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Threshold instability and inhomogeneous states in nonequilibrium superconductors with optical and tunnel quasiparticle pumping

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A theory of the threshold instability in nonequilibrium superconductors with narrow quasiparticle sources (the electromagnetic field frequency, ω, and the voltage, V, across the junction satisfy the condition $\omega - 2\Delta < \Delta$, $V - 2\Delta < \Delta$) that was predicted in an earlier paper by the present author [Sov. Phys. JETP 39, 862 (1974)] is developed. It is shown that, as a result of the development of the instability, a transition into an inhomogeneous state, which comprises regions with different finite (nonzero) order parameter values, is possible. The individual regions are separated by transition layers of width of the order of the quasiparticle diffusion length. In the case of a fixed voltage, V, across the junction (or of a given electromagnetic-field frequency ω), the inhomogeneous state is nonstationary. When the current through the junction is fixed (for a given level of absorption), the inhomogeneous state becomes stationary since the relative phase volume is fixed by the current. A broad range of experimental phenomena are well described by the theory.

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INTRODUCTION

It has been observed in a number of experimental investigations that nonequilibrium superconductors with optical and tunnel injection of quasiparticles go over into a new inhomogeneous state.[1-4] The nature of the inhomogeneous state of superconductors that are far from being in thermodynamic equilibrium has been intensively discussed in recent years. Two inhomogeneous-state models are known. The first model, first considered by Chang and Scalapino[5] and Baru and Sukhanov[6] (see also Ref. 7) is based on possible anomalous diffusion of nonequilibrium quasiparticles (the diffusion model). It is assumed that the diffusion proceeds from a region with a large value of the order parameter Δ (consequently, with a low quasiparticle concentration $\bar{n}$) into a region with a low Δ value (and a high $\bar{n}$), owing to the gradient $\partial\Delta/\partial r$. The instability condition in this model is extremely sensitive to the energy distribution of the nonequilibrium quasiparticles. Computations carried out with distribution functions satisfying a kinetic equation with a wide[8-10] and a narrow[11,12] quasiparticle source showed that the diffusion instability is apparently not realized under these conditions. This, of course, does not eliminate the possibility of realizing the diffusion instability in appropriate situations.

The second inhomogeneous-state model, which was proposed by the present author,[9] is based on the existence of a nonunique dependence of the order parameter on the quasiparticle-pump power β (or another corresponding parameter). In this model, the stratification of a homogeneous superconductor into regions of the normal ($\Delta = 0$) and superconducting ($\Delta \neq 0$) phases (or into regions with different values of Δ) can occur at a definite value, β_0, of β. For $\beta = \beta_0$, the energies of the phases with different Δ are equal, and the existence of a stationary phase boundary of width of the order of the quasiparticle-diffusion length or the coherence length is possible. This model is applied in Ref. 13 to nonequilibrium superconductors with tunnel injection [the superconductor-insulator-superconductor (SiS) junction].

According to Refs. 9 and 13, the inhomogeneous state is a nonstationary state for both optical pumping (fixed electromagnetic-field frequency) and tunnel injection in a fixed-voltage regime (fixed voltage, V, across the junction) with the exception of the $V = V_0$ regime. The phase boundary moves with a velocity proportional to $\beta - \beta_0$. If we neglect the time it takes the new phase to fill the sample, then the transition occurs discontinuously (first-order transition).

A stationary inhomogeneous state is attained in a regime in which a constant current flows through the

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SiS junction. In this case the given current fixes the relative phase volume, and the phase boundaries will be stationary and stable at $V \geq V_0$.[13] A change induced in the current by an external source is accompanied by the motion of the phase interface. These phenomena, which are similar to the "filamentation" phenomena in semiconductors, give rise to S-shaped current-voltage characteristics (CVC), hysteresis, etc. The theory expounded in Refs. 9 and 13 allows us to account for a significant portion of the experimental data given in Refs. 1–4.

It should be emphasized that a necessary condition for the stratification into the phases in the model of Ref. 9 is that the function $\Delta(\beta)[\Delta(V)]$ be many-valued, i.e., that there be three such solutions, which can, generally speaking, differ in nature. However, on the mechanism underlying the nonunique dependence depends the answer to the question of the types of phases that arise in the stratification. In Refs. 9 and 13 the mechanism, proposed in Ref. 8, in which the many-valuedness is due to the coherent nature of the quasiparticle recombination process involving the emission of phonons is considered. For this mechanism, one of the phases is the normal phase. The coherence mechanism is realized in optical pumping $(\omega \gg \Delta)$[1,2] and, apparently, under conditions of tunnel injection involving sufficiently large injection parameters.[13] However, the experimental data of Dynes *et al.*[3] and, especially, the data recently published by Gray and Williamson[4] indicate that phases with different finite values of the order parameter, Δ, can be realized in SiS junctions at voltages close to the threshold 2Δ.

This gives grounds for supposing that the cause of the many-valued dependence of Δ on V in a SiS junction near $V = 2\Delta$ is the threshold instability predicted in Ref. 14 for a narrow source, and consisting in the following. Let us first recall that, for a narrow quasiparticle source, i.e., for $\bar{\delta} \ll 1$ $[\bar{\delta} = (\omega - 2\Delta)/2\Delta = (V - 2\Delta)/2\Delta]$, the rate of quasiparticle production, which is porportional to the width $(\omega - 2\Delta)$ of the source, depends on Δ and, consequently, on the quasiparticle concentration $\bar{n}$. Let at the initial moment $\bar{n} = 0$ and $\delta = (\omega - 2\Delta_0)/2\Delta_0 \leq 0$ (Δ_0 is the gap in the absence of pumping). Then the fluctuation of $\bar{n}$, which leads to the decrease of Δ, increases the range of the action of the source, which, in its turn, leads to the increase of $\bar{n}$, and so on.

In the present paper we consider the theory of threshold instability in superconductors with optical and tunnel injection of quasiparticles. It is shown that there exist in the interval $\delta_p < \delta \leq 0$, $\delta_p \sim \alpha^m$ (where α is the quasiparticle-injection parameter) three branches of the solution $\Delta(\delta)$: $\Delta_1 = \Delta_0$, Δ_2, which grows with δ, and Δ_3, which decreases with δ. The CVC of a homogeneous superconductor are S-shaped. As a result of the threshold instability, a transition into an inhomogeneous state whose phases have different finite Δ values, namely, $\Delta_1 = \Delta_0$ and $\Delta = \Delta_3$, is possible. We show in the paper that both nonstationary (in the case of fixed ω and V) and stationary (in the case of a fixed current) inhomogeneous states are possible. The theory developed allows us to explain all the available experimental data,[3,4] in particular, the CVC of a "generator" and a "detector," the dependence of the transition voltage V_0, the order parameters of the phases, and other characteristic quantities on the junction resistance (the injection parameter).

§1. THE BASIC EQUATION

Let us consider a thin superconducting film at the temperature $T = 0$. In this case the thermal and nonequilibrium phonons can be ignored. The corresponding system of equations for Δ and the quasiparticle distribution function $n(\mathrm{p}, \mathbf{r}, t)$ can be written in the form[9]:

$$\xi_0^2 \partial^2\Delta/\partial\mathbf{r}^2 = -(\partial/\partial\Delta)U(\Delta), \tag{1}$$

$$U = -\int^{\Delta} \Delta' d\Delta' \left(\frac{1}{\lambda} - \int_0^{\omega_D} d\xi \frac{1-2n(\varepsilon,\mathbf{r},t)}{\varepsilon(\Delta')}\right), \quad \varepsilon = (\xi^2+\Delta^2)^{1/2}, \tag{2}$$

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}}\frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}}\frac{\partial \varepsilon}{\partial \mathbf{r}} = \left(\frac{\partial n}{\partial t}\right)_{im} + \left(\frac{\partial n}{\partial t}\right)_f + Q_{\omega,V}(\varepsilon), \tag{3}$$

$$\left(\frac{\partial n}{\partial t}\right)_f = (1-n)\int_{\xi}^{\omega_D} d\xi' n' \left(1 - \frac{\Delta^2}{\varepsilon\varepsilon'}\right) - n\int_0^{\xi} d\xi'(1-n')\left(1-\frac{\Delta^2}{\varepsilon\varepsilon'}\right) - n\int_0^{\omega_D} d\xi' n'(1+\Delta^2/\varepsilon\varepsilon'), \tag{4}$$

$$Q_\omega(\varepsilon) = \alpha_\omega[\rho^\omega(\omega-\varepsilon)(1-n_\varepsilon-n_{\omega-\varepsilon}) + \rho^\omega(\varepsilon-\omega)(n_{\varepsilon-\omega}-n_\varepsilon) + \rho^\omega(\omega+\varepsilon)(n_{\varepsilon+\omega}-n_\varepsilon)], \tag{5}$$

$$\rho^\omega(\omega-\varepsilon) = \frac{(\omega-\varepsilon)-(\Delta^2/\varepsilon)^{(1+K_\omega)/2}}{((\omega-\varepsilon)^2-\Delta^2)^{1/2}}\theta(\omega-\varepsilon+\Delta), \quad K_{\omega,V} = \pm 1, \quad \hbar = 1, \tag{6}$$

$$\alpha_\omega = \frac{E^2e^2L^2}{2m\omega^2\pi}, \quad \alpha_V = \frac{\sigma\tau_f}{de^2N(0)}, \quad L^2 = \frac{v_0^2\tau\tau_f}{3}. \tag{7}$$

Here ξ_0 is the coherence length; U is the energy of the system; $(\partial n/\partial t)_{im}$, $(\partial n/\partial t)_f$ are the collision integrals for the impurities and phonons[15]; Q_ω describes the interaction with the electromagnetic field of frequency ω (Ref. 15) and Q_V describes the injection of quasiparticles into the SiS junction[11]; the terms ρ^V are obtained from ρ^ω by setting $K_V = -1$ in them; α_ω and α_V are parameters characterizing the optical and tunnel injections; E is the field amplitude; L is the quasiparticle-diffusion length; v_0 is the velocity at the Fermi surface; τ and τ_f are the relaxation times of the quasiparticles in their interaction with the impurities and the phonons; σ is the conductivity of the junction in the normal state per unit area; d is the junction thickness; $N(0)$ is the density of states.

For a tunnel junction, it is further necessary to add the Kirchoff equation for the external circuit:

$$\mathscr{E} = V + R\bar{I}, \quad \bar{I} = \int d^2r \quad I(V,\Delta(\mathbf{r})), \tag{8}$$

$$I(V,\Delta) = \alpha_V \int_{\Delta}^{V-\Delta} d\varepsilon\, \rho(\varepsilon)\rho^V(V-\varepsilon)(1-n_\varepsilon-n_{V-\varepsilon}) + 2\alpha_V\int_{\Delta}^{\infty}\rho(\varepsilon)\rho^V(V+\varepsilon)(n_\varepsilon-n_{\varepsilon+V})d\varepsilon, \tag{9}$$

where $\mathscr{E}$ is the emf, R is the external resistance, I is the current in dimensionless form, and $\rho(\varepsilon) = \varepsilon/(\varepsilon^2 - \Delta^2)^{1/2}$.

Below we shall be interested in the situation in which the field frequency and the voltage V satisfy the condition

$\omega-2\Delta\ll\Delta, \quad V-2\Delta\ll\Delta.$

In this case the quasiparticle source is narrow, and the quasiparticles are localized in the narrow energy interval $\Delta<\varepsilon<V-\Delta$. The last property leads to a universal relation between Δ and $\bar{n}$, a relation which follows from Eq. (1)[14]:

$$\Delta=\Delta_0(1-2\bar{n}), \quad \bar{n}=\frac{1}{\Delta_0}\int_0^\infty n\,d\xi. \tag{10}$$

Here and below it is assumed that ξ_0 is small compared to L.

A similar relation is derived in Refs. 16 and 17 for a wide source, but under the assumption that the energy distribution of the quasiparticles is narrow.

The localized nature of $n(\varepsilon)$ allows the simplification of the equation, (3), for $\bar{n}$. Integrating (3) over the momentum, and using, in the usual fashion, the fact that τ is small compared to τ_f, we obtain

$$\frac{\partial\bar{n}}{\partial t}-\frac{\partial}{\partial\mathbf{r}}\left[\int_0\frac{\partial n}{\partial\mathbf{r}}L^2(\xi)\,d\xi-L^2\frac{\partial\Delta}{\partial\mathbf{r}}\int_0^\infty\frac{\Delta\xi}{\varepsilon^2}\frac{\partial n}{\partial\varepsilon}d\xi\right]$$

$$=-2\bar{n}^2+Q_{\omega,V}, \quad Q_{\omega,V}=\alpha_{\omega,V}\int_\Delta^{\omega-\Delta}\rho(\varepsilon)\rho^\omega(\omega-\varepsilon)(1-n_\varepsilon-n_{\omega-\varepsilon})\,d\varepsilon. \tag{11}$$

Notice that the second term in the square brackets in (11) describes the contribution made to the diffusion as a result of the order-parameter gradient, a contribution which can, in principle, lead anomalous diffusion. However, for a narrow source, this term is small. Indeed, setting $\Delta/\varepsilon\approx1$ in this equation, we obtain

$$\int_0^\infty\frac{\xi}{\varepsilon}\frac{\partial n}{\partial\varepsilon}d\xi=-n(\varepsilon=\Delta)\approx0,$$

since the equality $n(\varepsilon=\Delta)=0$ follows from (3). The conclusion about the smallness coincides with the conclusions arrived at through other means in Refs. 11 and 12.

Assuming further that the quasiparticle-diffusion length $L^2(\xi)=L^2\xi/\varepsilon$ does not depend on ξ, we can write (11) in the form

$$\frac{\partial\bar{n}}{\partial t}-L^2\frac{\partial^2\bar{n}}{\partial\mathbf{r}^2}=\frac{\partial}{\partial\bar{n}}U_{\omega,V}(\bar{n},\delta), \tag{12}$$

$$U_{\omega,V}=-\int^{\bar{n}}d\bar{n}'\,(2\bar{n}'^2-Q_{\omega,V}(\bar{n}',\delta)), \tag{13}$$

where U is the energy expressed in terms of the variables of $\bar{n}$, and coinciding with (2) up to a constant factor.

§2. DEPENDENCE OF THE QUASIPARTICLE CONCENTRATION AND THE ORDER PARAMETER OF THE SUPERCONDUCTOR ON THE ELECTROMAGNETIC-FIELD FREQUENCY

Let us consider a homogeneous superconductor with a narrow optical source and a small injection parameter $\alpha_\omega\ll1$ (everywhere below we shall assume α to be small).

Setting the left-hand side of (12) equal to zero, and computing Q_ω, we arrive at the following relation[14]:

$$\bar{n}^2=\alpha_\omega\frac{\omega-2\Delta}{2\Delta_0}\theta(\omega-2\Delta). \tag{14}$$

In computing Q_ω, we neglected the contribution from n_ε, which is justified for small α_ω. Substituting Δ from (10) into (14), we find an equation for $\bar{n}$:

$$\bar{n}^2=\alpha_\omega(\delta+2\bar{n})\theta(\delta+2\bar{n}), \quad \delta=\frac{\omega-2\Delta_0}{2\Delta_0}, \tag{15}$$

the solution to which has the form

$$\bar{n}=\begin{cases}0, \quad \delta<\delta_p, \quad \delta_p=-\alpha_\omega\\ \bar{n}_1=0, \quad \bar{n}_{2,3}=\alpha_\omega\mp(\alpha_\omega^2+\alpha_\omega\delta)^{1/2}, \quad \delta_p<\delta<0.\\ \alpha_\omega+(\alpha_\omega^2+\alpha_\omega\delta)^{1/2}, \quad \delta>0\end{cases} \tag{16}$$

It follows from (16) that there exist in the region $\delta_p<\delta<0$ three solutions for $\bar{n}$ and, consequently, for $\Delta[\Delta_1=\Delta_0, \Delta_{2,3}=\Delta_0(1-2\bar{n}_{2,3})]$. These solutions are shown in Fig. 1. For small α_ω, the values of Δ_2 and Δ_3 are close to Δ_0.

In Fig. 1 we show the full behavior of Δ with allowance for the nonuniqueness region resulting from the coherence instability.[8] This region begins at a high frequency $\omega>\omega_c=\beta_c/\alpha_\omega$. The value of ω_c decreases as α_ω increases, and can, in principle, become less than $2\Delta_0$.

The relation (14) can also be obtained by explicitly solving the kinetic equation.[12,14] Allowance should in this case be made for the fact that, for a narrow source, the rate of recombination should be much higher than the rate of scattering of the quasiparticles. Therefore, Eq. (3) can be written for $n(\varepsilon)$ in the form

$$n(z)\bar{n}=\alpha_\omega(\tilde{\delta})^{k\omega/2}\frac{(1-n(z)-n(1-z))}{(1-z)^{1/2}}\theta(z)\theta(1-z), \qquad z=(\varepsilon-\Delta)/\tilde{\delta}. \tag{17}$$

In Eq. (17) we have dropped the $\rho(\varepsilon\mp\omega)$ terms, which describe the redistribution of the quasiparticles over energy as a result of the absorption of photons, which is possible at low T and α. The exact solution to (17) is found in Ref. 18:

$$n(z)=\frac{A_\omega(z)^{1/2}\theta(z)\theta(1-z)}{(z(1-z))^{1/2}+A_\omega(z^{1/2}+(1-z)^{1/2})}, \quad A_\omega=(\alpha_\omega)^{1/2}. \tag{18}$$

Substituting (18) into (17), and integrating over z, we arrive at the relation (14).

§3. DEPENDENCE OF THE QUASIPARTICLE CONCENTRATION AND THE ORDER PARAMETER OF THE SUPERCONDUCTOR ON THE VOLTAGE ACROSS THE SiS JUNCTION

Let us consider an SiS junction with a narrow quasiparticle source, i.e., for which $\tilde{\delta}\ll1$. The distribu-

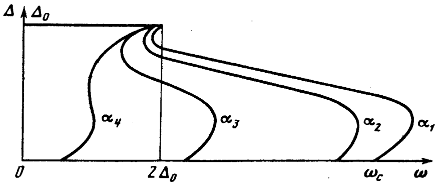

FIG. 1. Dependence of the order parameter Δ on the electromagnetic-field frequency ω for different values of the injection parameter $\alpha_1<\alpha_2<\alpha_3<\alpha_4$.

tion function n_ε satisfies Eq. (17) with the coefficient on the right-hand side replaced by $\tilde{\delta}^{-1/2}$ (in the case of optical pumping by $\tilde{\delta}^{1/2}$). This significant difference is due to the absence of the coherence factor, and plays an important role in the phenomena being described. (This leads, as is well known, to a current jump in tunnel junctions at $V > 2\Delta_0$ with $\alpha \to 0$.)

Let us seek the solution to (17) in the form of (18) with a constant A_V satisfying the equation

$$A_V^2 \int_0^1 \frac{dz}{(z(1-z))^{1/2} + A_V(z^{1/2} + (1-z)^{1/2})} = \frac{\alpha_V}{4\tilde{\delta}}. \tag{19}$$

In the limiting cases of high and low values of the ratio $\alpha_V/\tilde{\delta}$, we have from (19) in place of (14)

$$\begin{aligned} \bar{n}^2 &= 0{,}56(\Delta/\Delta_0)\tilde{\delta}\theta(\tilde{\delta}), \quad \tilde{\delta} \ll \alpha_V, \\ \bar{n}^2 &= \pi\alpha_V(\Delta/4\Delta_0)\theta(\tilde{\delta}), \quad \tilde{\delta} \gg \alpha_V. \end{aligned} \tag{20}$$

It follows from this that the quasiparticle source is switched on abruptly at $\tilde{\delta} > \alpha_V$, and varies slowly with $\tilde{\delta}$ (i.e., similar to the situation with very small α_V). If $\tilde{\delta} < \alpha_V$, then the source smoothly increases the injection rate, but does not depend on the injection parameter α_V.

Substituting (10) into (20), and solving it, we find for $\bar{n}$ in the nonuniqueness region $\delta_p < \delta < 0$ the expressions:

$$\bar{n} = \begin{cases} \bar{n}_1 = 0 \\ \bar{n}_2 = -(V - 2\Delta_0)/4\Delta_0, \quad \tilde{\delta} \ll \alpha_V. \\ \bar{n}_3 = (\pi\alpha_V\Delta/4\Delta_0)^{1/2}, \quad \tilde{\delta} \gg \alpha_V, \quad \delta_p = -\zeta\alpha_V^m \end{cases} \tag{21}$$

For the approximate solution shown in Fig. 2a, $\zeta = 2\pi^{1/2}$ and $m = \frac{1}{2}$. Thus, there also exists for the SiS junction a voltage region where there are three solutions: $\bar{n}_1 = 0$, $\bar{n}_2$, which decreases with V, and $\bar{n}_3$, which slowly grows with V. Although their behavior is qualitatively similar to that of (16), there are significant differences. Indeed, the solution $\bar{n}_3$ almost does not depend on the voltage, while $\bar{n}_2$ does not depend on α_V.

Figure 2a shows the corresponding plots for the order parameter $\Delta(V)$. The "universality" of the dependence $\Delta(V)$ for $V \leqslant 2\Delta_0$ should be noted. We have plotted in

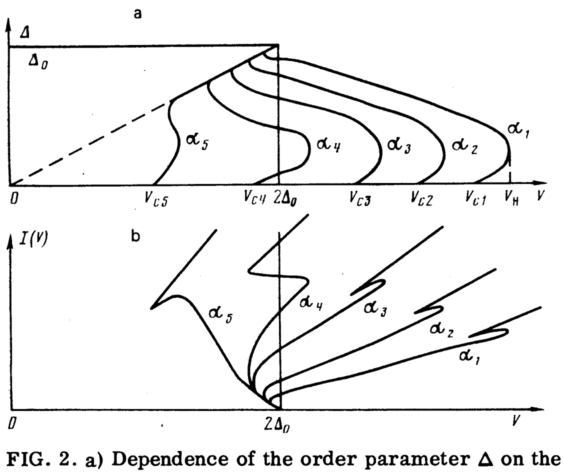

FIG. 2. a) Dependence of the order parameter Δ on the voltage across the junction for different values of the injection parameter $\alpha_1 < \alpha_2 < \alpha_3 < \alpha_4 < \alpha_5$; b) dependence of the current, $I(V)$, flowing through the tunnel junction on the voltage for different α_i.

the same figure the expected dependence of Δ on V for $V > 2\Delta_0$. When the voltage V attains the value $V_c = 1/4\alpha_V$, there appears another region, $V_c < V < V_n$, where there are three solutions for $\Delta(\Delta_3, \Delta_4, \Delta_5 = 0)$.[13] As α_V increases, the quantity V_c shifts to the left, and can attain values smaller than $2\Delta_0$. Therefore, the situation can arise at sufficiently large values of α_V when the two mechanisms underlying the nonuniqueness phenomena—and coherence and threshold—are superimposed on each other.

§4. THRESHOLD ELECTROMAGNETIC-FIELD ABSORPTION AND THE CVC OF A TUNNEL JUNCTION

Let us find the frequency dependence of the power absorbable by the superconductor in the vincinity of the absorption threshold. Substituting (16) into (14), we obtain

$$\begin{aligned} Q_\omega &= 0, \quad \delta < \delta_p, \\ Q_\omega &= 2\alpha_\omega[\delta + 2\alpha_\omega \mp 2(\alpha_\omega^2 + \alpha_\omega\delta)^{1/2}], \quad \delta_p < \delta < 0, \\ Q_\omega &= 2\alpha_\omega[\delta + 2\alpha_\omega + 2(\alpha_\omega^2 + \alpha_\omega\delta)^{1/2}], \quad \delta > 0. \end{aligned} \tag{22}$$

In the frequency interval $\delta_p < \delta < 0$ the absorbable power-versus-frequency plot is S-shaped. In order for this phenomenon to become observable, it is necessary that α_ω attain a value equal to at least a few parts of ten. The corresponding power has a value of the order of 10 W/cm^2.

It is natural to expect that the many-valuedness of $\Delta(V)$ will make the voltage dependence of the tunnel current $I(V)$ many-valued. Substituting (18) into (9), we find after simple calculations

$$I(V) = \frac{\alpha_V^2}{8\tilde{\delta}A_V^2}(1 + 3(2\tilde{\delta})^{1/2}A_V). \tag{23}$$

The first term in (23) describes the current in which the Cooper pairs are ruptured; the second term, the current of quasiparticles (in the present case only nonequilibrium quasiparticles, since $T = 0$). The second term is small compared to the first ($\sim\alpha_V^{3/2}$), except in the small region $\tilde{\delta} \ll \alpha_V^2$.

In the limiting cases $I(V)$ is given by the expressions

$$I(V) = \begin{cases} {}^9/_4\tilde{\delta}^2, & \tilde{\delta} \ll \alpha_V \\ (\alpha_V\Delta\pi)/8\Delta_0, & \tilde{\delta} \gg \alpha_V. \end{cases}$$

Figure 2b depicts the behavior of $I(V)$. The CVC of a homogeneous SiS junction has a negative slope near $2\Delta_0$ and then an inflection at δ_p, i.e., the CVC has an S-shaped character. It should be noted that CVC with a negative slope are used in Ref. 4 for a qualitative explanation of the experimental data, and have been experimentally observed in tin.[19]

Figure 2b shows CVC for different α_V in the entire voltage range. The second section of the S-shaped CVC is due to the coherence instability (see §3).

§5. INSTABILITY OF THE HOMOGENEOUS STATE OF A SUPERCONDUCTOR WITH THRESHOLD INJECTION OF QUASIPARTICLES

Let us consider the stability of the homogeneous state of a superconductor described by Eq. (12) with $\partial^2 n/\partial r^2$

$=0$:

$$\partial\bar{n}/\partial t=-\partial U(\bar{n},\ \delta)/\partial\bar{n}. \tag{24}$$

It is known from the general theory of differential equations of the type (24)[20] that the trajectories of $\bar{n}(t)$ lie between singular "trajectories," $\bar{n}_1=\text{const}$, $\bar{n}_2=\text{const}$, $\bar{n}_3=\text{const}$, of Eq. (24). Depending on the initial conditions, $\bar{n}(t)$ tends to $\bar{n}_1$ [if $\bar{n}(0)<\bar{n}_2(\delta)$], or to $\bar{n}_3(\delta)$ [if $\bar{n}(0)>\bar{n}_2(\delta)$] as $t\to\infty$. Thus, if the $\bar{n}$ fluctuation is sufficiently high [for $\delta\to 0, \bar{n}_2(\delta)\to 0$], then the superconductor is unstable against a transition into the $\bar{n}_3$ state when $\delta>\delta_p$, in accord with the prediction made in Ref. 14 (this is valid if the absorbable power or the current is not fixed; see below).

Let us investigate the stability of the homogeneous state against weak homogeneous perturbations in the case of small fluctuations:

$$\bar{n}(t)=\bar{n}+\bar{n}'e^{\gamma t},\quad \delta(t)=\delta+\delta'e^{\gamma t}. \tag{25}$$

Substituting (25) into (8) and (24), we find the decrement

$$\gamma=-\frac{1+RdI/dV}{\bar{n}_\delta(1+RI_V)},\quad \bar{n}_\delta=\frac{\partial\bar{n}}{\partial\delta},$$
$$I_V=\left(\frac{\partial I}{\partial V}\right)_{\bar{n}},\quad I_{\bar{n}}=\left(\frac{\partial I}{\partial\bar{n}}\right)_V,\quad \frac{dI}{dV}=I_V+I_{\bar{n}}\bar{n}_\delta. \tag{26}$$

In the regime of fixed voltage (frequency, in the case of optical pumping), the resistance can be set equal to zero, so that

$$\gamma=-\partial\bar{n}/\partial\delta.$$

It follows from this that the decreasing branch, $\bar{n}_2$, is unstable, and that out of the three homogeneous solutions only two—$\bar{n}_1=0$ and $\bar{n}=\bar{n}_3(\Delta_0,\Delta_3)$—are stable against weak perturbations. If initially the superconductor is in the $\bar{n}_1=0$ state, then this state is preserved as δ increases from δ_p to zero, but undergoes a first-order transition into the $\bar{n}_3$ state when $\delta\geq 0$, the transition being accompanied by a jump in Q_ω and $I(V)$. When δ is decreased, the state $\bar{n}_3$ is preserved up to the point δ_p, where it goes over discontinuously into the $\bar{n}_1=0$ state with a corresponding decrease in Q_ω, $I(V)$, i.e., there is hysteresis.

The situation is different in the fixed-current regime ($R\to\infty$). For a fixed current the homogeneous solution $\bar{n}_2$ is the only solution, and is stable. Indeed, since $I_V>0$ and $dI/dV<0$, if $R>|dI/dV|^{-1}$, then $\gamma<0$, and the homogeneous state is a stable state.

However, the state $\bar{n}_2$ on the decreasing branch $dI/dV<0$ is unstable against inhomogeneous perturbations, $\bar{n}'\exp(i\mathbf{q}\cdot\mathbf{r}+\gamma t)$,

$$\int d^2r\,\bar{n}'(\mathbf{r},t)=0, \tag{27}$$

that do not change the total current $\bar{I}$. In this case the decrement becomes positive for sufficiently smooth spatial perturbations:

$$\gamma=-1/\bar{n}_\delta-q^2=2-q^2>0,\quad q_0<\sqrt{2}. \tag{28}$$

Thus, a partitioning of the superconductor into regions with different values of Δ occurs. The regions with smaller Δ (higher currents) alternate with the regions with larger Δ (lower currents). The described phenomenon is similar to the filamentation phenomenon in semiconductors with S-shaped CVC.[21,22]

§6. THE STRUCTURE AND STABILITY OF THE INHOMOGENEOUS STATES OF SUPERCONDUCTORS

The growth of the weak perturbations is limited by the nonlinear effects. As a result, the inhomogeneous state described by (12) with $\partial\bar{n}/\partial t=0$ is established. The one-dimensional equation for the thin film in the dimensionless coordinate $x=x/L$ (the x axis is oriented along the film in a direction perpendicular to the current) has the form

$$d^2\bar{n}/dx^2=-\partial U/\partial\bar{n}. \tag{29}$$

The first integral of (29) is easy to find[21,22]:

$$\frac{1}{2}\left(\frac{d\bar{n}}{dx}\right)^2=C-U(\bar{n}),\quad C=U(n^{(i)}), \tag{30}$$

where the $\bar{n}^{(i)}$ are the roots of the equation $U(\bar{n}^i)=C$. Figure 3b shows a plot of the energy $U[\Delta(\bar{n})]$. The singularities of U coincide qualitatively with the singularities of U considered in Refs. 9, 21, and 22, where the possible inhomogeneous solutions are analyzed. It can be shown (see, for example, Ref. 22) that only the solutions with monotonic variation of $\bar{n}$ can be stable. There exists among the monotonic solutions a singular solution (joining two singular points, $\bar{n}_1$ and $\bar{n}_{30}$), called a layer solution. It represents a layer of width L separating the homogeneous regions with $\bar{n}_1=0$ and $\bar{n}=\bar{n}_{30}$. The layer solution is realized at a definite δ value, δ_0, corresponding to the equality of the phase energies:

$$U(\bar{n}_{10})=U(\bar{n}_{30},\ \delta_0). \tag{31}$$

In particular, for optical pumping, U_ω is equal to

$$U_\omega(\bar{n})=\begin{cases}-{}^2/{}_3\bar{n}^3, & \bar{n}/2<-\delta/2\\ -[{}^2/{}_3\bar{n}^3-\alpha_\omega\bar{n}(2\delta+2\bar{n})], & \bar{n}/2>-\delta/2\end{cases} \tag{32}$$

and from (31) we find the frequency and the order parameter of the Δ_3 phase and its connection with the injection parameter:

$$\delta_0=-\frac{3}{4}\alpha_\omega,\quad \frac{\omega_0}{2\Delta_0}=1-\frac{3}{4}\alpha_\omega,\quad \frac{\Delta_0-\Delta_{30}}{\Delta_0}=3\alpha_\omega. \tag{33}$$

Since in the phase with Δ_{30} the difference $\omega_0-2\Delta_{30}=9\alpha_\omega/2$, the optical source in this phase produces quasiparticles. In contrast, quasiparticle production does not occur in the Δ_0 phase, since $\omega_0<2\Delta_0$.

The corresponding layer solution for U_ω has the form

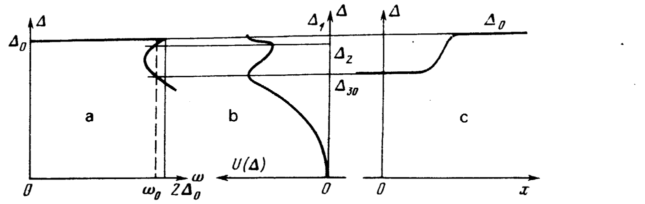

FIG. 3 a) Dependence of the order parameter Δ on the frequency ω for a narrow quasiparticle source; b) dependence of the energy, U, of the system on the order parameter for $\omega=\omega_0$; c) dependence of the order parameter on the coordinate x (the layer solution).

(see Fig. 3c):

$$\bar{n}_0(x)=\begin{cases}\dfrac{3}{(x-x_0-4(2\alpha_\omega)^{-1/2})^2}, & x\leqslant x_0,\\ \dfrac{3}{2}\alpha_\omega \operatorname{th}^2\left[\left(\dfrac{\alpha_\omega}{2}\right)^{1/2}(x-x_0)+C_2\right], & x\geqslant x_0,\end{cases} \tag{34}$$

where $\tanh C_2=\frac{1}{2}$ and x_0 is the coordinate of the junction boundary, whose position is determined by the total absorbed power (by the current).

Qualitatively similar results are obtained for the case of tunnel injection. If we use the approximation (21), then we obtain for V_0

$$\frac{V_0}{2\Delta_0}=1-\left(\frac{\pi\alpha_V}{4}\right)^{1/2},\quad \frac{\Delta-\Delta_{30}}{\Delta_0}=(\pi\alpha_V)^{1/2}. \tag{35}$$

Since the SiS junction is symmetric, in the phase with the smaller Δ_3, the gap from either side of the junction is Δ_{30}, and the difference $V_0-2\Delta_{30}=\Delta_0(\alpha_V\pi)^{1/2}>0$. Thus, quasiparticle injection goes on in this region, since $V_0>2\Delta_{30}$, but does not occur in the $\bar{n}_1$ region, since $V_0<2\Delta_0$.

It is also easy to find the monotonic solutions for $\delta \neq \delta_0$. Using these solutions, we can show that the CVC of an inhomogeneous superconductor possesses the following singularities (see also Refs. 21 and 22):

$$\left.\frac{d\bar{I}}{dV}\right|_{V<2\Delta_0}>0,\quad \left.\frac{d\bar{I}}{dV}\right|_{V\to V_0}\to-\infty, \tag{36}$$

where $\bar{I}$ is the total current flowing through the inhomogeneous junction (or $\bar{Q}_\omega$). It follows from (36) that, initially, $\bar{I}$ decreases as V decreases in the region below $2\Delta_0$, and then abruptly begins to increase, approaching a vertical asymptote as $V\to V_0$.

The investigation of the stability of the inhomogeneous solutions can be carried out, following Refs. 21 and 22. It turns out that, except for the layer solution, the solutions in the fixed-current regime are stable provided

$$\frac{dI}{dV}<0,\quad \left|\frac{dI}{dV}\right|<R. \tag{37}$$

It can be seen from (37) that, in the fixed-voltage (frequency) regime, when $R=0$, only the layer solution can be a stable inhomogeneous solution at $\delta=\delta_0$. It can be realized (at $R=0$) if there are seed regions of the new phase (nucleating centers), or if large fluctuations are possible. However, if δ begins to exceed δ_0, then the new phase (in the present example the phase $\bar{n}_3$) grows at a rate proportional to $\delta-\delta_0$. After an interval of time equal to the time it takes the new phase to fill the sample, the system goes over wholly into the state with $\bar{n}_3$. If we neglect the transition time, then we can speak of a discontinuous first-order transition.

Thus, the inhomogeneous states in superconductors with varying current (absorption) are essentially nonstationary (with the exception of the point $\delta=\delta_0$), and can be observed in nonstationary regimes (as, for example, in the experiments of Hu *et al.*[1] and Golovashkin *et al.*[2]).

Let us consider the fixed-current regime. When the current attains a value at which $dI/dV<0$, which occurs near the $V<2\Delta_0$ threshold, the superconductor undergoes a transition into an inhomogeneous state. According to (36), the derivative $d\bar{I}/dV$ is positive when $V<2\Delta_0$, and, in accordance with (37), such a state is also unstable. Therefore, there occurs a discontinuous transition to a voltage close to V_0 (which thus turns out to be fixed) and into a state in which the superconductor is partitioned into two phases with $\Delta_1=\Delta_0$ and $\Delta=\Delta_{30}$. If the current is increased further, then this leads to the growth of the volume of the Δ_{30} phase, the gap values for the phases and the voltages V_0 being preserved in the process. The growth of the current continues until the sample is completely filled by the Δ_{30} phase.

Further growth of the current is accompanied by the growth of the voltage and the decrease of Δ_3.

CONCLUSION

The developed theory allows us to explain the main experimental data on superconductors with tunnel injection[3,4] on the basis of the threshold-instability concept[14] and the inhomogeneous-state model.[9,13] The current-voltage characteristics computed on the basis of this theory for the "detector" and "generator"[3,4] are in good agreement with the experimental curves.

Unfortunately, the published experimental investigations do not contain sufficient quantitative data for a detailed comparison of the theory with experiment to be made. Therefore, we shall restrict ourselves to the following.

We have computed the value of the injection parameter α_V on the basis of the formula (8) and the experimental data on the junction resistance. The orders of magnitude turned out to be the same, and attained a value of 0.2 (for the sample described on p. 915 of Ref. 4). This value is, apparently, the highest attained in Ref. 4, and yet it remains small as compared to unity (this guarantees the applicability of the theory in the entire observable range of α_V). The substitution of the values of α_V into the formula (35) yields for the voltage V_0 and the order parameter, Δ_{30}, of the new phase values that agree with the experimental values to a high degree of accuracy. On the whole, the dependence of the difference $\Delta_0-\Delta_{30}$ on the square root of the injection parameter is well followed within the limits of applicability of the theory.

The generalization of the theory to the case of finite temperatures and the consideration of the effect of energy redistribution of the quasiparticles[15] are two remaining important problems. It is known that, as the temperature increases, the influence of the energy-redistribution effect can lead to an appreciable increase in the gap[15] (the Éliashberg effect). An increase in the gap was recently observed in tunnel junctions[23] (see Ref. 11). It may be conjectured that the growth of $\Delta(T)$ is the cause of the V_0-voltage shift observed by Gray and Williamson[4] at high temperatures.

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Contribution to the theory of ferromagnets with admixture of antiferromagnetic bonds

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It is shown that replacement of one ferromagnetic bond in a ferromagnet by an antiferromagnetic bond leads to appearance of local energy levels. The spectrum of these levels and their contribution to the thermodynamic functions are investigated. For a spin $s = 1/2$, the problem is solved exactly and an expression is obtained for the only energy level existing in this case and for the wave function. At $s > 1/2$ the problem is solved approximately. In this case several local levels can exist. Their energies are calculated. Turning on an external magnetic field causes these levels to cross, and this leads in turn to strong bursts of the susceptibility and of the heat capacity in magnetic field corresponding to the level crossing.

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1. INTRODUCTION

This paper deals with the properties of a Heisenberg ferromagnet with low concentration of randomly disposed antiferromagnetic bonds. Many recent papers are devoted to models of this kind in connection with the spin-glass problem (a detailed bibliography can be found in the reviews[1,2]). In practice, however, in all the papers the problem was solved in the molecular-field approximation or for the case of an infinite interaction radius (see, e.g., Refs. 3–5 and the references therein), and for the most part, furthermore, for an Ising magnet. No attention was paid whatever to all the phenomena connected with the presence of localized spin-wave excitations in the ferromagnet. It is these phenomena to which the present paper is devoted. We confine ourselves here to the case of low concentration of the antiferromagnetic bonds.

It must be noted first that the cited papers[1-5] dealt with a problem with a Gaussian distribution of the exchange integrals, i.e., with a purely model problem. On the other hand, there exist magnets in which the exchange integrals can be randomly both ferromagnetic and antiferromagnetic. This situation arises when the interaction is via indirect exchange and the crystal contains two sorts of atoms that effect the indirect exchange, the first leading to ferromagnetic exchange the second to the antiferromagnetic one. Examples of such substances are the alloys $CrTe_{1-x}Sb_x$ (Ref. 6) and $Co(S_x\text{-}Se_{1-x})_2$,[7,8] where CrTe and CoS_2 are ferromagnets and CrSb and $CoSe_2$ are antiferromagnets. We consider therefore a ferromagnet with nearest-neighbor interaction and with a ferromagnetic exchange integral J, con-

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