we have confined ourselves to the case of s capture, corresponding to isotropy of the process in a plane perpendicular to the dislocation axis. The results of the numerical calculations show that this limitation is justified, since the probabilities of the capture of the particle with $l \neq 0$, owing to the repulsive nature of the centrifugal potential, turns out to be much less than the s-capture probability.

According to the results of Ref. 3, the potential of the charged dislocation can be approximated by expression (2) only at sufficiently high temperatures $\alpha < 1$. At the same time, for the quasiclassical approximation to be valid it is necessary that many wavelengths of the captured carrier be spanned by the barrier thickness that the following inequality be satisfied

$$kr_0 = 2\alpha\pi^{-1/2} \ln^{1/2}(A/2\alpha) \gg 1,$$

which imposes an upper bound on the temperature interval. A joint analysis of the obtained criteria shows that for Ge with $E_c = 10$ eV, $\hbar\omega_D = 0.03$ eV, $m \sim 10^{-28}$ g, $M \sim 2.5 \cdot 10^{-22}$ g, $\varepsilon = 16$, $s \sim 5 \cdot 10^5$ cm/sec, with dnoro density $n_d \sim 10^{13}$ cm^{03}, and $0.3 < \alpha < 1$ the results of the present paper are qualitatively correct in the temperature interval 200 K $< T <$ 500 K. The absolute value of the capture radius turns out in this case to be of the order of 10^{-15} cm $< \rho_t(T) < 10^{-9}$ cm. If the dislocation density is $N_D = 10^6$ cm^{-2}, then according to (27) we have for the lifetime of the excess electrons $\tau_t \sim (10^{-6} - 1)$ sec.

The stronger temperature dependence of the capture radius than in the case of the point centers[10] can be explained by the fact that when the temperature changes a change takes place not only in the energy of the most effectively captured electrons, but also in the charged state of the dislocation, i.e., the shape of the electrostatic barrier that surrounds the edge dislocation.

The author thanks I. B. Levinson and Yu. A. Osip'yan for interest in the work and for a discussion of the results.

[1]V. A. Grazhulis, V. V. Kveder, and V. Yu. Mukhina, Phys. Stat. Sol. **43**, 407 (1977); **44**, 107 (1977).
[2]Yu. V. Gulyaev, Phys. Tverd. Tela (Leningrad) **3**, 1094 (1961) [Sov. Phys. Solid State **3**, 796 (1961)].
[3]R. A. Vardanyan, Zh. Eksp. Teor. Fiz. **73**, 2313 (1977)[Sov. Phys. JETP **46**, 1210 (1977)].
[4]V. L. Bonch-Bruevich and E. G. Landsberg, Phys. Stat. Sol. **29**, 9 (1968).
[5]M. Lax, Phys. Rev. **119**, 1502 (1960).
[6]Yu. E. Perlin, Usp. Fiz. Nauk **80**, 553 (1963) [Sov. Phys. Usp. **6**, 542 (1964)].
[7]G. Rickayzen, Proc. Roy. Soc. A241, 480 (1957).
[8]L. D. Landau and E. M. Lifshitz, Kvantovaya mekhanika, Nauka, 1974 [Quantum Mechanics, Nonrelativistic Part, Pergamon, 1977].
[9]N. Fröman and Per O. Fröman, JWKB Approximation: Contribution to the Theory, North Holland, 1965.
[10]V. A. Kovarskiĭ, Kinetika bezyzluchatel'nykh protsessov (Kinetics of Nonradiative Processes), Moldavian Academy of Sciences, 1968.

Translated by J. G. Adashko

Localized degrees of freedom in a ferromagnet with resonant impurities

M. V. Feĭgel'man and A. M. Tsvelik

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR
(Submitted 26 January 1979)
Zh. Eksp. Teor. Fiz. **76**, 2249–2259 (June 1979)

The Heisenberg model of a ferromagnet with an admixture of negative exchange integrals of a small, but definite magnitude is investigated. It is shown that this system exhibits low-temperature properties of the "glass" type along with the occurrence of a spontaneous moment close to the norminal moment. Long-wave spin waves are found to be abnormally strongly damped. The problem of elementary excitations in spin glasses is discussed.

PACS numbers: 75.10.Jm, 75.30.Ds

1. INTRODUCTION

There has been in recent years quite an intensive study of the properties of the models of disordered magnetic substances with random alternating-sign exchange interaction between the spins located on the regular lattice.[1,2] It is assumed that there is no spontaneous moment when the mean exchange value is close to zero, and the low-temperature phase in such models is similar to the spin-glass phase observed in disordered solutions of the Cu-Mn type.[2] The thermal capacity of spin glasses has a maximum at some temperature T_f, and is linear at $T \ll T_f$, while the magnetic susceptibility assumes a constant value at $T \ll T_f$.

In the present paper we show that the low-temperature, "glass" type properties are not necessarily due to the absence of a spontaneous moment, but can also be observed in a ferromagnet with a moment of magnitude close to the maximum value. Below we propose for the Heisenberg ferromagnet a model in which the magnitude and sign of a small part of the exchange integrals are

 0038-5646/79/061136-06$02.40

changed. For a small negative-coupling concentration $c \ll 1$, the system possesses a spontaneous moment that differs little from the nominal moment, yet its low-temperature properties turn out to be similar to those observed in spin glasses. This effect is explained by the appearance in the ferromagnet of a quasi-independent subsystem of localized states whose behavior determines all the low-temperature properties of the ferromagnet.

The most important distinctive feature of our model is the presence in it of a resonance level with energy $|\varepsilon_0| \ll JS$ (J is the exchange integral and S is the magnitude of the spin; below we set $JS = 1$) for a spin wave scattered by a negative coupling. We carry out in the case of exact resonance (i.e., for $\varepsilon_0 = 0$) at $T \gg c$ a virial expansion similar to the one performed in Ref. 3. In the case when $c \ll |\varepsilon_0| \ll 1$, the virial expansion allows us to find the properties of the system for arbitrarily low temperatures. The thermal capacity C_v turns out in this case to be linear in the temperature: $C_v \sim c^2 \varepsilon_0^{-2} T$; at $T = 0$ the susceptibility is finite; $\chi \sim c^2/\varepsilon_0^2$. We have computed the spin-wave damping constant:

$$\gamma(k, \omega) \sim \omega_0(k)\,(c^2/\varepsilon_0^2)\,\mathrm{th}\,(\omega/2T).$$

Our results also allow us to draw qualitative conclusions about the absence of well-defined quantum quasiparticles in spin glasses.

2. THE SINGLE-MAGNON APPROXIMATION

The Hamiltonian of our model is the Heisenberg Hamiltonian with only the nearest-neighbor interaction taken into account and part of the exchange integrals J replaced by $J' = -I + 1$. We write down our Hamiltonian, using for the spin operators the $1/S$ expansion of the Holstein-Primakoff formulas

$$S_i^+ = (2S)^{1/2}(\hat{a}_i - \hat{a}_i\hat{a}_i^+\hat{a}_i/4S + \ldots),$$
$$S_i^- = (2S)^{1/2}(\hat{a}_i^+ - \hat{a}_i^+\hat{a}_i^+\hat{a}_i/4S + \ldots).$$

As will become clear from what follows, the problem gets simplified in the case of large spin magnitudes, i.e., in the $S \gg 1$ case. It is this case that we shall consider. In this representation the Hamiltonian has the form

$$\mathcal{H} = \sum_{\mathbf{k}} \omega_0(\mathbf{k})\, a_{\mathbf{k}}^+ a_{\mathbf{k}} - \frac{1}{N}\sum_{\mathbf{k},\mathbf{k}'} a_{\mathbf{k}}^+ a_{\mathbf{k}'} \sum_{\mathbf{n},\mathbf{m}} I_{\mathbf{nm}} e^{i(\mathbf{k}'-\mathbf{k})\mathbf{n}} (1 - e^{i\mathbf{k}(\mathbf{n}-\mathbf{m})})$$
$$\times (1 - e^{-i\mathbf{k}'(\mathbf{n}-\mathbf{m})}) + W = H_1 + W. \qquad (1)$$

The W term contains products of four or more Bose operators and

$$\omega_0(\mathbf{k}) = \sum_{(\mathbf{a})} (1 - \cos \mathbf{ka})$$

is the spin-wave spectrum of a "pure" ferromagnet. The second term in (1) contains a sum over all the lattice sites linked by negative couplings (via the "impurities").

Let us show that, at $T = 0$, the ferromagnetic state with the maximum spontaneous moment turns out, for a definite I, to be unstable in the presence of any arbitrarily small impurity concentration. For this purpose, let us consider the problem of the scattering of one spin wave on the negative couplings. The W term in the Hamiltonian (1) will not be needed by us in the present case.

The Green function, $G(\mathbf{k}, \mathbf{k}')$, of the spin wave scattered on the potential,

$$V_{\mathbf{kk}'} = -I(1 - e^{i\mathbf{ka}})(1 - e^{-i\mathbf{k}'\mathbf{a}}),$$

of one impurity can be determined from the equation

$$G(\omega; \mathbf{k}, \mathbf{k}') = \delta_{\mathbf{kk}'} G^{(0)}(\omega, \mathbf{k}) + G^{(0)}(\omega, \mathbf{k}) \int V_{\mathbf{kp}} G(\omega; \mathbf{p}, \mathbf{k}') \frac{d\mathbf{p}}{(2\pi)^3} \Omega_0, \qquad (2)$$

where $G^{(0)}(\omega, \mathbf{k}) = [\omega - \omega_0(\mathbf{k}) + i\delta]^{-1}$ and Ω_0 is the unit-cell volume.

The solution to Eq. (2) has the following form:

$$G(\mathbf{k}, \mathbf{k}') = G^{(0)}(\mathbf{k})\,\delta_{\mathbf{kk}'} + \frac{V_{\mathbf{kk}'} G^{(0)}(\mathbf{k}) G^{(0)}(\mathbf{k}')}{1 + I\Omega_0 \int 2(1 - \cos \mathbf{pa}) G^{(0)}(\mathbf{p}, \omega)\, d\mathbf{p}/(2\pi)^3}. \qquad (3)$$

For $|\omega| \ll 1$, $|2I/Z - 1| \ll 1$ (Z is the number of nearest neighbors), we can approximately write:

$$1 + I\Omega_0 \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{(1 - \cos \mathbf{pa})}{\omega - \omega_0(\mathbf{p}) + i\delta} \approx 1 - \frac{2I}{Z} - \alpha\omega - i\omega^{1/2}\frac{\theta(\omega)}{8\pi},$$
$$\alpha = \Omega_0 \int \frac{d\mathbf{p}}{(2\pi)^3} \frac{1}{\omega_0(\mathbf{p})}. \qquad (4)$$

Substituting (4) into (3), we find that the spin-wave scattering amplitude has the following form:

$$f^{\mathbf{kk}'}(\omega) = -\frac{1}{\alpha} \frac{V_{\mathbf{kk}'}}{\omega - \varepsilon_0 + i\omega^{1/2}\theta(\omega)/8\pi\alpha}. \qquad (5)$$
$$\varepsilon_0 = \alpha^{-1}(1 - 2I/Z).$$

Everywhere below we restrict ourselves to the consideration of the simple cubic lattice, for which $Z = 6$ and $\varepsilon_0 = \alpha^{-1}(1 - I/3)$.

In order to investigate the stability of the system, it is necessary to find the Green function of a solitary spin wave for a finite impurity concentration. For our purposes, it is sufficient to use the effective-medium approximation[4] in this case. All the results of this section can be obtained with the aid of the standard impurity diagram technique,[5] but it will be convenient for us, bearing further generalizations in mind, to use another method.

Notice that if we neglect the imaginary part of (5), the scattering amplitude can be represented in terms of the Green function of the magnon bound state in the impurity:

$$D_{\mathbf{na}}^{(0)}(\omega) = \langle\langle b_{\mathbf{na}}(t)\, b_{\mathbf{na}}^+(0)\rangle\rangle_{I\omega} = \frac{1}{\omega - \varepsilon_0 + i\delta}. \qquad (6)$$

Here $b_{\mathbf{na}}$ and $b_{\mathbf{na}}^+$ are the creation and annihilation operators for the bound state in the (n, a) coupling. Below $D_{\mathbf{na}}^{(0)}$ will be represented in diagrams by a wavy line.

Thus, the Hamiltonian (1) assumes the form

$$H_1 = \sum_{\mathbf{k}} \omega_0(\mathbf{k})\, a_{\mathbf{k}}^+ a_{\mathbf{k}} + \varepsilon_0 \sum_{\mathbf{n},\mathbf{a}} b_{\mathbf{na}}^+ b_{\mathbf{na}} + \frac{1}{N^{1/2}} \sum_{\mathbf{n},\mathbf{a},\mathbf{k}} (g_{\mathbf{k}}^{\mathbf{a}} e^{i\mathbf{kn}} a_{\mathbf{k}} b_{\mathbf{na}}^+ + \mathrm{H.c.}), \qquad (7)$$

where $g_{\mathbf{k}}^{\mathbf{a}} = (I/\alpha)^{1/2}(1 - e^{i\mathbf{k}\cdot\mathbf{a}})$ is denoted by a small cross. In constructing the diagram technique with the Hamiltonian (7), account should be taken of the fact that the sites n and m should be different in diagrams of the type shown in Fig. 1 [here a thin line represents the func-

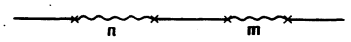

FIG. 1.

tion $G^{(0)}(\mathbf{k},\omega)$], since the contributions from multiple scattering on the same site have already been taken into account in the derivation of the formula (5). After averaging over the impurities, we obtain for the complete Green functions D and G the diagram equations shown in Fig. 2.

The oval in Fig. 2 denotes averaging over the impurities, which amounts to multiplication by c. The effective-medium approximation consists in the assumption that we can neglect the diagrams in which two or more Green's functions of bound states belonging to different couplings are simultaneously averaged. In the language of the conventional impurity diagram technique, this corresponds to the discarding of diagrams with intersecting dashed lines (see Fig. 3).

It is easy to see that the effective-medium approximation yields the first term of the expansion in powers of $c/(\omega-\varepsilon_0)$. It will be further shown that, in the single-magnon approximation, only $\varepsilon_0-\omega\sim\sqrt{c}$ is important to us and, thus, $c/(\omega-\varepsilon_0)\sim\sqrt{c}\ll 1$, and our approximation is justified. For the same reason, we can neglect the frequency dependence of $G^{(0)}(\mathbf{k})$ and $G(\mathbf{k})$ in the analytic expression for the diagram of Fig. 2b and, moreover, set $G(\mathbf{k})\approx G^{(0)}(\mathbf{k})$. Thus, the analytic expression for the diagram of Fig. 2b has the following form:

$$D(\omega)=D^{(0)}+D^{(0)}c\sum_{a'}\int\frac{d\mathbf{p}}{(2\pi)^3}\Omega_0 g_a^2 g_{a'}^2[G^{(0)}(p)]^2D^2(\omega). \tag{8}$$

Its solution is

$$D(\omega)=(\omega-\varepsilon_0)\frac{\alpha^2}{2c}\left(1-\left[1-\frac{4c}{\alpha^2(\omega-\varepsilon_0)^2}\right]^{1/2}\right). \tag{9}$$

It follows from the diagram in Fig. 2a that the self-energy part of the function G can be expressed in terms of D:

$$\Sigma(\omega,\mathbf{k})=\omega_0(\mathbf{k})(c/\alpha)D(\omega), \tag{10}$$

$$-\operatorname{Im}\Sigma=\tfrac{1}{2}\alpha\omega_0(\mathbf{k})[4c/\alpha^2-(\omega-\varepsilon_0)^2]^{1/2}.$$

It can be seen from (10) that, in the effective-medium approximation, for $\varepsilon_0<2c^{1/2}/\alpha$, the density of states at negative energies $\rho(\omega<0)>0$, which indicates the instability of the ground state with the maximum moment, which is what we wanted to show. As we shall see below, when the correlation effects are taken into account, negative levels arise virtually at any c/ε_0 (with a concentration containing higher powers of c).

3. THE EFFECTIVE HAMILTONIAN

As shown above, the state of the system with the maximum ferromagnetic moment is unstable. In the true

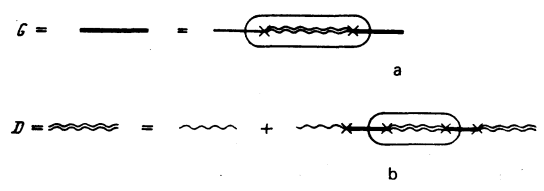

FIG. 2.

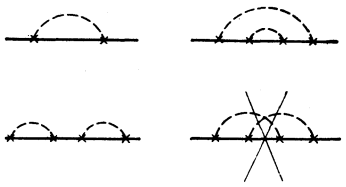

FIG. 3.

ground state the negative levels produced as a result of the overlap of the wave functions of the "single-impurity" bound states are filled.

Thus, for the investigation of the properties of the system near the ground state, it is necessary to take into consideration the correlations between the magnons at the impurity levels. It is easy to see that two magnons located on the same impurity strongly repel each other, since the wave function of the impurity state is highly localized ($|\Psi|^2\sim 1/r^4$); the repulsive energy $U\sim 1/S\gg c$ (below it will be shown that, in contrast to the results of the preceding section, the characteristic energies near the ground state are $\sim c$), and therefore the states with more than one magnon per coupling can be regarded as being unrealized. Since the mean number of magnons bound to one impurity is $\lesssim 1$, the spontaneous moment of the system at $T=0$ is $M(0)\gtrsim S-c$.

The magnon eigenstates on one impurity are exhausted by the set $|0\rangle$ and $|1\rangle$, where $|0\rangle$ corresponds to the vacant, and $|1\rangle$ to the filled, ε_0 level. Projecting the operators $b_{\mathbf{na}}$ and $b^+_{\mathbf{na}}$ onto this narrower basis, we obtain operators, $X^{\mathbf{na}}_{01}$ and $X^{\mathbf{na}}_{10}$, similar to the operators first introduced by Habbard[6] for systems of strongly interacting Fermions.

The algebra of the X operators is determined by the relations

$$X_{\lambda_1\lambda_2}|\lambda_3\rangle=|\lambda_1\rangle\delta_{\lambda_2\lambda_3},$$
$$X_{\lambda_1\lambda_2}X_{\lambda_3\lambda_4}=\delta_{\lambda_2\lambda_3}X_{\lambda_1\lambda_4}, \tag{11}$$
$$\lambda_i=0,\ 1.$$

The interaction between magnons in different couplings is $\sim c/S\ll c$ (let us recall that in our case $S\gg 1$), and we shall not consider it. In the X-operator representation the Hamiltonian of the system can be written as follows:

$$\mathcal{H}=\sum_{\mathbf{k}}\omega_0(\mathbf{k})a_{\mathbf{k}}^+a_{\mathbf{k}}+\frac{1}{N^{1/2}}\sum_{\mathbf{k},\mathbf{n},\mathbf{a}}(g_{\mathbf{k}}^{\mathbf{a}}e^{-i\mathbf{kn}}a_{\mathbf{k}}^+X_{01}^{\mathbf{na}}+\text{H.c.})+\varepsilon_0\sum_{\mathbf{n},\mathbf{a}}X_{11}^{\mathbf{na}}. \tag{12}$$

This is the Anderson Hamiltonian for Bose particles. For the study of the properties of the bound states, it is convenient to eliminate the spin-wave variables $a_{\mathbf{k}}$, $a^+_{\mathbf{k}}$, and go over to the Habbard Hamiltonian for bound states:

$$H_{\text{св}}=\sum_{\mathbf{n}\neq\mathbf{m}}t_{\mathbf{nm}}(X_{01}^{\mathbf{na}}X_{10}^{\mathbf{ma}'}+\text{H.c.})+\varepsilon_0\sum_{\mathbf{n},\mathbf{a}}X_{11}^{\mathbf{na}}, \tag{13}$$

$$t^{\mathbf{aa}'}(\mathbf{R})=\sum_{\mathbf{k}}g_{\mathbf{k}}^{\mathbf{a}}g_{\mathbf{k}}^{\mathbf{a}'}G^{(0)}(\mathbf{k},0)e^{i\mathbf{kR}}=\frac{3}{4\pi\alpha}\left(-\frac{\mathbf{aa}'}{R^3}+\frac{3(\mathbf{aR})(\mathbf{a}'\mathbf{R})}{R^5}\right), \tag{14}$$

where $G^{(0)}$ is the bare Green function for the magnons.

In writing down (13) and (14), we neglected the frequency dependence of $G^{(0)}(\omega,\mathbf{k})$. Allowance for this dependence would have led to the appearance in $t(\mathbf{R})$

of an additional factor $\sim\exp(i\omega^{1/2}R)$, which is important for $|\omega|^{1/2}R \gtrsim 1$. It will subsequently become clear that of importance for the computation of the thermodynamic properties are $\omega \sim T$ and $t(R) \sim T$, i.e., $R \sim T^{-1/3}$, and, consequently, in the region of interest to us $|\omega|^{1/2}R \sim T^{1/6} \ll 1$.

In the general form, the problem of finding the Green function $D_{nm}(\omega)$ with the Hamiltonian (13) is extremely complex. We shall restrict ourselves to the study of the cases in which it is sufficient to take only the pair correlations between the impurities into consideration. This means that we shall carry out the virial expansion (see, for example, Ref. 3) in powers of the impurity concentration only up to second order in c. For this purpose, it is necessary to find the Green functions for the bound states on an isolated impurity and on an impurity pair. The corresponding computations are given in the Appendix. The Green function for a magnon on one impurity has the form

$$D(\omega_n)=\int_0^\beta \langle\langle X_{01}(\tau)X_{10}(0)\rangle\rangle \exp(i\omega_n\tau)\,d\tau=\frac{\operatorname{th}(\varepsilon_0/2T)}{i\omega_n-\varepsilon_0}. \tag{15}$$

For two impurities (denoted by 1 and 2) we shall have (see the Appendix)

$$\begin{aligned} D_{11}(\omega_n)=D_{22}(\omega_n)&=\frac{1}{2}\left(\frac{\operatorname{th}(\varepsilon_+/2T)}{i\omega_n-\varepsilon_+}+\frac{\operatorname{th}(\varepsilon_-/2T)}{i\omega_n-\varepsilon_-}\right),\\ D_{12}(\omega_n)&=\frac{t\bar{K}(t^2,\varepsilon_0)}{(i\omega_n-\varepsilon_0)^2-t^2},\quad \varepsilon_\pm=\varepsilon_0\pm t(R). \end{aligned} \tag{16}$$

4. THE DENSITY OF STATES AND THE THERMODYNAMIC CHARACTERISTICS

As follows from the form of the Hamiltonian (12), in first order of the virial expansion, the self-energy part of the magnon Green function $G=\langle\langle a_k(t)a_k^+(0)\rangle\rangle_{imp}$, averaged over the impurities, can be expressed in terms of the solitary-impurity Green function $D(\omega)$, (15):

$$\Sigma^{(1)}=c\langle g_k^a D(\omega) g_k^{a*}\rangle_a=c\frac{\omega_0(\mathbf{k})}{\alpha}\frac{\operatorname{th}(\varepsilon_0/2T)}{\omega-\varepsilon_0+i\delta}. \tag{17}$$

The angular brackets denote averaging over the directions of the impurity couplings a.

In the second virial approximation, $\Sigma^{(2)}$ is expressed in terms of the pair Green functions (16), and the answer should be averaged over the distance, R, between the impurities and over the orientations of a and a′:

$$\begin{aligned} \Sigma^{(2)}=c^2\frac{I}{\alpha}\Big[&\left\langle (\mathbf{ka})^2\int (D_{11}(\omega)-D(\omega))\,d\mathbf{R}\right\rangle_{aa'}\\ &+\left\langle (\mathbf{ka})(\mathbf{ka}')\int D_{12}(\omega)\cos\mathbf{kR}\,d\mathbf{R}\right\rangle_{aa'}\Big]. \end{aligned} \tag{18}$$

In the first term of (18), we have, as we should in a virial expansion, subtracted the contribution, $D(\omega)$, of the solitary impurities. Notice that the contribution of the off-diagonal Green function $D_{12}(\omega)$ vanishes on being averaged because of its oddness. In that region where we can restrict ourselves to the first orders of the virial expansion, $|\Sigma| \ll \omega_0(\mathbf{k})$, and, consequently, we can use the approximate expression $G \approx G_0 + \Sigma G_0^2$ in the density-of-states calculation. Then the contribution of the impurity states to the density of states of the system will have the form

$$\rho(\omega)=-\frac{1}{\pi}\sum_{\mathbf{k}} G_0^2(\mathbf{k},0)\operatorname{Im}\Sigma(\mathbf{k},\omega).$$

Bearing (17) and (18) in mind, we obtain

$$\rho^{(1)}(\omega)=c\operatorname{th}(\omega/2T)\,\delta(\omega-\varepsilon_0), \tag{19}$$

$$\begin{aligned} \rho^{(2)}(\omega)=-\frac{c^2}{2}\operatorname{th}\frac{\omega}{2T}\int d\mathbf{R}\langle\delta(\omega-\varepsilon_0-t^{aa'}(\mathbf{R}))\\ +\delta(\omega-\varepsilon_0+t^{aa'}(\mathbf{R}))-2\delta(\omega-\varepsilon_0)\rangle_{aa'}=\frac{2c^2}{3\pi\alpha}\frac{\operatorname{th}(\omega/2T)}{(\omega-\varepsilon_0)^2}\left(1+\frac{\pi}{3^{3/2}}\right). \end{aligned} \tag{20}$$

The total density of states is represented by a virial series in powers of $c/(\omega-\varepsilon_0)$. In this case there appears in each term the factor $\tanh(\omega/2T)$, which arises as a result of the two-levelness of the system of impurity states. Therefore, the density of states $\rho(\omega)$ can be represented in the form $\rho(\omega)=\tilde{\rho}(\omega)\operatorname{th}(\omega/2T)$, where $\tilde{\rho}(\omega)$ is the density of states corresponding to an effective Fermi statistics for the impurity states.

We shall be able to carry out further computations in those cases when the parameter $|c/(\omega-\varepsilon_0)| \ll 1$ in the important energy (ω) region. In this case we shall be able to limit ourselves to the second virial approximation. We shall investigate the low-temperature properties of the system for $c \ll |\varepsilon_0|$ (the case A) and compute the thermal capacity and the susceptibility of the system at "high" temperatures $T \gg c$ for the on-resonance case (case B).

Case A: $c \ll |\varepsilon_0|$, $T \ll |\varepsilon_0|$

When $\varepsilon_0 < 0$, the calculations can be carried out at any T; when $\varepsilon_0 > 0$, we are limited to $T \ll \varepsilon_0$, since the important energies $\omega \sim T$. We shall in both cases investigate the low-temperature properties ($T \ll \varepsilon_0$). The virial expansion is in powers of the parameter c/ε_0.

For the thermal capacity of the impurity states we obtain

$$C_v=\frac{\pi^2}{3}\tilde{\rho}(0)T=\frac{2\pi}{9\alpha}\left(1+\frac{\pi}{3^{3/2}}\right)\frac{c^2}{\varepsilon_0^2}T. \tag{21}$$

The spontaneous moment $M(0)$ at $T=0$ is less than the nominal moment by the quantity

$$\Delta M(0)=\begin{cases}\dfrac{2c^2}{3\pi\varepsilon_0\alpha}\left(1+\dfrac{\pi}{3^{3/2}}\right) & \text{for } c\ll\varepsilon_0,\ \varepsilon_0>0\\ \sim c & \text{for } c\sim\varepsilon_0 \text{ at } \varepsilon_0<0\end{cases}.$$

It is easy to show that the introduction of a weak magnetic field H is equivalent to raising the energy of the single-impurity state by a value H (we set $g\mu_B+1$). Therefore, the impurity part of the susceptibility at $T \ll \varepsilon_0$ will be

$$\chi=-\int_{-\infty}^{+\infty}\left(\frac{d}{d\varepsilon_0}\tilde{\rho}(\omega)\right)\frac{d\omega}{e^{\omega/T}+1}\approx\tilde{\rho}(0)=\frac{c^2}{\varepsilon_0^2}\frac{2}{3\pi\alpha}\left(1+\frac{\pi}{3^{3/2}}\right). \tag{22}$$

An important characteristic of the system is the existence of well-defined quasiparticles. In our case the decrement of the spin waves is determined with the aid of (18), and turns out to be equal to

$$\gamma(k,\omega)=\frac{c^2}{(\omega-\varepsilon_0)^2}\frac{2}{3\alpha^2}\left(1+\frac{\pi}{3^{3/2}}\right)\omega_0(k)\operatorname{th}\frac{\omega}{2T}. \tag{23}$$

Thus, there exists at low temperatures a wide frequency range $T \lesssim \omega \ll \varepsilon_0$ in which the ratio of the wave-damping constant to the wave frequency turns out to be frequency

independent:

$$\frac{\gamma(k)}{\omega_0(k)}=\frac{c^2}{\varepsilon_0^2}\frac{2}{3\alpha^2}\left(1+\frac{\pi}{3^{3/2}}\right).$$

Case B: $\epsilon_0 = 0; c \ll T$

In this case the virial-expansion parameter is $c/T \ll 1$. With the aid of the expressions, (19) and (20), for the density of states, we obtain the thermal capacity C_v and the susceptibility χ connected with the impurity states:

$$C_v=\frac{c^2}{T}\frac{2}{3\pi\alpha}\left(1+\frac{\pi}{3^{3/2}}\right), \tag{25}$$

$$\chi=\frac{c}{4T}-\frac{c^2}{3\pi\alpha}\frac{(1+\pi/3^{3/2})}{T^2}\beta, \quad \beta=\frac{1}{2}\int_0^\infty\frac{\mathrm{th}^2 y}{y^2}dy\sim 1. \tag{26}$$

We draw attention to the complete analogy between the expressions (25), (26) and the corresponding results obtained by Larkin and Khmel'nitskiĭ[3] for a system of magnetic impurities with the RKKY exchange. This is, of course, not accidental, since we have reduced our problem to a system of impurities with an alternating-sign $1/R^3$ interaction.

5. DISCUSSION OF THE RESULTS

We have investigated the effect of a small concentration of negative exchange integrals of definite magnitude on the low-temperature properties of a ferromagnet. It turned out that the behavior of the system is determined by the presence in it of localized excitations connected with the possibility of occupation by magnons of low-energy "impurity" states. Since these states admit of only single-magnon occupation, we actually obtain a set of two-level (in other words, spin-$\frac{1}{2}$) systems with a dipole interaction between them.

Let us note that our treatment can be carried over wholly to a slightly disordered crystal with a certain number of resonance levels for phonons. This system would be an intermediary between the normal crystal and glass, just as the "spoiled" ferromagnet considered by us is an intermediate state between the normal ferromagnet and a spin glass. There is, however, a difference between spin glass and ordinary glass: if spin glass can be regarded as a thermodynamic-equilibrium system, ordinary glass is basically a nonequilibrium system. In our model, this means that not all the negative resonance levels are correctly filled; therefore, there should remain at zero frequency residual phonon attenuation, with decrement $\gamma(k, \omega=0)\sim\omega(k)$, much like our results of Sec. 2.

The idea that two-level systems are important in spin- and ordinary-glass theory was first expressed by Anderson *et al.*[7] (see also Ref. 8). They showed that the phenomenological introduction of a certain density of two-level states allows us to give a qualitative account of the experimental data on the thermodynamics of, and ultrasound absorption in, glasses.[9] As we can see, two-level systems are important even in a relatively weakly disordered system with preserved ferromagnetic long-range order. Let us emphasize, however, that the two-level systems in our case manifest themselves differently than in Ref. 7, which accounts for the significant difference between the results; as can be seen from the formulas (21)–(24), the thermodynamic quantities and the spin-wave damping constant are determined by the same quantity $\tilde{\rho}(0)$, whereas in the mechanism considered in Ref. 7 only a small fraction of the states determining the thermodynamics participates in ultrasound absorption.

The formulas (21)–(24) were derived for $c/\varepsilon_0 \ll 1$, when the "impurity" part, $\tilde{\rho}(0)$, of the density of states is small compared to unity. For $c \gtrsim \varepsilon_0, T$, all the terms of the virial expansion of the density of states are important, and therefore a quantitative consideration is impossible. The problem for the impurity subsystem turns out to be similar to the problem that arises in the investigation of Cu-Mn-type spin glasses with the RKKY interaction between the magnetic impurities; the difference between the dipole and RKKY interactions is insignificant. It seems to us to be most probable that the density, $\tilde{\rho}(0)$, of the "Fermion" states approaches a constant value ~1 at $c \gtrsim \varepsilon_0$. Then at low temperatures $C_v \sim T$ and $\chi \sim 1$, as in spin glasses. If in this region, as in the $c/\varepsilon_0 \ll 1$ region, the magnon-damping constant is proportional to $\tilde{\rho}(0)$, then we obtain $\gamma(\mathbf{k}, \omega) \sim \omega_0(\mathbf{k}) \tanh(\omega/2T)$. In other words, no weakly damped excitations with energy $\omega \gtrsim T$ exist in the system, and at $T = 0$ there are no well-defined long-wave quasiparticles at all, despite the presence of long-range ferromagnetic order. Notice that in the region investigated by us [see Eqs. (21)–(24)] the ratio of the magnon-damping constant to the magnon frequency is small only to the extent that c/ε_0 is small, and does not contain powers of the momentum, as obtains in the case of a normal ferromagnet. At the same time, in the region of classical fluctuations the system behaves like a ferromagnet; in particular, there should occur in this region a ferromagnetic phase transition with exponents that are characteristic of slightly disordered systems.[10]

Thus far we have talked about a highly model problem with a definite value of "impurity" exchange integrals J and, consequently, of the level energy ε_0. It would have been more natural to consider a distribution of ε_0 values with some width Δ. We should then have obtained the relation $\tilde{\rho}(0) \sim c/\Delta$ for $c/\Delta \ll 1$ and, probably, $\tilde{\rho}(0) \sim 1$ for $c/\Delta \gtrsim 1$.

Let us now make an important qualitative deduction about the properties of spin glasses. Normally, it is assumed that the magnetic moments in this phase are frozen in some random directions determined by the potential-energy minimum. This means that each moment should be directed along the local field, which is the resultant of the fields produced by the spins interacting with the spin in question. A fundamental spin-glass property, called frustration,[2,8,11] is that some of these spins necessarily produce a field directed in opposition to the resultant field. The above-considered problem of a ferromagnet with a small admixture of negative exchange integrals constitutes an extreme simplification of this situation.

Let us consider from this standpoint the question of elementary excitations in glasses.

Macroscopic symmetry arguments show that three long-wave vibrational modes with the linear spectrum $\omega(q) = sq$ could exist in a spin glass.[12] Let us now note that the frustration phenomenon will necessarily lead to the appearance, for the quanta of these modes, of resonance levels similar to the magnon levels considered by us. Therefore, that same mechanism will lead to the appearance of damping of these modes: $\gamma(q) \sim sq \tanh(\omega/2T)$ (naturally, for spin glass $c \sim \Delta \sim 1$). This means that, at $T = 0$, the long-wave excitations are damped out over their wavelength. Notice that a similar result has been obtained by Shender[13] for spin glass in a strong magnetic field. At the same time, it has been established by Sherrington[14] that the attenuation of long-wave excitations in the model problem of spin glass without frustration (the Mattis model) is weak [$\omega(q) \sim q$; $\gamma(q) \sim q^2$].

In conclusion, we wish to thank D. E. Khmel'nitskiĭ for numerous useful discussions.

APPENDIX

Let us find the Green function for a pair of impurities. Let us take the Hamiltonian of the pair in the form

$$\mathcal{H} = \varepsilon_0 (X_{11}^1 + X_{11}^2) + t (X_{01}^1 X_{10}^2 + \text{H.c.}). \tag{A.1}$$

The equation of motion for the Green function $D_{ij}(\tau) = \langle\langle X_{01}^1(\tau) X_{10}^j(0) \rangle\rangle$ is:

$$\frac{dD_{ij}}{d\tau} = -\delta_{ij}\delta(\tau)(1-2n) - t(L_{2j}^{(0)} - L_{2j}^{(1)}) - \varepsilon_0 D_{ij}. \tag{A.2}$$

Here $n = \langle X_{11} \rangle = -T \sum_n D_{11}(\omega_n)$ and we have introduced the functions $L_{2j}^{(0,1)} = \langle\langle X_{01}^2 X_{00(11)}^1(\tau) X_{10}^j(0) \rangle\rangle$. The equations of motion for the L functions have the following form:

$$\begin{aligned} (i\omega_n - \varepsilon_0) L_{2j}^{(0)}(\omega_n) &= (\delta_{2j} K^{(0)} - \delta_{1j}\Delta) + t L_{1j}^{(0)}(\omega_n), \\ (i\omega_n - \varepsilon_0) L_{1j}^{(0)}(\omega_n) &= (\delta_{1j} K^{(0)} - \delta_{2j}\Delta) + t L_{2j}^{(0)}(\omega_n). \end{aligned} \tag{A.3}$$

Here $K^{(0)} = \langle X_{00}^2 - X_{11}^2, X_{00}^1 \rangle$, $\Delta = \langle X_{01}^2 X_{10}^1 \rangle$.

Similar equations can be written down for the $L_{2j}^{(1)}$ functions. Solving them, we obtain

$$L_{2j}^{(0)} - L_{2j}^{(1)} = \frac{(i\omega_n - \varepsilon_0)[\delta_{2j}(K^{(0)} - K^{(1)}) - 2\delta_{1j}\Delta] + t\delta_{1j}(1-2n)}{(i\omega_n - \varepsilon_0)^2 - t^2}. \tag{A.4}$$

Substituting (A.4) into the formula (A.2), we obtain

$$\begin{aligned} D_{11} &= \frac{(1-2n)(i\omega_n - \varepsilon_0)}{(i\omega_n - \varepsilon_0)^2 - t^2} - \frac{2t\Delta}{(i\omega_n - \varepsilon_0)^2 - t^2}, \\ D_{12} &= \frac{t\tilde{K}}{(i\omega_n - \varepsilon_0)^2 - t^2}, \end{aligned} \tag{A.5}$$

where $\tilde{K} = K^{(0)} - K^{(1)} = \langle (X_{00}^1 - X_{11}^1)(X_{00}^2 - X_{11}^2) \rangle$. From (A.5) and the definition of the quantity Δ, we have

$$\Delta = -T \sum_n D_{12}(\omega_n) = \frac{1}{2} \tilde{K}(n(\varepsilon_+) - n(\varepsilon_-)). \tag{A.6}$$

Here and below $n(x) = (e^{x/T} - 1)^{-1}$, $\varepsilon_\pm = \varepsilon_0 \pm t$. In order to determine $\tilde{K}$, let us investigate the Green function

$$P(\tau) = \langle\langle X_{01}^1 X_{01}^2(\tau) X_{10}^1 X_{10}^2(0) \rangle\rangle.$$

It is evident that $P(-0) = -\langle X_{11}^1 X_{11}^2 \rangle$. The equation of motion for $P(\tau)$ yields

$$P(\omega_n) = \frac{1-2n}{i\omega_n - 2\varepsilon_0}. \tag{A.7}$$

Substituting (A.7) into the defining expression for $\tilde{K}$, we obtain

$$\tilde{K} = 1 - 4n + 4n(2\varepsilon_0)(1-2n). \tag{A.8}$$

Substituting (A.6) and (A.8) into (A.5), we obtain after simple, but long calculations the expression

$$D_{11} = \frac{1}{2}\left(\frac{\text{th}(\varepsilon_+/2T)}{i\omega_n - \varepsilon_+} + \frac{\text{th}(\varepsilon_-/2T)}{i\omega_n - \varepsilon_-} \right). \tag{A.9}$$

Letting $t \to 0$, we obtain the Green function of the solitary impurity:

$$D_{11} = \frac{\text{th}(\varepsilon_0/2T)}{i\omega_n - \varepsilon_0}. \tag{A.10}$$

From the formula (A.9) we obtain

$$n = \frac{1}{2}\left(\frac{1}{e^{\varepsilon_+/T} + 1} + \frac{1}{e^{\varepsilon_-/T} + 1} \right);$$

this quantity does not change when the sign of t is changed. Consequently, as can be seen from (A.8), the quantity $\tilde{K}$ is also invariant under a change of sign of t. Casting a glance at the formula (A.5), we see that the Green function D_{12} should thus be odd in t.

[1] S. F. Edwards and P. W. Anderson, J. Phys. **F5**, 965 (1975). D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**, 1792 (1975). D. J. Thouless, P. W. Anderson, and R. G. Palmer, Philos. Mag. **35**, 593 (1977).

[2] P. W. Anderson, in: Amorphous Magnetism II, Plenum Press, New York, 1977, p. 1.

[3] A. I. Larkin and D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **58**, 1789 (1970) [Sov. Phys. JETP **31**, 958 (1970)].

[4] S. Kirkpatrick, in: Teoriya i svoĭstva neuporyadochennykh materialov (The Theory and Properties of Disordered Materials), edited by V. L. Bonch-Bruevich, Mir, 1978, p. 271.

[5] A. Abrikosov, L. Gor'kov, and I. Dzyaloshinskiĭ, Metody kvantovoĭ teorii polya v statisticheskoĭ fizike (Methods of Quantum Field Theory in Statistical Physics), Fizmatgiz, 1962 (Eng. Transl., Pergamon, New York, 1965).

[6] J. Habbard, Proc. Roy. Soc. London Ser. A **285**, 542 (1965).

[7] P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. **25**, 1 (1972).

[8] J. Villain, J. Phys. **C10**, 4793 (1977); **C11**, 745 (1978).

[9] R. C. Zeller and R. O. Pohl, Phys. Rev. **B4**, 2029 (1971).

[10] D. E. Khmel'nitskiĭ, Zh. Eksp. Teor. Fiz. **68**, 1960 (1975) [Sov. Phys. JETP **41**, 981 (1976)].

[11] G. Toulouse, Commun. Phys. **2**, 115 (1977). J. Villain, J. Phys. C10, 1717 (1977).

[12] B. I. Halperin and W. M. Saslow, Phys. Rev. **B16**, 2154 (1977). A. F. Andreev, Zh. Eksp. Teor. Fiz. **74**, 785 (1978) [Sov. Phys. JETP **47**, 411 (1978)].

[13] E. F. Shender, J. Phys. **C11**, L423 (1978).

[14] D. Sherrington, Phys. Rev. Lett. **41**, 1321 (1978).

Translated by A. K. Agyei