Hyperfine structure of x-ray lines that accompany K capture

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It is shown that interference of the amplitudes of a transition of the Fermi type and of a transition of the Gamow-Teller type in allowed K capture with $\Delta I = 0$ leads to a nonstatistical excitation of the hyperfine structure states of the final atoms. Measurement of the ensuing hyperfine shift of the x-ray lines can be used as a new method of determining the ratio of the Fermi and Gamow-Teller matrix elements.

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1. Borchert *et al.*[1] and Egorov *et al.*[2] have shown that the laws of angular-momentum conservation in K capture[1] or in internal K conversion[2] with a unity change of the nuclear spin cause the atom to be excited in a definite state of the hyperfine structure, namely in a state with total angular momentum $F^{(+)} = I_f + \frac{1}{2}$ at $\Delta I = I_f - I_i = -1$ or $F^{(-)} = I_f - \frac{1}{2}$ at[1)] $\Delta I = 1$ (I_i and I_f are respectively the spins of the initial and final nucleus). The ensuing x-ray K line is shifted away from the fluorescent line[2)] by an amount equal to the hyperfine shift $\Delta E^{(+)} = (A/2)I_f$ or $\Delta E^{(-)} = -(A/2)(I_f + 1)$, which exceeds the hyperfine broadening by several orders of magnitude. Measurement of these shifts[1,2] uncovers new ways of determining the magnetic moments of excited states of nuclei. Borchert *et al.*[1] have also noted that the x-ray line shift mechanism is connected with the nonstatistical character of the population of the hyperfine-structure levels because of the spin dependence of the amplitude of the allowed K capture with $I = 0$ for a pure Gamow–Teller transition.

2. We discuss in this paper a mechanism that is connected with the Weak-interaction $V-A$ structure and leads to the nonstatistical population of the hyperfine-structure levels in allowed K capture with $\Delta I = 0$. It is shown that the interference between amplitudes of the Fermi type and of the Gamow–Teller type can lead to a predominant excitation of one of the components of the hyperfine structure of the final atoms. The reason is that the sign of the Gamow–Teller amplitude depends on the relative orientation of the spins produced in K capture of a neutron in the nucleus and of a hole in the K shell, and since the neutron spin correlates with the nuclear spin, the sign depends on the relative orientation of the spins of the nucleus and of the hole, i.e., on the state of the hyperfine structure.[3)]

We illustrate the foregoing using the very simple example of an "ideal" specular transition. Let the K capture be by an odd proton that is converted into a neutron, where the angular momentum of the latter determines the spin of the final nucleus. The process can be described as follows: the initial state $|i\rangle = |IK\rangle$ (a nucleus with spin I and a fully compensated K shell with zero angular momentum) breaks up into a nucleus and a hole in a shell with total angular momentum $F^{(\pm)} = I \pm \frac{1}{2}$ and a neutrino with spin $\frac{1}{2}$, and the total angular momentum of the system must again be equal to I. The matrix element of the transition takes in the nonrelativistic approximation the form

$$M_{fi}^{(F)} = \mathscr{M} \sum_{\substack{M,\nu \\ K',\mu'}} C^{IK}_{FM\,{}^1\!/_2\nu} C^{FM}_{IK'\,{}^1\!/_2\mu'} \langle \psi_{IK'}\chi_{{}^1\!/_2\mu'} | \mathscr{H}_w | \psi_{IK}\chi_{{}^1\!/_2-\nu} \rangle (-1)^{{}^1\!/_2-\nu}, \tag{1}$$

where ψ_{IK} is the angular part of the wave function of the nucleus, $\chi_{\frac{1}{2}\mu}$ is the spin function of the lepton, $\mathscr{H}_w = (G_V/2)[1 - \alpha(\sigma_N\sigma_e)]$, $\alpha = G_A/G_V = 1,25$, $\mathscr{M}$ is the K-capture radial matrix element (assumed for the time being to be the same for the two types of transition). Using the relation

$$(\sigma_N\sigma_e) = \frac{4(\mathbf{s}_N\mathbf{I})(\mathbf{I}\mathbf{s}_e)}{I(I+1)}, \tag{2}$$

we get

$$|M_{fi}^{(F)}|^2 = \frac{2F+1}{2I+1} |\mathscr{M}|^2 |\langle FM|\mathscr{H}_w|FM\rangle|^2, \tag{3}$$

where $|FM\rangle$ is a state of the hyperfine structure of the atom. Since

$$\langle FM|2(\mathbf{I}\mathbf{s}_e)|FM\rangle = \begin{cases} -(I+1), & F=I-{}^1\!/_2 \\ I, & F=I+{}^1\!/_2 \end{cases}, \tag{4}$$

we have for the ratio δ of the hyperfine-structure component populations

$$\delta = \frac{|M_{fi}^{(-)}|^2}{|M_{fi}^{(+)}|^2} = \frac{I}{I+1} \left[\frac{1+2\alpha(\mathbf{I}\mathbf{s}_N)/I}{1-2\alpha(\mathbf{I}\mathbf{s}_N)/(I+1)} \right]^2. \tag{5}$$

Since the spin of the neutron resulting from the K capture can be either parallel or antiparallel to the spin of the nucleus (the orbital angular moment is in this case $l = I \mp \frac{1}{2}$), there are two variants of the relative population:

$$\delta = \begin{cases} \dfrac{I}{I+1} \left(\dfrac{1+\alpha(I+1)/I}{1-\alpha} \right)^2 & \text{at} \quad l = I - \dfrac{1}{2} \\ \dfrac{I}{I+1} \left(\dfrac{1-\alpha}{1+\alpha I/(I+1)} \right)^2 & \text{at} \quad l = I + \dfrac{1}{2} \end{cases}. \tag{6}$$

It is seen from (6) that since α is close to unity either one hfs component or the other is predominantly populated, depending on the orbital angular momentum of the neutron.

An exact expression for the population ratio is obtained with the aid of the Wigner–Eckart theorem. It takes the form

$$\delta = \frac{I}{I+1} \left(\frac{1+\alpha\alpha_N[(I+1)/I]^{1/2}}{1-\alpha\alpha_N[I/(I+1)]^{1/2}} \right)^2, \tag{7}$$

where $\alpha_N = M_{GT}/M_F$, and M_{GT} and M_F are respectively the reduced Gamow–Teller and Fermi matrix elements.

Thus, measurement of the hyperfine shift, whose value is connected with δ by

$$\Delta E=\frac{\Delta E^{(+)}+\delta\Delta E^{(-)}}{1+\delta}=\frac{A}{2}\frac{I-\delta(I+1)}{1+\delta}, \tag{8}$$

yields direct information on the relative values of the Gamow–Teller and Fermi matrix elements.

3. For single-particle transitions, α_N can be expressed in the form

$$\alpha_N=\alpha_N^{\text{Teop}}\xi=\pm[(I+1)/I]^{\pm 1/2}\xi, \qquad l=I\mp{}^1/_2, \tag{9}$$

where ξ is a correction that takes into account the fact that the transition is not single-particle (usually $\xi<1$). We then have

$$\delta=\begin{cases}\dfrac{I}{I+1}\left(\dfrac{1+\alpha\xi(I+1)/I}{1-\alpha\xi}\right)^2, & l=I-{}^1/_2\\[2ex] \dfrac{I}{I+1}\left(\dfrac{1-\alpha\xi}{1+\alpha\xi I/(I+1)}\right)^2, & l=I+{}^1/_2\end{cases}. \tag{10}$$

At $\xi=1$ formula (10) coincides with (6).

Thus, for single-particle transitions, depending on the parity and spin of the nuclear state, we get predominant population of one of the hfs sublevels that result from the K capture. In fact, consider, for example, the transitions $\frac{3}{2}^+\to\frac{3}{2}^+$, $\frac{3}{2}^-\to\frac{3}{2}^-$ and let $\alpha\xi\approx 0.6$,; we then have $\delta\approx 15$ for the first transition and $\delta\approx 0.052$ for the second. The formula for the shift then simplifies for $l=I=\frac{1}{2}$ and $\delta\gg 1$ the form

$$\Delta E\approx-\frac{A}{2}(I+1)\left(1-\frac{2I+1}{I+1}\frac{1}{\delta}\right); \tag{8'}$$

and for $lI+\frac{1}{2}$ and $\delta\ll 1$

$$\Delta E\approx\frac{A}{2}I\left(1-\frac{2I+1}{I}\delta\right). \tag{8''}$$

The sign of the shift is determined, given the spin, by the level parity, and the deviation of the shift from the value $\Delta E^{(+)}$ (or $\Delta E^{(-)}$) contains information on the value $\alpha\xi$.

4. Interest attaches also to the possibility of studying (in addition to the heretofore known methods of angular $\beta-\gamma_{cp}$ correlations[4)] and to the measurement of the angular distribution of the leptons in β decay of oriented nuclei, see, e.g., Refs. 7–9) the violation of isotopic invariance in nuclei with the aid of allowed K captures with $\Delta I=0$ and unity change of the isospin of the nucleus. In this case the Fermi transition is strongly suppressed and measurement of the shift of the x-ray line that accompanies the K capture makes it possible to determine the admixture of the Fermi amplitude to the Gamow–Teller amplitude.

In fact, at $\alpha_N\gg 1$ we have

$$\delta\approx\frac{I+1}{I}\left[1+\frac{2}{\alpha\alpha_N}\left(\left(\frac{I}{I+1}\right)^{1/2}+\left(\frac{I\mp 1}{I}\right)^{1/2}\right)\right]. \tag{11}$$

The formula for the shift (8) takes correspondingly the form

$$\Delta E\approx-\frac{A}{2}\left(1+\frac{2[I(I+1)]^{1/2}}{\alpha\alpha_N}\right). \tag{12}$$

As $\alpha_N\to\infty$ (pure Gamow–Teller transition), Eq. (12) yields the shift $\Delta E=-A/2$ cited in Ref. 1. The quantities $1/\alpha\alpha_N$ reach values[7] $\sim 4\times 10^{-2}$, so that at sufficiently high spins (e.g., $I=4.5$), the shifts can differ from $-A/2$ by several dozen per cent.

On the other hand, since $A/2$ can have values $\sim(0.1-1)$ eV, and the errors in modern methods of measuring the x-ray line shifts amount to several millielectron volts,[1,3,4] it is seen from (12) that the measurement of the admixture $1/\alpha\alpha_N$ can be performed with an absolute error $\sim 10^{-3}$.

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1)We have in mind the case when a zero orbital momentum is carried away by the neutron in K capture or by the electron in conversion.

2)In photoexcitation, owing to the statistical population of the hyperfine-structure sublevels [proportional to $(2F+1)$], the hyperfine interaction does not lead to a displacement but only to a broadening of the line; this was observed experimentally in Refs. 3 and 4.

3)A similar situation obtains in μ capture and leads to different level widths in the hyperfine structure of the μ-mesic atom.[5]

4)The subscript "cp" stands for circular polarization.

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