On the theory of nonlinear surface waves in a plasma

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The problem of the propagation of a high-frequency nonlinear ionizing surface wave in a plasma with permittivity $\varepsilon_0 \geq 0$ is considered in the normal skin-effect approximation. It is shown that the change of the permittivity, ε, of the plasma in the wave field from positive to negative values occurs abruptly. In the region of positive values the permittivity and the electric field of the wave behave nonmonotonically. The magnitudes of the electric-field and permittivity jumps are found, and their effect on the phase characteristics is assessed.

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The study of the processes of propagation in a weakly ionized plasma of electromagnetic waves of sufficiently high power (and of frequency lying in the optical, microwave, or any other frequency range) should be related to the problem of taking into account the effect of the ionization of the medium on the behavior of the electromagnetic field.

The properties of a nonlinear high-frequency surface wave ionizing a supercritical plasma have been studied by Prokopov and the present author.[1] Of greater interest from the standpoint of applications is the problem of the propagation of an ionizing surface wave in a transparent plasma (with an electron concentration in zero field, n_0, lower than the critical concentration $n_c = m\omega^2/4\pi e^2$). This case includes as a particular case the problem of neutral-gas ($n_0=0$) ionization by a wave, which is a typical problem in the study of self-maintained gas discharges. However, theoretically, this problem is a more difficult problem, since the existence of a surface wave in such a plasma necessarily requires, as a result of the nonlinear (ionizing) effect of the field, that the permittivity be negative in some layer of the plasma, i.e., that the permittivity of the plasma in the field of the surface wave should necessarily change its sign.

Because of the presence in the wave of an electric-field component along the gradient of ε that is singular at the point $\varepsilon = 0 (E_z \sim 1/\varepsilon)$ and the nonlinear coupling between the permittivity and the field, $\varepsilon = \varepsilon(|\mathbf{E}|^2)$, the transition from the region $\varepsilon < 0$ to the region $\varepsilon > 0$ can occur only abruptly, and, consequently, that solution to the problem which describes the wave field should be constructed with allowance for this discontinuity. The existence of the jump of the nonlinear field was first pointed out by Gurevich and Pitaevskiĭ,[2] and the properties of the jump in an inhomogeneous plasma are studied in Refs. 3 and 4. The solution of the problem in a waveguide channel in an opaque plasma with allowance for the field jump is considered in Ref. 5.

Below we construct in the normal skin-effect approximation the exact solution to the problem of the propagation of a high-frequency ionizing surface wave in a plasma with $n_0 \leqslant n_c$. The spatial structure of the field is investigated, and it is shown that, in the region of admissible positive values, the permittivity of the plasma and the electric field of the wave vary nonmonotonically. The change of the permittivity of the plasma in the wave field from positive to negative values occurs abruptly. The magnitude of the jump is found, and its effect on the phase velocity and the existence domain of the surface wave is estimated.

1. Let there exist a half-space, $z \geqslant 0$, filled by a plasma with permittivity in zero field $0 \leqslant \varepsilon \leqslant 1$, and bordering on a dielectric (permittivity ε_2). Along the boundary of the plasma in the direction of the y axis propagates a surface electromagnetic wave with electric- and magnetic-field components $\mathbf{E}\{0, E_y, E_z\}$ and $\mathbf{H}\{H, 0, 0\}$ respectively. The wave frequency $\omega \gg \nu$, the effective collision rate. We shall neglect the dissipation of the wave energy; the conditions under which this can be done will be discussed below.

We shall assume that the penetration depth, L_E, of the field significantly exceeds the diffusion length L_D and the length characterizing the redistribution of the density of the electrons as a result of their being heated up. Then the dependence of the plasma-electron concentration n on the electric-field intensity will be localized. Let us choose it in the form

$$n = \begin{cases} n_0 + n_s(|\mathbf{E}|^2 - E_g^2) & \text{for} \quad |\mathbf{E}| \geqslant E_g, \\ n_0 & \text{for} \quad |\mathbf{E}| \leqslant E_g, \end{cases} \tag{1.1}$$

where n_s is some characteristic concentration determined by the ionization mechanism. The dependence (1.1) takes account of the possibility of the existence of a nonzero threshold field, E_g, connected, for example, with the allowance for the effect of adhesion of electrons to neutral particles in the balance equation for the number of plasma particles.[6] When the electric-field intensity of the wave is less than E_g, no perturbations exist in the plasma, and the behavior of the field is described by the linear problem. The field dependence of the concentration has been chosen to be quadratic for simplicity; its complication has no effect on the qualitative behavior of the solution.

That permittivity of the plasma which corresponds to the dependence (1.1) has the form

$$\varepsilon = \begin{cases} \varepsilon_0[1 - p(|\mathbf{E}|^2 - E_g^2)] & \text{for} \quad |\mathbf{E}| \geqslant E_g \\ \varepsilon_0 & \text{for} \quad |\mathbf{E}| \leqslant E_g, \end{cases} \tag{1.2}$$

where $\varepsilon_0 = 1 - n_0/n_c$, $\varepsilon_s = 1 - n_s/n_c$, and

$$p = (1 - \varepsilon_s)/\varepsilon_0 > 0, \quad \varepsilon_0 \neq 0. \tag{1.3}$$

If the change in the wave amplitude over a wavelength

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along the y axis is small, then, seeking the solution in the form $\sim\exp[i(hy-\omega t)]$, we arrive at the following problem for the field in the plasma:

$$
\begin{gathered}
\varepsilon\frac{d}{d\zeta}\left[\frac{1}{\varepsilon}\frac{db}{d\zeta}\right]=(\eta-\varepsilon)b,\\
E_\perp=\frac{\eta^{1/2}}{\varepsilon}b,\quad E_\parallel=\frac{i}{\varepsilon}\frac{db}{d\zeta},\\
E^2=E_\perp^2+E_\parallel^2=\frac{1}{\varepsilon^2}\left\{\eta b^2+\left(\frac{db}{d\zeta}\right)^2\right\},
\end{gathered}
\tag{1.4}
$$

where

$$b=H/E_g,\quad E_\perp=E_z/E_g,\quad E_\parallel=E_y/E_g,$$
$$\zeta=\omega z/c,\quad \eta=(hc/\omega)^2.$$

The field in the plasma should join the field in the dielectric at the boundary $z=0$:

$$b=B_2,\quad \frac{1}{\varepsilon_1}\frac{db}{d\zeta}=\frac{(\eta-\varepsilon_2)^{1/2}}{\varepsilon_2}B_2, \tag{1.5}$$

where $\varepsilon_1=\varepsilon(+0)$ is the value of the permittivity of the plasma at the boundary and B_2 is the field amplitude at the boundary of the dielectric. For the field decreasing with increasing distance into the interior $(db/d\zeta<0)$, conditions (1.5) will be fulfilled only when $\varepsilon_1<0$.

Deep inside the plasma, at some point $\zeta=\zeta_0$, where $E^2=1$ and $\varepsilon=\varepsilon_0$, the solution of the nonlinear problem should join the solution of the linear problem:

$$b=b_g=B_0\exp[-(\eta-\varepsilon_0)^{1/2}\zeta_0]\quad\text{for}\quad \varepsilon=\varepsilon_0. \tag{1.6}$$

For a given ζ_0 the quantity B_0 is determined by the relation

$$B_0^2\exp[-2\zeta_0(\eta-\varepsilon_0)^{1/2}]=\varepsilon_0^2/(2\eta-\varepsilon_0). \tag{1.7}$$

The solution to Eq. (1.4) that satisfies the condition (1.6) has the form[1]

$$b^2=\frac{\varepsilon[\varepsilon_0^2(1+2p)-\varepsilon^2]}{2(2\eta-\varepsilon)(1-\varepsilon_s)}, \tag{1.8}$$

where ε as a function of ζ is given by the formula

$$\zeta(\varepsilon)=\int_{\varepsilon}^{\varepsilon_1}\frac{db}{d\varepsilon}\left\{\varepsilon^2\left[\frac{\varepsilon_0(1+p)-\varepsilon}{1-\varepsilon_s}\right]-\eta b^2(\varepsilon)\right\}^{-1/2}d\varepsilon. \tag{1.9}$$

The dependence $b^2(\varepsilon)$ as given by (1.8) is represented in Fig. 1 (the curve 1) by the partly dashed, partly solid, curve. Here the interval $-\varepsilon_*=\varepsilon_0(1+2p)^{1/2}<\varepsilon<0$ is forbidden $(b^2(\varepsilon)<0)$, and therefore the variation of the permittivity in the wave field from positive to negative values in a continuous fashion is impossible. In view of this, we shall construct the solution to the problem for positive and negative values of ε separately, and then join the solutions, using the continuity conditions for the tangential components of the fields.

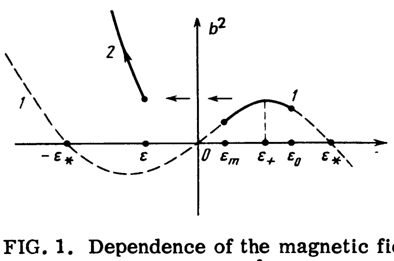

FIG. 1. Dependence of the magnetic field on the permittivity of the plasma: curve 1) $b^2(\varepsilon)$ computed from (1.8); 2) $b^2(\varepsilon)$ computed from (3.1).

2. As the solution for positive ε, let us choose the solution (1.8). It is nonnegative for $0\leqslant\varepsilon\leqslant\varepsilon_0$, and satisfies the boundary condition (1.6) inside the plasma.

The dependence $b^2(\varepsilon)$, (1.8), for $\varepsilon>0$ is nonmonotonic; at $\varepsilon=\varepsilon_+$ (Fig. 1), which satisfies the equation

$$\varepsilon_+^3+3\eta\varepsilon_+^2-\eta\varepsilon_0^2(1+2p)=0, \tag{2.1}$$

the derivative $db^2/d\varepsilon$ changes its sign.

It is not difficult to verify that $0\leqslant\varepsilon_+\leqslant\varepsilon_0$ if $p\leqslant 1+\varepsilon_0/2\eta$, and $b^2(\varepsilon)$ decreases as the point ε_0 is approached. When $p>1+\varepsilon_0/2\eta$ and $\varepsilon_0\neq 0$, the peak of the function $b^2(\varepsilon)$ lies more to the right of ε_0, and the region of positive values degenerates into the point ε_0.

For a sufficiently slowed-down wave $[\varepsilon_0(1+2p)^{1/2}\ll 6\eta^2]$ the solution to Eq. (2.1) that lies in the interval $(0,\varepsilon_0)$ can be written in the form

$$\varepsilon_+=\varepsilon_0[(1+2p)/3]^{1/2},\quad p\leqslant 1. \tag{2.2}$$

The requirement that the radicand under the integral sign in (1.9) should be negative reduces to the following inequality for ε:

$$F\equiv\varepsilon\{2\varepsilon^3-\varepsilon^2[3\eta+2\varepsilon_0(1+p)]+4\eta\varepsilon_0(1+p)\varepsilon-\eta\varepsilon_0^2(1+2p)\}\geqslant 0. \tag{2.3}$$

To ascertain the spatial dependence $\varepsilon(\zeta)$, it is important to know the location of the least value of the permittivity, ε_m, starting from which the inequality (2.3) will be fulfilled relative to that root, ε_+, of Eq. (2.1) which determines the location of the maximum of the function $b^2(\varepsilon)$.

If $\varepsilon_m<\varepsilon_+$ (Fig. 1), then during the variation of ε from ε_m to ε_0, the derivative $d\varepsilon/d\zeta$ will change its sign at the point ε_+ and the permittivity will attain a local minimum at $\zeta_m=\zeta(\varepsilon_m)$. If $\varepsilon_m\geqslant\varepsilon_+$, then $d\varepsilon/d\zeta$ will be positive when $\varepsilon>\varepsilon_m$, and the permittivity will increase monotonically into the plasma.

It is convenient to carry out the investigation of the positions of the roots of Eqs. (2.1) and (2.3) and, consequently, of the sign of $F(\varepsilon)$ as a function of the parameters p and η after solving these equations for η. Let us first consider the case when $\varepsilon_0\neq 0$ and $p<1+\varepsilon_0/2\eta$. The analysis shows that in the region of variation $1\leqslant\eta<\infty$, which is characteristic of a surface wave, we always have

$$\varepsilon_m<\varepsilon_+.$$

For example, for $p=4\times10^{-2}$ and $\varepsilon_0=1$, as η varies within the interval $1\leqslant\eta<\infty$, the roots ε_m and ε_+ vary in the intervals

$$0.334<\varepsilon_m\leqslant 0.362,\quad 0.55\leqslant\varepsilon_+<0.6.$$

Since $db/d\varepsilon>0$ in the interval $(\varepsilon_m,\varepsilon_+)$, we have, according to (1.9), that

$$\zeta_+=\zeta(\varepsilon_+)<\zeta(\varepsilon_m)\equiv\zeta_m,$$

and therefore the plasma having a positive permittivity in the wave field will be concentrated in the region of space $\zeta_+\leqslant\zeta<\infty$. Here ε will vary nonmonotonically in space; at the point ζ_m it will have a minimum equal to

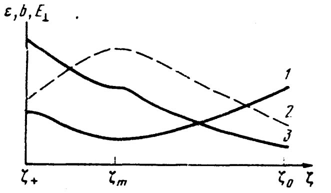

FIG. 2. The spatial structure of the field and the permittivity in the positive-value region: curve 1) the permittivity ε; 2) the electric-field component $E_\perp$; 3) the magnetic field b.

ε_m. At the point $\zeta=\zeta_+, db/d\varepsilon=0$. The magnetic field decreases with increasing distance into the plasma, having a point of inflection at $\zeta=\zeta_m$.

An investigation of the behavior of the modulus of the normal component of the electric field as a function of ζ shows that $E_\perp$ has the point ζ_m a maximum equal to

$$E_m=\frac{1}{\varepsilon_m}\left\{\frac{\eta[\varepsilon_0^2(1+2p)-\varepsilon_m^2]}{2(1-\varepsilon_s)(2\eta-\varepsilon_m)}\right\}^{1/2}.$$

This value of E_m is at the same time the maximum value of the modulus, $E=(E_\perp^2+E_\parallel^2)^{1/2}$, of the electric-field vector of the wave, since the tangential component, $E_\parallel$, of the electric field vanishes at $\zeta=\zeta_m$.

Figure 2 shows the qualitative behavior of $\varepsilon(\zeta)$ (the curve 1), the magnetic field $b(\zeta)$ (the curve 2), and the normal component, $E_\perp(\zeta)$, of the electric field (the curve 3).

The transition of ε from the region of positive into the region of negative values will occur when the wave amplitude increases from the point $\zeta=\zeta_+$ in space, and therefore in the plane (b^2,ε) the jump of ε into the region of negative values should be accomplished from the point ε_+. Thus, the value ε_+, which is a root of Eq. (2.1), is the positive limit of the jump of the permittivity of the plasma. Notice that Eq. (2.1) is similar to the equation obtained in Ref. 3 for the limit of the jump in the region of negative ε.

3. We shall seek the solution to the problem in the region of negative values in the form

$$b^2=\frac{\varepsilon[\varepsilon_0^2(1+2p)-\varepsilon^2+C]}{2(2\eta-\varepsilon)(1-\varepsilon_s)},\quad \varepsilon<0, \tag{3.1}$$

where C is for the present an arbitrary constant. We find the value of this constant and the largest negative permittivity value ε_-, which is the second limit of the jump, from the continuity conditions for the tangential components of the fields:

$$b^2(\varepsilon_-)=b_+^2,\quad E_\parallel^2(\varepsilon_-)=E_\parallel^2(\varepsilon_+), \tag{3.2}$$

where b_+ is the magnetic-field value given by (1.8) for $\varepsilon=\varepsilon_+$.

As a result, we obtain

$$C=\frac{(2\eta-\varepsilon_-)\varepsilon_+}{(2\eta-\varepsilon_+)\varepsilon_-}[\varepsilon_0^2(1+2p)-\varepsilon_+^2]-[\varepsilon_0^2(1+2p)-\varepsilon_-^2], \tag{3.3}$$

$$\frac{\varepsilon_-}{\varepsilon_+}=\frac{(1-\varepsilon_s)\eta b_+^2}{2\varepsilon_+^3}-\left\{\left[\frac{(1-\varepsilon_s)\eta b_+^2}{2\varepsilon^{+3}}\right]^2+\frac{(1-\varepsilon_s)\eta b_+^2}{\varepsilon_+^3}\right\}^{1/2}. \tag{3.4}$$

It can be seen that C is negative together with ε_-. It is not difficult to verify that in the region of variation $-\infty<\varepsilon\leqslant\varepsilon_-$ we have

$$b^2(\varepsilon)\geqslant b^2(\varepsilon_-)>0,\quad E_\parallel^2(\varepsilon)>0.$$

Figure 1 shows the $b^2(\varepsilon)$ dependence as given by (3.1) for C given by (3.3) (the curve 2). The arrows in the same figure indicate the direction of variation of the field in crossing the jump.

For a sufficiently slowed-down wave ($2\eta\gg\varepsilon_+,\varepsilon_-$), we find from (3.3) and (3.4) that

$$C=-{}^9/_4\varepsilon_0^2(1+2p),$$
$$\varepsilon_-=-{}^1/_2\varepsilon_+=-{}^1/_2\varepsilon_0[(1+2p)/3]^{1/2}. \tag{3.5}$$

The magnitude of the permittivity jump in this case is equal to

$$\langle\varepsilon\rangle=\varepsilon_+-\varepsilon_-={}^3/_2\varepsilon_0[(1+2p)/3]^{1/2}.$$

The normal component of the electric field and, as a result, the modulus of the electric field of the wave undergo an abrupt change in value when the permittivity changes its sign. In this case

$$\langle E_\perp^2\rangle=E_\perp^2(\varepsilon_+)-E_\perp^2(\varepsilon_-)=\langle\varepsilon\rangle\eta b_+^2/\varepsilon_+|\varepsilon_-|.$$

It is easy to verify that in the region of negative ε the field in, and the permittivity of, the plasma decreases monotonically with increasing ζ.

The solution to the problem in the $\varepsilon_0\neq0, p>1+\varepsilon_0/2\eta$ case, when the maximum of the function $b^2(\varepsilon)$ lies to the right of ε_0, can be constructed in completely similar fashion. In this case the region of positive values of the permittivity degenerates into the point ε_0. The jump into the region of negative values then occurs directly from the point ε_0. To find the constant C from (3.1) and ε_-, we should replace ε_- and b_+^2 in (3.2) by ε_0 and b_ε^2 from (1.6) and (1.7). As a result, we obtain

$$\varepsilon_-=\frac{(1-\varepsilon_s)\eta}{2(2\eta-\varepsilon_0)}-\left\{\left[\frac{(1-\varepsilon_s)\eta}{2(2\eta-\varepsilon_0)}\right]^2+\frac{(1-\varepsilon_s)\eta\varepsilon_0}{2\eta-\varepsilon_0}\right\}^{1/2},$$
$$C=\frac{(2\eta-\varepsilon_-)\varepsilon_0^3}{(2\eta-\varepsilon_0)\varepsilon_-}2p-[\varepsilon_0^2(1+2p)-\varepsilon_-^2]. \tag{3.6}$$

For sufficiently large values of η, the magnitude of the permittivity jump is given by the formula

$$\langle\varepsilon\rangle=\varepsilon_0-\varepsilon_-=\varepsilon_0\left\{1-\frac{p}{4}+\left[\left(\frac{p}{4}\right)^2+\frac{p}{4}\right]^{1/2}\right\}.$$

If the permittivity of the unperturbed plasma $\varepsilon_0\to0$, then the quantity b_ε given by (1.6) tends to zero, and the solution has the form

$$b^2=-\varepsilon^3/2(1-\varepsilon_s)(2\eta-\varepsilon). \tag{3.7}$$

The region of admissible ε values that satisfy the inequality $E_\parallel^2\geqslant0$ is the semiaxis $\varepsilon\leqslant0$. The electric and magnetic fields and the permittivity of the plasma decrease monotonically with increasing distance into the plasma. For $\varepsilon\to0$

$$b^2\sim E_\perp^2\sim\varepsilon\to0,\quad E_y^2\sim|E|^2\to E_g^2,$$

and, consequently, the field will vanish in the unperturbed plasma if the threshold field in the ionization law (1.1) is equal to zero.

If the field E_g is nonzero, then there gets excited in the region of unperturbed plasma a traveling—along the y axis—electrostatic wave of constant amplitude equal to E_g:

$$H=E_\perp=0,\quad E_y=E_z\exp\{i(hy-\omega t)\}. \tag{3.8}$$

In respect to the structure of the field, this wave is equivalent to a plasma wave, but the characteristics of its propagation are determined not by the thermal motion of the electrons, but by the characteristics of the propagation of the nonlinear field. In this sense, this wave is, as it were, an intermediate wave between an electrostatic, and an electromagnetic, surface plasma wave.

The Eqs. (3.2), which constitute joining conditions for the fields at the jump point, determine only the ratio $\varepsilon_-/\varepsilon_+$ and C as functions of ε_0 and η, and are insufficient for the unique determination of the values of ε_+ and ε_-. For this purpose, we need to know the solutions in the regions outside the region of the jump. Analysis of these solutions and the joining of them with the aid of the conditions (3.2) enable us to eliminate the arbitrariness, and make the solution to the problem unique.

4. The dispersion equation for the wave can be obtained from the boundary conditions (1.5), with $b^2(\varepsilon)$ given by the formula (3.1). It has the following form:

$$\eta=\frac{\varepsilon_1^2\varepsilon_2\{2\varepsilon_2(\varepsilon_0-\varepsilon_1)-[\varepsilon_0^2(1+2p)-\varepsilon_1^2+C]\}}{4(\varepsilon_0-\varepsilon_1)\varepsilon_1\varepsilon_2^2-(\varepsilon_1^2+\varepsilon_2^2)[\varepsilon_0^2(1+2p)-\varepsilon_1^2+C]}. \tag{4.1}$$

This equation determines the phase velocity of the wave as a function of the permittivity of the plasma at the ε_1 boundary (and, as a result, of the wave amplitude), the ionization-law parameter p, and the quantities ε_1 and ε_2.

The existence domain of the surface wave is determined by the condition $\varepsilon_2\leqslant\eta<\infty$. If $|\varepsilon_1|\gg\varepsilon_2$ (strong field), then $\eta\approx\varepsilon_2$, and the phase velocity of the wave is close to the velocity of light in the dielectric. The values of the permittivity ε_1 at which the phase velocity of the wave vanishes ($\eta\to\infty$) are given by the equation

$$4(\varepsilon_0-\varepsilon_1)\varepsilon_1\varepsilon_2^2-(\varepsilon_1^2+\varepsilon_2^2)\times[\varepsilon_0^2(1+2p)-\varepsilon_1^2+C]=0. \tag{4.2}$$

Since a surface wave does not exist in a plasma with only a positive permittivity, there should exist a certain threshold field necessary for the creation at the boundary of a negative permittivity. The strength of this field and the permittivity values determined by it are given by Eq. (4.2) as functions of the parameters ε_0, ε_2, and p.

Figure 3 (curve 1) shows the ε_0 dependence of the threshold permittivity of the plasma at the boundary for $p\ll1$. A surface wave exists in the region of $|\varepsilon_1|$ values higher than the boundary value for the given ε_0. The same figure (curve 2) shows the dependence of the threshold permittivity as found from the analogous

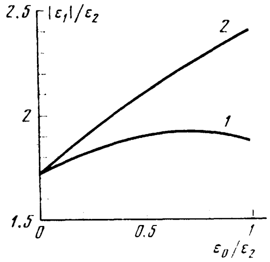

FIG. 3. Threshold values of the permittivity of the plasma at the boundary: curve 1) according to (4.2); 2) according to the solution of Ref. 1.

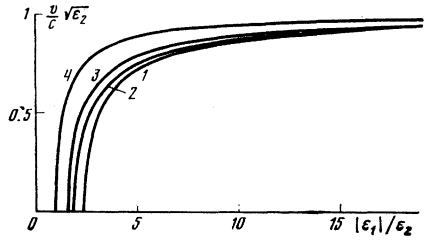

FIG. 4. The phase velocity of the wave: curve 1) $\varepsilon_0=1$ (according to the solution of Ref. 1); 2) $\varepsilon_0=1$; 3) $\varepsilon_0=0$; 4) the linear theory.

equation of Ref. 1, an equation which is certainly not applicable in the case of positive ε values, since it is constructed from a solution that does not take account of the jump in, and the existence of a region of positive values of, the permittivity.

In Fig. 4 we show the dependence of the phase velocity of the wave on the value of the permittivity of the plasma at the boundary for different values of ε_0: $\varepsilon_0=1$ (the curve 2) and $\varepsilon_0=0$ (the curve 3). We show in the same figure for comparison the dependence of the phase velocity of the wave as computed from: a) the formulas of Ref. 1 for $\varepsilon_0=1$ (the curve 1) and b) the linear theory (the curve 4). It can be seen that, qualitatively, all the curves behave in like manner; they are only shifted differently along the $|\varepsilon_1|$ axis as a result of the difference in the threshold values (the threshold permittivity value in the linear theory is $\varepsilon_1=\varepsilon_0=-1$).

A comparison of the curves in Fig. 2 and the curves 1) and 2) in Fig. 4 shows that the phase characteristics of the wave depend quite weakly on the details of the spatial structure of the field in the plasma, which is explained by the relatively small magnitude of the jump in ε and the relatively small ε_0 values. This fact can be used to construct approximate solutions.

In conclusion, let us discuss the conditions under which we can neglect the dissipation of the surface-wave energy.

The requirement that the ratio ν/ω be small is not sufficient for this purpose, since dissipation mechanisms connected with spatial dispersion (collisionless damping, transformtaion into a plasma wave in the plasma-resonance region) are possible in the case of surface waves.

The collisionless damping can be neglected if the phase velocity, v_f, of the wave exceeds the thermal velocity, v_T, of the electrons and the depth, L_E, of penetration of the field into the plasma exceeds the mean free path, i.e.,

$$L_E\gg v_T/\nu. \tag{4.3}$$

On account of the sharp increase in the phase velocity v_f and the penetration depth L_E in the vicinity of the threshold value of the permittivity ($L_E^{-1}=k|\varepsilon_1|^{1/2}$), the conditions (4.3) will be fulfilled if

$$|\varepsilon_1|_{min}<|\varepsilon_1|\ll\left(\frac{\nu}{\omega}\frac{c}{v_T}\right)^2. \tag{4.4}$$

Using the results of Refs. 4 and 1, we can write the relative change that occurs in the flux density of the wave over a unit length along the propagation direction as a result of the conversion of the wave into a plasma wave in the region of the jump in the form

$$\frac{\delta S_p}{S_w}\sim\frac{v_T}{c}\frac{(E_\perp)_1^2}{E_g^2}\left(\frac{\varepsilon_+}{\varepsilon_2}\right)^{1/2}\left(1+\frac{\varepsilon_+}{|\varepsilon_-|}\right)\frac{1}{|\varepsilon_1|}.$$

This ratio is small under the conditions of the inequality (4.4).

The obtained solution can be used to construct a more complex solution—of the type of a plasma facula (layer) in an unbounded plasma with $\varepsilon_0>0$–having even two jumps. Treating the $z=0$ plane as a middle plane, and seeking the solution that describes the decreasing field for $|z|\to\infty$, we arrive on account of the symmetry of the problem to a solution of the surface-wave type. However, the dispersion properties of such a wave ($\infty>\eta\geqslant\varepsilon_0$) will now be determined by another factor: the law of conservation of the wave-energy flux along the axis of the facula:

$$S_0=\frac{c}{2\pi}E_g^2\frac{\eta^{1/2}}{k}\int_0^\infty\frac{b^2(\zeta;\eta)}{|\varepsilon|}d\zeta.$$

Introducing the effective skin depth

$$\delta=\frac{1}{kB_2}\int_0^\infty b(\zeta)d\zeta,$$

where B_2 is the amplitude of the field at $z=0$, we obtain for a sufficiently large wave amplitude[1]

$$S_0\approx\frac{c}{2\pi}E_g^2\eta^{1/2}\Delta,$$

where Δ does not depend on the wave amplitude.

[1]A. G. Boev and A. V. Prokopov, Zh. Eksp. Teor. Fiz. **69**, 1208 (1975) [Sov. Phys. JETP **42**, 617 (1975)].
[2]A. V. Gurevich and L. P. Pitaevskiĭ, Zh. Eksp. Teor. Fiz. **45**, 1243 (1963) [Sov. Phys. JETP **18**, 855 (1964)].
[3]V. B. Gil'denburg, Zh. Eksp. Teor. Fiz. **46**, 2156 (1964) [Sov. Phys. JETP **19**, 1456 (1964)].
[4]V. B. Gil'denburg and G. M. Fraĭman, Zh. Eksp. Teor. Fiz. **69**, 1601 (1975) [Sov. Phys. JETP **42**, 816 (1975)].
[5]A. G. Litvak and G. M. Fraĭman, Zh. Eksp. Teor. Fiz. **68**, 1288 (1975) [Sov. Phys. JETP **41**, 640 (1975)].
[6]V. E. Golant, Usp. Fiz. Nauk **65**, 39 (1958).

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Linear wave interaction in a plasma with an inhomogeneous magnetic field

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The linear interaction of waves in the region where the geometrical-optics approximation is violated is analyzed qualitatively on the basis of the Budden–Kravtsov equations that describe the propagation of the electromagnetic waves in a smoothly inhomogeneous magnetoactive plasma. It is shown that the interaction sets in when the polarization of the geometrical-optics waves is substantially altered over the spatial period of the beats between these waves. Conditions are obtained under which the efficiency of the interaction is characterized by a single parameter whose form can be established without solving the equations that describe this phenomenon. The plasma-parameter regions in which the interaction is the most effective at a specified scale of the inhomogeneity of the magnetic field are determined. The exact solution of the standard problem that describes the linear interaction in plasma layers of the transition type is analyzed.

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The study of the sources of cosmic radio emission calls for an exact account of those changes that the emission undergoes in the plasma located on the path from the source to the observer. Because of the weak inhomogeneity of the electron density and of the magnetic fields in the cosmic plasma (over scales comparable with the wavelength), these changes can usually be described in the geometrical-optics approximation, so that it is rather a simple matter to take them into account. However, if the emission passes through regions where the geometrical-optics approximation does not hold, the situation becomes much more complicated: a linear interaction arises between the waves and causes the amplitude and phases of the emerging ordinary and extraordinary waves to differ substantially from those expected in the geometrical-optics approximation.

If the polarization characteristics of the radiation source are known, then the parameters of the cosmic plasma in the interaction region can be evaluated from the observed polarization that is produced as a result of the linear wave transformation. Diagnostics of this type uncovers new possibilities of studying the plasma near the earth and between the planets by reception of radiowaves from spacecraft. It is of interest also for the study of processes in a laboratory plasma.

In a tenuous plasma ($\omega_L\ll\omega$, where ω_L is the plasma

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