The asymmetrical rotator as a detector of monochromatic gravitational radiation

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The interaction of a rotating asymmetric (different principal moments of inertia) body with a gravitational wave is considered. A resonant rotational detector of monochromatic gravitational radiation is proposed in which the rotations due to the incident wave and the rotation that ensures resonance of the detector with the wave correspond to different rotational degrees of freedom. This greatly facilitates the development of a detector. The noise due to the gradient of the earth's gravitational force and to the rotation of the detector as a whole is estimated.

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Rapidly rotating neutron stars may turn out to be sources of intense monochromatic gravitational radiation. The most powerful among them is considered to be[1] the pulsar PSRO531 + 21 in the Crab nebula, which has the shortest period. The cyclic frequency of its radiation is $\omega_{gr} \approx 380$ rad/sec, and the dimensionless amplitude can be $h \approx 10^{-25}$. These parameters will be used below for estimates.

To detect monochromatic gravitational radiation, Braginskiĭ, Zel'dovich, and Rudenko[2] have proposed a rotational "heterodyne" detector. The measured quantity in this detector is the change of the angle (phase difference) between the rotators (dumbbells) that rotate freely around an axis pointing to the source of the radiation. The angular velocities of the rotator should be the same and equal to $\omega_{gr}/2$ and the initial phases should differ by $\pi/2$. At a favorable ratio of the phases of the rotators to the phase of the wave, after an accumulation time t, the phase difference of the rotators can change by

$$\sim \omega_{gr}^2 t^2 h \approx 10^{-12}\ \text{rad/sec}^{-1} \cdot \left(\frac{\omega_{gr}}{380\ \text{rad/sec}^{-1}}\right)^2 \left(\frac{t}{10^4\,\text{sec}}\right)^2 \frac{h}{10^{-25}}.$$

In general, such a phase shift can be registered.[3]

1. The realization of the proposed rotation-detector scheme entails certain difficulties.

In order for the wave to produce a relative rotation of the rotators, the latter must be able to rotate freely. The average difference between the random and dissipative angular accelerations must not exceed

$$\sim 3\cdot 10^{-20}\ \text{rad/sec}^2 \left(\frac{\omega_{gr}}{380\ \text{rad/sec}^{-1}}\right)^2 \frac{t}{10^4\ \text{sec}} \frac{h}{10^{-25}}$$

(relative rotation ~1″ per year), otherwise in a measurement time ~10^4 sec their random relative rotation turns out to be $\gtrsim 10^{-12}$ rad. The accelerations themselves must be

$$\lesssim 3\cdot 10^{-8}\ \text{rad/sec}^{-2} \cdot (t/10^4\ \text{sec})^2$$

(complete stoppage in $\gtrsim 300$ years). Otherwise after ~10^4 sec the phases of the rotators will change by $\pi/2$ and the gravitational wave will begin to apply to them accelerations of opposite sign. The relative difference $\Delta\omega/\omega$ between the initial velocities of the rotators must not exceed

$$\sim 10^{-18} (\omega_{gr}/380\text{rad/sec}^{-1})(t/10^4\ \text{sec})(h/10^{-25}),$$

otherwise the rotators will turn relative to each other, within time t, through an angle larger than the measured angle.

These stringent requirements are aggravated by the fact that the angular velocity of the rotation of the neutrons star of the pulsar decreases.[4] The effect of this factor on the rotation detector constructed in accordance with the scheme proposed in Ref. 2 was considered in Ref. 5, from which it follows that the conditions for the accumulation of the signal become substantially worse. This is seen also from the following simple estimate. For the PSR 0531 + 21, the rate of slowing down of the rotation of the neutron star is ~2.5×10^{-10} rad·sec^{-2} (Ref. 4), so that after ~3×10^4 sec the phase of the gravitational wave changes by $\pi/2$. The relative angular acceleration of rotators will then reverse sign and the accumulation of the signal will stop. The coherence of the phases of the rotators and of the gravitational wave is violated also because of the Doppler shift due to the rotation of the earth.[6] At an accumulation time $\gtrsim 10^4$ sec, this effect is substantial. In order to provide a detector-sensitivity margin, (the estimate $h \approx 10^{-25}$ is optimistic) and to be able to accumulate the signal for a long time (3×10^4 sec), it is desirable to tune the detector to the varying frequency of the gravitational wave.

In the discussed scheme,[2] one and the same degree of freedom of the dumbbell rotation is connected with two other rotations: "resonant" (with frequency $\omega_{gr}/2$), which ensures resonance with the incident gravitational wave, and "signal," which is produced by this wave. This makes it impossible to monitor and to correct the resonant rotation in the course of the reception, and simultaneously requires that the parameters of the resonant rotation be maintained with an accuracy much higher than needed for the resonance conditions. In the present paper we consider the behavior, in a monochromatic gravitational wave, of an asymmetrical rotator with account taken of the three rotational degrees of freedom. It turns out that the resonant and signal rotations can be separated, since the incident wave causes rotation of the rotator around an axis perpendicular to the axis of the resonant rotation. Under definite conditions this effect is of the same

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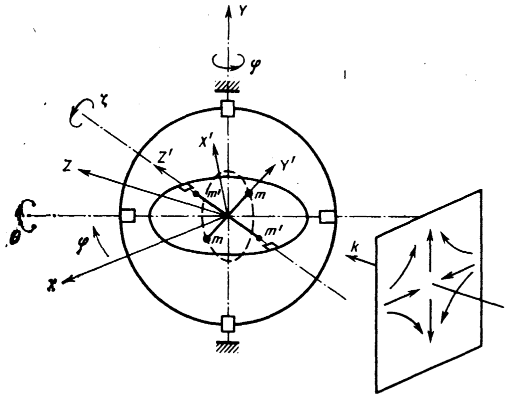

FIG. 1. Relative positions of the asymmetrical rotator in a Cardan suspension, of the fixed coordinate system XYZ with axis Z along the wave vector of the gravitational wave k, of the moving system $X'Y'Z'$ with axis Z' along the rotation axis of the rotator.

order as that considered in Ref. 2. Therefore we can use the rotation of the resonant-rotation axis itself as the signal motion.

The separation of the rotating motion from the signal motion lowers the requirements on the accuracy of synchronization of the rotation of the rotators by several orders of magnitude. In addition, it becomes possible to use a servomechanism control with feedback to synchronize the rotation of the rotators with the variable frequency of the gravitational wave. Because of this, the development of a rotational detector seems in our opinion more realistic.

2. We consider now an asymmetrical rotator. The simplest form of such a rotator, which we shall use for the sake of clarity (see Fig. 1), is a "dumbbell" with masses m on the ends and with an "axis" on which are located masses m'. We assume first that this rotator is in a Cardan suspension, i.e., it has three degrees of freedom. We introduce an immobile coordinate system XYZ, which is connected with the platform on which the Cardan suspension is mounted (Fig. 1). We connect with the rotator a coordinate frame $X'Y'Z'$ with Z' axis along the rotation axis and with Y' axis along the dumbbell axis. The transformation from the XYZ system with the $X'Y'Z'$ will be specified by three rotations: through an angle φ around the Y axis (which coincides with the external axis of the Cardan suspension), through an angle θ around the internal axis of the suspension, and through an angle ζ around the rotation axis of the rotator.

We assume that a monochromatic gravitational wave is incident along the Z axis. If the origin of the coordinate system coincides with the mass center of the rotator, then the wave exerts on a massive point with coordinates x and y the accelerations

$$a_x = px + qy, \tag{1}$$

$$a_y = qx - py, \tag{2}$$

where

$$p = -{}^1/{}_2\omega_{gr}{}^2 h^+ \cos(\omega_{gr}t + \beta^+), \tag{3}$$

$$q = -{}^1/{}_2\omega_{gr}{}^2 h^\times \cos(\omega_{gr}t + \beta^\times). \tag{4}$$

The symbols + and × pertain to the two possible polarizations with dimensionless amplitudes $h^{+,\times}$ and phases $\beta^{+,\times}$. In the case of the + polarization, the purely radial accelerations are directed along the axis X and Y.

The rotator in the gravitational wave is acted upon by a moment of forces with components

$$M_x = {}^1/{}_2 I[q(\gamma \sin 2\varphi \cos^2\theta - \sin 2\varphi \sin^2\zeta + \cos^2\varphi \sin\theta \sin 2\zeta - \sin^2\varphi \cos\theta \sin 2\zeta + \sin 2\varphi \sin^2\theta \cos^2\zeta) + p(-\gamma \cos\varphi \sin 2\theta - \sin\varphi \cos\theta \sin 2\zeta + \sin 2\theta \sin^2\zeta)] - m_0 \sin\varphi \cos\theta, \tag{5}$$

$$M_y = {}^1/{}_2 I[q(-\gamma \cos\varphi \sin 2\theta - \sin\varphi \cos\theta \sin 2\zeta + \cos\varphi \sin 2\theta \cos^2\zeta) + p(-\gamma \sin 2\varphi \cos^2\theta + \sin 2\varphi \sin^2\zeta - \cos^2\varphi \sin\theta \sin 2\zeta + \sin^2\varphi \sin\theta \sin 2\zeta - \sin 2\varphi \sin^2\theta \cos^2\zeta)] - m_0 \sin\theta, \tag{6}$$

$$M_z = {}^1/{}_2 I[q(2\gamma \sin^2\varphi \cos^2\theta - 2\gamma \sin^2\theta + 2\cos^2\varphi \sin^2\zeta + \sin 2\varphi \sin\theta \sin 2\zeta + 2\sin^2\varphi \sin^2\theta \cos^2\zeta - 2\cos^2\theta \cos^2\zeta) + p(-2\gamma \sin\varphi \sin 2\theta + \cos\theta \cos\varphi \sin 2\zeta + \sin\varphi \sin 2\theta \cos^2\zeta + \cos\varphi \cos\theta \sin 2\zeta + \sin\varphi \sin 2\theta \cos^2\zeta)] + m_0 \cos\varphi \cos\theta. \tag{7}$$

Here I is the moment of inertia of the rotator dumbbell, and γ is the ratio of the moment of inertia of the massive axis of the rotator and of the dumbbell. The presented expressions include also the moment m_0, which can act along the rotation axis and change the rotation velocity $\dot{\zeta}$ so as to be able to follow the pulsar frequency.

It is seen from (5)–(7) that the moment of the forces exerted by the wave on the rotator is oscillatory. An exception is the case when the circular velocity of rotation $\dot{\zeta} = \omega$ is equal to $\omega_{gr}/2$. In this case secular terms appear in the expressions (5)–(7) for the components and cause a regular motion of the rotator axis and a change in its angular velocity. If we retain in the equations of motion only the non-oscillating components, then these equations take the form

$$I\omega \cos\theta d\varphi/dt = M_x \cos\varphi + M_z \sin\varphi, \tag{8}$$

$$I\omega d\theta/dt = M_x \sin\theta \sin\varphi - M_y \cos\theta - M_z \sin\theta \cos\varphi, \tag{9}$$

and

$$I d\omega/dt = -M_x \cos\theta \sin\varphi - M_y \sin\theta + M_z \cos\theta \cos\varphi + m_0. \tag{10}$$

It is seen from (10) that, just as for the detector considered in Ref. 1, at $m_0 = 0$ the expression for the angle of rotation of the free rotator ζ should contain terms proportional to t^2. However, as already mentioned, the measurement of the contributions of these terms to the running value of the angle

$$\zeta = \int_0^t dt' \omega(t')$$

is difficult, since it changes rapidly. If the detector operates in a regime wherein it follows the frequency $\omega_{gr}/2$, then $m_0 \neq 0$ and expression (10) should be regarded as an equation for this quantity.

It follows from (8) and (9) that the angles θ and φ vary linearly with t, i.e., the rotator precesses. After a measurement time $t \sim 10^4$ sec, these changes

$$\Delta\theta \sim \Delta\varphi \sim \omega_{gr} t h$$

$$\approx 4 \cdot 10^{-19}\ \text{rad} \cdot \frac{\omega_{gr}}{380\ \text{rad/sec}^{-1}} \frac{t}{10^4\ \text{sec}} \frac{h}{10^{-25}}$$

are very small and can hardly be measured at present. Thus we arrive at the conclusion that a rotator whose axis is freely suspended in space can hardly be used effectively to detect monochromatic gravitational radiation.

3. The situation changes substantially if one of the degrees of freedom of the rotation of the rotator axis and the Cardan suspension is eliminated. It is immaterial from the structural point of view which of the angles, θ or φ, remains constant. Let $\dot{\varphi}=0$. This means that the rotator is acted upon by an additional moment of the reaction

$$\mathbf{m}=m(\sin\varphi\sin\theta,\ \cos 2\varphi\cos\theta,\ \cos\varphi\sin\theta). \qquad (11)$$

Its value m is determined from the equations of motion, which in our case take the form

$$\frac{1}{2}I\frac{d}{dt}\left[(1+2\gamma+\cos 2\zeta)\frac{d\theta}{dt}\right]=M_x\cos\varphi+M_z\sin\varphi+m\sin 2\varphi\sin\theta, \qquad (12)$$

$$\frac{1}{2}I\left[-\frac{d}{dt}\left(\sin 2\zeta\frac{d\theta}{dt}\right)+2\omega\frac{d\theta}{dt}\right]=M_x\sin\varphi\sin\theta-M_y\cos\theta$$
$$-M_z\cos\varphi\sin\theta-m\cos 2\varphi, \qquad (13)$$

$$\frac{1}{2}I\left[\sin 2\zeta\left(\frac{d\theta}{dt}\right)^2-2\frac{d\omega}{dt}\right]=M_x\sin\varphi\cos\theta+M_y\sin\theta-M_z\cos\varphi\cos\theta-m_0. \qquad (14)$$

If $\varphi=0$ or $\pi/2$ is chosen, the unknown quantity m is eliminated from (12) and this equation turns out to be independent of the two others. From expressions (5) and (7) for M_x and M_z it is easily seen that at $\varphi=0$ the right-hand side of (12) is proportional to $\sin\theta$ and is small near $\theta=0$. In contrast, at $\varphi=\pi/2$, the right-hand side of (12) is proportional to $-q\cos 2\theta+p\sin 2\theta$ and both polarizations of the wave enter in this expression symmetrically. For detection it is preferrable to choose $\varphi=\pi/2$. Integrating (12) at this value of φ and averaging the results over a time $t\ll|\omega-1/2\omega_{gr}|^{-1}$, we find that the change of the angle θ under the influence of the gravitational wave is

$$\Delta\theta\approx\frac{t^2\omega_{gr}^2}{16[\gamma(1+\gamma)]^{1/2}}(h^\times\cos 2\theta\cos(2\zeta_0-\beta^\times)-h^+\sin 2\theta\cos(2\zeta_0-\beta^+)), \qquad (15)$$

where ζ_0 is the initial value of the phase of rotation of the rotator. Thus, it follows from (15) that under the influence of a monochromatic gravitational wave, under resonance conditions $\omega=\omega_{gr}/2$, the axis of the asymmetrical rotator rotates with equal acceleration in a direction perpendicular to the wave vector of the wave.

4. It is usually assumed that the pulsar radiation is due to rotation of a neutron star with a quadrupole a moment of the mass distribution. If the rotation axis makes an angle ϑ with the direction to the observer (the Z axis) and lies in the same plane as the Y axis, then

$$h^+(t)={}^1/_2h(1+\cos^2\vartheta)\cos(\omega_{gr}t+2\delta), \qquad (16)$$
$$h^\times(t)=h\cos\vartheta\sin(\omega_{gr}t+2\delta), \qquad (17)$$

where $\delta=kR/2+\delta_0$, k is the length of the wave vector of the gravitational wave, δ_0 is the phase of rotation of the star at the instant $t-R/c$, R is the distant to the pulsar, $\omega_{gr}/2$ is the angular velocity of the star, and h is the amplitude of the gravitational wave.

Substituting (16) and (17) in expression (15) for $\Delta\theta$ and recognizing that $\omega_{gr}=4\pi/P$(P is the period of the pulsar), we obtain

$$\Delta\theta\approx\frac{\pi^2t^2h}{P^2[\gamma(1+\gamma)]^{1/2}}\left[\frac{1}{2}(1-\cos\vartheta)^2\cos 2\theta\cos 2(\zeta_0-\delta)\right.$$
$$\left.+\cos\vartheta\cos 2(\theta-\zeta_0+\delta)\right]. \qquad (18)$$

After a measurement time ~10^4 sec, the axis of the rotator will turn through an angle

$$\Delta\theta\approx 10^{-13}\ \text{rad}\cdot(t/10^4\ \text{sec})^2(h/10^{-25})(0.033\ \text{sec}/P)^2.$$

If its length is ~10^2 cm, that is a result of this rotation the positions of the end of the axis will be displaced by ~10^{-11} cm. Such displacements can be readily registered at the present time.[2] On the basis of the measurements we can estimate the amplitude of the gravitational wave and consequently determine the quadrupole moment of the neutron star. Measuring $\Delta\theta$ at different values of ζ_0 and θ, we can determine the angle ϑ between the rotation axis of the neutron star and the line of sight. If sufficiently accurate account is taken of the change of the propagation velocity of the electromagnetic radiation of the pulsar in the interstellar medium, we can determine the location of the star at the instant when it emits the electromagnetic pulses.

5. In the case analyzed above ($\omega\perp\mathbf{k}$), observation of a gravitational wave is possible in principle in the presence of only one rotator, in contrast to the detector with $\omega\parallel\mathbf{k}$. However, the influence of the Coriolis accelerations (to be discussed below) makes it advantageous to use two synchronously rotating rotators and to measure the relative rotations of their axes. If the initial phases ζ_0 of the rotators differ by $\pi/2$, then the moments applied to them by the gravitational wave have opposite signs and the relative rotation of the axis is $2\Delta\theta$ [see (18)].

From the condition $|2\omega-\omega_{gr}|t\ll 1$ it follows that the angular rotations of the rotators should follow during the course of time the changing frequency $\omega_{gr}(t)$ with accuracy $\Delta\omega/\omega\lesssim 3\cdot 10^{-7}\cdot(10^4\ \text{sec}/t)$. In the synchronization it is necessary to take into account the Doppler shift of ω_{gr} due to the earth's rotation. It is very important that the errors of the synchronization of ω and $\omega_{gr}/2$, which do not go beyond these limits, have practically no influence on the relative rotation of the axis of the rotations of the rotator, i.e., on the observations.

To estimate the influence of the Coriolis accelerations, we assume that the platform with the detector rotates relative to the fixed stars with angular velocity Ω. Each of the dumbbells is acted upon by a moment of forces $2m\mathbf{r}\times\omega(\Omega\cdot\mathbf{r})$, where $\mathbf{r}$ is the radius vector of the masses m on the end of the dumbbells relative to the inertia center, and ω is their angular velocity. At $\varphi=\pi/2$, the component of the moment of the Coriolis forces, which changes the angle θ that is measured in the course of the detection, turns out to be

$$M_\theta\approx I\left([\Omega\times\omega]\frac{\mathbf{k}}{k}\right)\cos^2(\omega t+\zeta_0), \qquad (19)$$

where $\mathbf{k}$ is the wave vector of the gravitational wave. The Coriolis acceleration does not influence the observations of the difference between the regular parts of the moments (19) acting on the dumbbells $\Delta M_\theta\approx I\Delta\omega\Omega$ is much less than the moment of the forces applied by the wave. This corresponds to the condition

$$\Omega \frac{\Delta\omega}{\omega} < 4h\omega \approx 10^{-22} \frac{h}{10^{-25}} \frac{\omega}{190 \text{ rad/sec}^{-1}}. \qquad (20)$$

One can expect (see, e.g., Ref. 7) that the mean value during the time ~10^4 sec of the random velocity of rotation of a stabilized platform can be decreased to ~10^{-10} rad/sec. In this case the influence of the Coriolis accelerations on the relative rotation of the rotator axes is small, if the degree of synchronization of their rotation is $\Delta\omega/\omega \lesssim 10^{-12}$. The use of a frequency standard with superconducting resonators can in principle attain the required accuracy.[7]

6. The field gradient of the force of gravity interferes strongly with rotational detectors.[1)] We shall show that if the rotation axis of the rotators are perpendicular $\omega \perp \mathbf{k}$ this interference can be eliminated with sufficient accuracy.

A point located at a distance $\mathbf{r}$ from the mass center of the rotator is acted upon, because of the inhomogeneity of the field of the force of gravity, by an acceleration

$$\Delta \mathbf{g}(\mathbf{r}) \approx g\left\{\frac{r}{L}[\mathbf{e}_r(3\cos^2\mu-1)-\mathbf{e}_\mu\cdot 3\cos\mu\sin\mu] + \left(\frac{r}{L}\right)^2\left[\mathbf{e}_r\frac{3}{2}\cos\mu(3-5\cos^2\mu)+\mathbf{e}_\mu\frac{1}{2}\sin\mu(15\cos^2\mu-3)\right]\right\}. \qquad (21)$$

Here $\mathbf{g}$ is the acceleration due to gravity, L is the distance from the center of inertia of the rotator to the earth's center,

$$\mathbf{e}_\mu \sin\mu = \left[\frac{\mathbf{L}}{L}\times\left[\frac{\mathbf{r}}{r}\,\frac{\mathbf{L}}{L}\right]\right],$$

where μ is the angle between the vectors $\mathbf{r}$ and $\mathbf{L}$. When this acceleration acts on the rotator it produces moments

$$M_\eta = (3gI/4L)\sin 2\mu[(1-2\gamma)-\cos 2\zeta], \qquad (22)$$

$$M_\xi = (3gI/2L)\sin\mu\sin 2\zeta, \qquad (23)$$

which are directed respectively along the vectors $\eta = [\omega\times\mathbf{L}]/\omega L$ and $\xi = [\mathbf{L}\times\eta]/L$. These moments can greatly exceed the moment produced by the gravitational wave. The moment M_η contains a term that can cause a regular rotation of the rotator axis. This rotation is less than the rotation induced by the gravitational wave if $\gamma = 1/2$ with relative accuracy ~10^{-14} (this should be ensured by the tuning of the detector). The second term of the moment M_η and the moment M_ξ are proportional to $\cos 2(\omega t + \zeta_0)$ and $\sin 2(\omega t + \zeta_0)$. They cause forced oscillations of the rotator axis with frequency 2ω and with amplitude $\lesssim 10^{-11}$ rad, but, being averaged out, do not hinder the observation of a slow systematic displacement due to the monochromatic wave if the reception time is long enough.

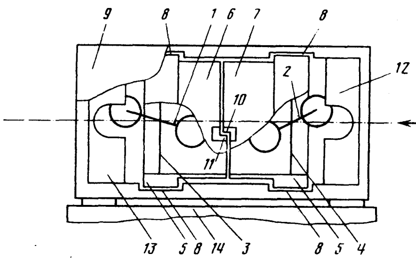

FIG. 2. Possible schematic diagram of the detector of monochromatic gravitational radiation with transverse arrangement of asymmetrical rotators (the callouts are explained in the text).

7. A possible schematic diagram of a detector with transversely placed asymmetric rotators is shown in Fig. 2. The asymmetric rotators 1 and 2 rotate about the axes 3 and 4, which are secured with the aid of "frictionless" bearings 5 in the measuring cylinders 6 and 7. The latter are fixed with the aid of bearings 8 relative to the longitudinal displacements along the cylindrical vessel 9, but can rotate about its axis. The relative rotation of axes 3 and 4 of the rotators causes a relative rotation of the cylinders 6 and 7, and a change in the gaps 10, which are measured by the pickups 11. The synchronism of the rotators and satisfaction of the resonance condition $\omega = \omega_{gr}/2$ are ensured by angular-velocity synchronizers 12 and 13, which are the actuating elements of the server mechanism. The platform 14 makes it possible to specify the optimal position of the detector relative to the source of the gravitational waves ($\omega \perp \mathbf{k}$) and ensures stabilization of the detector.

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[1)]This circumstance was pointed out to us by G. Ya. Lavrent'ev.

[1]M. Reese, R. Ruffini, and J. Wheeler, Black Holes, Gravitational Waves, and Cosmology [Russ. trans.], Mir, 1977.
[2]V. B. Braginskiĭ, Ya. B. Zel'dovich, and V. N. Rudenko, Pis'ma Zh. Eksp. Teor. Fiz. 10, 437 (1969) [JETP Lett. 10, 280 (1969)].
[3]V. B. Braginskiĭ and A. B. Manukin, Izmereniya malykh sil v fizicheskikh éksperimentakh (Measurement of Weak Forces in Physical Experiments), Nauka, 1974.
[4]J. H. Taylor and R. N. Manchester, Astron. J. 80, 794 (1975).
[5]V. B. Braginskiĭ and V. S. Nazarenko, in: Gravitatsiya (Gravitation), Naukova dumka, Kiev, 1972, pp. 9-16.
[6]B. Bertotty, General Relativity and Gravitation 7, 847 (1976).
[7]V. B. Braginsky, C. Caves, and K. S. Thorne, Phys. Rev. D 15, 2047 (1977).

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