the order parameter depends little on the coordinates: $a > \xi_0/(\Delta\tau)^{1/2}$, where $\xi_0 = (D/T_c)^{1/2}$ is the pair dimension. On the other hand, the time of spatial diffusion of the electrons in the contact must be small compared with the period of the field, $a < (D/\omega)^{1/2}$, so as to be able to neglect the disequilibrium of the electrons with energies $\omega > \Delta_0$. We see that at $\omega < T\Delta\tau$ both conditions are satisfied for a contact with dimension $\omega < Da^{-2} < T\Delta\tau$.

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Fluctuations in layered superconductors in a parallel magnetic field

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The effect of fluctuations on the properties of layered superconductors in a magnetic field parallel to the layers is considered. The fluctuations lead to a phase transition with respect to the field. In strong fields the long-range order is destroyed in both the longitudinal and transverse direction. The pair correlation function falls off in a power-law manner along the layer and exponentially across the layer. In this state the superconductivity is retained along the layers but disappears in the direction perpendicular to the layers.

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In certain layered superconductors Josephson interaction of the layers evidently occurs. The intercalation compounds TaS_2 and NbS_2 can serve as examples. The spectrum of the one-electron energies in the normal state of such compounds can be described by the dependence

$$\varepsilon(\mathbf{p}) = \mathbf{p}_\parallel^2/2m - 2W\cos p_z d, \tag{1}$$

where $\mathbf{p}_\parallel$ is the quasi-momentum along the layers, m is the effective mass, p_z is the quasi-momentum in the direction perpendicular to the layers, and d is the distance between the layers.

Josephson interaction of the layers occurs when the electrons can move over a distance of the order of the size of a Cooper pair without once hopping to neighboring layers. This situation obtains if the condition

$$W \ll T_c \tag{2}$$

is fulfilled, where T_c is the superconducting-transition temperature, calculated in the BCS approximation.

In a paper by Bulaevskii,[1] Ginzburg-Landau differential-difference equations were derived to describe layered superconductors. These equations go over into the ordinary Ginzburg-Landau equations for anisotropic superconductors if the temperature is close to the critical temperature. In the opposite limiting case

$$(T_c - T)/T_c = \tau \gg W^2/T_c^2 \tag{3}$$

such a transition is impossible. The most important differences arise in a magnetic field parallel to the layers. The Josephson interaction of the layers leads to the result that the diamagnetic currents are limited in magnitude and cannot destroy the superconducting order parameter. It was shown in Ref. 1 that only a paramagnetic effect can lead to suppression of the superconductivity in a parallel magnetic field. If the magnetic field is not very strong ($\mu H \ll T_c$, where μ is the Bohr magneton), or if the Chandrasekhar-Clogston paramagnetic limit is absent for any of the reasons in Refs. 2–4, the modulus $|\Delta|$ of the order parameter is close to the value obtained in the BCS approximation. In this case all the magnetic properties are described by the changes in the phase.

In a purely two-dimensional superconductor, phase fluctuations are important and lead to destruction of the long-range order.[5] However, even a very small probability of hops from layer to layer leads to restoration of the long-range order.[6] This result is obtained in the absence of a magnetic field. A magnetic field parallel to the layers weakens the interaction of the layers and enhances the fluctuations. In fields $\mu H \gg W/p_0 d$ the layers cease to interact and the fluctuations become purely two-dimensional. In this region of fields the long-range order is destroyed. The pair correlation function within the layers falls off in a power-law manner. The superconductivity is retained within the layers but vanishes in the direction perpendicular to the layers.

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1. CHOICE OF MODEL

We shall consider a system of conducting planes. Let the one-electron energy be described by formula (a) with W satisfying the condition (2). Below we shall consider the region of temperatures not very close to T_c, so that the condition (3) is assumed to be fulfilled. The paramagnetic limit will be disregarded. This can be done if the fields are not very strong: $\mu H \ll T_c$. In strong fields the paramagnetic limit is unimportant if electrons from different layers are paired,[2] if strong spin-orbit interaction exists,[3] or if there is a strong Kohn anomaly in the phonon spectrum.[4] Under these conditions we can neglect changes in the modulus of the order parameter. If triplet pairing occurs,[2,4] the direction of the spin also fluctuates. However, these fluctuations are weakly coupled with the magnetic field and are small in all fields for all reasonable values of W. Only the fluctuations in the phase of the order parameter are important. The free-energy functional F describing the superconductor in an external field H_0 parallel to the layers can be written in the form

$$F[\varphi,\mathbf{A}]=\frac{d}{8\pi}\sum_n\int\Big[\frac{1}{\lambda_L^2}\left(\frac{\Phi_0}{2\pi}\nabla_\parallel\varphi_n-\mathbf{A}_n\right)^2$$
$$+2H_c^2\left(1-\cos\left(\varphi_{n+1}-\varphi_n-\frac{2\pi d}{\Phi_0}A_z(n)\right)\right)\Big]\,d^2\mathbf{r}_n+\frac{1}{8\pi}\int(\mathbf{H}^2-2\mathbf{H}\mathbf{H}_0)\,dV. \tag{4}$$

In this expression,

$$\Phi_0=\pi c/e,\quad \lambda_L^2=mc^2/4\pi e^2N_s(T),\quad H_c^2=2\pi N_s(T)W^2/\varepsilon_F, \tag{5}$$

where Φ_0 is the quantum of flux, λ_L is the London penetration depth, $N_s(T)$ is the density of superconducting electrons, and the symbol $\nabla_\parallel$ denotes the gradient in the plane. Below we consider the case $d \ll \lambda_L$.

In the first term in (4) the integral is taken over the plane and a summation over all the planes is then performed, while in the second term the integral is taken over the whole volume. In the derivation of formula (4) it was assumed that the variation of $A_z(n)$ with the coordinate n is slow (this is fulfilled for $H_0 \ll \Phi_0/d^2$). The second term in the first integral of (4) is the Josephson energy. The expression (4) is written for clean superconductors and band motion of the electrons between layers. It can also be used for superconductors with impurities if we understand the quantity $N_s(T)$ to be the corresponding quantity in dirty superconductors and replace W by W_{eff}. In sufficiently dirty superconductors W_{eff} is determined by the hopping mechanism of the motion of electrons between layers.

Using the free-energy functional (4) we can calculate the free energy $\mathcal{F}$:

$$\mathcal{F}=-T\ln\int e^{-F[\varphi]/T}D\varphi, \tag{6}$$

and also all the thermodynamic quantities. In the following sections the calculations will be performed in the limits of weak ($\mu H \ll W/p_0 d$) and strong ($\mu H \gg W/p_0 d$) fields.

2. WEAK FIELDS

In the region of weak fields $\mu H \ll W/p_0 d$ the energy (6) can be calculated by the method of steepest descents. The zeroth approximation was considered in Ref. 1. The effect of the fluctuations can be estimated by calculating the corrections to the zeroth approximation.

Let the external field H_0 be directed along the X axis. Minimizing the free-energy functional (4), we obtain equations determining the vector potential $\mathbf{A}$ and phase φ:

$$-\frac{1}{\lambda_L^2}\frac{\Phi_0}{2\pi}\frac{\partial^2\varphi_n}{\partial y^2}+H_c^2\Big[\sin\left(\varphi_n-\varphi_{n+1}+\frac{2\pi d}{\Phi_0}A_z(n)\right)$$
$$+\sin\left(\varphi_n-\varphi_{n-1}-\frac{2\pi d}{\Phi_0}A_z(n)\right)\Big]=0,$$
$$\frac{\partial H}{\partial z}=-\frac{1}{\lambda_L^2}\left(\frac{\Phi_0}{2\pi}\frac{\partial\varphi_n}{\partial y}-A_y(n)\right), \tag{7}$$
$$\frac{\partial H}{\partial y}=\frac{2\pi d}{\Phi_0}H_c^2\sin\left(\varphi_{n+1}-\varphi_n-\frac{2\pi d}{\Phi_0}A_z(n)\right).$$

The exact solution of Eqs. (7), minimizing the free energy, is difficult to find. However, in fields $\mu H \ll W/p_0 d$ we can find a good approximation. This approximation is a lattice of anisotropic vortices, analogous to the lattice of vortices in ordinary type-II superconductors. The solution for an individual vertex at large distances can be found by expanding the sine in Eqs. (7) and replacing the finite differences by derivatives. This solution has the form[1]

$$\varphi=\operatorname{arctg}\frac{u}{v},\quad H=\frac{\Phi_0}{2\pi\lambda_j\lambda_L}K_0(\rho). \tag{8}$$

In the formulas (8),

$$\lambda_j=\Phi_0/2\pi dH_c,\quad u=y/\lambda_j,\quad v=z/\lambda_L,\quad \rho=(u^2+v^2)^{1/2},$$

and K_0 is the zeroth-order Bessel function of imaginary argument.

At short distances $\rho\sim d/\lambda_L$ the sines in (7) become of order unity, and the expressions (8) are inapplicable. The region $\rho\sim d/\lambda_L$ is the nonlinear "core" of the vortex. We emphasize that inside this core, unlike in ordinary type-II superconductors, the superconductivity is not destroyed. The other properties of the state obtained are analogous to those of the mixed state of type-II superconductors. All the quantities can be calculated if in the corresponding expressions we change all the scales along the y axis by a factor of λ_j/λ_L and replace ξ by d. In particular, the field H_{c1} at which vortices first penetrate into the sample is determined from the relation

$$H_{c1}=\frac{\Phi_0}{4\pi\lambda_L\lambda_j}\ln\frac{\lambda_L}{d}. \tag{9}$$

The vortex structure gives a good description of the state so long as the spacing between the vortices is much greater than the size of the core. The nonlinear cores of the vortices begin to overlap in fields of the order of H_{c2}:

$$H_{c2}=\Phi_0\lambda_L/2\pi d^2\lambda_j. \tag{10}$$

However, a more exact solution of Eqs. (7) in fields $H \gtrsim H_{c2}$ is meaningless, since in this region fluctuations are important. Fluctuations are small only the region $H \ll H_{c2}$. We shall estimate their contribution in the region of fields $H_{c1} \ll H \ll H_{c2}$.

We represent the phase φ in the expression (4) in the form

$$\varphi=\varphi^{(0)}+\varphi^{(1)}, \tag{11}$$

where $\varphi^{(0)}$ is the solution of Eqs. (7). Regarding $\varphi^{(1)}$ as a small correction we expand the cosine in (4) to terms quadratic in $\varphi^{(1)}$ and represent the free-energy functional F (4) in the form

$$F=F'+\frac{d}{8\pi}\sum_n\int\left[\frac{\Phi_0^2(\nabla\varphi_n^{(1)})^2}{(2\pi\lambda_L)^2}\right.$$
$$\left.+H_c^2\cos(\varphi_{n+1}^{(0)}-\varphi_n^{(0)})(\varphi_{n+1}^{(1)}-\varphi_n^{(1)})^2\right]d^2\mathbf{r}_n. \tag{12}$$

In this expression the energy F' is equal to the energy of the vortex state with neglect of fluctuations. For the calculation of the fluctuations, F' is not important.

In the second term the vector potential A_z has been omitted in the cosine. This can be done in fields $H_{c1} \ll H \ll H_{c2}$. Using the expression (12) we can calculate the mean square deviation of the phase, $\langle(\varphi^{(1)})^2\rangle$. For this we expand $\varphi^{(1)}$ in a complete set of functions ψ_q:

$$\varphi^{(1)}(n,\mathbf{r})=\sum_q C_q\psi_q(n,\mathbf{r}).$$

As the functions ψ_q we choose the eigenfunctions of the "Schrödinger equation"

$$-\Delta\psi_q+\left(\frac{\lambda_L}{\lambda_j d}\right)^2\cos(\varphi_{n+1}^{(0)}-\varphi_n^{(0)})(2\psi_q(n)-\psi_q(n+1)-\psi_q(n-1))=E_q\psi_q(n). \tag{13}$$

After this it is not difficult to express the mean square deviation in terms of the energy eigenvalues E_q of Eq. (13):

$$\langle(\varphi^{(1)})^2\rangle=\frac{4mT}{N_s(T)}\sum_q E_q^{-1}. \tag{14}$$

In the derivation of this expression the formulas (5) have been used.

In the region of fields $H \ll H_{c2}$ the vortex cores overlap weakly, and the cosine in Eq. (13) can be replaced by unity. In this case the solutions of Eq. (13) are plane waves.

The energy $E_q^{(0)}$ in the zeroth approximation is equal to

$$E^{(0)}=k_\parallel^2+4\sin^2\frac{k_z d}{2}\left(\frac{\lambda_L}{\lambda_j d}\right)^2.$$

Regarding $1-\cos(\varphi_{n+1}^{(0)}-\varphi_n^{(0)})$ as a perturbation and calculating the correction $E^{(1)}$ to the energy, we obtain for small k_z

$$E=E^{(0)}+E^{(1)}=k_z^2\left(\frac{\lambda_L}{\lambda_j}\right)^2\left(1-\frac{\pi H}{H_{c2}}\ln\frac{H_{c2}}{H}\right)+k_\parallel^2.$$

Substituting the energy E into (14) and performing the integration we find the mean square deviation of the phase:

$$\langle(\varphi^{(1)})^2\rangle=\frac{mT}{\pi d N_s(T)}\ln\left[\frac{\lambda_j d}{\lambda_L\xi}\left(1-\frac{\pi H}{H_{c2}}\ln\frac{H_{c2}}{H}\right)^{-1/2}\right]. \tag{15}$$

This formula shows that fields $H \ll H_{c2}$ have only a weak effect on the fluctuations. The fluctuations in the absence of a magnetic field were estimated in the paper by Dzyaloshinskii and Kats.[6] In the region $H \ll H_{c2}$ the quantity (15) is small for all reasonable values of the hopping integral.

For $H \gtrsim H_{c2}$ fluctuations become important. In this region the approximations used above are inapplicable.

3. PAIR CORRELATION FUNCTION IN THE REGION OF STRONG FIELDS

In the region of strong external fields $H \gg H_{c2}$ fluctuations have a substantial influence on the properties of the superconductor. In weak fields long-range order exists, since the phase fluctuations are small. It is of interest to elucidate the behavior of the pair correlation function in the region of strong fields too. Strong fields penetrate almost wholly between the layers. Therefore, the free-energy functional (4) can be simplified. Discarding the unimportant constant terms, we obtain

$$F[\varphi]=F_0[\varphi]+F_1[\varphi], \tag{16}$$

where F_0 and F_1 are determined by the following expressions:

$$F_0[\varphi]=\frac{d}{8\pi}\left(\frac{\Phi_0}{2\pi\lambda_L}\right)^2\sum_n\int(\nabla\varphi_n)^2d^2\mathbf{r}_n,$$
$$F_1[\varphi]=-\frac{dH_c^2}{4\pi}\sum_n\int\cos(\varphi_{n+1}-\varphi_n-\mathbf{rh})\,d^2\mathbf{r}_n. \tag{17}$$

In formula (17),

$$\mathbf{h}=(0, 2\pi dH_0/\Phi_0, 0). \tag{18}$$

In strong fields the cosine in F_1 oscillates rapidly. The contribution from F_1 can be regarded as a perturbation. Neglecting the energy F_1 it is not difficult to obtain the pair correlation function $\Pi_0(\mathbf{R})$ within the layer:

$$\Pi_0(\mathbf{R})=\frac{\int\exp\{i\varphi_n(\mathbf{R})-i\varphi_n(0)\}\exp\{-F_0[\varphi]/T\}D\varphi}{\int\exp\{-F_0[\varphi]/T\}D\varphi}. \tag{19}$$

Calculating the Gaussian integral in (19), we find

$$\Pi_0(R)=(\xi/R)^\alpha. \tag{20}$$

In formula (20) $\xi=v/T_c$ is the Cooper-pair size. The index α can be written in the form

$$\alpha=Tm/2\pi dN_s(T). \tag{21}$$

The formulas (20), (21) show that in a strong field the correlation function in the layers falls off in a power-law manner, though very slowly. It is not difficult to convince oneself that there is no correlation between different layers. These results are obtained with neglect of F_1. To calculate the exact correlator it is necessary to replace F_0 by $F[\varphi]$ in (19). Then, expanding the exponentials in F_1/T, we can bring the first nonvanishing correction $\Pi^{(1)}(\mathbf{R})$ to the form

$$\Pi^{(1)}(\mathbf{R})=\frac{1}{2T^2}\left(\frac{\xi}{R}\right)^\alpha\left(\frac{dH_c^2}{4\pi}\right)^2\int\left(\frac{|\mathbf{n}+\mathbf{m}-\mathbf{R}|^\alpha|\mathbf{m}|^\alpha}{|\mathbf{n}+\mathbf{m}|^\alpha|\mathbf{m}-\mathbf{R}|^\alpha}-1\right)$$
$$\times\frac{\cos\mathbf{hn}}{|\mathbf{n}|^{2\alpha}}d^2\mathbf{m}\,d^2\mathbf{n}. \tag{22}$$

It is sufficient to perform the subsequent calculations in (22) in the limit of small α. Performing the integration in (22) in this limit and using the formulas (10), (18), and (21), we obtain

$$\Pi(\mathbf{R})=\left(\frac{\xi}{R}\right)^{\alpha}\left[1-4\left(\frac{H_{c2}}{H}\right)^{4}-8\left(\frac{H_{c2}}{H}\right)^{4}\alpha\ln Rh\right]. \quad (23)$$

This expression is applicable for not very large distances. As R tends to infinity the third term in (23) becomes of order unity. In this region we cannot confine ourselves to the first corrections.

The principal contribution for large R is made by terms of the form

$$(\alpha(H/H_{c2})^{4}\ln Rh)^{n}. \quad (24)$$

Calculation shows that allowance for these terms leads to renormalization of the index in formula (20). We must suppose that a power-law decrease also remains when the next corrections are taken into account. In this case the correlator $\Pi(\mathbf{R})$ at large distances can be written in the form

$$\Pi(\mathbf{R})=C\left(\frac{H_{c2}}{H}\right)\left(\frac{\xi}{R}\right)^{\alpha}(hR)^{-\beta}. \quad (25)$$

In the limit of strong fields $H\gg H_{c2}$ the functions $C(H_{c2}/H)$ and $\beta(H_{c2}/H)$ can be obtained by comparing (25) with (23):

$$C(H_{c2}/H)=1-4(H_{c2}/H)^{4},\quad \beta=8(H_{c2}/H)^{4}. \quad (26)$$

The correlation function between different layers falls off much faster than the correlation function in one layer. In the zeroth approximation in $(H/H_{c2})^{-1}$ the layers do not interact with each other at all. Assuming that the replacement $F_0[\varphi]\to F[\varphi]$ has been made in (19), and expanding the exponentials in F_1/T, we can convince ourselves that the correlations between layers at a distance nd apart have order of magnitude $(H_{c2}/H)^{2n}$. For example, for $n=1$ a simple calculation leads to the following formula:

$$\langle\exp\{i\varphi_0(\mathbf{R})-i\varphi_1(\mathbf{R})\}\rangle=2(H_{c2}/H)^{2}\exp\{-i\mathbf{hr}\}. \quad (27)$$

The forms of the correlation function (25) along the layers and the correlation function across the layers in the region of strong fields differ from those of the correlation functions in the region of weak fields, in which there is long-range order. This difference permits us to conclude that, at a certain field of order H_{c2}, a phase transition occurs. We note that this critical field does not depend on the temperature, so long as the condition (3) is fulfilled.

The pair correlation function permits us to conclude that there is a phase transition, but does not itself appear in any physical quantities. The next section is devoted to calculating some of the physical quantities, and also to elucidating the question of the superconductivity in the region of strong fields.

4. SUPERCONDUCTIVITY IN THE REGION OF STRONG FIELDS

It was shown above that a strong field parallel to the layers causes the interaction between the layers to vanish. Electron hops from layer to layer cease to be important. The motion of the electrons can be assumed to be two-dimensional. To elucidate the possibility of superconductivity in one layer it is necessary to consider the possibility of formation of a vortex in one layer. If the probability of a fluctuation with formation of such a vortex is nonzero the superconductivity is destroyed, since the motion of the vortex leads to a decrease of the current. If the probability of such a fluctuation is equal to zero a supercurrent is possible. The latter occurs in the two-dimensional XY model.[7] In layered superconductors the problem of a vortex in one layer requires a special analysis, since even for two-dimensional motion of the electrons a three-dimensional magnetic field is produced. Neglecting the hopping probability and minimizing the energy (4) we obtain equations determining the vector potential:

$$\sum_n \mathbf{A}_\parallel\delta(z-nd)-\lambda_{\text{eff}}\Delta\mathbf{A}_\parallel=\mathbf{\Phi}(\mathbf{r})\delta(z),$$
$$A_z=0,\quad \partial\mathbf{A}_\parallel/\partial x+\partial\mathbf{A}_\parallel/\partial y=0. \quad (28)$$

The equations (28) are written for a vortex at $r=0$ in the plane with coordinate $n=0$; $\lambda_{\text{eff}}=\lambda^2_L/d$. In the cylindrical system of coordinates the vector $\mathbf{\Phi}$ has components

$$\Phi_r=\Phi_z=0,\quad \Phi_\theta=\Phi_0/2\pi r.$$

The right-hand side of the first equation (28) is obtained from the condition that the phase φ changes by 2π when we go round the vortex. We emphasize that the second term in the first equation (28) contains the three-dimensional Laplacian. Applying a Fourier transformation to both sides of Eq. (28), we obtain

$$\tilde{A}_{\parallel\mathbf{k}q}+\lambda_{\text{eff}}(\mathbf{k}^2+q^2)\mathbf{A}_{\parallel\mathbf{k}q}=\mathbf{\Phi}_\mathbf{k}, \quad (29)$$

where

$$\mathbf{A}_{\parallel\mathbf{k}q}=\int e^{-i\mathbf{kr}-iqz}\mathbf{A}_\parallel(r,z)\,d^2\mathbf{r}\,dz,$$
$$\tilde{\mathbf{A}}_{\parallel\mathbf{k}q}=\sum_n\int \mathbf{A}_\parallel(\mathbf{r},z)\delta(z-nd)e^{-i\mathbf{kr}-iqz}d^2\mathbf{r}\,dz$$
$$=\frac{1}{d}\sum_{n=-\infty}^{\infty}\mathbf{A}_{\parallel\mathbf{k}_q-2\pi n/d}. \quad (30)$$

Expressing $\mathbf{A}_{\parallel\mathbf{k}q}$ from Eq. (29) in terms of $\tilde{A}_{\parallel\mathbf{k}q}$ and substituting into (30) we obtain a formula for $\tilde{A}_{\mathbf{k}q}$:

$$\tilde{\mathbf{A}}_{\mathbf{k}q}=\mathbf{\Phi}_\mathbf{k}\left[1+2k\lambda_{\text{eff}}\frac{\operatorname{ch}kd-\cos qd}{\operatorname{sh}kd}\right]^{-1}. \quad (31)$$

To find the vector potential in a plane at distance nd from the plane in which the vortex is situated we make use of the formula for transformation to the coordinate representation with respect to the Z axis:

$$\mathbf{A}_{\parallel\mathbf{k}}(nd)=d\int\tilde{\mathbf{A}}_{\parallel\mathbf{k}q}e^{-iqnd}\frac{dq}{2\pi}. \quad (32)$$

Substituting the expression (31) into (32) and performing the integration, we obtain

$$\mathbf{A}_{\parallel\mathbf{k}}(nd)=\frac{\mathbf{\Phi}_\mathbf{k}\operatorname{sh}kd}{2k\lambda_{\text{eff}}}\frac{[f-(f^2-1)^{1/2}]^n}{(f^2-1)^{1/2}}, \quad (33)$$

where

$$f=\operatorname{ch}kd+\operatorname{sh}kd/2k\lambda_{\text{eff}}.$$

Using the formula (33) for the vector potential of the nth plane, we can obtain an expression for the current $\mathbf{J}_n$ in this plane:

$$\mathbf{J}(nd)=\frac{c}{4\pi\lambda_{\text{eff}}}[\Phi\delta(n)-\mathbf{A}(nd)]. \qquad (34)$$

For $n=0$ and $d\to\infty$ the expressions (33) and (34) go over into the corresponding expressions for a film.[8]

The energy of one vortex can be written in the form

$$E_0=\sum_n\int\left[\frac{2\pi\lambda_{\text{eff}}}{c^2}\mathbf{J}_k^2(nd)+\frac{1}{2c}\mathbf{J}_k(nd)\mathbf{A}_{\parallel k}(nd)\right]\frac{d^2\mathbf{k}}{(2\pi)^2}. \qquad (35)$$

Substituting the formulas (33) and (34) for $d\ll\lambda_L$ into (35) and calculating the integral, we obtain the energy of the vortex:

$$E_0=\frac{\Phi_0^2}{(4\pi)^2\lambda_{\text{eff}}}\ln\frac{R}{\xi}, \qquad (36)$$

where R is the size of the system. Formula (36) shows that the vortex energy increases logarithmically with the size of the system and is not cut off at distances of the order of the penetration depth. This dependence of the vortex energy is characteristic for the XY model. In this respect the situation differs from the case of a superconducting film, in which the vortex energy is finite.

The fact that the vortex energy is infinite leads to the result that a fluctuation with formation of a vortex cannot occur, since the probability of such a fluctuation is proportional to $\exp(-E_0/T)$. In the absence of hops from plane to plane, arguments analogous to those for the XY model[7] show that the superconductivity along the layers is not destroyed. The response to a weak vector potential $A_\parallel$ has the usual form:

$$\mathbf{j}_\parallel=-\frac{c}{4\pi\lambda_L^2}\mathbf{A}_\parallel. \qquad (37)$$

The quantity λ_L in (37) is expressed by formula (5). Only in the very close neighborhood of the BCS temperature $(T_c-T)/T_c\sim T_c/\varepsilon_F$ is the contribution from vortex-antivortex configurations important. In this region the temperature dependence of λ_L is more complicated.

To estimate the influence of hops we write down the general formula for the current in an external potential:

$$\mathbf{j}_\parallel=\frac{c}{4\pi\lambda_L^2}\left\langle\frac{\Phi_0}{2\pi}\nabla_\parallel\varphi-\mathbf{A}_\parallel\right\rangle. \qquad (38)$$

In this expression the averaging is performed with the free-energy functional (4). In the region of strong parallel fields we expand the statistical weight $\exp(-F/T)$ in the term describing the hops. One can convince oneself that, in all orders of perturbation theory, there is no contribution from slow fluctuations. Therefore, the formula (37) with λ_L described by the expression (5) is applicable in the entire region of fields parallel to the layers.

More interesting is the behavior of the response in the direction perpendicular to the layers. In the region of weak parallel fields superconductivity exists in all directions. To examine the question of the superconductivity in the region of strong fields we write the formula for the current perpendicular to the layers:

$$j_z(r)=\frac{dH_c^2}{2\Phi_0}\left\langle\sin\left(\varphi_{n+1}-\varphi_n-\mathbf{hr}-\frac{2\pi d}{\Phi_0}A_z(\mathbf{r})\right)\right\rangle. \qquad (39)$$

In this formula, A_z is the slowly varying external vector potential. The averaging is performed with the energy functional (17), in which the replacement $Hy\to Hy+A_z$ has been made. Expanding the statistical weight in the term with the hops and averaging, we obtain in the first approximation

$$j_z=\frac{(dH_c)^2}{8\pi\Phi_0 T}\int\Pi^2(\mathbf{r}-\mathbf{r}')\sin(\mathbf{h}(\mathbf{r}-\mathbf{r}')+A_z(\mathbf{r})-A_z(\mathbf{r}'))d^2\mathbf{r}'. \qquad (40)$$

Going over to Fourier components in (40) and expanding to linear terms in A_z, after calculating the integral we obtain

$$j_{zk}=\frac{2\lambda_L^2}{\lambda_j^2\Phi_0}\left(\frac{1}{\mathbf{h}^2}-\frac{1}{2(\mathbf{h}+\mathbf{k})^2}-\frac{1}{2(\mathbf{h}-\mathbf{k})^2}\right)A_{zk}. \qquad (41)$$

This formula shows that for $k\to 0$ the response to the vector potential tends to zero and there is no superconductivity in the direction perpendicular to the layers. Analysis of higher orders of perturbation theory shows that for $k=0$ the response is equal to zero in any order.

The absence of superconductivity in the perpendicular direction has the result that, in a parallel magnetic field, a rather small magnetic moment is produced. Calculating the free energy in the first nonvanishing approximation in the hopping integral using the formulas (6), (16), and (17), and using the usual formulas of thermodynamics, we obtain an expression for the magnetic moment M in an external field $H\gg H_{c2}$:

$$M=-\frac{3}{16\pi}H_c\left(\frac{H_cH_{c2}}{H}\right)^2. \qquad (42)$$

In fields $H\ll H_{c2}$ the magnetic moment can be found from the usual formulas for the mixed state in anisotropic superconductors.

CONCLUSION

The analysis carried out above shows that fluctuations in layered superconductors in a magnetic field parallel to the layers lead to a phase transition with respect to the field. In order of magnitude the critical field is equal to $W/p_0d\mu$ and does not depend on the temperature. In fields lower than the critical field the fluctuations are small and the mixed-state picture proposed by Bulaevskii is applicable.[1] The cores of the vortices are not destroyed in this state. The vortices occupy fixed positions between the layers and cannot move across the layers. This leads to the result that the superconducting current along the layers does not decrease. The motion of the vortices along the layers does not affect the longitudinal current. Neither does it act on the transverse current, since the motion of a vortex causes the phase in the current state to change by 2π and this change of phase does not change the Josephson current. Therefore, the state with small external fields is superconducting in all directions.

In fields greater than the critical field the superconductivity along the layers remains, although long-range order is absent. The pair correlation function falls off in a power-law manner. An analogous situation obtains in the two-dimensional XY model.[7] The correlation between different layers is weak. There is no superconductivity in the direction perpendicular to the layers.

The theory developed above can be applied to the intercalation compound $TaS_2(Py)_{1/2}$, which displays superconducting properties at low temperatures. For $W < T_c$ and $d \sim p_0^{-1}$, which, apparently, is fulfilled for this compound, the critical field should not exceed 50–60 kOe. Measurements in a parallel field with such values have already been performed,[9] but only the longitudinal resistivity was measured. It was discovered that at low temperatures it is equal to zero even in very strong fields $H \sim 150$ kOe. Unfortunately, it is not possible to detect the phase transition considered above by measuring the longitudinal conductivity. Measurements of the transverse conductivity might clarify the situation.

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Study of orientation-statistical properties of liquid crystals of the tolane class by optical methods

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The methods of Raman light scattering and optical probing are used to study the orientation-statistical properties of some new nematic liquid crystals of the tolane class: methoxy-amyltolane, ethoxy-hexyltolane, ethoxy-octyltolane. An improved method of Raman-scattering spectroscopy is proposed, which makes it possible to eliminate the effects of multiple scattering and to develop the spectroscopy of specimens with a thickness comparable with the scattered wavelength. On the basis of the experimental results, single-particle orientational distribution functions are constructed for the three crystals. It is shown that the negative order parameter observed experimentally is due to neglect of the anisotropy of the local field of the light wave. The effect of elongation of semiflexible segments of molecules and of increase of the mobility of their end groups on the orientation-statistical properties of a rigid molecular nucleus is explained. It is shown that the Maier-Saupe mean-field theory agrees satisfactorily with experiment far from the phase transition to an isotropic liquid, but that it does not give a satisfactory description of the orientational statistics of the mesophase in the pretransition temperature range.

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1. INTRODUCTION

The physical properties of nematic liquid crystals (NLC) are determined to a significant degree by the orientation-statistical properties of the structure of the mesophase. These in turn are connected with the character of the intermolecular interactions and depend on the chemical nature and conformational properties of the molecules. The orientation-statistical degree of order of molecules in a NLC is described by the single-particle orientational distribution function [1]$F(\varphi, \theta, \psi)$, which gives the probability of finding the orientation of a molecule within a small solid angle $d\Omega$ close to the corresponding Euler angles φ, θ, ψ. The latter determine the orientation of the molecular system of coordinates with respect to the laboratory system (x, y, z). The z axis coincides with the direction of the director **r**, and the x and y axes lie in the plane perpendicular to it. On the basis of x-ray structural data it can be concluded[2,3] that the directions **r** and −**r** in a NLC are equivalent and that the projections of the long molecular axes on the xy plane are randomly distributed.

The local uniaxiality of the nematic mesophase, the presence of rotation of the molecules about the long axes, and the closeness of their form to cylindrical permit us to regard a NLC as an ensemble of uniaxial structural units with an orientational distribution function $F(\theta)$. This function can be represented as a series of even Legendre polynomials $P_l(\cos\theta)$[4]:

$$F(\theta)=\sum_l \frac{2l+1}{2}\langle P_l(\cos\theta)\rangle P_l(\cos\theta), \qquad (1)$$

where the coefficients of the series are determined by the expression

$$\langle P_l(\cos\theta)\rangle=\int_0^\pi P_l(\cos\theta)F(\theta)\sin\theta d\theta. \qquad (2)$$

The first three coefficients have the form

$$\langle P_0\rangle=1, \quad \langle P_2\rangle={}^1/_2\langle 3\cos^2\theta-1\rangle,$$
$$\langle P_4\rangle={}^1/_8\langle 35\cos^4\theta-30\cos^2\theta+3\rangle. \qquad (3)$$