is of the order of 10^3–10^7 cm/sec). We also note that the smallness of the function ν itself still does not mean the smallness of its derivatives: according to Ref. 1, a rapid increase of ν with increase in temperature of the electron gas is entirely possible.

At $V\neq 0$, it is more convenient to apply[2] the boundary condition on the function Φ (see also the Appendix) not on the irradiated surface itself but on the boundary of the space charge layer, considering this layer as a plane (this procedure is justified by the left hand inequality of (2)); then the condition (9a) changes to the following:

$$\frac{\partial\Phi}{\partial z}=\frac{\varphi_s'[\xi(0)-\xi_s(0)]}{1+\xi_s(0)}. \tag{9a'}$$

Here $\varphi_s'=\varphi_s'(V)$ is the field intensity on the irradiated surface of the sample in the static regime, before the free convection has yet set in.

Under certain conditions, the relation (9b) is subject to a similar modification. As is shown in the Appendix, we have here, instead of (9b),

$$\varkappa\frac{\partial\xi}{\partial z}+\left\{\Phi+\frac{5+2r}{3}-\left[\frac{2}{3}F_0+\ln(1+\xi)\right]\xi\right\}\mu\frac{\partial\Phi}{\partial z}=\nu\xi. \tag{9b'}$$

However, such a modification is justified only if all the electrons incident on the near-surface space charge layer take part in the exchange of energy with the surrounding medium (or recombine on the surface). This means, in particular, that the mean free path length in the momentum, l_p, should exceed the screening radius. Then the quantity ν becomes, strictly speaking, not a function of the relative heating, taken at $z=0$, but a function of $\xi(x,y,z)$. For this reason, we shall in what follows consider the case $l_p\leq r_0$, using the boundary condition (9b).

Finally, as $z\to\infty$, all the quantities of interest to us should be bounded, and standard periodicity conditions should be imposed on the lateral faces of the sample, as in Refs. 1–5.

Taking into account the first of the formulas (3), we can conveniently introduce the variable

$$\zeta=(1+\xi)^{r+2} \tag{10}$$

in place of ξ. Then Eqs. (4) and (5), with account of (7) and (8), take the form

$$\zeta^{-(r+1)/(r+2)}\frac{\partial\zeta}{\partial t}-(r+2)\mu_0\zeta^{r/(r+2)}(\nabla\Phi)^2 -\nabla^2\zeta+\mu_0\zeta^{-1/(r+2)}\left(\frac{2}{3}F_0+\frac{\ln\zeta}{r+2}\right)\nabla\Phi\cdot\nabla\zeta +(r+2)(\zeta^{(1-p)/(r+2)}-\zeta^{-p/(r+2)})=(r+2)\gamma I_m e^{-\tau z}, \tag{11}$$

$$\frac{r}{r+2}\nabla\Phi\cdot\nabla\zeta+\zeta\nabla^2\Phi=0. \tag{12}$$

The term with $\ln\zeta$ in Eq. (11) appeared because of account of the relation between F_0 and n_0 in a nondegenerate semiconductor. The boundary conditions also transform in obvious fashion at $z=0$.

We note two identities which the solutions of the stated problem should satisfy. Integrating Eqs. (4) and (11) over the volume of the sample with account of the boundary conditions given above, and using the relations (5) and (12), we obtain, in the case (9b),

$$\frac{\partial A_1}{\partial t}+A_2+\frac{1}{S}\int dx\,dy\left\{\nu[\xi]\xi-\frac{5+2r}{3}(\xi-V)\mu[\xi]\frac{\varphi_s'(\xi-\xi_s)}{1+\xi_s}\right\}=I_m, \tag{13}$$

$$A_1=S^{-1}\int dx\,dy\,dz\,\xi(x,y,z),\qquad A_2=S^{-1}\int dx\,dy\,dz\,\xi\tau^{-1}[\xi],$$

$$\frac{1}{r+2}\frac{\partial B_1}{\partial t}+B_2-\mu_0B_3+S^{-1}\int dx\,dy\left\{\nu[\zeta](\zeta^{1/(r+2)}-1) +\mu_0\frac{\varphi_s'(\zeta-\zeta_s^{1/(r+2)}\zeta^{(r+1)/(r+2)})}{\zeta_s^{1/(r+2)}}\left(\frac{2}{3}F_0-1+\frac{\ln\zeta}{r+2}\right)\right\}=I_m. \tag{14}$$

Here S is the area of the surface $z=0$,

$$B_1=S^{-1}\int dx\,dy\,dz\,\zeta^{1/(r+2)}(x,y,z),$$

$$B_2=S^{-1}\int dx\,dy\,dz(\zeta^{1/(r+2)}-1)\tau^{-1}[\zeta],$$

$$B_3=S^{-1}\int dx\,dy\,dz\,\zeta^{r/(r+2)}(\nabla\Phi)^2.$$

The functions ξ and ζ in the double integrals on the left sides of (13) and (14) are taken at $z=0$.

2. STATIC SOLUTION

Setting $\nabla\Phi=0$ and $\zeta=\zeta_s(z)$, we get from (11)

$$-\zeta_s''+(r+2)(\zeta_s^{(1-p)/(r+2)}-\zeta_s^{-p/(r+2)})=(r+2)\gamma I_m e^{-\tau z}. \tag{15}$$

The solution of Eq. (15) is easily found by using the smallness of γ. We denote by $\bar\zeta_s$ the solution of the equation obtained from (15) by discarding the second derivative and we set $\zeta_s=\bar\zeta_s+x$, where $x\ll\bar\zeta_s$. Linearizing Eq. (15) in x, we find

$$-\chi''+\bar U(z)\chi=0, \tag{16}$$

where

$$\bar U=(1-p)\bar\zeta_s^{-(p+r+1)/(r+2)}+p\bar\zeta_s^{-(p+r+2)/(r+2)} \tag{17}$$

The values $p=\pm\frac{1}{2}$ are in fact of interest. Here $\bar{U}\geqslant 0$, and for $\bar{\xi}_s$ we obtain

$$\bar{\xi}_s = x^{-(r+2)/p}, \tag{18}$$

$$x=-v/2+(v^2/4+1)^{1/2}, \quad p=1/2,$$

$$x=[v/2+(v^2/4-1/27)^{1/2}]^{1/3}+[v/2-(v^2/4-1/27)^{1/2}]^{1/3} \tag{19a}$$

At $v \geqslant 2\cdot 3^{-3/2}$ and $p=-1/2$;

$$x=3^{-1/2}\cos\left[\tfrac{1}{3}\arccos\left(\tfrac{1}{2}3^{3/2}v\right)\right] \tag{19b}$$

at $v \leqslant 2\cdot 3^{-3/2}$, $p=-\frac{1}{2}$, where $v=\gamma I_m e^{-\gamma z}$.

Since $\bar{\xi}_s$ depends on z only through the argument v, the derivatives $d\bar{U}/dz$ and $d^2\bar{U}/dz^2$ are small in comparison with $\bar{U}$ in view of the smallness of the parameter γ, and the solution of Eq. (16) is easily determined by the WKB method. Taking into account the boundary conditions at $z=0$, we find

$$\chi=\frac{(r+2)\left[\xi_s^{1/(r+2)}(0)-1\right]\bar{\nu}_s-\xi_s'(0)}{\bar{\sigma}(0)\left[1+\nu_1\bar{\sigma}^{-1}(0)\right]}\exp\left[-\int_0^z\bar{\sigma}(z')\,dz'\right]. \tag{20}$$

Here $\bar{\sigma}(z)=\bar{U}^{1/2}(z)$, $\bar{\nu}_s=\nu[\bar{\xi}_s]$,

$$\overline{\nu_1}=\left\{\nu[\xi_s]+[\xi_s^{1/(r+2)}(0)-1]\frac{d\nu(\xi_s)}{d\xi_s}\right\}\xi_s^{-(r+1)/(r+2)}(0). \tag{21}$$

At $\bar{\xi}_s-1\ll 1$ (i.e., $\xi_s\ll 1$) we naturally obtain the result of the linear theory.[1]

We see that the function x is actually small in comparison with $\bar{\xi}_s$ at small γ and $\bar{\nu}_s$. However, we note that (as also in the theory that is linear in ξ_s) the derivative x' at $z\to 0$ is comparable with ξ_s'; along with this, both $\bar{\xi}_s''$ and x_s'' are small in comparison with $\bar{\xi}_s$.

3. DEVIATION FROM THE STATIC SOLUTION

We shall seek the standard solution describing the free convection regime ($u\neq 0$). It should be expected that it occurs at a light intensity exceeding some critical value I_{cr}. In accord with the method of Sorokin[8] (see also the book of Gershuni and Zhukhovitskiĭ[9]) we set[1)]

$$(I_m-I_{cr})/I_{cr}=\varepsilon^2 \tag{22}$$

and

$$\zeta=\zeta_s+\varepsilon\zeta_1+\varepsilon^2\zeta_2+\ldots, \quad \Phi=\varepsilon\Phi_1+\varepsilon^2\Phi_2+\ldots \tag{23}$$

Here we must use for ζ_s the formulas of the previous section, replacing I_m in them by I_{cr}.

In addition, setting

$$\zeta_1=f_1(z)\cos\mathbf{kr}, \quad \Phi_1=\chi_1(z)\cos\mathbf{kr}, \tag{24}$$

$$f_1(z)=f_1(0)\psi_1(z), \quad \mathbf{k}=\{k_x,k_y\}, \quad \mathbf{r}=\{x,y\},$$

$$\zeta_2=f_2(z)\cos 2\mathbf{kr}+f_2(z), \quad \Phi_2=\chi_2(z)\cos 2\mathbf{kr}+\chi_2(z), \tag{25}$$

we find, in first-order approximation (omitting terms that are small because of the smallness of γ):

$$\chi_1'=\frac{f_1(0)\varphi_s'}{(r+2)\zeta_s(0)}e^{-kz}, \tag{26}$$

$$-\psi_1''+U(z)\psi_1=a\psi(z), \tag{27}$$

where the function $U(z)$ is obtained from $\bar{U}$ (17) by the replacement of $\bar{\xi}_s$ by ζ_s and by the addition of the component k^2,

$$a=\frac{2\mu_0F_0\varphi_s'}{3(r+2)\zeta_s(0)},$$

$$y=-\zeta_s'(z)\zeta_s^{-1/(r+2)}(z)\left[1+\frac{3\ln\zeta_s(z)}{2(r+2)F_0}\right]e^{-kz}. \tag{28}$$

At $\zeta_s-1\ll 1$, Eq. (27) transforms into the corresponding equation of linear theory.[1,2] However, it is seen that the parameter of expansion here is not γI_m but the quantity $(2p+r+1)\gamma I_m$. In the absence of a distortion band ($\varphi_s'=0$) this quantity is equal to unity, i.e., at $I_m\geqslant I_{cr}$ this expansion is not valid.[2)] Actually, however, it is not necessary because, as is also the case of (16), Eq. (27) is easily solved by the WKB method (see the next section).

The boundary conditions to Eq. (27) follow from the relations (9) and (24):

$$\psi_1'(0)=\nu_1, \quad \psi_1(0)=1, \tag{29}$$

where ν_1 is given by Eq. (21) with the replacement[3)] of $\bar{\xi}_s$ by ζ_s. In the second approximation, we obtain

$$\nabla^2\Phi_2+\frac{r}{(r+2)\zeta_s}\nabla\Phi_1\cdot\nabla\zeta_1=0, \tag{30}$$

$$\begin{aligned}&-\nabla^2\zeta_2-(r+2)\mu_0\zeta_s^{r/(r+2)}(\nabla\Phi_1)^2\\&+\mu_0\zeta_s^{-(r+3)/(r+2)}\zeta_s'\frac{1-{}^2/_3F_0-(r+2)^{-1}\ln\zeta_s}{r+2}\zeta_1\frac{\partial\Phi_1}{\partial z}\\&+\mu_0\zeta_s^{-1/(r+2)}\left(\frac{2}{3}F_0+\frac{\ln\zeta_s}{r+2}\right)\nabla\Phi_1\cdot\nabla\zeta_1\\&+\mu_0\zeta_s^{-1/(r+2)}\left(\frac{2}{3}F_0+\frac{\ln\zeta_s}{r+2}\right)\zeta_s'\frac{\partial\Phi_2}{\partial z}\\&+[U(z)-k^2]\zeta_2+U_1(z)\zeta_1^2=\gamma I_{cr}(r+2)e^{-\gamma z},\end{aligned} \tag{31}$$

$$U_1=-\frac{1}{2(r+2)}\{(1-p)(p+r+1)\zeta_s^{-(p+2r+3)/(r+2)}+p(p+r+2)\zeta_s^{-(p+2r+4)/(r+2)}\}. \tag{32}$$

We note that $U-k^2=\bar{U}$ with accuracy to within small quantities.

The boundary conditions at $z=0$ take the form

$$\frac{\partial\Phi_2}{\partial z}=\varphi_s'\left\{\frac{\zeta_2}{(r+2)\zeta_s}-\frac{(r+1)\zeta_1^2}{2(r+2)^2\zeta_s^2}\right\}, \tag{33}$$

$$\frac{\partial\zeta_2}{\partial z}=\nu_1\zeta_2+\nu_2\zeta_1^2, \tag{34}$$

where

$$\begin{aligned}\nu_2=\frac{1}{r+2}\Big\{&\frac{1}{2}(\zeta_s^{1/(r+2)}-1)\left[\left(\frac{d^2\nu}{d\xi^2}\right)_s\zeta_s^{-2(r+1)/(r+2)}\right.\\&\left.-(r+1)\left(\frac{d\nu}{d\xi}\right)_s\zeta_s^{-(2r+3)/(r+2)}\right]\\&+\zeta_s^{-2(r+1)/(r+2)}\left(\frac{d\nu}{d\xi}\right)_s+\frac{r+1}{2}\zeta_s^{-(2r+3)/(r+2)}\nu_s\Big\},\end{aligned} \tag{35}$$

the subscripts of ν and of the derivatives of ν mean that the corresponding quantity is taken at $\zeta=\zeta_s$.

4. FIRST APPROXIMATION. CRITICAL LIGHT INTENSITY

Solving the inhomogeneous equation (27) and taking into account the boundary conditions (29), we obtain the following equation for the determination of k^2:

$$-\sigma(0)-\frac{\sigma^{-2}(0)}{4}U'(0)-a\sigma^{1/2}(0)\int_0^z\frac{y(z')}{\sigma^{1/2}(z')}e^{-I(z')}\,dz'|_{z=0}=\nu_1, \tag{36}$$

here

$$\sigma(z)=U^{1/2}(z), \quad I(z')=\int_0^{z'}\sigma(z'')\,dz''.$$

We first consider the case of low light intensity, when $(2p+r+1)\gamma I_{cr}\ll 1$. The integrals in Eq. (36) are then

easily calculated, and at $\varepsilon \ll 1$ we get

$$k=k_0(I_m-I_{cr})/I_{cr}, \tag{37}$$

where

$$k_0=2(\gamma-\vartheta_s)/\gamma I_{cr}(3\gamma-\vartheta_s), \tag{38}$$

$$\gamma I_{cr}=3(1+\vartheta_1+\nu_1)/(\gamma-\vartheta_s)F_0\varphi_s'\mu_0. \tag{39}$$

We retain here the terms of order ν_1, keeping in mind the possible role of the boundary condition (9b'). They can be neglected upon use of the condition (9b).

Equations (38) and (39) have meaning at $\gamma > \bar{\nu}_s$ and

$$a \gg \frac{1+\vartheta_s}{\gamma-\nu_s}\frac{2p+r+1}{r+2}. \tag{40}$$

In essence they represent nothing else than Eqs. (39) and (40) of Ref. 2 (in different notation). The only difference is that here we do not use the effective value of ν_e' that enters in these equations, expressing the result in terms of the initial function $\nu(\xi)$. As is seen, it is this which manifests itself as the limitation on the rate of energy loss at the surface.

In accord with the estimates in Ref. 2, the inequality (40) imposes a rather stringent limitation on the quantity φ_s'. For this reason, it is natural to consider also the case of higher light intensity, $\gamma I_m \gtrsim 1$. We can make use here of the fact that $\bar{\zeta}_s$ is a slowly changing function of z and calculate approximately the integral in (36), setting $\bar{\zeta}_s \approx \bar{\zeta}_s(0)$ and $\bar{\zeta}_s' \approx -\gamma^2 I_m c$, where $c = d\bar{\zeta}_s/d(\gamma I_m)$. Then, with accuracy to within small quantities, Eq. (36) reduces to the form

$$k=k_1(1-b_{cr}/b); \tag{41}$$

$$b=\frac{3\gamma^2 I_m\varphi_s'}{(2r+5)(r+2)}\left[F_0+\frac{3}{2}\frac{\ln\xi_s}{r+2}\right], \tag{42}$$

$$b_{cr}=\frac{2\bar{\sigma}^2(1+\vartheta_1)}{c(1+2\vartheta_1\bar{\sigma}^{-1})-(r+2)\nu_s\gamma^{-2}I_{cr}^{-1}(\xi_s^{1/(r+2)}(0)-1)}, \tag{43}$$

$$k_1=\frac{2\bar{\sigma}^2 b_{cr}(\bar{\sigma}+\nu_1)}{cb_{cr}+\bar{\sigma}(\bar{\sigma}+\nu_1)}. \tag{44}$$

The quantities $\bar{\zeta}_s$ and $\bar{\sigma}$ in Eqs. (42)–(44) are taken at $x=0$ and (as is also ν_1) at $I_m=I_{cr}$.

Equation (43) has meaning only if the right side of it is positive. It is easy to demonstrate that at $\gamma I_{cr} \sim 1$ (as also at $\gamma I_{cr} \ll 1$) this condition is satisfied so long as $\gamma > \bar{\nu}_s$ (the case $\gamma I_{cr} \gg 1$ under conditions of small supercriticality is apparently of little interest).

At $\gamma I_{cr} \ll 1$, Eqs. (41)–(44) transform into (37)–(39). In the general case, however, the quantity γI_{cr} enters into the right side of (43) in rather complicated fashion. Therefore, the equality $b=b_{cr}$ is more simply regarded as the definition of the critical field intensity $\varphi_{s\,cr}'$ as a function of γI_{cr}:

$$3\gamma^2 I_{cr}\varphi_{s\,cr}' = \frac{2(2r+5)(r+2)\bar{\sigma}^2(F_0+{}^3/_2(r+2)^{-1}\ln\xi_s)-\xi_s^{(r+3)/(r+2)}}{c(1+2\vartheta_1\bar{\sigma}^{-1})-(r+2)\vartheta_s\gamma^{-2}I_{cr}^{-1}(\xi_s^{1/(r+2)}-1)}. \tag{45}$$

As also in the case of low light intensity, what is found uniquely are the individual quantities I_{cr} and $\varphi_{s\,cr}'$ but rather their ratio. In order words, either of these two quantities can be varied, selecting the other in accord with (45). However, the freedom of variation is not unlimited. Thus, at $\bar{\nu}_1 > 0$, the presence of bending of the bands turns out to be mandatory: as $\varphi_s' \to 0$, Eq. (45) is either not satisfied or requires that I_{cr} become infinite.

5. SECOND APPROXIMATION. STABILITY OF A TEMPERATURE SUPERLATTICE

Solving the second-approximation equation with the boundary conditions (33) and (34), we verify that the functions $\tilde{f}_2$ and $\bar{x}_2$ are determined here with accuracy to within a multiplicative constant. The role of the latter, as in the first approximation, can be played by the quantity $\tilde{f}_2(0)$; it can be found only by considering the next approximation.

On the other hand, the function f_2 is determined completely: the boundary conditions (33) and (34) yield

$$f_2(0)=\frac{1-r}{4(r+2)}\zeta_s^{-1}(0)f_1^2(0)\equiv\rho f_1^2(0) \tag{46}$$

(the quantity ρ is defined by this equation), and

$$f_1^2(0)=(r+2)\gamma I_{cr}R^{-1}. \tag{47}$$

Here, with accuracy to within small quantities,

$$R=\bar{\sigma}[(\bar{\sigma}+\vartheta_1)\rho+\nu_2/2]-a_1+a_2(\bar{\sigma}-8\gamma)-\frac{U_1(0)}{18}\left(1+2\sigma^2-\frac{8}{3}\bar{\sigma}^{-1}\right), \tag{48}$$

$$a_1=\frac{9\varphi_{s\,cr}'^2\xi_s^{-(r+1)/(r+3)}}{2(5r+2)(r+2)}$$

$$a_2=\frac{3\varphi_{s\,cr}'(F_0+{}^3/_2(r+2)^{-1}\ln\xi_s)}{2(5r+2)(r+2)\xi_s^{(r+3)/(r+2)}}. \tag{49}$$

By $\varphi_{s\,cr}'$ we mean the critical value of (45), and $\zeta_s = \zeta_s(0)$. The quantities ν, and 8γ in the first and third terms are left because, as we shall see, at $\gamma I_{cr} \gtrsim 1$ the quantity $\bar{\sigma}$ turns out to be much less than unity.

The quantity R can have any sign. Obviously, the stationary formulation of the problem that we have chosen, under the conditions of small supercriticality, is valid only at $R > 0$; the state described by the functions ζ_1, Φ_1 is stable here in the small. At $R < 0$, the considered stationary state is not realized and the superlattice turns out to be unstable—the superheating increases with time (we can establish this by using, for example, the identity (14)). The problem is, to what does this instability lead? This will be considered below in Sec. 6.

We first consider the case of low light intensity, $\gamma I_{cr} \ll 1$. Then, in accord with Eqs. (17)–(19), (47) and (49), we have

$$\bar{\sigma} \approx 1,\ -U_1 \approx \frac{r+1+2p}{2(r+2)},\ \xi_s - 1 \ll 1$$

and the condition $R > 0$ reduces to

$$x_2 < \varphi_{s\,cr}' < x_1;$$

$$x_{1,2}=\frac{F_0}{24}\pm\left[\left(\frac{F_0}{24}\right)^2+\frac{5r+2}{9}\left(\frac{5(2p+r+1)}{27}-\frac{1-r}{2}+\nu_2\right)\right]^{1/2}. \tag{50}$$

In the nondegenerate system $\exp F_0 \gg 1$, but, since the electron concentration under conditions of interest to us must still be not too small, F_0 cannot exceed several units. On the other hand, according to (39), $\varphi_{s\,cr}' \gg 1$. Thus, $R > 0$ and the system is stable only if $\nu(\xi)$

increases improbably rapidly with the heating of the electron gas: at $\nu_s \ll 1$, the inequality $\nu_2^{1/2} > 1$ should be satisfied. Apparently this possibility is not realized.

At $\gamma I_{cr} \geq 1$, the cases of acoustic ($p = -\frac{1}{2}$) and piezoelectric ($p = \frac{1}{2}$) scattering of the energy should be considered separately. According to (19b), at $p = -\frac{1}{2}$ and $\gamma I_m \geq 2$, with accuracy to within ~10–15%, we have $x \approx (\gamma I_m)^{1/2}$ and, consequently, $\bar{\zeta}_s(0) \approx (\gamma I_{cr})^{7/3}$ at $r = 3/2$ (impurity scattering of the momentum) and $\zeta_s(0) \approx \gamma I_{cr}$ at $r = -\frac{1}{2}$ (acoustic scattering of the momentum. Under the given conditions, (45) yields

a) at $r = 3/2$

$$\varphi'_{s\,cr} \approx \frac{4(\gamma I_{cr})^{-1/3}[1 - 1/3(\gamma I_{cr})^{-1/3}]}{\gamma(F_0 + \ln \gamma I_{cr})[1 + 1{,}6\nu_1(\gamma I_{cr})^{1/3}]}, \tag{51a}$$

b) at $r = -\frac{1}{2}$

$$\varphi'_{s\,cr} \approx \frac{6(\gamma I_{cr})^{1/3}[1 - 1/3(\gamma I_{cr})^{-2}]}{\gamma(F_0 + \ln \gamma I_{cr})[1 - 3\nu_s((\gamma I_{cr})^{1/3} - 1)/2\gamma^2 I_{cr}]}. \tag{51b}$$

The third and fourth terms on the right side of (48) turn out to be small in comparison with the second, and the condition $R > 0$ reduces to the form

a) at $r = 3/2$

$$\nu_2 > 38\frac{(\gamma I_{cr})^{-11/3}[1 - 1/3(\gamma I_{cr})^{-1/3}]^{1/3}}{\gamma^2(F_0 + \ln \gamma I_{cr})^2[1 + 1{,}6\nu_1(\gamma I_{cr})^{1/3}]}, \tag{52a}$$

b) at $r = -\frac{1}{2}$

$$\nu_2 \geq 44\frac{(\gamma I_{cr})^{-1}}{\gamma^2(F_0 + \ln \gamma I_{cr})^2}. \tag{52b}$$

The condition (52b) is unrealistic. However, it must be remarked that in the materials of interest to us (carrier concentration of the order of 10^{15}–10^{16} cm^{-3} and at not too high temperatures, see the estimates in Refs. 1 and 2) the impurity scattering of the momentum changes to acoustic only at very intense heating of the electron gas. The corresponding asymptotic form is investigated in the next section.

On the other hand, the inequality (52a) is not impossible. Thus, at $\gamma = 10^{-2}$, $\nu_2 = 10^{-1}$ and $F_0 = 4.5$, it is satisfied at $\gamma I_{cr} \approx 10$. According to (51a), the corresponding value of $\varphi'_s \approx 17$ or, in ordinary units,

$$\varphi_s' = 6600\frac{T_0}{300[K]}\frac{\lambda_0}{10^4[\text{cm}^{-1}]}.$$

At $T_0 = 75$ K and $\lambda_0 = 10^4$ cm^{-1}, this corresponds to a value of $V = 0.022$ volts.

The value of γI_{cr} given above is still rather large: in dimensional units, at a photon absorption cross section equal to 10^{-14} cm^2 and $\tau_0 = 10^{-10}$ sec, we obtain $I_{cr} \approx 800$ W/cm^2. By decreasing γI_{cr} at the same voltage on the sample we cannot satisfy the critical condition (51a) (i.e., the free convection does not arise); but if we increase the voltage correspondingly, then not only the one-dimensional distribution of the electron temperature becomes unstable, but also the three-dimensional one (with the superlattice).

We now turn to the case $p = \frac{1}{2}$ and $r = 3/2$ (piezoelectric scattering of the energy and impurity scattering of the momentum). It can be realized at not too intense heating of the electron gas; the solution can be investigated numerically. The condition $R > 0$ here reduces to the form

$$\nu_2 > \frac{m}{\gamma^2(F_0 + 3\ln x^{-1})^2}. \tag{52c}$$

The quantity m is shown in the table. We see that at $\gamma \sim 10^{-2}$ acceptable values of ν_2 and $\varphi'_{s\,cr}$ are obtained beginning with $\gamma I_{cr} \approx 1.5$ and at $\gamma = 10^{-1}$—beginning with $\gamma I_{cr} \approx 0.7$.

TABLE I. Dimensionless values of the critical light intensity, of the critical stress on the irradiated surface, and of the number m in the case $p = \frac{1}{2}$, $r = \frac{3}{2}$

γI_{cr}	$\gamma\varphi'_{s\,cr}(F_0+3\ln x^{-1})$	m	γI_{cr}	$\gamma\varphi'_{s\,cr}(F_0+3\ln x^{-1})$	m
0.1	21.8	90	1.1	0.46	0.0012
0.2	16	40	1.2	0.35	$5.1\cdot10^{-4}$
0.3	8.6	7.7	1.3	0.27	$2.2\cdot10^{-4}$
0.4	5.1	1.8	1.4	0.22	$1.0\cdot10^{-4}$
0.5	3.3	0.53	1.5	0.17	$4.7\cdot10^{-5}$
0.6	2.2	0.17	1.6	0.14	$2.3\cdot10^{-5}$
0.7	1.5	0.057	1.7	0.11	$1.0\cdot10^{-5}$
0.8	1.1	0.022	1.8	0.094	$5.9\cdot10^{-6}$
0.9	0.805	0.0075	1.9	0.078	$3.3\cdot10^{-6}$
1.0	0.60	0.0030	2.0	0.065	$1.5\cdot10^{-6}$

6. CHANGE IN THE SCATTERING MECHANISM. ASYMPTOTIC CASE OF INTENSE SUPER HEATING

At $R < 0$ there are two possibilities. First, there could exist, in principle, a strange attractor in the phase space of the considered dynamical system.[11] Then turbulent motion of the electron liquid would develop. Second, the superheating (with the framework of the formulation of the problem given above) could increase "without limit." Clarification of the question as to which of these possibilities is actually realized is connected with a number of mathematical difficulties. However, we note that overcoming them can be less necessary than it would seem, because upon increase of the superheating a new factor comes into play—the interchange of the mechanisms of energy and momentum scattering. The conditions of this interchange are easy to find by using the expressions for the energy and momentum relaxation times.[7,12,13]

As before, we limit ourselves to a simple parabolic model of the bands. Here the impurity scattering of the momentum changes to deformation acoustical scattering so long as

$$1 + \xi \geq \frac{2\pi Ze^3[Ms^2W_BN_ta_B^3\ln(12mT_0r_0^2\hbar^{-2})]^{1/2}}{E_1T_0^{1/2}\varepsilon^{1/2}V_0^{1/2}}. \tag{53}$$

Here Z is the absolute value of the charge of the impurity center in units of e; M and V_0 are the mass and volume of the elementary cell, s is the sound velocity, N_t is the impurity concentration, W_B and a_B are the Bohr energy and the Bohr radius in the crystal, m is the effective mass of the carrier, E_1 is strain-potential constant, and ε is the permittivity of the material. In addition, the piezoelectric (acoustical) mechanism of energy scattering changes to deformation acoustics at

$$1 + \xi \geq 7\beta^2e^2\hbar^2/m\varepsilon^2E_1^2T_0, \tag{54}$$

where β is the piezoelectric modulus (for simplicity, the case is considered in which only one component of the corresponding tensor is significant).

For an estimate, we set $W_B = 6.4\times10^{-4}$ eV, $a_B^{-3} = 4\times10^{15}$ cm^{-3}, $Ms^2 = 1$ eV, $T_0 = 3.5\times10^{-4}$ eV, $\varepsilon = 16.8$, $V_0^{1/3} = 6.5\times10^{-8}$ cm, $E_1 = 20$ eV, $\beta = 10^{-5}$ cal/cm^2 and $m = 10^{-2}m_0$ (m_0 is the mass of a free electron), and we obtain the following result from Eqs. (53) and (54): $1 + \xi \geqslant 0.8\,(10^{-3}N_t\ [\text{cm}^{-3}])^{1/2}$ and $1 + \xi \geqslant 1.25$. It is seen that as applied to the energy scattering mechanism, the expression "sufficiently great superheating" actually means $\xi \gtrsim 0.25$. The theory developed above still has meaning here. On the other hand, for the interchange of the momentum scattering mechanism under conditions of interest ot us ($N_t \approx 10^{15}$–10^{16} cm^{-3}), values of $\xi \approx 10$–20 are required. Here the study of asymptotically great superheating, when $\xi \gg 1$, has significance. Neglecting unity in the expressions (3) in comparison with ξ, and setting $p = r = -\frac{1}{2}$, we get, in place of (11),

$$-\nabla^2\zeta + {}^3/_2\zeta = {}^3/_2\gamma I_m e^{-\gamma z}. \tag{55}$$

At $\gamma I_{cr} \lesssim 1$, we can write on the right side of (55), in place of γI_m the difference $\gamma(I_m - I_{cr})$.

With accuracy to a factor of 3/2 in the second term, and on the right side, Eq. (55) is the same as the equation (15) linearized in ξ. It has a solution that depends only on z and is completely analogous with the solution for ξ_s at small superheating (Eqs. (17) of Ref. 1):

$$\zeta = \zeta_* = \frac{3\gamma I_m}{3-2\gamma^2}\{e^{-\gamma z} + Ce^{-z(1.5)^{1/2}}\}. \tag{56}$$

Here C is a constant determined from the boundary condition at $z = 0$. The latter obviously follows directly from the relation (9b) and, generally speaking, is a transcendental equation. It is simplified in two limiting cases of almost impenetrable and completely penetrable surfaces. The first of these corresponds to a very small (at any superheating) value of $\nu(\ll\gamma)$, while the second corresponds to a practically constant value of $\nu\xi^{-1/2}$. In both cases, the quantity γI_m drops out of the boundary condition and, consequently, C does not depend on γI_m. Here $\xi(z) \sim (I_m - I_{cr})^{3/2}$.

It is easy to establish the fact that the solution (56) is stable in the small, so long as $\nu_1(\xi_s) > 0$. The investigation by methods of ordinary branching theory also shows that the solutions which depend not only on z but also on x and y, and correspond to nonvanishing velocity u do not exist in the present case.[4)]

The distribution (56) completes to a certain degree the process of heating up of the electrons in the considered case, arising either as a result of development of an instability or at a sufficiently great supercriticality. It remains in force until scattering processes with participation of optical phonons become significant. It can be shown that the corresponding critical value is $\xi \sim \hbar\omega_0/2T_0$, where ω_0 is the frequency of the optical phonon. However, we note that, thanks to the inhomogeneity of heating, the interchange of the scattering mechanisms will not take place simultaneously over the entire sample, but initially only near the irradiated surface. Thus the problem arises of the distribution of the electron temperature in two- and three-dimensional systems (the number of layers obviously depends on which scattering mechanisms had predominated at low superheating). It is easy to show that such a system is unstable under the considered conditions. Actually, according to (52a)–(52b), the conditions of stability of the newly formed states turn out—at the given value of I_m—to be more and more rigorous. Thus the regions corresponding to the lower-temperature scattering mechanisms will gradually be displaced by the higher temperature ones, as long as the distribution of ξ does not take the asymptotic form (56).

APPENDIX
EFFECTIVE BOUNDARY CONDITIONS TO THE EQUATION OF CONTINUITY AND ENERGY TRANSPORT

The condition (9a′) was obtained and used (at $\xi_s \ll 1$) in Ref. 2. Here we shall give a somewhat more general derivation.

The general expression for the drift velocity, which is valid also in the space-charge region, has the form

$$u = -\mu\left(\nabla\varphi + \alpha\nabla T + \frac{D}{\mu}\nabla\ln n\right), \tag{A.1}$$

where D is the diffusion coefficient. Here the potential φ can be represented in the form of the sum of two components, $\varphi = \varphi_1 + \varphi_2$, the first of which is connected with the change in carrier concentration in the space-charge layer, while the second is due to inhomogeneity of the temperature of the electrons. It is obvious that φ_1 changes considerably over the distance r_0 (the fast component), while φ_2 changes over the length λ_0^{-1} (the slow component). In the body of the paper, the quantity φ_2 has been designated simply as φ; it is the only one that enters in the definition (8). Thus,

$$u = -\mu\nabla\Phi - \mu\left(\nabla\varphi_1 + \frac{D}{\mu}\nabla\ln n\right). \tag{A.2}$$

At $z = 0$, we have $u_z = 0$, i.e.,

$$\frac{\partial\Phi}{\partial z} = -\left(\frac{\partial\varphi_1}{\partial z} + \frac{D}{\mu}\frac{\partial\ln n}{\partial z}\right) \tag{A.3}$$

because of (2), the quantity $\partial\varphi/\partial z$ remains practically the same at $z = r_0$ as at $z = 0$ while the right side of (A.3) goes to zero. Then

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=r_0} \approx -\left(\frac{\partial\varphi_1}{\partial z} + \frac{D}{\mu}\frac{\partial\ln n}{\partial z}\right). \tag{A.4}$$

Under static conditions, this is the identity $0 = 0$. With the appearance of convection, both sides of Eq. (A.4) become different from zero (the right side, basically because of the dependence of the ratio D/μ on the electron temperature); here

$$\left(\frac{\partial\ln n}{\partial z}\right)_* = \left(\frac{\mu}{D}\right)_* \varphi_*', \quad \varphi_*' = \left(\frac{\partial\varphi_1}{\partial z}\right)_{*,z=0}.$$

Thus, Eq. (A.4) can conveniently be written in the form

$$\left.\frac{\partial\Phi}{\partial z}\right|_{z=r_0} = \left[\left(\frac{D}{\mu}\right)_{t=t_*+\delta t} - \left(\frac{D}{\mu}\right)_{t=t_*}\right] \Big/ \left(\frac{D}{\mu}\right)_{t=t_*}. \tag{A.5}$$

In the absence of Fermi degeneracy the condition (9a′) then follows.

In general expression for the energy flux density is

$$q+ue\varphi n=-nx\nabla T+nuT(^{5}/_{2}+r)+eun\varphi.$$

Transforming here to the nondimensional variable given in the body of the paper, and using the equality $n\approx n_0$, we obtain the condition (9b').

[1)]This method has been used in a number of researches (see, for example, Ref. 10).

[2)]Thus, although the ideological content of Ref. 1 and the statement of the problem given there are not subject to doubt, the result expressed by Eqs. (43) and (44) of this paper apparently does not have a region of applicability.

[3)]We note that the quantity v_1 could in principle turn out to be negative. As is easily shown, under certain conditions this circumstance can itself cause the instability of the static distribution of the electron temperature in the heated electron gas. In what follows, we shall nevertheless assume $v_1 > 0$.

[4)]This result should not produce surprise, because (55) is normally nothing other than the equation of transport of energy under conditions in which an external voltage is lacking and the kinetic coefficients do not depend on the temperature of the electrons.

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Translated by R. T. Beyer

Polarization of recombination radiation of multiparticle exciton–impurity complexes in silicon under uniaxial deformation

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(Submitted 18 December 1978)
Zh. Eksp. Teor. Fiz. **76**, 1727–1738 (May 1979)

The spectral distribution and the polarization of recombination radiation of free excitons and of multiparticle exciton–impurity complexes were investigated in silicon subjected to uniaxial compression and tension in the directions [111] and [001]. The agreement of the experimental and calculated degrees of polarization of all the exciton–emission bands was attained by choosing the values of two parameters that characterize the contributions of the different intermediate states in radiative recombination with participation of phonons. The degree of polarization of different emission lines of multiparticle exciton–impurity complexes is calculated on the basis of the choice of the symmetry of the initial and final electronic states in accordance with the shell model of the complexes and in accordance with the data on the polarization of the exciton radiation. A quantitative comparison of the results of this calculation with the experimental values of the degree of polarization of the phonon and no-phonon radiation components of complexes bound on phosphorus and boron atoms in silicon show that the shell model describes correctly the main properties of multiparticle exciton complexes.

PACS numbers: 78.60. – b, 71.35. + z, 62.20.Fe

1. INTRODUCTION

The substantial progress made recently in the investigation of multiparticle exciton–impurity complexes (MEIC)[1] are due to a considerable degree to the use of the "shell" model.[2,3] According to this model, the electrons and holes fill in succession the shells of the complexes in accordance with the Pauli principle. The cells are made up in this case of single-particle states and have the degeneracy multiplicity and the symmetry of a simple donor for electrons and of a simple acceptor for holes. The validity of this assumption has already been confirmed by the observation of new relatively weak lines in the emission spectra of complexes bound on donors of group V in silicon (the β series and some other lines[4-6]), as well as by the character of the splitting of the emission lines of the complexes in uniaxially deformed silicon[5-7] and in strong magnetic fields.

The polarization of the recombination radiation makes it possible to assess directly the symmetry of the initial and final electronic states in radiative decay of the

0038-5646/79/050878-07$02.40